Global Dynamics and Ephemerides

D. Hestroffer[1], P. David[1]

1. IMCCE/Paris observatory, 75014 Paris, France. Daniel.Hestroffer@imcce.fr

Introduction

When speaking of Gaia it is instructive to remember the first ESA mission dedicated to high precision astronometry HIPARRCOS launched in August 1989. HIPPARCOS can be considered as Gaia's precursor, indeed the observing strategy aspects are similar and both missions were designed specifically for high precision astronometry. After three and a half years of observations HIPPARCOS produced three catalogues. In 1997 the HIPARCOS catalogue of ~ 120000 stars and the first Tycho catalogue containing about one million stars. In 2000 the consolidated catalogue Tycho-2 was released. It contains 99% of all stars down to magnitude 11, approximately ~ 2.5 million objects.

For small solar system bodies (SSSB), HIPPARCOS observed only 48 asteroids, 5 satellites orbiting Jupiter and Saturn, and 2 planets. The accuracy for the HIPPARCOS astrometry was $\sigma \sim 10\,\mathrm{mas}$ and for the photometry $\sigma \sim 0.05$ magnitude [1] [2]. These results are certainly spectacular but Gaia is projected to achieve much more.

The expected performances for Gaia compared to HIPPARCOSS are listed in Table 1. It is clear that the performances of Gaia will greatly excede those of HIPPARCOS.

Table 1: Comparison between performance for HIPPARCOS and those expected for Gaia

	HIPPARCOS	Gaia
magnitude limit	12 mag	20 mag
completeness	7.3 – 9.0 mag	20 mag
number of objects	120000	$26\,10^6$ to $V = 15$ $250\,10^6$ to $V = 18$ 10^9 to $V = 20$
effective distance limit	1 kpc	50 kpc
quasars	1 (3 273)	$5\,10^5$
galaxies	none	10^6
accuracy (whole mission)	milliarcsec	$7\,\mu\mathrm{arcsec}$ at $V = 10$ $10 - 25\,\mu\mathrm{arcsec}$ at $V = 15$ $300\,\mu\mathrm{arcsec}$ at $V = 20$
photometry	2 – colour (B and V)	low-resolution spectra to $V = 20$
radial velocity	none	$15\,\mathrm{km}s^{-1}$ to $V = 17$
observing programme	pre–selected	complete unbiased survey

It should be underlined that all moving objects down to 20 magnitudes will be detected, so roughly 350000 asteriods will be observed, mainly from the main belt. Orbits are expected to be 30 times better than at present even if a century of previous observations are used in their determination. Spin–axis, rotation periods, and shape parameters will be detrmined for a majority of these. Taxonomical / mineralogical composition versus heliocentric distance will be available while diameters for ~ 1000 to 20 % and masses to 10 % for some 150 asteroids will be determined also (see Tanga et al., this conference).

1. Gaia SSSB data processing

The Gaia SSSB data processing is divided into two pipelines, a so called short–term pipeline and a long–term pipeline [3]. The short–term pipeline runs daily on the data collected over the previous 24 hours.

This processing characterises the moving source, sorting those which are already known from unknown objects. Basic CDD processing and astrometric reduction are performed. An attempt at threading unknown object positions onto one orbit is undertaken so that in some cases a preliminary orbit can be computed. Results from this pipeline are then stored in a main data base (MDB) for later reprocessing. The bulk of the accurate characterisation of the objects is achieved by the long–term pipeline. This pipeline improves the results of the short–term analysis by determining the SSSB parameters through dynamic considerations and requires as many observations of the same object as possible, at intervals covering a reasonable part of their trajectory. The long–term pipeline is run on a six month schedule taking as input the cumulated data which has been stored in the MDB. Ideally all SSSB data will be used in the long–term pipeline but this is not guaranteed. In any event, only positions from Gaia observations will be used in this proccessing. The initial conditions required for simulating an orbit are obtained from an auxiliary data base which will be updated at regular intervals during the mission. This data base is fed by the ASTORB, input from the Lowell Observatory, and consequently is indirectly updated through the GAIA–FUN–SSO. This pipeline provides the final, more accurate, orbit determination and parameter estimates which will be published in the final catalogue. Note that only identified asteroids and comets will be used in the parameter estimation. Unidentified sources will only be incorporated when an orbit is available after the short–term processing and only if the GAIA–FUN–SSO has made the source available through the MPC. Furthermore, some technical issues will need to be addressed to include any new object in the parameter estimation procedures (see 2.3).

