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# COMPARISON OF 3-RPR PLANAR PARALLEL MANIPULATORS WITH REGARD TO THEIR KINETOSTATIC PERFORMANCE AND SENSITIVITY TO GEOMETRIC UNCERTAINTIES

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**ABSTRACT:** *This paper deals with the sensitivity analysis of 3-RPR planar parallel manipulators. First, the manipulators under study as well as their degeneracy conditions are presented. Then, an optimization problem is formulated in order to obtain their maximal regular dexterous workspace. Moreover, the sensitivity coefficients of the pose of the manipulator moving platform to variations in the geometric parameters and in the actuated variables are expressed algebraically. Two aggregate sensitivity indices are determined, one related to the orientation of the manipulator moving platform and another one related to its position. Then, we compare two non-degenerate and two degenerate 3-RPR planar parallel manipulators with regard to their dexterity, workspace size and sensitivity. Finally, two actuating modes are compared with regard to their sensitivity.*

**KEYWORDS:** Sensitivity analysis, degenerate manipulators, regular dextrous workspace, transmission angle.

## 1 Introduction

Variations in the geometric parameters of PKMs can be either compensated or amplified. For that reason, it is important to analyze the sensitivity of the mechanism performance to variations in its geometric parameters. For instance, Wang et al. [1] studied the effect of manufacturing tolerances on the accuracy of a Stewart platform. Kim et al. [2] used a forward error bound analysis to find the error bound of the end-effector of a Stewart platform when the error bounds of the joints are given, and an inverse error bound analysis to determine those of the joints for the given error bound of the end-effector. Kim and Tsai [3] studied the effect of misalignment of linear actuators of a 3-Degree of

Freedom (DOF) translational parallel manipulator on the motion of its moving platform. Caro et al. [4] developed a tolerance synthesis method for mechanisms based on a robust design approach. Cardou et al. [5] proposed some kinematic-sensitivity indices for dimensionally nonhomogeneous jacobian matrices. Caro et al. [6] proposed two indices to evaluate the sensitivity of the end-effector pose (position + orientation) of Orthoglide 3-axis, a 3-DOF translational PKM, to variations in its design parameters. Besides, they noticed that the better the dexterity, the higher the accuracy of the manipulator. However, Yu et al. [7] claimed that the accuracy of a 3-DOF Planar Parallel Manipulator (PPM) is not necessarily related to its dexterity. Meng et al. [8] proposed a method to analyze the accuracy of parallel manipulators with joint clearances and ended up with a standard convex optimization problem to evaluate the maximal pose error in a prescribed workspace.

Some architectures of planar parallel manipulators are compared with regard to their sensitivity to geometric uncertainties in [9].

This paper deals with the comparison of the sensitivity of two degenerate and two non-degenerate 3-RPR PPMs. Likewise, the sensitivity of two actuating modes of the 3-RPR PPM, namely the 3-RPR PPM and the 3-RPR PPM, is analyzed. First, the degeneracy conditions of 3-RPR manipulators and the manipulators under study are presented. Then, the formulation of an optimization problem is introduced to obtain the regular dextrous workspace of those manipulators. Finally, a methodology is introduced to analyze and compare the sensitivity of the pose of their moving platforms to variations in their geometric parameters.

## 2 Manipulator Architecture

Here and throughout this paper,  $R$ ,  $P$  and  $\underline{P}$  denote revolute, prismatic and actuated prismatic joints, respectively. Figure. 1 illustrates the architecture of the manipulator under study. It is composed of a base and a moving platform (MP) connected with three legs. Points  $A_1$ ,  $A_2$  and  $A_3$ , ( $C_1$ ,  $C_2$  and  $C_3$ , respectively) lie at the corners of a triangle, of which point  $O$  (point  $P$ , resp.) is the circumcenter. Each leg is composed of a  $R$ , a  $P$  and a  $R$  joints in sequence. The three  $P$  joints are actuated. Accordingly, the manipulator is named 3- $\underline{RPR}$  manipulator.

$\mathcal{F}_b$  and  $\mathcal{F}_p$  are the base and the moving platform frames of the manipulator. In the scope of this paper,  $\mathcal{F}_b$  and  $\mathcal{F}_p$  are supposed to be orthogonal.  $\mathcal{F}_b$  is defined with the orthogonal dihedron  $(\vec{Ox}, \vec{Oy})$ , point  $O$  being its center and  $\vec{Ox}$  parallel to segment  $A_1A_2$ . Likewise,  $\mathcal{F}_p$  is defined with the orthogonal dihedron  $(\vec{PX}, \vec{PY})$ , point  $C$  being its center and  $\vec{PX}$  parallel to segment  $C_1C_2$ .

The pose of the manipulator MP, i.e., its position and its orientation, is determined by means of the Cartesian coordinates vector  $\mathbf{p} = [p_x, p_y]^T$  of operation point  $P$  expressed in frame  $\mathcal{F}_b$  and angle  $\phi$ , that is the angle between frames  $\mathcal{F}_b$  and  $\mathcal{F}_p$ . Finally, the passive joints do not have any stop.

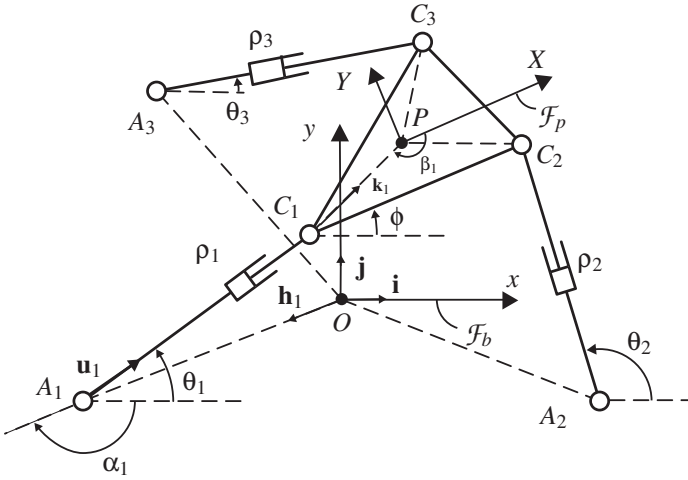


Figure 1. 3- $\underline{RPR}$  manipulator

## 3 Degenerate and Non-Degenerate Manipulators

In this section, we focus on the sensitivity analysis of two degenerate and two non-degenerate 3- $\underline{RPR}$  manipulators. First, the degeneracy conditions of such manipulators are given. Then, the architectures of the four manipulators under study are illustrated.

### 3.1 Degeneracy condition

The forward kinematic problem of a parallel manipulator often leads to complex equations and non analytic solutions, even

when considering 3-DOF PPMs [10]. For those manipulators, Hunt showed that the forward kinematics admits at most six solutions [11] and some authors proved that their forward kinematics can be reduced to the solution of a sixth-degree characteristic polynomial [12, 13].

As shown in [14], [15] and [16], a 3- $\underline{RPR}$  PPM is said to be degenerate when the degree of its characteristic polynomial becomes smaller than six. Six types of degenerate 3- $\underline{RPR}$  PPMs exists in the literature, namely,

1. 3- $\underline{RPR}$  PPMs with two coincident joints;
2. 3- $\underline{RPR}$  PPMs with similar aligned base and moving platforms;
3. 3- $\underline{RPR}$  PPMs with nonsimilar aligned base and moving platforms;
4. 3- $\underline{RPR}$  PPMs with similar triangular base and moving platforms;
5. 3- $\underline{RPR}$  PPMs with the three actuated prismatic joints satisfying a certain relationship;
6. 3- $\underline{RPR}$  PPMs with congruent base and moving platforms, of which the moving platform is rotated of 180 deg about one of its side.

