Leibniz's Notion of Conditional Right and the Dynamics of Public Announcement

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ABSTRACT. The main aim of our paper is to implement Leibniz's analysis of the conditional right in the framework of a dialogical approach to *Public Announcement Logic*.According to our view, on one hand: PAL furnishes a dynamic epistemic operator which models communication exchange between different agents that seems to be very close to Leibniz understanding of the dynamics between *the truth* of a proposition and *the knowledge of the truth* of that proposition (Leibniz calls the latter *certification* of its truth); on the other hand, the dialogical approach provides a semantics for the dynamic epistemic operator in the context of conditional right by means of which: (i) the exchange between agents leading to a public announcement amounts to the (contractual) interaction of commitments of both the *benefactor* and the *beneficiary* of a conditional right (ii) the notion of certification is understood as an action, namely as a move where the beneficiary asks the benefactor to stand to his commitments, (iii) some restrictions specific to the logical nature of the head and the tail of the conditional right can be implemented by combining PAL with some features of linear logic within the same theory of meaning.

KEYWORDS: Leibniz, law, conditional-right, dialogical logic, public-announcement, epistemic logic.

In some recent papers [Thi08], [Thi10] and [Thi11][1], A. Thiercelin underlines that the study of the notion of the conditional right of the young Leibniz already shows the start of the interaction of logical and epistemological perspectives that are characteristic of his mature work.

In this context, the originality of Leibniz' proposal is to provide a logical analysis of the notion of the conditional right. The idea behind this is that conditional rights can be understood as a special kind of conditional sentences.

The main aim of our paper is to implement Thiercelin's thorough insights on Leibniz's analysis of the conditional right in the framework of a dialogical approach to *Public Announcement Logic*.[2]

[1] Thoroughly developed in his PHD dissertation : A. Thiercelin, *La théorie juridique leibnizienne des conditions: ce que la logique fait au droit (ce que le droit fait à la logique)*, 318 p. 2009.

[2] See [vDvdHK08] for an overview of this logic.

According to our view, on the one hand:

- PAL furnishes a dynamic epistemic operator which models communication exchange between different agents that seems to be very close to Leibniz understanding of the dynamics between *the truth* of a proposition and *the knowledge of the truth* of that proposition (Leibniz calls the latter *certification* of its truth);

on the other hand, the dialogical approach provides:

- a semantics for the dynamic epistemic operator in the context of the conditional right by means of which:
  - (i) the exchange between agents leading to a public announcement amounts to the (contractual) interaction of commitments of both the *benefactor* and the *beneficiary* of a conditional right
  - (ii) the notion of certification is understood as an action, namely as a move where the beneficiary asks the benefactor to stand to his/her commitments,
  - (iii) some restrictions specific to the logical nature of the head and tail of the conditional right can be implemented by combining PAL with some features of linear logic within the same theory of meaning.

First we will present some definitions about the investigated notion of the conditional right; then we expose PAL semantics. We will end with its dialogical reconstruction and an example to illustrate how it works.

1. Preliminary Notions

As pointed out by A. Thiercelin [Thi10], the main point of the young Leibniz [Lei65] is to cast the juridical modality called "suspension" in the logical framework of conditional sentences. Indeed, Leibniz claims that a logical analysis of sentences of the form "*If a ship arrives from Asia; then Secundus receives 100 coins*" provides an appropriate approach to the meaning of juridical formulations, such as "*Secundus right to receive 100 coins is suspended until a ship arrives from Asia*".

The head of a conditional sentence expressing a conditional right is called the *suspensive condition*. It is the head of such kinds of conditional sentences that triggers the suspension. The notion of suspension is in fact what makes the up the conditional right conditional. According to Leibniz, the notion of suspension should be linked to the *knowledge* of the truth value of the condition of the proposition expressed by the conditional sentence at stake. Access to the truth value of that suspensive proposition is carried out by means of an act called by Leibniz *certification*. By means of the certification act, the incertitude about the truth value of the suspensive condition is lifted. Henceforth, it is known if the suspensive condition is true or false.

As already mentioned, we will implement Leibniz's analysis of the conditional right in the framework of a dialogical approach of PAL. In this context,

we will understand conditional rights as involving a specific *contract*[3] between a *benefactor* (*conditionator*)[4] and a *beneficiary* (*conditionarius*). Throughout our paper we will stick to Leibniz's example:

- *If a ship arrives from Asia, I, Primus, will give 100 coins to Secundus*.

Let us now display some definitions that should first build the bridge to the standard semantics of PAL (rather than to dialogical semantics).

