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1 Towards local identification of cohesive zone models
2 using digital image correlation

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10 **Abstract**

In recent years, cohesive-zone models have been formulated and used to numerically simulate the fracture of solid materials. Cohesive-zone models presented in the literature involve a 'jump' in the displacement field describing crack onset within a predefined interface network corresponding to interfaces between elements of the finite element (FE) mesh. The introduction of a virtual displacement jump is convenient to numerically manage micro-crack or void initiation, growth and coalescence. Until now, the forms of interface laws were mainly chosen semi-empirically in connection with the overall responses of specimens when subjected to standard loadings. In this study, a cohesive-zone model identification method is proposed based on the local material behavior derived from kinematical measurements obtained by digital image correlation (DIC). A series of tensile loadings were performed for several damageable elastic-plastic materials on standard tensile specimens. Kinematical data analysis enabled early detection and tracking of the zone where the crack will finally occur. The results of this study highlight the

potential of DIC to quantify damage and show how damage assessments can be inserted in cohesive-zone model identification.

11 *Keywords:* Computational Fracture Mechanics, Cohesive Zone Models,
12 Ductile Damage, Digital Image Correlation, Experimental Identification

13 **1. Introduction**

14 Cohesive-zone models (CZMs), which were first introduced through the
15 pioneering work of Dugdale (1960) and Barenblatt (1962), are suitable for
16 simulating fractures in a wide range of materials and to account for hetero-
17 geneities at various scales from the grain to the structure (see e.g. the review
18 of de Borst et al. (2006)). In the so-called '*cohesive-volumetric finite element*'
19 framework, CZMs are introduced at interfaces between adjacent elements of
20 a finite element discretization (see Figure 1). They have been successfully
21 used to simulate and predict the entire fracture process from crack onset to
22 rupture, including crack growth, propagation, potential bifurcation, multiple
23 fracturing, etc. (see e.g. [Perales et al. \(2010\)](#) for a review and references
24 therein).

25 In recent years, substantial progress has been achieved from a numerical
26 standpoint, along with regular improvement of CZM predictions. In par-
27 ticular, studies have been carried out to assess the impact of the shape of
28 CZMs (bilinear, exponential, trapezoidal, etc.). A range of information can
29 be extracted from recent literature. Chandra et al. (2002) and Tvergaard
30 and Hutchinson (1992) found that the shape only had a slight influence on
31 numerical predictions, with the main parameters being the ultimate stress
32 and fracture energy. On the contrary, Alfano (2006) noted that the numeri-

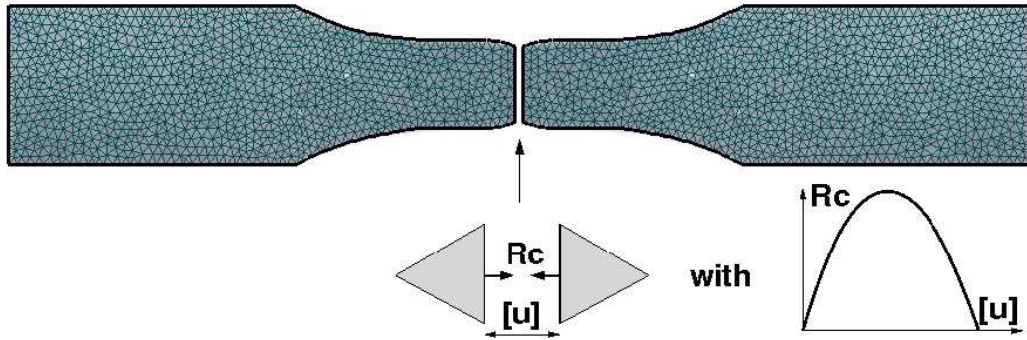


Figure 1: Sketch of a sample after a uniaxial traction test using the cohesive/volumetric finite element approach. Cohesive zone models are embedded along connected meshes and describe the traction-separation relationship as a softening curve. When the traction vanishes the cohesive bond is broken and the crack occurs (the resulting crack path can be complex; this figure exhibits a straight crack for sake of simplicity).

33 cal behavior of a metal matrix/composite was very sensitive to the shape of
 34 the interfacial constitutive relation of the CZM and the review of de Borst
 35 et al. (2006) underlines that the shape has to be chosen on the basis of an
 36 empirical rule that is popular throughout the numerical research commu-
 37 nity: 'triangle shapes' are convenient for modeling brittle materials, while
 38 'door-like shapes' are suitable for ductile materials. Moreover, Kubair and
 39 Geubelle (2003) show that the stability of cohesive models due to the sur-
 40 face softening behavior depends on the shape of the model: *extrinsic* CZMs
 41 (models with infinite initial stiffness, i.e. the cohesive traction is equal to the
 42 material strength) are more stable than *intrinsic* ones (models with finite ini-
 43 tial stiffness, i.e. the traction-separation relationship exhibits a finite slope).
 44 Concerning these intrinsic models, [Chaboche et al. \(2001\)](#); [Perales et al.](#)
 45 [\(2010\)](#) Tomar et al. (2004) underline the influence of an additional surface
 46 compliance on the overall response of a fully cohesive-volumetric formulation.