In the scope of this paper, we focus on the sensitivity analysis of the fourth and the sixth cases. For the fourth case, the forward kinematics is reduced to the solution of two quadratics in cascade. For the sixth case, the forward kinematics degenerates over the whole joint space and is reduced to the solution of a third-degree polynomial and a quadratic in sequence.

### 3.2 Manipulators under study

Figures 2(a)-(d) illustrate the four manipulators under study, named  $M_1$ ,  $M_2$ ,  $M_3$  and  $M_4$ , respectively.  $M_1$  and  $M_2$  are non-degenerate manipulators while  $M_3$  and  $M_4$  are degenerate manipulators. From Fig. 2(a), the base and moving platforms of  $M_1$  are equilateral. From Fig. 2(b), the base and moving platforms of  $M_2$  are identical but in a different geometric configuration for an orientation  $\phi = 0$ .  $M_3$  and  $M_4$  illustrate the fourth and the sixth degeneracy cases presented in Sec. 3.1. It is noteworthy that the base and moving platforms of  $M_2$ ,  $M_3$  and  $M_4$  have the same circumscribed circle, its radius being equal to  $\sqrt{2}/2$ . As far as  $M_1$  is concerned, the circumscribed circle of its moving platform is two times smaller than the one of the base platform.

To compare the sensitivity of these PPMs, the geometric parameters have to be normalized. Therefore, let  $R_1$  and  $R_2$  be the radii of the base and moving platforms of the PPM. In order to come up with finite values,  $R_1$  and  $R_2$  are normalized as

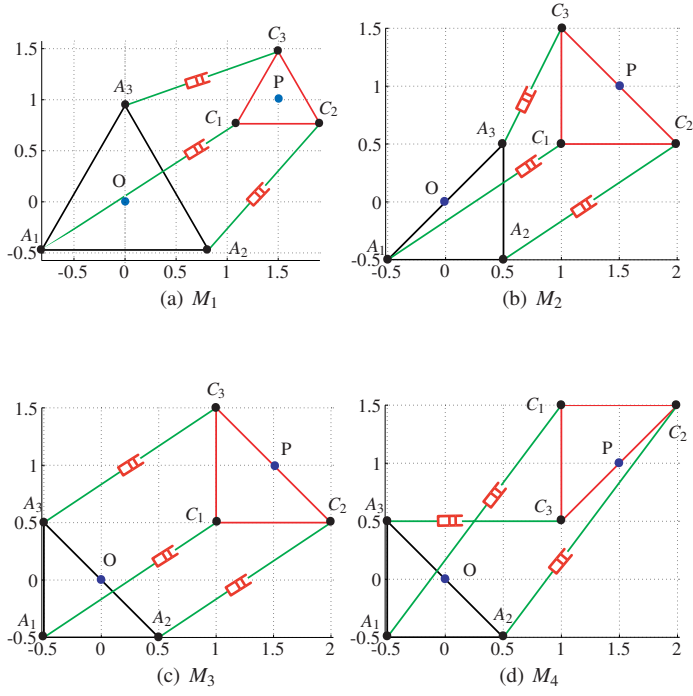


Figure 2. The four 3-RPR manipulators under study with  $\phi = 0$  and  $\mathbf{p} = [1, 1.5]^T$ : (a)-(b) non-degenerate manipulators, (c)-(d) degenerate manipulators

in [17–19]. For that matter, let  $N_f$  be a normalizing factor:

$$N_f = (R_1 + R_2)/2 \quad (1)$$

and

$$r_m = R_m/N_f, \quad m = 1, 2 \quad (2)$$

Therefore,

$$r_1 + r_2 = 2 \quad (3)$$

From Eqs.(2) and (3), we can notice that:

$$r_1 \in [0, 2], r_2 \in [0, 2] \quad (4)$$

As the former two-dimensional infinite space corresponding to geometric parameters  $R_1$  and  $R_2$  is reduced to a one-dimensional finite space defined with Eq.(3), the workspace analysis of the 3-RPR manipulator under study turns out to be easier.

## 4 Regular Dexterous Workspace

In order to compare the sensitivity of the foregoing manipulators, we first define their Regular Dexterous Workspace (RDW). Then, the sensitivity of  $M_1$ ,  $M_2$ ,  $M_3$  and  $M_4$  can be evaluated throughout their RDW and compared. The RDW of a manipulator is a regular-shaped part of its workspace with good and homogeneous kinetostatic performance. The shape of the RDW is up to the designer. It may be a cube, a parallelepiped, a cylinder or another regular shape. A good shape fits to the singular surfaces.

The kinetostatic performance of a manipulator is usually characterized by the conditioning number of its kinematic Jacobian matrix [20, 21]. From [22, 23], the transmission angle of a 3-DOF PPM can be also used to evaluate its kinetostatic performance. Here, we prefer to use the transmission angle as a kinetostatic performance index as it does not require the normalization of the kinematic Jacobian matrix. On the contrary, the kinematic Jacobian matrix of 3-DOF PPM has to be normalized by means of a normalizing length in order its conditioning number to make sense [24].

### 4.1 Transmission angle

The transmission angle  $\psi_i$  associated with the  $i$ th leg is defined as the angle between force vector  $\mathbf{F}_{c_i}$  and translational velocity vector  $\mathbf{V}_{c_i}$  at point  $C_i$  as illustrated in Fig. 3.

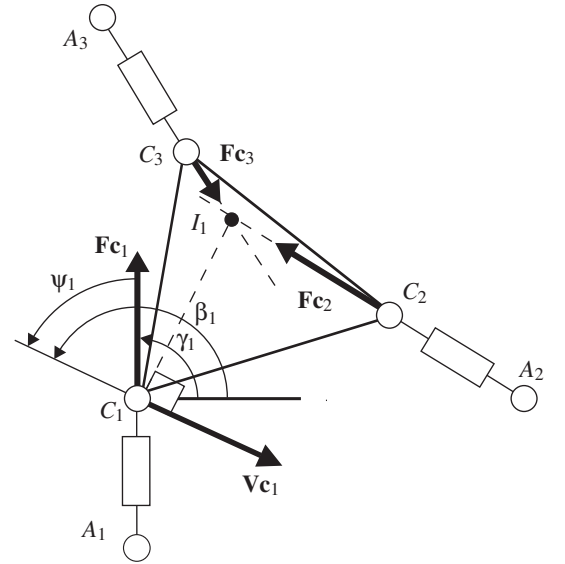


Figure 3. Transmission angle of the 3-RPR PPMs

The direction of force  $\mathbf{F}_{c_i}$  is the direction of leg  $A_i C_i$ , namely,

$$\gamma_i = \arctan \left( \frac{y_{C_i} - y_{A_i}}{x_{C_i} - x_{A_i}} \right), \quad i = 1, 2, 3 \quad (5)$$

The instantaneous centre of rotation depends on the leg under

study. For example, instantaneous centre of rotation  $I_1$  associated with leg 1 is the intersecting point of forces  $\mathbf{F}c_2$  and  $\mathbf{F}c_3$ .

Table 1 gives the Cartesian coordinates of instantaneous centre of rotation  $I_i$  associated with the  $i$ th leg of the 3-RPR PPM, expressed in frame  $\mathcal{F}_b$ , with  $b_i = y_{C_i} - x_{C_i} \tan \gamma_i$ ,  $i = 1, 2, 3$ .

The direction of  $\mathbf{V}c_i$  is defined as,

$$\beta_i = \arctan\left(\frac{y_{C_i} - y_{I_i}}{x_{C_i} - x_{I_i}}\right) + \frac{\pi}{2}, i = 1, 2, 3 \quad (6)$$

Accordingly,

$$\psi_i = |\gamma_i - \beta_i|, i = 1, 2, 3 \quad (7)$$

Finally, the transmission angle  $\psi$  of the overall mechanism is defined as,

$$\psi = \max(\psi_i), i = 1, 2, 3 \quad (8)$$

and the smaller  $\psi$ , the better the force transmission of the mechanism.