1.1. Terminology

Contract: the suspensive feature of a conditional sentence expressing the conditional right, such as *If φ, then ψ* is captured by the following interpretation of a public announcement*: If it is a public knowledge that φ is true; then the benefactor is committed to allow the beneficiary to make use of the right described by ψ*. Thus, the conditional right is understood as a kind of contract involving rights and duties.
Benefactor: is the one who formulates the contract in such a way that the fulfillment of the tail is made dependent on the fulfillment of a specific condition (called the *suspensive condition*).
Beneficiary: is the one who receives the benefits of the contract if he/she is able to show that the suspensive condition is fulfilled.
Suspension: Leibniz's logical analysis of the notion of suspension is linked to the knowledge of the truth value of a determined proposition. When we say that the truth value of the proposition ψ is suspended to the truth value of the proposition φ, we mean that we come to know the truth value of ψ when we know the truth value of φ.
Certification: the certification is the act which lifts the suspension. Certification arises when the head of the conditional of a public announcement is played. A suspensive proposition can be certified as being true or false.

As the next step we will present the list of requirements that Leibniz's notion of *suspensive conditional* assumes, as worked out by [Thi10].[5]

[3] The approach of public announcement as a contract has been proposed for the first time in [Mag11Mag12]. In fact it extends to the logic of public announcement Lorenzen's dialogical interpretation of the conditional: If a player **<X>**, promises (to player **<Y>**) β under the condition that α is the case, then when the condition is fulfilled **<X>** is committed to β, see [Lor78].
[4] See [Thi08] for that terminology.
[5] We slightly adapted Thiercelin's own formulation.

1.2. Suspensive Conditionality

A sentence expressing a conditional right should be understood as a specific kind of conditional sentence consisting of:

(i) a head and tail such that the (knowledge of the) truth of the tail is dependent on (the knowledge of) the truth of the head.

Moreover:

(ii) The tail of the conditional cannot be true if the head is not true (see condition VI).
(iii) The tail of the conditional cannot be its own condition.
(iv) The truth value of the tail can be known only when the head of the conditional is certified.
(v) If the head of the conditional is certified as being true, the tail of the conditional must be also true.
(vi) If the head of the conditional is certified as being false, the tail of the conditional must be *nil* (approximately: the truth value of (the deontic proposition involved by) the tail is indeterminate).
(vii) A logical contradiction cannot be the head of a suspensive conditional.
(viii) A tautology cannot be the tail of a suspensive conditional.

2. Public Announcement and Conditional Right: The First Steps

2.1 The Semantics of the Operator of Public Announcement

The main approach to PAL makes use of model theoretic semantics that is an extension of the standard Krikpe-semantics for modal logic. In this framework a model is understood as a theoretic construction which groups a set of points and relations between those points. Points represent different possible situations. Through these different situations the truth value of an atomic proposition can change.

A public announcement is a formula of the form w: $[\varphi]\psi$. The intended interpretation of such a formula is "in the situation w, after the announcement of φ, ψ holds". Thus, the only way to obtain ψ in situation w is to have φ first in w (i.e. $M, w \models \varphi$).

Furthermore, the truth of the content of the announcement triggers a reduction of the original model, namely the so-called restricted model that contains only those worlds where φ is true. In other words, after the public announcement that φ is true, φ is true in every possible situation in the model, that is not only in w. The possible situations in which φ is false must be removed from the model. This is exactly what the semantic condition for the public announcement describes:

$$M, w \models [\varphi]\psi \text{ if and only if } M, w \models \varphi \text{ implies } M|_{\varphi}, w \models \psi,$$

where "$M|_{\varphi}, w$" signalizes that the original model M has been reduced to that part of the model that only contains worlds where φ is true.

2.2 The Operator of Public Announcement and Suspensive Conditionality

Applying PAL to our case study (see 1.) amounts to the fact that if a ship arrives from Asia and this is a public knowledge, there is no situation in which *Primus* and *Secundus* can deny that a ship arrived.

Furthermore, if we interpret the fact that the head of the conditional right is true and that this is a public knowledge (this is the certification of the truth of the head of the conditional right at stake) the semantics of the public announcement operator (PA-operator) mentioned above amounts to establishing that the truth of ψ in *w* (*Primus* must give 100 coins to *Secundus*) follows from the certification of the truth of φ (*A ship arrives from Asia*).