2. Parameter estimation

Parameter estimation is an inverse problem, and in the case of the Solar System it is ill posed. For Gaia, the chosen method for deriving a solution, is a linear least squares procedure which can be formally written as :

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

Here $\mathbf{y}$ is a difference vector of the observed sky coordinnates, $\mathbf{A}$ is the Jacobian matrix formed with the partial derivative with respect to the parameters of the model and $\mathbf{x}$ is the vector of corrections. To obtain $\mathbf{y}$ the equations of motion modeling the orbits of the SSSB observed must be integrated. This is done by a N–body simulation. The matrix $\mathbf{A}$ can be obtained by simply including the variational equations in the system of ordinary differential equations to integrate.

Local and global parameters, i.e. those parameters which principally affect a given SSSB and those which affect the Solar System as a whole or a subset of SSSB respectively will be determined. These are summarised in Table 2.

Table 2: Parameters which will be determined using asteroid and comet orbits from Gaia

local parameter	description
$(x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0)$	initial conditions
(A_1, A_2, A_3)	non–gravitational coefficients for comets
(A_4)	non–gravitational coefficients for asteroids
global parameter	description
β	PPN nonlinearity in the superposition law for gravity
J_2	solar quadrupole moment
$(\omega_0, \omega_1, \omega_2, \dot{\omega}_0, \dot{\omega}_1, \dot{\omega}_2)$	rotation and rotation rate of Gaia reference frame
$\dot{G}/G$	variation of the gravitational constant
m_i	mass of the ith perturbing body

Most of these parameters appear in the equations of motion as specified below. The solution is obtained
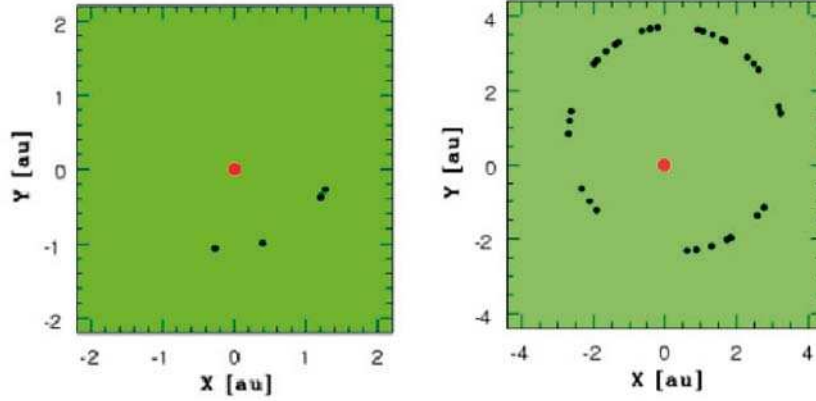
Figure 1: Simulations of the expected sightings of Piazzia, right panel, and the synthetic NEO asteroid 20075, left panel. The expected 63 observations of Piazza covering its whole orbit will allow the determination of all its local parameters whereas for 20075 the paucity in observations will lead to a statistically unreliable solution for the local parameters.

by an iterative process where the corrected parameters are reinjected into the model and a new N–body integration is performed, yielding new positions and Jacobian matrix. Convergence criteria must be given, $\|\mathbf{y}\| \ll 1$ for example. The iteration is stopped once these criteria are satisfied. Tests indicate that $2-4$ iterations are required when the data quality is good, ie sufficient number of observations with extended coverage of the trajectory (see below 2.1). The technical aspects of the implentation can be found in [4].

2.1 *Orbit improvement*

Although Gaia will observe for 5 years, simulations have shown that not all objects will be equally represented in the final data. Consequently all orbits cannot be improved to the same degree of reliability. In fact, any orbit improvement will narrowly depend on the distribution of observed positions along the trajectory of the SSSB under consideration. For example, using the Gaia simulator of rendez-vous, Aten 2062 will be observed 96 times in the course of the 5 year span of the Gaia mission. The observations for this object will cover the orbit sufficiently well to obtain a new improved set of initial conditions. To the contrary, 1994 BX, a Near Earth Object (NEO), will only be observed a few times (simulations give 4 observations over 5 years forming a short arc of its trajectory ony) due to its high magnitude. The result is that for this object not enough data is available for a reliable determination of the its local parameters. For those asteroids where the data is of good quality we can expect a 30 times better determination of the initial conditions based on Gaia observations only. In the case of poor quality data, the least squares inversion will be rank deficient and a full set of initial conditions cannot be determined. A main belt asteroid such as Piazzia will be seen 63 time by Gaia during, somewhat less than Aten 2062 but nevertheless well enough to determine its initial conditions to reasonable accuracy. This can be seen in Figure 1.