## 4.2 RDW determination

In the scope of this study, the RDW of the PPM is supposed to be a cylinder of  $\phi$ -axis with good kinetostatic performance, i.e., the transmission angle  $\psi$  is smaller than  $75^\circ$  throughout the cylinder. In order to obtain such a RDW, we can solve the following optimization problem:

$$Pb \begin{cases} \min_{\mathbf{x}} 1/R \\ s.t. \Delta\phi \geq 30^\circ \\ \psi \leq 75^\circ \end{cases}$$

$R$  being the radius of the cylinder and  $\Delta\phi$  the orientation range of the MP of the manipulator within its RDW. Here,  $\Delta\phi$  is equal to  $30^\circ$  arbitrarily. This optimization problem has five decision variables:

$$\mathbf{x} = [R \ I_x \ I_y \ \phi_{min} \ \phi_{max}]$$

$I_x$  and  $I_y$  being the Cartesian coordinates of the center of the cylinder,  $\phi_{min}$  and  $\phi_{max}$  being the lower and upper bounds of  $\phi$  angle ( $\Delta\phi = \phi_{max} - \phi_{min}$ ).

This optimization problem is solved by means of a Tabu

search Hooke and Jeeves algorithm [25]. As a result, the RDW of the manipulator is completely defined by means of the decision variables corresponding to the global minimum<sup>1</sup>.

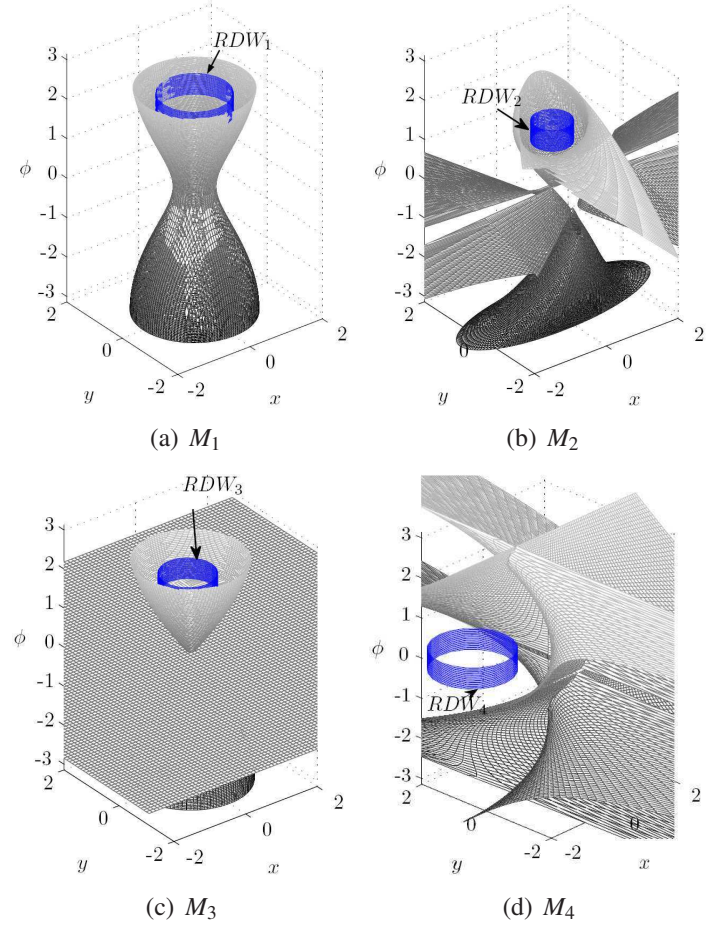


Figure 4. Maximal Regular Dexterous Workspace

Table 2. RDW radius of  $M_1, M_2, M_3$  and  $M_4$

$R_1$	$R_2$	$R_3$	$R_4$
1.21	0.62	0.75	2.69

Figures 4-(d) illustrate the workspace, the singularities and the maximal RDW of  $M_1, M_2, M_3$  and  $M_4$ . Their radii are given in Table 2. We can notice that  $M_4$  has the biggest RDW and  $M_2$  the smallest one.

## 5 Sensitivity Analysis

In this section, the sensitivity of  $M_1, M_2, M_3$  and  $M_4$  is evaluated throughout their RDW for a matter of comparison. The

<sup>1</sup>The solution obtained with a Tabu search Hooke and Jeeves algorithm will not be necessarily the global optimum. However, it will provide a solution that is close to the global one and satisfactory in the framework of this research work.

Table 1. Cartesian coordinates of instantaneous centres of rotation

	$I_1$	$I_2$	$I_3$
$x_i$	$\frac{b_3 - b_2}{\tan(\gamma_2) - \tan(\gamma_3)}$	$\frac{b_1 - b_3}{\tan(\gamma_3) - \tan(\gamma_1)}$	$\frac{b_2 - b_1}{\tan(\gamma_1) - \tan(\gamma_2)}$
$y_i$	$\frac{b_3 \tan(\gamma_2) - b_2 \tan(\gamma_3)}{\tan(\gamma_2) - \tan(\gamma_3)}$	$\frac{b_1 \tan(\gamma_3) - b_3 \tan(\gamma_1)}{\tan(\gamma_3) - \tan(\gamma_1)}$	$\frac{b_2 \tan(\gamma_1) - b_1 \tan(\gamma_2)}{\tan(\gamma_1) - \tan(\gamma_2)}$

sensitivity coefficients, and, two aggregate sensitivity indices are determined to analyze the sensitivity of the pose of the moving platform of a 3-RPR manipulator to variations in its geometric parameters. Then, the contours of these indices are plotted in  $M_1$ ,  $M_2$ ,  $M_3$  and  $M_4$  RDWs and the results are analyzed.

### 5.1 Sensitivity coefficients

From the closed-loop kinematic chains  $O - A_i - C_i - P - O$ ,  $i = 1, \dots, 3$  depicted in Fig. 1, the position vector  $\mathbf{p}$  of point  $P$  can be expressed in  $\mathcal{F}_b$  as follows,

$$\mathbf{p} = \begin{bmatrix} p_x \\ p_y \end{bmatrix} = \mathbf{a}_i + (\mathbf{c}_i - \mathbf{a}_i) + (\mathbf{p} - \mathbf{c}_i), \quad i = 1, \dots, 3 \quad (9)$$

$\mathbf{a}_i$  and  $\mathbf{c}_i$  being the position vectors of points  $A_i$  and  $C_i$  expressed in  $\mathcal{F}_b$ . Equation (9) can also be written as,

$$\mathbf{p} = a_i \mathbf{h}_i + \rho_i \mathbf{u}_i + c_i \mathbf{k}_i \quad (10)$$

with

$$\mathbf{h}_i = \begin{bmatrix} \cos \alpha_i \\ \sin \alpha_i \end{bmatrix}, \quad \mathbf{u}_i = \begin{bmatrix} \cos \theta_i \\ \sin \theta_i \end{bmatrix}, \quad \mathbf{k}_i = \begin{bmatrix} \cos(\phi + \beta_i + \pi) \\ \sin(\phi + \beta_i + \pi) \end{bmatrix}$$

where  $a_i$  is the distance between points  $O$  and  $A_i$ ,  $\rho_i$  is the distance between points  $A_i$  and  $C_i$ ,  $c_i$  is the distance between points  $C_i$  and  $P$ ,  $\mathbf{h}_i$  is the unit vector  $\vec{OA}_i / \|\vec{OA}_i\|_2$ ,  $\mathbf{u}_i$  is the unit vector  $\vec{A_i C_i} / \|\vec{A_i C_i}\|_2$  and  $\mathbf{k}_i$  is the unit vector  $\vec{C_i P} / \|\vec{C_i P}\|_2$ .