The semantics of the public announcement underlines the notion of suspensive conditionality. Without the certification of φ, the tail of the public announcement is suspended. But when the content of the announcement is certified as being true, this certification implies the truth of the suspended proposition. In fact, it is the semantics of the public announcement itself which satisfies the first general requirement for suspensive conditionality (i): *a sentence expressing a conditional right should be understood as consisting of a head and tail such that the truth of the tail is dependent on (the knowledge of) the truth of the head*. Indeed, the truth value of the tail of the announcement directly depends on the value of the head of the conditional. If φ is certified to be true, it follows that ψ is also true. In that sense, this first analysis of the notion of the conditional right in the context of the PA-operator renders a weaker form of the clause (iv): *the truth value of the tail can be known only when the head of the conditional is certified*. According to our approach, the point is that if the PA-proposition [φ]ψ is true, in the reduced model triggered by the certification of the head the conditional right (i.e. φ), the tail (i.e. ψ) is true if and only if the head is true.

In fact, there is a difficulty in Leibniz's own analysis, since it looks as if the clause (iii): *the tail of the conditional cannot be its own condition* and the clause (iv) that requires that ψ is true if and only if φ is true are in tension. Indeed, the biconditional analysis of the conditional right renders ψ as its own condition which contradicts (iii): *the tail of the conditional cannot be its own condition*. The issue on the biconditionality or *convertibility* (to use Leibniz' own words) is puzzling. [Thi11] suggests that the convertibility of the conditional right should fix a close relationship between the head and the tail of the conditional sentence in such a way that if the head is not true, the benefactor is not committed to the right established by the tail of the conditional right.

Once more, according to our approach, Leibniz's point is reconstructed as establishing that the tail must be true in the context of the reduced model triggered by the certification of the head of the conditional right. However, in this first analysis of the conditional right there is something still lacking, namely the precise

semantics of the deontic aspect in the tail of the conditional right and the precise description of the effects of the falsity of the head of such conditionals. In relation to the latter, Leibniz suggests that the tail should consider to be *nil* [Lei65] (see suspensive condition ii and vi). We will tackle these issues in the next section.

2.3 The Certification of the Conditional Right and its involved Obligation

It appears from our analysis that the crucial point concerning the suspensive conditionality is the certification. It is the certification that determines the continuation or not of the conditional. This certification can either represent the end of the suspension or the end of the contract established by the conditional right.

Let us consider once more the contract between *Primus* and *Secundus*. When *Primus* addresses *Secundus* the conditional sentence "if a ship arrives from Asia, I give you 100 coins", *Primus* and *Secundus* ignore if the head of such a conditional is true or not (if *Primus* knew that a ship would arrive from Asia, the very point of establishing a contract by means of the conditional right does not make sense). Consequently, the commitment involved in the tail of this conditional remains suspended as long as *Primus* and *Secundus* are ignorant about the value of the head. But, if the head of the conditional turns out to be true, then *Secundus'* right to receive 100 coins comes into effect. It means that it is obligatory for *Primus* to give to *Secundus* 100 coins in every situation following the arrival of the ship from Asia.

For this reason, we need to introduce the deontic operator $\mathbb{O}$, with the standard intended interpretation *the agent must…*[6] Let us consider that contract [A] $\mathbb{O}$B stands for *if a ship arrives from Asia, Primus must give to Secundus 100 coins*, where:

- A: means "*a ship arrives from Asia*"
- $\mathbb{O}$: stands for "*must*", and
- B: "*Primus gives to Secundus 100 coins*".

Only two cases are possible, 1. maybe *one day* it will be certified that a ship is arrived from Asia or 2. maybe *one day* it will be certified that no ship will come from Asia.

- What happens in the first case?

We already started to study this case: If the arrival of a ship from Asia is certified being true, then this certification strictly corresponds to the end of the suspension. In other words, it is the case that $M, w \models A$ implies $M|_A, w \models \mathbb{O}B$. So, *Primus* must give 100 coins to *Secundus*. This corresponds to requirement (v): "*If*

[6] Furthermore, without such a deontic operator, or more generally, without a modal operator in the scope of a PA-operator, formulas are equivalent to the standard *material implication*. Indeed, if the tail of a public announcement is Boolean we cannot evaluate the formula in other situations alternative to the one of the actual world. In that case the interest of a PA-operator with the dynamics involved in the construction of a reduced model is lost.

the head of the conditional is certified as being true, the tail of the conditional must be true too".

Now, the problem is that in fact there are two types of accessibility relations involved, one cares of the epistemic features of the semantics of the public announcement and the other cares for the accessibility relation for the deontic operator $\bigcirc$ that *in general* assumes seriality – though we need to assume reflexivity. However, since the reduction of the model triggered by a PA-operator, which is independent of the epistemic accessibility relation (the reduced model does include all the worlds where the head is true, accessible or not to the actual world), we can in fact in *principle* assume that the deontic accessibility relation is defined over worlds of the reduced model. We say in principle because the accessibility relation must stand for the update of the reduced model. In our case, seriality will be a problem since the update might eliminate the worlds necessary for seriality. However, our framework requires reflexivity and thus the use of such an operator is saved.[7] Therefore, we will not need to burden the notation with two accessibility relations.