2.2 *Non–gravitational forces*

In the N–body simulation, non–gravitational forces are parameterized in a simplified manner (see for example [5] and [6]). The force acting on an comet close to the Sun can be written as :

$$\mathbf{f}_{p|noGrav} = A_1 \frac{g(r)}{r} \mathbf{r} + A_2\, g(r)\, \mathbf{t} + A_3\, g(r)\, \mathbf{n} \qquad (1)$$

47

where the transverse unit vector $\mathbf{t} = \langle \mathbf{v} - (\mathbf{v} \cdot \mathbf{r}) . \mathbf{r}/r^2 \rangle$ is normal to the heliocentric position vector $\mathbf{r}$, in the osculating orbital plane, and directed toward the motion, and $\mathbf{n} = (\mathbf{r}/r) \times \mathbf{t} = \langle \mathbf{r} \times \dot{\mathbf{r}} \rangle$ completes the right-handed frame. The coefficients (A_1, A_2, A_3) are the parameters to be adjusted. For an asteroid a similar model is used with only one non–zero component, the transverse one.

For how many comets these coefficients can be determined with a strong degree of confidence remains to be determined. Present estimates give only ~ 5 long period comets observed during the Gaia campaign, which is a minimal number. For the Yarkovsky effect on asteroids we expect more candidates, of the order of 60 (see [7]). For consistency the processing includes only Gaia observations, however after the mission it will be possible to combine the Gaia data with available ground based observation in order to improve these values. Note also that some objects (unexpected degassing for example) may issue an alert for ground based observation by the Gaia–FUN–SSO.

2.3 *Asteroid masses*

Masses for ~ 150 asteroids should be available after processing. It is expected that about 100 with $\sigma \leq 50\%$ and about 50 with $\sigma \leq 10\%$ ([8]). To determine an asteroid's mass, in addition to those measured in binary systems, its effect on another body must be measurable. In the main belt, about 100000 asteroids will suffer a close encounters with some perturbing asteroids. However, mutual perturbations between all these objects will not be strong enough to yield a mass determination. Thus not all these masses can be recovered. A list of asteroids for which a mass determination is indeed possible has been established using the Gaia simulator, there are roughly 2000 candidate bodies. Although in principle then some 2000 masses of perturbing asteroids could be determined this number is greatly reduced because the data quality is an important factor (see 2.1). A novel implementation for the determination of binary asteroid masses is in the process of being implemented. The method is based on Markov Chain Monte Carlo approach [9].

2.4 *Fundamental physics*

The equations of motion include the relativistic corrections through the Parametrized Post Newton formulation. The more important correction terms are written below :

$$\mathbf{f}_{p|relat} = \frac{m_\odot}{r^3} \left\{ \left[2 \left(\gamma + \beta \right) \frac{GM_\odot}{r} - \gamma \dot{r}^2 \right] . \mathbf{r}_i + 2 \left(\gamma + 1 \right) \left(\mathbf{r} \cdot \dot{\mathbf{r}} \right) . \dot{\mathbf{r}} \right\} + o(c^{-3}) \tag{2}$$

where $\mathbf{r}$ and $\dot{\mathbf{r}}$ are the heliocentric position and velocity, respectively.

Of the 10 PPN parameters only β which measures the degree of non–linearity in the superposition law of gravity, and γ which measures how much space curvature is produced by unit rest mass appear as parameters in the above formula. The parameter γ is to be determined by other means and will be assumed to equal to one in the long–term pipeline, consequently only β will be determined. The NEO will be strongly affected.

The Solar quadrupole also affects object which approach the Sun. The resulting force is written :

$$\mathbf{f}_{p|j_2} = \frac{-3Gm_\odot a_e^2 J_2}{2r^7} \left\{ 2(\mathbf{K} \cdot r) r^2 K_i + \left[r^2 - 5(\mathbf{K} \cdot r^2) \right] r_i \right\} \tag{3}$$