Upon differentiation of Eq.(10), we obtain:

$$\begin{aligned} \delta \mathbf{p} = & \delta a_i \mathbf{h}_i + a_i \delta \alpha_i \mathbf{E} \mathbf{h}_i + \delta \rho_i \mathbf{u}_i + \rho_i \delta \theta_i \mathbf{E} \mathbf{u}_i \\ & + \delta c_i \mathbf{k}_i + c_i (\delta \phi + \delta \beta_i) \mathbf{E} \mathbf{k}_i \end{aligned} \quad (11)$$

with matrix  $\mathbf{E}$  defined as

$$\mathbf{E} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad (12)$$

$\delta \mathbf{p}$  and  $\delta \phi$  being the position and orientation errors of the MP. Likewise,  $\delta a_i$ ,  $\delta \alpha_i$ ,  $\delta \rho_i$ ,  $\delta c_i$  and  $\delta \beta_i$  denote the variations in  $a_i$ ,  $\alpha_i$ ,  $\rho_i$ ,  $c_i$  and  $\beta_i$ , respectively.

The idle variation  $\delta \theta_i$  is eliminated by dot-multiplying Eq.(11) by  $\rho_i \mathbf{u}_i^T$ , thus obtaining

$$\begin{aligned} \rho_i \mathbf{u}_i^T \delta \mathbf{p} = & \rho_i \delta a_i \mathbf{u}_i^T \mathbf{h}_i + \rho_i a_i \delta \alpha_i \mathbf{u}_i^T \mathbf{E} \mathbf{h}_i + \rho_i \delta \rho_i \\ & + \rho_i \delta c_i \mathbf{u}_i^T \mathbf{k}_i + \rho_i c_i (\delta \phi + \delta \beta_i) \mathbf{u}_i^T \mathbf{E} \mathbf{k}_i \end{aligned} \quad (13)$$

Equation (13) can now be cast in vector form, namely,

$$\begin{aligned} \mathbf{A} \begin{bmatrix} \delta \phi \\ \delta \mathbf{p} \end{bmatrix} = & \mathbf{H}_a \begin{bmatrix} \delta a_1 \\ \delta a_2 \\ \delta a_3 \end{bmatrix} + \mathbf{H}_\alpha \begin{bmatrix} \delta \alpha_1 \\ \delta \alpha_2 \\ \delta \alpha_3 \end{bmatrix} + \mathbf{B} \begin{bmatrix} \delta \rho_1 \\ \delta \rho_2 \\ \delta \rho_3 \end{bmatrix} + \\ & \mathbf{H}_c \begin{bmatrix} \delta c_1 \\ \delta c_2 \\ \delta c_3 \end{bmatrix} + \mathbf{H}_\beta \begin{bmatrix} \delta \beta_1 \\ \delta \beta_2 \\ \delta \beta_3 \end{bmatrix} \end{aligned} \quad (14)$$

with

$$\mathbf{A} = \begin{bmatrix} m_1 & \rho_1 \mathbf{u}_1^T \\ m_2 & \rho_2 \mathbf{u}_2^T \\ m_3 & \rho_3 \mathbf{u}_3^T \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \rho_1 & 0 & 0 \\ 0 & \rho_2 & 0 \\ 0 & 0 & \rho_3 \end{bmatrix} \quad (15a)$$

$$\mathbf{H}_a = \text{diag} \left[ \rho_1 \mathbf{u}_1^T \mathbf{h}_1 \quad \rho_2 \mathbf{u}_2^T \mathbf{h}_2 \quad \rho_3 \mathbf{u}_3^T \mathbf{h}_3 \right] \quad (15b)$$

$$\mathbf{H}_\alpha = \text{diag} \left[ \rho_1 a_1 \mathbf{u}_1^T \mathbf{E} \mathbf{h}_1 \quad \rho_2 a_2 \mathbf{u}_2^T \mathbf{E} \mathbf{h}_2 \quad \rho_3 a_3 \mathbf{u}_3^T \mathbf{E} \mathbf{h}_3 \right] \quad (15c)$$

$$\mathbf{H}_c = \text{diag} \left[ \rho_1 \mathbf{u}_1^T \mathbf{k}_1 \quad \rho_2 \mathbf{u}_2^T \mathbf{k}_2 \quad \rho_3 \mathbf{u}_3^T \mathbf{k}_3 \right] \quad (15d)$$

$$\mathbf{H}_\beta = \text{diag} \left[ \rho_1 c_1 \mathbf{u}_1^T \mathbf{E} \mathbf{k}_1 \quad \rho_2 c_2 \mathbf{u}_2^T \mathbf{E} \mathbf{k}_2 \quad \rho_3 c_3 \mathbf{u}_3^T \mathbf{E} \mathbf{k}_3 \right] \quad (15e)$$

and

$$m_i = -\rho_i c_i \mathbf{u}_i^T \mathbf{E} \mathbf{k}_i, \quad i = 1, \dots, 3 \quad (16)$$

Let us notice that  $\mathbf{A}$  and  $\mathbf{B}$  are the direct and the inverse Jacobian matrices of the manipulator, respectively. Assuming that  $\mathbf{A}$  is non singular, i.e., the manipulator does not meet any Type II sin-

gularity [26], we obtain upon multiplication of Eq.(14) by  $\mathbf{A}^{-1}$ :

$$\begin{bmatrix} \delta\phi \\ \delta\mathbf{p} \end{bmatrix} = \mathbf{J}_a \begin{bmatrix} \delta a_1 \\ \delta a_2 \\ \delta a_3 \end{bmatrix} + \mathbf{J}_\alpha \begin{bmatrix} \delta\alpha_1 \\ \delta\alpha_2 \\ \delta\alpha_3 \end{bmatrix} + \mathbf{J} \begin{bmatrix} \delta\rho_1 \\ \delta\rho_2 \\ \delta\rho_3 \end{bmatrix} + \mathbf{J}_c \begin{bmatrix} \delta c_1 \\ \delta c_2 \\ \delta c_3 \end{bmatrix} + \mathbf{J}_\beta \begin{bmatrix} \delta\beta_1 \\ \delta\beta_2 \\ \delta\beta_3 \end{bmatrix} \quad (17)$$

with

$$\mathbf{J} = \mathbf{A}^{-1}\mathbf{B} \quad (18a)$$

$$\mathbf{J}_a = \mathbf{A}^{-1}\mathbf{H}_a \quad (18b)$$

$$\mathbf{J}_\alpha = \mathbf{A}^{-1}\mathbf{H}_\alpha \quad (18c)$$

$$\mathbf{J}_c = \mathbf{A}^{-1}\mathbf{H}_c \quad (18d)$$

$$\mathbf{J}_\beta = \mathbf{A}^{-1}\mathbf{H}_\beta \quad (18e)$$

and

$$\mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})} \begin{bmatrix} v_1 & v_2 & v_3 \\ v_1 & v_2 & v_3 \end{bmatrix} \quad (19a)$$

$$v_i = \rho_j \rho_k (\mathbf{u}_j \times \mathbf{u}_k)^T \mathbf{k} \quad (19b)$$

$$v_i = \mathbf{E}(m_j \rho_k \mathbf{u}_k - m_k \rho_j \mathbf{u}_j) \quad (19c)$$

$$\det(\mathbf{A}) = \sum_{i=1}^3 m_i v_i \quad (19d)$$

$$\mathbf{k} = \mathbf{i} \times \mathbf{j} \quad (19e)$$

$j = (i+1)$  modulo 3;  $k = (i+2)$  modulo 3;  $i = 1, 2, 3$ .  $\mathbf{J}$  is the kinematic Jacobian matrix of the manipulator whereas  $\mathbf{J}_a$ ,  $\mathbf{J}_\alpha$ ,  $\mathbf{J}_c$  and  $\mathbf{J}_\beta$  are named *sensitivity Jacobian matrices* of the pose of the MP to variations in  $a_i$ ,  $\alpha_i$ ,  $c_i$  and  $\beta_i$ , respectively. Indeed, the terms of  $\mathbf{J}_a$ ,  $\mathbf{J}_\alpha$ ,  $\mathbf{J}_c$  and  $\mathbf{J}_\beta$  are the sensitivity coefficients of the position and the orientation of the moving platform of the manipulator to variations in the Polar coordinates of points  $A_i$  and  $C_i$ . Likewise,  $\mathbf{J}$  contains the sensitivity coefficients of the pose of the MP of the manipulator to variations in the prismatic actuated joints. It is noteworthy that all these sensitivity coefficients are expressed algebraically.