- What happens in the second case?

Consider that instead of $M, w \models A$, we have $M, w \not\models A$. Otherwise, in Leibniz's language, we have to consider the case that it is certified that no ship arrives from Asia. The problem is that we should restrict the model to all situations in which φ is true but it is $\neg A$ which is true! We should then evaluate the value of $\bigcirc B$ in a situation which has vanished. If we follow [vDvdHK08][8], then the truth value of $\bigcirc B$ is said to be undefined because we should evaluate a formula in a situation which does not hold in the remaining model, so the formula cannot be evaluated!

This fits nicely with Leibniz's requirements (ii): "*the tail of the conditional cannot be true if the head is not true*" and (vi): "*if the head of the conditional is certified as being false, the tail of the conditional must be nil*".

2.4 The Certification-Act as the Climax of the Conditional Right

In the dialogical framework the notion of certification involved in the conditional right is understood as an act. Indeed, certification is understood as a move of the beneficiary **<X>** who demands to the benefactor **<Y>** to stand up to his commitments, since he/she (**<X>**) claims that the head of the conditional is the case.

Consider that *Primus* and *Secundus* are the two players of the dialogue. Now, consider that *Primus* says to *Secundus* "if a ship arrives from Asia, I give you 100 coins". In the eyes of *Secundus* and in respect of the conditional right, *Primus* takes here a commitment. Indeed, if *Secundus* is able to certificate that a ship is arrived from Asia, *Primus* is forced to give the 100 coins to *Secundus*. But, as long as

[7] This has been pointed out by Tiago de Lima in a personal conversation.
[8] See p. 106.

Secundus is not able to certificate any arrival of a ship from Asia, he/she cannot receive the 100 coins. In fact, it is in *Secundus*' best interest to (publically) claim that the head of the conditional is the case if he/she wants to obtain its benefits (the tail of the conditional). In other words, in an argumentative process where a contract is uttered by *Primus* in favor of *Secundus*, *Secundus* has to take charge of the certification of the head of the conditional right and in that case with regard to the certification, *Primus*, his opponent, must satisfy the suspended condition. As we will see (section 3.2.3), it is exactly what the dialogical rule of public announcement captures.

We are not still at the end of our analysis but nearly. In fact we must still deal with the following requirements (vii): "*a logical contradiction cannot be the head of a suspensive conditional*", (viii): "*a tautology cannot be the tail of a suspensive conditional*" and (iii): "*the tail of the conditional cannot be its own condition*". In relation to the last criterion $[\varphi]\varphi$ is not in general a tautology in PAL, particularly not if $\varphi = : (p \wedge \neg Kp)$.[9] The next section is devoted to the two other requirements.

2.5 What is a Logic with Limited Resources?

Henceforth, our problem is the following: we must satisfy requirements (vii) and (viii) that clearly have considerations on relevance, but PAL has no consideration on relevance! Moreover, the logic underlying PAL is insensitive to limited resources: the repetition of a public announcement does not change the situation, but in the context of conditional law it looks that limited resources can play an important role. Indeed, let us go back once more to our case of the conditional right: *Secundus* will receive 100 coins if a ship arrives from Asia. But, if a second ship arrives from Asia, it is sensible to assume that the conditional right does not intend to ascribe *Secundus* the right to gather once more 100 coins.[10] Hence we must seriously take into account the question of limited resources.

It has been said quite often that linear logic is the logic where there is a consciousness of limited resources.[11] This can be illustrated by considering logical expressions, such as describing transitions between actions. For example, the conditional *If* φ*, then* ψ signals that if you perform an action of type φ then you can perform an action of type ψ. To use Girard's favorite example: if you spend one dollar you get a packet of cigarettes.[12] Now, in every

[9] See [vDK06] and [vD10] for more details concerning this point.

[10] In fact, this is also linked with a more general issue tackled by [End11]: since the number of applicable cases of a law is in general undetermined, they ought to be formulated vaguely, but this does not mean under normal circumstances that the resources involved are considered to be unlimited.

[11] See [RGR04] and [Ra02].