where $\mathbf{K}$ is the solar north pole. For these parameters all objects will be involved in the inversion procedure however NEO will be most affected as they approach the Sun. Finally the determination of $\dot{G}$, the link between the ICRF (optical Gaia) and the dynamical reference frame (Ecliptic and equinox) is also projected.
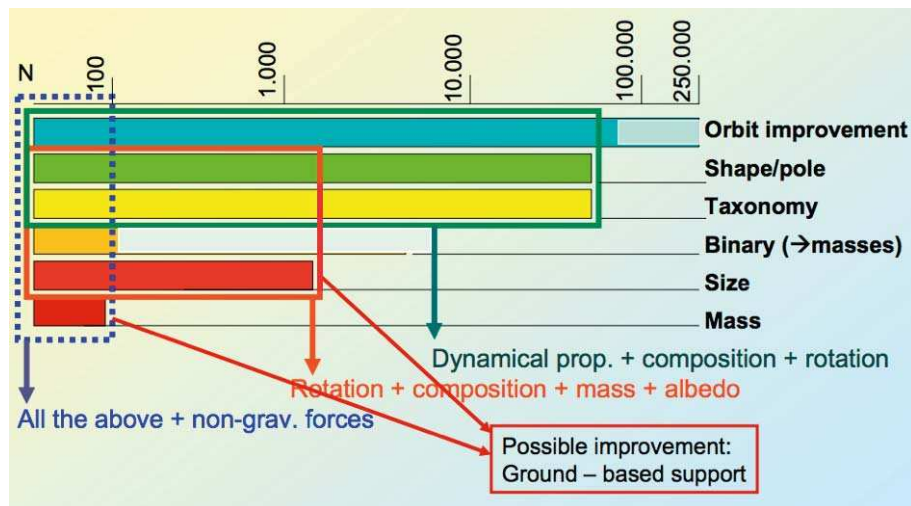
Figure 2: Anticipated improvement with the Gaia data after five years (courtes of Paolo Tanga Observatoire de la Cote d'Azur, Nice, France.

3. Near Earth Objects

NEOs are an important class for confirming the predictions of general relativity. Some 1500 of these are expected to be detected during the projected mission lifetime. As NEO pass close to the Sun non–gravitational effects will be felt, it will then be possible to decouple these forces from the general relativistic corrections (see 2.2 and 2.4). These are fast moving objects, however, and may not be seen in all the field of view and thus escape detection on all of the CCDs of the focal plane. If the identification of the object fails, then it may not be used in the long–term pipeline. Still, there remains a chance that an orbit reconstruction was successful in the short–term pipeline. In this latter case, the object can perhaps participate in the parameter estimation. Evidently it is in our interest to determine accurate parameters for these object because of their potential danger. For a more specific contributions on these objects please refer to the talks by D. Bancelin and B. Carry in these procedings.

Conclusion

Gaia will provide unprecedent high precision data for the objects of the solar system. We emphasise that the Gaia data processing will use only the data colected by the mission, no other data will be imported into the reduction pipeline. This provides a clean and faithful and statistically significant sample for determining the local and global parameters of our Solar System. Figure 1 resumes what we anticipate as possible with Gaia after five years of surveying. The orbits of some 100000 astrids will be improved along with their shape and taxanomical characteristics. For about 1000 objects we will be able to recover masses and ground based data will perhaps be injected into the processing albeit in an indirect fashion. Finally for 100 objects we will be able to determine their non–gravitaional parameters also.

Later, once the mission has completed its course, it will be possible to use data from other source to estimate again these partameters, but even now we can safely state that the Gaia data alone will improve the value of these parametrs at least 30 fold. Also the astrometric stellar catalogue will revolutionise future astronomy of SSSBs. During the mission the Gaia–FUN–SSO will indirectly contribute to the processing by providing ground baesed observations through the MPC, which periodically provides Gaia with the initial conditions and some estimated parameters of thre known asteroid.

References

[1] Perryman, M., 2009, Astronomical Applications of Astrometry: Ten Years of Exploitation of the Hipparcos Satellite Data, Cambridge University Press, ch. 11

[2] Hestroffer D., Morando B., Hog E., Kovalevsky J., Lindegren L., Mignard F., 1998, The HIPPAR-COS solar system objects catalogues, *A & A*, 334, 325

[3] Frezouls, B., Prat, G., Pham, K.-C., Poujoulet, E., 2012, CU4 Software Design Description, *GAIA-C4-SP-CNES-BF-007*, 2.1

[4] Hestroffer, D., Fouchard, M., David, P., 2011, DU457 Software Design Description, *GAIA-C4-SP-IMC-DHE-002-0*, D.1

[5] Marsden, B.G., Sekanina, Z., Yeomans, D.K. 1973 Comets and nongravitational forces. V *AJ*, 78, 211–225

[6] Chesley, S.R., Vokrouhlický, D., Ostro, S.J, Benner, L.A.M., Margot, J.L., Matson, R.L., Nolan, M.C, Shepard, M.K., 2008, Direct Estimation of Yarkovsky Accelerations on Near–Earth Asteroids *LPI Contributions*, 1405, 8330

[7] Mouret S., Mignard F., 2011, Detecting the Yarkovsky effect with the Gaia mission: list of the most promising candidates *MNRAS*, 413, 2, 741 – 748

[8] Mouret, S., Hestroffer, D., Mignard, F., 2008. Asteroid mass determination with Gaia mission *P&SS*, 56, 14, 1819 – 1822

[9] Oszkiewicz D.A., private communication