Let  $\delta a_{ix}$  and  $\delta a_{iy}$  denote the position errors of points  $A_i$ ,  $i = 1, 2, 3$ , along  $\vec{Ox}$  and  $\vec{Oy}$ , namely, the variations in the Cartesian coordinates of points  $A_i$ . Likewise, let  $\delta c_{iX}$  and  $\delta c_{iY}$  denote the position errors of points  $C_i$  along  $\vec{PX}$  and  $\vec{PY}$ , namely, the variations in the Cartesian coordinates of points  $C_i$ .

From Fig. 1,

$$\begin{bmatrix} \delta a_{ix} \\ \delta a_{iy} \end{bmatrix} = \begin{bmatrix} \cos \alpha_i & -a_i \sin \alpha_i \\ \sin \alpha_i & a_i \cos \alpha_i \end{bmatrix} \begin{bmatrix} \delta a_i \\ \delta \alpha_i \end{bmatrix} \quad (20a)$$

$$\begin{bmatrix} \delta c_{iX} \\ \delta c_{iY} \end{bmatrix} = \begin{bmatrix} \cos \beta_i & -c_i \sin \beta_i \\ \sin \beta_i & c_i \cos \beta_i \end{bmatrix} \begin{bmatrix} \delta c_i \\ \delta \beta_i \end{bmatrix} \quad (20b)$$

Accordingly, from Eq.(17) and Eqs.(20a)-(b), we obtain the following relation between the pose error of the MP and variations in the Cartesian coordinates of points  $A_i$  and  $C_i$ :

$$\begin{bmatrix} \delta\phi \\ \delta\mathbf{p} \end{bmatrix} = \mathbf{J}_A \begin{bmatrix} \delta a_{1x} \\ \delta a_{1y} \\ \delta a_{2x} \\ \delta a_{2y} \\ \delta a_{3x} \\ \delta a_{3y} \end{bmatrix} + \mathbf{J} \begin{bmatrix} \delta\rho_1 \\ \delta\rho_2 \\ \delta\rho_3 \end{bmatrix} + \mathbf{J}_C \begin{bmatrix} \delta c_{1X} \\ \delta c_{1Y} \\ \delta c_{2X} \\ \delta c_{2Y} \\ \delta c_{3X} \\ \delta c_{3Y} \end{bmatrix} \quad (21)$$

$\mathbf{J}_A$  and  $\mathbf{J}_C$  being named *sensitivity Jacobian matrices* of the pose of the MP to variations in the Cartesian coordinates of points  $A_i$  and  $C_i$ , respectively. Indeed, the terms of  $\mathbf{J}_A$  and  $\mathbf{J}_C$  are the sensitivity coefficients of the pose of the MP to variations in the Cartesian coordinates of points  $A_i$  and  $C_i$ .

In order to better highlight the sensitivity coefficients, let us write the  $3 \times 6$  matrices  $\mathbf{J}_A$  and  $\mathbf{J}_C$  and the  $3 \times 3$  matrix  $\mathbf{J}$  as follows:

$$\mathbf{J}_A = \begin{bmatrix} \mathbf{J}_{A_1} & \mathbf{J}_{A_2} & \mathbf{J}_{A_3} \end{bmatrix} \quad (22a)$$

$$\mathbf{J}_C = \begin{bmatrix} \mathbf{J}_{C_1} & \mathbf{J}_{C_2} & \mathbf{J}_{C_3} \end{bmatrix} \quad (22b)$$

$$\mathbf{J} = \begin{bmatrix} \mathbf{j}_1 & \mathbf{j}_2 & \mathbf{j}_3 \end{bmatrix} \quad (22c)$$

the  $3 \times 2$  matrices  $\mathbf{J}_{A_i}$  and  $\mathbf{J}_{C_i}$  and the three dimensional vectors  $\mathbf{j}_i$  being expressed as:

$$\mathbf{J}_{A_i} = \begin{bmatrix} \mathbf{j}_{A_i\phi} \\ \mathbf{j}_{A_i p} \end{bmatrix}, \quad i = 1, 2, 3 \quad (23a)$$

$$\mathbf{J}_{C_i} = \begin{bmatrix} \mathbf{j}_{C_i\phi} \\ \mathbf{j}_{C_i p} \end{bmatrix}, \quad i = 1, 2, 3 \quad (23b)$$

$$\mathbf{j}_i = \begin{bmatrix} j_{i\phi} \\ \mathbf{j}_{ip} \end{bmatrix}, \quad i = 1, 2, 3 \quad (23c)$$

with

$$\mathbf{j}_{A_i\phi} = \frac{1}{\det(\mathbf{A})} \begin{bmatrix} v_i q_i & v_i r_i \end{bmatrix} \quad (24a)$$

$$\mathbf{j}_{C_i\phi} = \frac{1}{\det(\mathbf{A})} \begin{bmatrix} v_i s_i & v_i t_i \end{bmatrix} \quad (24b)$$

$$j_{i\phi} = \frac{\rho_i v_i}{\det(\mathbf{A})} \quad (24c)$$

$$\mathbf{J}_{A_i p} = \frac{1}{\det(\mathbf{A})} \begin{bmatrix} q_i \mathbf{v}_i^T \mathbf{i} & r_i \mathbf{v}_i^T \mathbf{i} \\ q_i \mathbf{v}_i^T \mathbf{j} & r_i \mathbf{v}_i^T \mathbf{j} \end{bmatrix} \quad (24d)$$

$$\mathbf{J}_{C_i p} = \frac{1}{\det(\mathbf{A})} \begin{bmatrix} s_i \mathbf{v}_i^T \mathbf{i} & t_i \mathbf{v}_i^T \mathbf{i} \\ s_i \mathbf{v}_i^T \mathbf{j} & t_i \mathbf{v}_i^T \mathbf{j} \end{bmatrix} \quad (24e)$$

$$\mathbf{j}_{i p} = \frac{1}{\det(\mathbf{A})} \begin{bmatrix} \rho_i \mathbf{v}_i^T \mathbf{i} \\ \rho_i \mathbf{v}_i^T \mathbf{j} \end{bmatrix} \quad (24f)$$

$q_i$ ,  $r_i$ ,  $s_i$  and  $t_i$  taking the form:

$$q_i = \rho_i \mathbf{u}_i^T \mathbf{i} \quad (25a)$$

$$r_i = \rho_i \mathbf{u}_i^T \mathbf{j} \quad (25b)$$

$$s_i = \rho_i \mathbf{u}_i^T \mathbf{k}_i \cos \beta_i - \rho_i \mathbf{u}_i^T \mathbf{E} \mathbf{k}_i \sin \beta_i \quad (25c)$$

$$t_i = \rho_i \mathbf{u}_i^T \mathbf{k}_i \sin \beta_i + \rho_i \mathbf{u}_i^T \mathbf{E} \mathbf{k}_i \cos \beta_i \quad (25d)$$

$\mathbf{j}_{A_i\phi}$ ,  $\mathbf{j}_{C_i\phi}$  and  $j_{i\phi}$  contain the sensitivity coefficients of the orientation of the MP of the manipulator to variations in the Cartesian coordinates of points  $A_i$ ,  $C_i$  and prismatic actuated variables, respectively. Similarly,  $\mathbf{J}_{A_i p}$ ,  $\mathbf{J}_{C_i p}$  and  $\mathbf{j}_{i p}$  contain the sensitivity coefficients related to the position of the MP.