[12] See [Gir95].

new application of this conditional by means of a token of the corresponding type (spending one dollar to get cigarettes) you actually have a new token of the corresponding action (spending another dollar to get another package of cigarettes). That is, once an action of type φ has been performed, the corresponding token is no longer available (the dollar has been spent) and if you want to repeat the transition expressed by the conditional, you really should produce another token. In fact, linear logic can be seen as combining resources consciousness with a horror of irrelevance (that is, a horror of introducing unused assumptions in a proof). Relevance logic avoids the careless introduction of unused possible assumptions in classical and intuitionistic logic by disallowing weakening. Linear logic disallows contraction too: if a formula is to be used *n* times it must be assumed *n* times.

Now should we then simply combine PAL with a logic sensitive to the issues mentioned above? Well the problem is that while the semantics of PAL is a standard model-theoretical framework of normal modal logic, the model-theoretic semantics of relevant logics includes impossible worlds and the semantics of a logic aware of limited resources (some varieties of linear logic) are proof-theoretical. Thus, combination seems to confront us with tough semantic difficulties.

The task of combining PAL with a logic which is sensitive to limited resources confronts us to a choice: should we extend the language of PAL with new logical constants or should we assume that the local meaning of the constants of PAL is the same but globally it changes by the use of structural rules? The second simpler strategy is the one we will follow in this present paper. In such an investigation, the dialogical approach to semantics displays all its force and beauty. Indeed the dialogical approach allows one to develop the semantics of different logics within the same framework. In fact, in [Mag12] a dialogical semantics for PAL is developed and there are already dialogical semantics for linear and relevance logic. And as in dialogical logic we think of proofs as strategies consisting of actions (moves) where formulas have been uttered, then every move can naturally be seen as another action (even when uttering the same formula). Let us start by briefly presenting the dialogical principle.

3. Dialogical Logic

Dialogical Logic was suggested at the end of the 1950's by P. Lorenzen and then worked out by K. Lorenz.[13] Inspired by Wittgenstein's *meaning as use* the

[13] The main original papers are collected in [Lor78]. A detailed account of recent developments can be found in [Fel85], [RaKe04], [RaCleKe09], [FiRüRa10], [RT06], [Rüc01]. For a textbook presentation (in French), see [FonRed08]. The most thorough completeness proof for dialogical FOL and that points out the differences between dialogical strategies and Tableaux has been just penned by Clerbout in [Cle12].

basic idea of the dialogical approach to logic is that the meaning of the logical constants is given by the norms or rules for their use. Anyway, the point is that those rules that fix meaning may be of more than one type, and that they determine the kind of reconstruction of an argumentative and/or linguistic practice that certain kind of language games, called dialogues, provide.

In a dialogue, two parties argue about a *thesis* respecting certain fixed rules.[14] The player who states the thesis is called proponent **<P>**; his/her rival, called opponent **<O>**, puts in question the thesis which is called opponent **<O>**. In its original form, dialogues were designed in such a way that each of the plays ends after a finite number of moves with one player winning, while the other loses. Actions or moves in a dialogue are often understood as *utterances*[15] or *speech-acts*.[16] The point is that the rules of the dialogue do not operate on expressions or sentences isolated from the act of uttering them.[17] The rules are divided into particle rules or rules for logical constants and structural rules. The structural rules determine the general course of a dialogue game, whereas the particle rules regulate those moves (or utterances) which are requests (to the moves of a rival) and those moves which are answers (to the requests). The particle rules determine *the local meaning* (what is at stake is only the request and the answer corresponding to the utterance of a given logical constant, rather than the whole context where the logical constant is embedded), and the structural rules determine the *global meaning*.

3.1 Labels

Since we will embed the standard logical particles in PAL we will need of labels that in a model-theoretical context will correspond to names of worlds. Labels are finite sequences of positive integers, such as 1.1.1 and 1.1.2 which indicate (from the model-theoretical point of view and via an interpretation function) that the worlds named by 1.1.1 and 1.1.2 are accessible from 1.1.

- A *label* is a finite sequence of positive integers. A *labelled formula* is an expression of the form $i : \varphi$, where i is the label of the formula φ.
- If the label i is a sequence of length >1, the positive integers of the sequence will be separated by periods. Thus, if i is a label and n is a positive integer, then $i.n$ is a new label, called *an extension of i*. The label is then an *initial segment* of $i.n$.

[14] The set of rules determines the dialogical system.
[15] See [RR01a], [RR01aRR01b] and [Rüc01].
[16] See [Kei07].
[17] See [Tul10].

3.2 Particle rules for logical constants

Particle rules should be read in the following way: if a player **<X>** utters *something*, then player **<Y>** challenges this utterance with another utterance or a question, hence **<X>** must give an appropriated defense.

3.2.1 The Propositional Connectives

Connectives	**<X> Utterance**	**<Y> Challenge**	**<X> Defence**
$\neg$ if player Y challenges a negation at *i*, **there is no available defense** for X.	$i : \neg \varphi$	$i : \varphi$	$\times$ (there is no possible defense)
$\wedge$, if player Y challenges a conjunction, **Y can choose the conjunct** that X has to defend.	$i : \varphi \wedge \psi$	$i : ?1$ *and* $i :?2$	$i : \varphi$ *respectively* $i : \psi$
$\vee$, if player Y challenges a disjunction, **X can choose the disjoint** to defend	$i : \varphi \vee \psi$	$i : ?$	$i : \varphi$ *or* $i : \psi$
$\rightarrow$, if player X utters a conditional, then player **Y concedes** the head and player **X has to defend** the tail.	$i : \varphi \rightarrow \psi$	$i : \varphi$	$i : \psi$

For rules concerning the public announcement and deontic operator, a technical advice is required for labels. Until now, labels have beenwere constituted by integer numbers *i, i.n,* …, and so on. Now, in order to keep track of public announcements challenged, we must assume that the labels have two parts, namely a – possibly empty list $\mathscr{A}$ of announced formulas,[18] that is constituted by the announcement(s) involved and the integer number. *E.g.*, after a challenge to *i*: $[\varphi]\psi$, the defense ψ will be labeled $\varphi \mid i$ – since in this example the list is constituted by only one element; this assumes that no other public announcement was played before. Notice that as already mentioned, if no public announcement has been played, $\mathscr{A}$ will be empty and in that particular case the label just consists of the integer number *i* (or *i.n*…)[19].

[18] The idea of the list in order to keep track of announcement was firstly introduced in [BavDHeLi10].

[19] Since there is no challenge before the utterance of the thesis, any thesis is always uttered in *i* and not in $\Sigma \mid i$.

3.2.2 The Deontic Operator

Operator	<X> Utterance	<Y> Challenge	<X> Defense
$\mathscr{A} \mid i : \mathsf{O}\varphi$ If a player **X** utters $\mathsf{O}\varphi$ at *i*, player **Y challenges the utterance by choosing a label** *i.n* at which X must carry out his/her defense.	$\mathscr{A} \mid i : \mathsf{O}\varphi$	? *i.n*	$\mathscr{A} \mid i.n : \varphi$

By uttering $\mathsf{O}\varphi$ at *i* player **<X>** is committed to defend φ at any label chosen by its adversary **<Y>**. Now, consider a challenge on the deontic operator where the challenger chooses a label *i.n*. This attack commits the challenger to utter at *i.n* the last formula of the list $\mathscr{A}$. This feature of the challenge to a deontic operator in the dialogical context of PAL opens the possibility to the defender of the deontic operator to counterattack the last formula of the list $\mathscr{A}$ uttered by the challenger.

3.2.3 The Public Announcement Operator[20]

Operator	<X> Utterance	<Y> Challenge	<X> Defense
$\mathscr{A} \mid i : \mathbf{[\varphi]}\,\psi$ **X has the choice,** he can perform φ and utter ψ at label $\mathscr{A} \bullet \varphi \mid i$ or reject the burden of φ by uttering $\neg\,\varphi$ at $\mathscr{A} \mid i$	$\mathscr{A} \mid i : [\varphi]\psi$	$\mathscr{A} \mid i : ?$	$\mathscr{A} \mid i : \neg\,\varphi$ $\mathscr{A} \bullet \varphi \mid i : \psi$

By uttering $[\varphi]\psi$ at *i* player **<X>** has the choice. He can reject the burden of φ or accept to be committed to the utterance of ψ in the label $\mathscr{A} \bullet \varphi \mid i$[21] triggered by a challenge. During the challenge, φ is being uttered.[22] If **<X>** defends with $\neg\,\varphi$, φ can be used by **<Y>** at the label *i* when he/she challenges *i*: $[\varphi]\psi$for challenging the negation. This challenge corresponds to what Leibniz called *certification*.

[20] See [Mag11Mag12], note: a thorough detailed version with soundness and completeness is being developed in a forthcoming paper by S. Magnier and T. Dede Lima.
[21] The symbol · is used as concatenation symbol, i.e. φ is concatenated to the list Σ.
[22] The symbol · is used as concatenation symbol, *i.e.*, φ is concatenated to the list A.

3.2 Structural Rules

As already mentioned, the idea of linear logic can be seen as combining resource consciousness with a horror of introducing redundant steps. This amounts to the following structural linear rule:

(SR-LP) Linearity principle:

1. In order to defend the thesis at stake, every formula uttered in the game (=every move) has to be used.
2. In order to defend the thesis at stake, no move can be used more than once – once a move has been used, we say that the move has been *used up*.