Accordingly, the designer of such a planar parallel manipulator can easily identify the most influential geometric variations to the pose of its MP and synthesize proper dimensional tolerances from the previous sensitivity coefficients. Two aggregate sensitivity indices related to the geometric errors of the moving and base platforms are introduced thereafter.

## 5.2 Global sensitivity indices

The pose errors of the manipulator MP depend on variations in the geometric parameters as well as on the manipulator configuration. In order to analyze the influence of the manipulator configuration on those errors, let us first formulate some indices in order to assess the aggregate sensitivity of the MP pose to variations in the geometric parameters for a given manipulator configuration. To this end, let Eq.(21) be expressed as:

$$\begin{bmatrix} \delta\phi \\ \delta\mathbf{p} \end{bmatrix} = \mathbf{J}_s \begin{bmatrix} \delta\mathbf{a}_i & \delta\rho_i & \delta\mathbf{c}_i \end{bmatrix}^T \quad (26)$$

with

$$\mathbf{J}_s = \begin{bmatrix} \mathbf{J}_A & \mathbf{J} & \mathbf{J}_C \end{bmatrix} \quad (27)$$

and

$$\delta\mathbf{a}_i = \begin{bmatrix} \delta a_{1x} & \delta a_{1y} & \delta a_{2x} & \delta a_{2y} & \delta a_{3x} & \delta a_{3y} \end{bmatrix} \quad (28a)$$

$$\delta\rho_i = \begin{bmatrix} \delta\rho_1 & \delta\rho_2 & \delta\rho_3 \end{bmatrix} \quad (28b)$$

$$\delta\mathbf{c}_i = \begin{bmatrix} \delta c_{1X} & \delta c_{1Y} & \delta c_{2X} & \delta c_{2Y} & \delta c_{3X} & \delta c_{3Y} \end{bmatrix} \quad (28c)$$

The  $3 \times 15$  matrix  $\mathbf{J}_s$  is named ‘‘sensitivity Jacobian matrix’’ and can be written as follows:

$$\mathbf{J}_s = \begin{bmatrix} \mathbf{j}_{s\phi} \\ \mathbf{J}_{s p} \end{bmatrix} \quad (29)$$

with

$$\mathbf{j}_{s\phi} = \begin{bmatrix} \mathbf{j}_{A_1\phi} & \mathbf{j}_{A_2\phi} & \mathbf{j}_{A_3\phi} & j_{1\phi} & j_{2\phi} & j_{3\phi} & \mathbf{j}_{C_1\phi} & \mathbf{j}_{C_2\phi} & \mathbf{j}_{C_3\phi} \end{bmatrix} \quad (30a)$$

$$\mathbf{J}_{s p} = \begin{bmatrix} \mathbf{J}_{A_1 p} & \mathbf{J}_{A_2 p} & \mathbf{J}_{A_3 p} & \mathbf{j}_{1 p} & \mathbf{j}_{2 p} & \mathbf{j}_{3 p} & \mathbf{J}_{C_1 p} & \mathbf{J}_{C_2 p} & \mathbf{J}_{C_3 p} \end{bmatrix} \quad (30b)$$

From Eq.(30a), we can define an aggregate sensitivity index  $v_\phi$  of the orientation of the MP of the manipulator to variations in its geometric parameters and prismatic actuated joints, namely,

$$v_\phi = \frac{\|\mathbf{j}_{s\phi}\|_2}{n_v} \quad (31)$$

$n_v$  being the number of variations that are considered. Here,  $n_v$  is equal to 15.

Likewise, from Eq.(30b), an aggregate sensitivity index  $v_p$  of the position of the MP of the manipulator to variations in its geometric parameters and prismatic actuated joints can be defined as follows:

$$v_p = \frac{\|\mathbf{J}_{s p}\|_2}{n_v} \quad (32)$$

For any given manipulator configuration, the lower  $v_\phi$ , the lower the overall sensitivity of the orientation of its MP to variations in the geometric parameters. Similarly, the lower  $v_p$ , the



lower the overall sensitivity of the MP position to variations in the geometric parameters. As a matter of fact,  $v_\phi$  ( $v_p$ , resp.) characterizes the intrinsic sensitivity of the MP orientation (position, resp.) to any variation in the geometric parameters. Let us notice that  $v_p$  as well as the sensitivity coefficients related to the MP position defined in Sections 5.1 are frame dependent, whereas  $v_\phi$  and the sensitivity coefficients related to the MP orientation are not.

In order to evaluate the sensitivity of the manipulator over its workspace or part of it, four global sensitivity indices are defined as follows:

$$\overline{v_\phi} = \frac{\int_W v_\phi dW}{\int_W dW} \quad (33a)$$

$$v_{\phi_{max}} = \max(v_\phi) \quad (33b)$$

$$\overline{v_p} = \frac{\int_W v_p dW}{\int_W dW} \quad (33c)$$

$$v_{p_{max}} = \max(v_p) \quad (33d)$$

$\overline{v_\phi}$  and  $\overline{v_p}$  are the average values of  $v_\phi$  and  $v_p$  over  $W$ ,  $W$  being the manipulator workspace or part of it. Likewise,  $v_{\phi_{max}}$  and  $v_{p_{max}}$  are the maximum values of  $v_\phi$  and  $v_p$  over  $W$ .

Finally,  $v_\phi$ ,  $\overline{v_\phi}$  and  $v_{\phi_{max}}$  are expressed in [rad/L], whereas  $v_p$ ,  $\overline{v_p}$  and  $v_{p_{max}}$  are dimensionless, [L] being the unit of length.

### 5.3 Comparison of two non-degenerate and two degenerate 3-RPR PPMs

In this section, the sensitivity of  $M_1$ ,  $M_2$ ,  $M_3$  and  $M_4$  is evaluated within their RDW for a matter of comparison based on aggregate sensitivity indices  $v_\phi$  and  $v_p$  defined in Eqs.(31) and (32) and global sensitivity indices  $\overline{v_\phi}$ ,  $v_{\phi_{max}}$ ,  $\overline{v_p}$  and  $v_{p_{max}}$  defined in Eqs.(33a)-(d).

Figures 5(a)-(d) (Figures 6(a)-(d), resp.) illustrate the isocontours of the maximum value of  $v_\phi$  ( $v_p$ , resp.) for a given orientation  $\phi$  of the MP throughout the RDW of  $M_1$ ,  $M_2$ ,  $M_3$  and  $M_4$ , respectively.

Table 3. Values of  $\overline{v_\phi}$ ,  $v_{\phi_{max}}$ ,  $\overline{v_p}$  and  $v_{p_{max}}$  for  $M_1$ ,  $M_2$ ,  $M_3$  and  $M_4$

	$M_1$	$M_2$	$M_3$	$M_4$
$\overline{v_\phi}$	0.292	0.254	0.233	0.192
$v_{\phi_{max}}$	0.426	0.365	0.386	0.322
$\overline{v_p}$	0.171	0.231	0.194	0.316
$v_{p_{max}}$	0.263	0.327	0.284	0.441

Table 3 and Fig. 7 illustrate the values of  $\overline{v_\phi}$ ,  $v_{\phi_{max}}$ ,  $\overline{v_p}$  and  $v_{p_{max}}$  for the four manipulators under study. It is apparent that