Moreover:

1. An atomic **<O>** formula can be used if and only if the proponent utters this formula in order to utter an aggressive (=challenge) or a defensive move.
2. A complex formula α can be used if and only if all the possible aggressive and defensive moves related to α have been uttered.

As a graphic device we will cross-out formulas that cannot be used again.

- **(SR-0) Starting rule**: the initial formula is uttered by **<P>**. It provides the topic of the argumentation. After the thesis has been set, **<O>** and then **<P>** respectively, choose a natural number *n* and *m* (termed their repetition ranks). After that, moves are numbered and alternatively uttered by **<O>** and **<P>**.
- **(SR-1)** Players act alternately. Each move which follows **P**'s choice about his repetition rank is either a challenge or a defense in reaction to a previous challenge. Whenever he has a turn to play, a player <**X**> can challenge any previous **Y**'s move or defend against himself any previous **Y**'s challenge up to his repetition rank.
- **(SR-2) Formal use of atomic formulas**: **<P>** is allowed to utter an atomic formula only if <**O**> has uttered it first.
- **(SR-3) Winning rule**: player **<X>** wins the dialogue if and only if it is **<Y>**'s turn but he/she cannot move (either challenge or defense).
- **(SR-4) Round closure rule**: in any move, **<X>** may challenge a (complex) formula uttered by **<Y>** or he/she may defend themselves against any challenge (including those challenges that have already been defended once).
- **(SR-5L) Choice of labels**: to challenge a deontic operator at *i*, **<P>** can choose any label *i'* such that *i'* is not new[23] in the dialogue.

[23] Remarks: "not new" means that either **<O>** has already chosen this label or it is the label where the thesis has been uttered.

- **(SR-PA) Public announcement**: If there is a PA-operator, the game must continue only in a <**X**>'s move such that $\mathscr{A}$ is non-empty, then <**Y**> can compel <**X**> to utter the last element of the list $\mathscr{A}$ in the label *i* or in *i.n* if <**X**>'s move = $?_{i.n}$.

3.3 Example:

Now it is time to look at how to play with the dialogical rules shown above. For this we will test two formulas which seem to be equivalent and in fact they are. If we follow our analysis of the conditional right and the certification, when *Primus* says: "If a ship arrives from Asia, I give 100 coins to *Secundus*", we can understand it in two equivalent ways, namely:

1. "After the certification of the arrival of a ship from Asia, *Primus* must give 100 coins to *Secundus*": [A] OB
2. "If ship is arrived from Asia, then it is obligatory that after its certification, *Primus* gives 100 coins to *Secundus*": A → O[A] B

We can show that both are equivalent. Moreover, we can prove that the cases of a second arrival of a ship and the case of where B is a tautology do not hold:

[A] OB → (A → O[A] B)

O			**P**		
				1 : ~~[A] O B~~ → (~~A → O [A] B~~)	0
1	1 : ~~[A] O B~~	0		1 : ~~A~~ → ~~O[A] B~~	2
3	1 : ~~A~~	2		1 : ~~O[A] B~~	4
5	1 : ? 1.1	4		1.1 : ~~[A]~~ B	6
7	1.1 : ?	6		1.1 : ¬ A	8
9	1.1 : ~~A~~	8		1 : A	14
11	A|1: ~~O B~~		1	1 : ?	10
13	A|1.1: ~~B~~		11	A|1 : $?_{1.1}$	12

Short comment: in move 1, **<O>** challenges the material implication by conceding the head and asking **<P>** to defend the tail (with move 2). At move 3 **<O>** carries out the same kind of challenge. After the proponents' defense of the material implication **<O>** introduces, at move 5, a new label 1.1 in order to challenge the deontic operator. At move 7 **<O>** challenges the PA-operator (this move corresponds to the certification act). <**P**> rejects the burden of A at move 8, but <**O**> takes it at move 9.From now on, it is certified

that a ship is arrived from Asia, and then the game must continue only in those situations where this arrival is a fact. <P> can change his previous defense but, due to **(SR-2)**, and since **<O>** has not introduced the proposition B yet, **<P>** cannot produce his/her defense it for the moment. Hence, **<P>** challenges what **<O>** has previously conceded in move 1. **<O>** has uttered the (atomic) proposition A in label 1. This allows **<P>** to challenge the PA-operator uttered by **<O>** in move 1. **<O>** has also uttered A at label 1.1; it means that 1.1 can always be chosen from label 1. Consequently, **<O>** is now obliged to defend that B in label 1.1. Thus, **<P>** can defend B in the very same situation and by doing that he wins the dialogue.

$(A \rightarrow \bigcirc[A]\, B) \rightarrow [A]\, \bigcirc B$

O			**P**		
				1 : ~~(A → ○[A] B)~~ → ~~[A] ○B~~	0
1	1 : ~~A~~ → ~~○[A] B~~	0		1 : ~~[A] ○B~~	2
3	1 : ?	2		1 : ¬ ~~A~~	4
5	1 : ~~A~~	4		A|1 : ~~○~~B	6
7	1 : $?_{1.1}$	6		1 : A	14
9	1 : ~~○[A] B~~		1	1 : A	8
11	1.1 : ~~B~~		9	$1.1_{.1}$	10
13	A|1.1 : ~~B~~		11	1.1 : ?	12

Short comment: After **<O>**'s challenge in move 7, due to **(SR-2)**, **<P>** cannot produce the defense B for the moment but he/she can counter-attack move 1 by uttering A at label 1. Thereafter (move 10), **<P>**challenges the deontic operator reusing label 1.1. He/She is allowed to carry out this challenge because **<O>** has previously introduced A at label 1.1 (with move 5). Then (move 12), the proponent challenges the PA-operator and **<O>** defends B in A|1.1. Consequently, **<P>** reuses this utterance and closes the dialogue with move 14.

The proponent wins in both dialogues, thus these formulas are indeed equivalent. It means that when *Primus* says to *Secundus* that "if a ship arrives from Asia, then I must give you 100 coins after the certification of that arrival", it has, strictly speaking," means the same thing than if he says "after the certification of the arrival of the ship from Asia, I must give you 100 coins".

4. Concluding Remarks

Throughout this paper we tried to show on one hand how close is Leibniz's analysis of the notion of the conditional right to the PAL-approach; and on the other hand how the dialogical approach furnishes an accurate framework to capture the dynamics involved in Leibniz's notion of the conditional right and suspensive conditionality. Indeed, according to our view, the dialogical approach:

- Allows for one to understand what the logical nature of a conditional right is during its suspension. It is a kind of non-executed action as long as there is no certification act. The act of certification triggers the act involved in the tail of the conditional right: It is this act that triggers further actions.
- Allows for one to understand the actions above mentioned as a display of the exchanges of rights and duties involved in the contract established between the benefactor and beneficiary.
- allows for one to extend Leibniz's analysis to multi-agent systems.

The general picture we can draw from these reflections of the young Leibniz is that not only did healready show showed the logical and philosophical insights that will make him a master of the art in his mature work, but also that already from the start of his intellectual life he knew that the real challenge is the dynamic nature of the argumentative practices in the processes of acquisition of knowledge and decision making, not only in science but also in our everyday life, when acting and interacting as members of a society.

References

Fontaine, M. & Redmond, J., *How to Play Dialogues – An Introduction to Dialogical Logic* College Publications: Dialogues and Games of Logic, London, 2011. http://plato.stanford.edu/entries/logic-dialogical/

Keiff, L., "Dialogical Logic", in *Stanford Encyclopedia of Philosophy*, 2007.

Leibniz, G. W., *De Conditionibus*, tr. P. Boucher *Des Conditions*, coll. « Bibliothèque des Textes Philosophiques », Paris, Vrin, 2002.

Lorenzen, P., "Logik und Agon", 1960. Reprinted in Lorenzen & Lorenz Dialogische Logik, Darmstadt: Wissenschaftliche Buchgesellschaft, 1978.

Magnier, S., "PAC *vs* DEMAL", to appear in *Logic of Knowledge. Theory and Applications*, College Publications, London.

Rahman, S. & Keiff, L., "On how to be a dialogician", in D. Vanderveken (Ed.) *Logic, Thought and Action*, Springer, Dordrecht, pp. 359-408, 2004.

Rahman, S., Clerbout, N. & Keiff, L., "Dialogues and Natural Deduction", in *Acts of Knowledge, History, Philosophy, Logic*. G. Primiero (Ed.), College Publications, London, chapter 4, pp. 301-336, 2009.

Rückert, H., *Dialogues as a Dynamic Framework for Logic*, Collegue Publications: Dialogues and Games of Logic, London, 2011. van Ditmarsch, H. P., van der Hoek, W. & Kooi, B., *Dynamic Epistemic Logic*, Springer Coll. Synthese Library: Studies in Epistemology, Logic, Methodology, and Philosophy of Science, vol. 337, Dordrecht, 2008, pp. 11-42 and pp. 67-108.