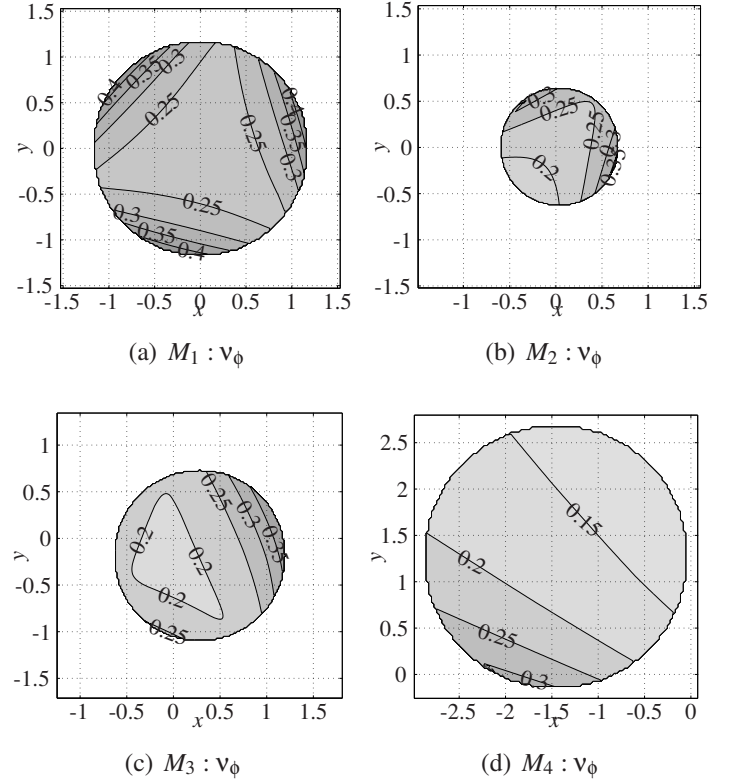


Figure 5.  $v_\phi$  isocontours of: (a)  $M_1$ , (b)  $M_2$ , (c)  $M_3$  and (d)  $M_4$

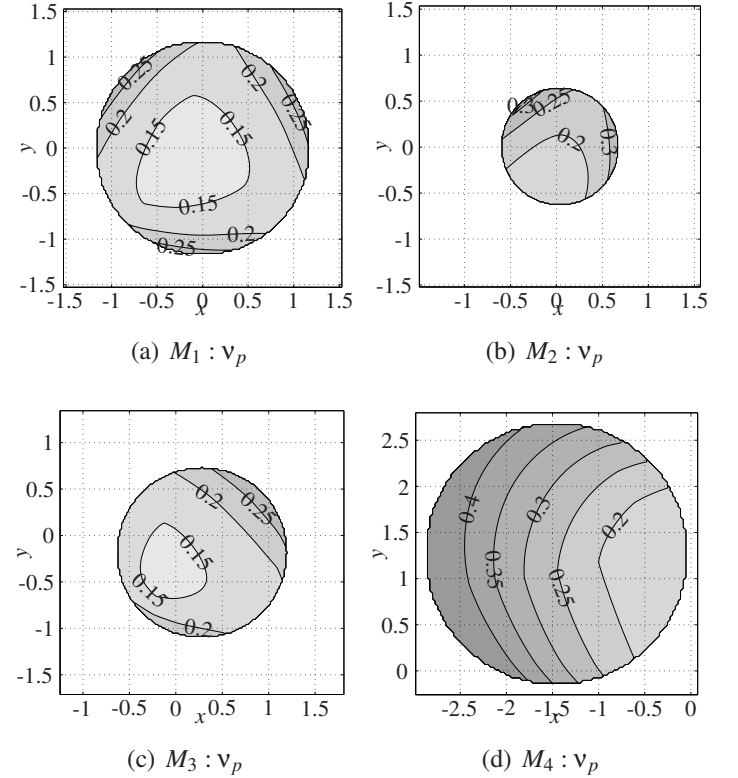


Figure 6.  $v_p$  isocontours of: (a)  $M_1$ , (b)  $M_2$ , (c)  $M_3$  and (d)  $M_4$

$M_4$  has the least sensitive orientation of its MP and that  $M_1$  has the least sensitive position of its MP. On the contrary,  $M_4$  has the most sensitive position of its MP and  $M_1$  has the most sensitive orientation of its MP.

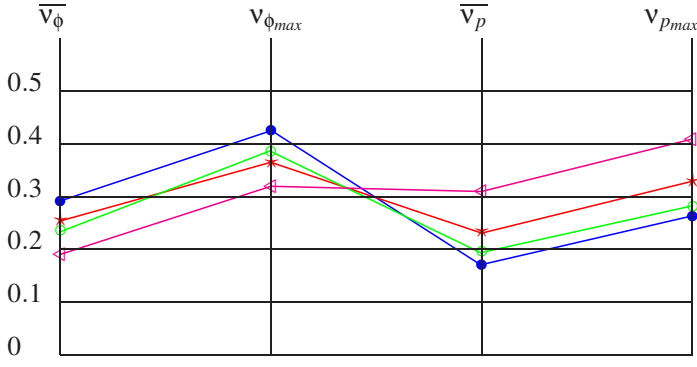


Figure 7. Comparison of  $\bullet-\bullet : M_1$ ,  $\star-\star : M_2$ ,  $\square-\square : M_3$ ,  $\triangle-\triangle : M_4$

## 6 Sensitivity Comparison of Two Actuating Modes

In this section, two actuating modes of the 3-RPR PPM, namely the 3- $\underline{R}$ PR PPM and the 3- $\underline{R}$ PR PPM, are compared with regard to their sensitivity to variations in geometric parameters<sup>2</sup>.

Table 4 shows the eight actuating modes of the 3-RPR PPM. For instance, the first actuating mode corresponds to the 3- $\underline{R}$ PR PPM, also called  $\underline{R}PR_1-\underline{R}PR_2-\underline{R}PR_3$  PPM in the scope of this paper, as the first revolute joints (located at points  $A_i$ ) of its limbs are actuated. Likewise, the eighth actuating mode corresponds to the 3- $\underline{R}$ PR PPM, also called  $\underline{R}PR_1-\underline{R}PR_2-\underline{R}PR_3$  PPM, as the prismatic joints of its limbs are actuated. For the fourth actuating mode, the prismatic joint of the first limb is actuated while the first revolute joints of the two other limbs are actuated.

Table 4. The eight actuating modes of the 3-RPR PPM

Actuating mode number		active angles
1	$\underline{R}PR_1-\underline{R}PR_2-\underline{R}PR_3$	$\theta_1, \theta_2, \theta_3$
2	$\underline{R}PR_1-\underline{R}PR_2-\underline{R}PR_3$	$\theta_1, \theta_2, \rho_3$
3	$\underline{R}PR_1-\underline{R}PR_2-\underline{R}PR_3$	$\theta_1, \rho_2, \theta_3$
4	$\underline{R}PR_1-\underline{R}PR_2-\underline{R}PR_3$	$\rho_1, \theta_2, \theta_3$
5	$\underline{R}PR_1-\underline{R}PR_2-\underline{R}PR_3$	$\theta_1, \rho_2, \rho_3$
6	$\underline{R}PR_1-\underline{R}PR_2-\underline{R}PR_3$	$\rho_1, \rho_2, \theta_3$
7	$\underline{R}PR_1-\underline{R}PR_2-\underline{R}PR_3$	$\rho_1, \theta_2, \rho_3$
8	$\underline{R}PR_1-\underline{R}PR_2-\underline{R}PR_3$	$\rho_1, \rho_2, \rho_3$

Table 5.  $\underline{R}PR_1-\underline{R}PR_2-\underline{R}PR_3$  RDW radius of  $M_1$  and  $M_5$

$R_1$	$R_5$
1.21	1.60

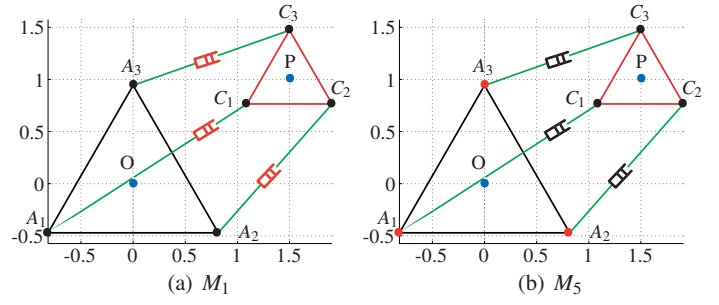


Figure 8. Two actuating modes: (a) 3- $\underline{R}$ PR manipulator, (b) 3- $\underline{R}$ PR manipulator,  $\phi = 0$  and  $\mathbf{p} = [1.5, 1]^T$

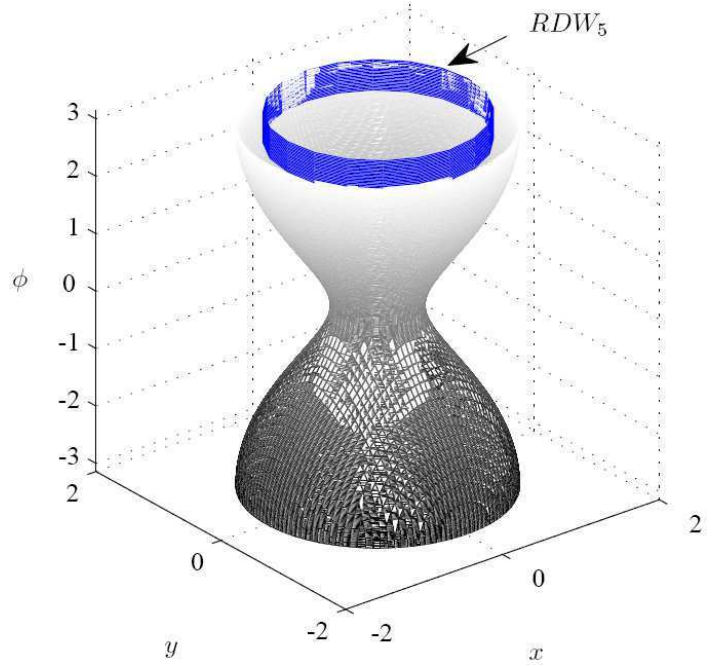


Figure 9.  $M_5$  Regular Dexterous Workspace

Let  $M_1$  and  $M_5$  denote the 3- $\underline{R}$ PR and the 3- $\underline{R}$ PR PPMs, respectively, as shown in Figs. 8(a)-(b). The RDW of  $M_5$  is illustrated in Fig. 9. From Table 5, we can notice that the RDW of  $M_5$  is larger than the one of  $M_1$ .

Figures 10(a)-(b) show the isocontours of the maximum value of  $v_{\phi M_1}$  and  $v_{\phi M_5}$  throughout the RDW of  $M_1$  and  $M_5$ . Likewise, Figs. 11(a)-(b) show the isocontours of the maximum value of  $v_{p M_1}$  and  $v_{p M_5}$ . As a matter of fact, those isocontours correspond to the maximal global positioning and orientation errors with regard to the orientation  $\phi$  of the moving platform of the manipulator.

Table 6 and Fig. 12 illustrate the values of  $\overline{v_\phi}$ ,  $v_{\phi_{max}}$ ,  $\overline{v_p}$  and  $v_{p_{max}}$  for the two actuating modes under study. It is apparent that  $M_1$  is better than  $M_5$ , both in terms of orientation and positioning errors of its moving platform due to variations in geometric parameters.

<sup>2</sup>As the actuators are not of the same type for the two manipulators (revolute actuators for the first one and prismatic actuators of the second one), their variations are not considered in order the sensitivity comparison of the two manipulators to make sense.

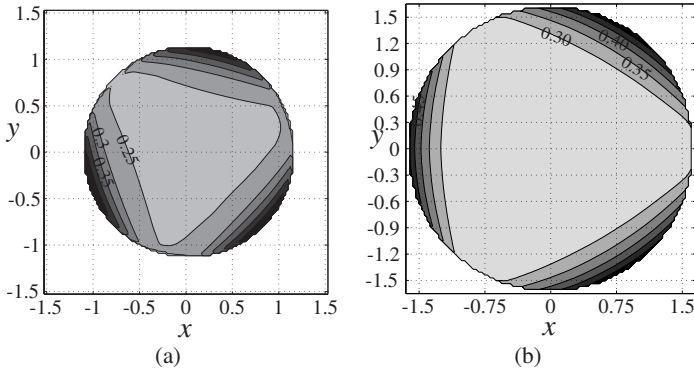


Figure 10. (a)  $M_1 v_\phi$  and (b)  $M_5 v_\phi$  isocontours

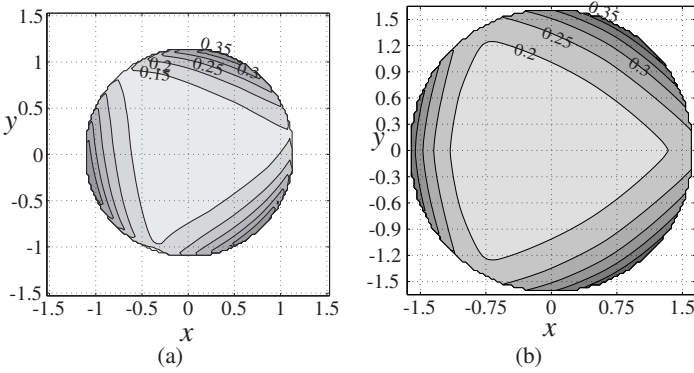


Figure 11. (a)  $M_1 v_p$  and (b)  $M_5 v_p$  isocontours

Table 6. Values of  $\overline{v_\phi}$ ,  $v_{\phi_{max}}$ ,  $\overline{v_p}$  and  $v_{p_{max}}$  for  $M_1$  and  $M_5$

	$M_1$	$M_5$
$\overline{v_\phi}$	0.251	0.289
$v_{\phi_{max}}$	0.448	0.501
$\overline{v_p}$	0.163	0.222
$v_{p_{max}}$	0.369	0.423

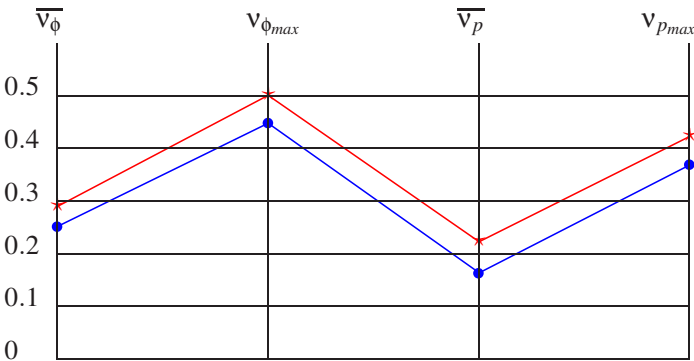


Figure 12. Comparison of  $\bullet\text{---}\bullet$ :  $M_1$  and  $\star\text{---}\star$ :  $M_5$

## 7 Conclusions

This paper dealt with the sensitivity analysis of 3-RPR planar parallel manipulators (PPMs). First, the manipulators under study as well as their degeneracy conditions were presented. Then, an optimization problem was formulated in order to obtain

their maximal regular dexterous workspace (RDW). Accordingly, the sensitivity of the pose of their moving platform to variations in geometric parameters was evaluated within their RDW. Then, a methodology was proposed to compare PPMs with regard to their dexterity and sensitivity. Four 3-RPR PPMs were compared as illustrative examples. Moreover, two actuating modes were compared with regard to their sensitivity to geometric uncertainties. Finally, four global sensitivity indices were introduced in order to evaluate the sensitivity of PPMs over their Cartesian workspace. Those indices characterize the intrinsic sensitivity of the moving platform pose to any variation in the geometric parameters. They are like amplification factors of errors in geometric parameters. Their values remain always lower than one for the manipulators under study. It means that there is no amplification of errors in geometric parameters. The proposed indices can also be used to help the designer of PPMs select a good manipulator architecture at the conceptual design stage.

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