47 Although the CZM technique is becoming increasingly robust, its applica-
48 tion is somewhat arbitrary and there are still several difficulties, notably re-
49 garding the identification of constitutive equations. It seems that, in addition
50 to the mechanical parameters of CZMs, the shape also has to be determined.

51 Some recent attempts have been proposed in the literature to derive cohe-
52 sive laws from experimental measurements. At the nanoscale, these attempts
53 often concern the identification of a potential from which the cohesive model
54 derived (Jiang, 2010; Ngo et al., 2010). At higher scales, most studies have
55 been based on the assumption of a general shape for the constitutive equa-
56 tions and a predefined crack path, and the material parameters have been
57 derived from experimental strain data (see e.g. Andena et al. (2006) among
58 others for pure mode I conditions). Hong and Kim (2003) propose an inverse
59 identification of cohesive laws without any assumptions on the shape of the
60 traction-separation relationship. This approach is based on some eigenfunc-
61 tion expansion of *elastic* far-fields surrounding a cohesive crack tip and a field
62 projection method. This inverse method is, by construction, limited to elas-
63 tic bulk behaviors but was successfully used by Arias et al. (2007) to validate
64 cohesive-based simulations of dynamic fractures and was recently extended
65 to elastic-plastic bulk behaviors through a hybrid numerical-experimental
66 approach. Full experimental identification of cohesive laws without any as-
67 sumptions on the bulk behavior or on the shape of the CZM was proposed
68 by (Tan et al., 2005) using some digital image correlation (DIC) techniques.
69 This identification was, however, limited to predefined crack paths. To our
70 best of knowledge, **all recent works dealing with CZM identification**
71 **with DIC techniques are limited to: (a) an identification of the**

72 cohesive parameters, i.e. the shape of the cohesive law is a priori
73 fully given (Fedele et al., 2009; Valoroso and Fedele, 2010) or partly
74 given (Shen and Paulino, 2011), and/or (b) predefined crack paths,
75 i.e. debond of interfaces (Valoroso and Fedele, 2010; Zhu et al.,
76 2009) or precracked samples (Fuchs and Major, 2011). In conclu-
77 sion, no direct experimental method is currently available for identifying
78 cohesive laws – *shape* and parameters – in elastoplastic materials where a
79 crack emerges *naturally* from the overall loading (no predefined crack path).

80 The main questions addressed in this paper are thus the following.

- 81 1. Is there any experimental evidence of the cohesive-volumetric decom-
82 position relevance, in particular for elastoplastic damageable materials?
- 83 2. If so, can the associated CZM be directly deduced from the surrounding
84 bulk material behavior?
- 85 3. If so, can the shape and the mechanical parameters of the CZM be
86 simultaneously identified even if the locus of the crack onset is not
87 known *a priori*?
- 88 4. And, as a consequence of the previous question: is early detection of
89 the locus of the main crack possible during a standard mechanical test
90 such as uniaxial tension test on homogeneous materials without any
91 precracks?

92 Here we propose answers to these questions based on experimental measure-
93 ments and the proposed experimental methodology is founded on full-field
94 measurement techniques which, as pointed out by Tan et al. (2005), seem
95 promising to enhance choices of CZM constitutive equations and improve
96 their identification.

97 Over the last ~~10~~ten years, there has been substantial development in
98 quantitative imaging techniques, with widespread applications in the me-
99 chanics of materials. Digital image correlation (DIC), for instance, has be-
100 come a powerful technique that provides reliable kinematic measurement
101 fields (displacement, **and with the help of some numerical differen-**
102 **tiation**, strain, strain-rate, acceleration, etc.). DIC is a fully non-intrusive
103 and sensitive measurement tool that can be used to monitor material sur-
104 face displacements in a wide range of engineering materials such as metals,
105 polymers, ceramics and concretes (Corr et al., 2007; Daly et al., 2007; Fang
106 et al., 2006; Mekky and Nicholson, 2006).

107 As already announced, the goal of this paper is to propose a CZM iden-
108 tification based on full-field measurements (DIC). This identification does
109 not assume neither any particular shape nor any predefined crack path, but
110 focuses on the experimental validity of the projection of volumic (micro) dam-
111 age onto a simple surface. Because of the difficulty of this task, the study
112 is restricted to standard metallic materials subjected to overall uniaxial ten-
113 sion. In particular, the local effect of the stress triaxiality on the volumic
114 damage is neglected and we propose a simple and pragmatic method to iden-
115 tify the normal part of the cohesive law (1D approach). The experiment
116 methodology is composed of three steps.

117 First, the zone where a crack initiates and propagates in mode I is care-
118 fully identified. As previously shown in other situations, we underline that
119 this zone corresponds to the strain localization locus, and can be early de-
120 tected and tracked by image analysis (inflection point of the longitudinal
121 velocity profile). Moreover, the 1D approach can be warranted by the kine-

122 matical data which show very low elastoplastic rotations, even in the local-
123 ization zone. This zone is, as expected, the region where necking occurs and
124 damage preferentially develops.

125 Second, the kinematic data may also give fields of void fraction since
126 isotropic straining of the specimen cross sections is supposed. A simple model
127 of spherical voids is here employed, but any convenient micromechanical
128 model can be used instead. This spherical voids model was selected for its
129 simplicity and to illustrate the experimental protocol.

130 Third, the method proposes to construct a local stress-strain correspon-
131 dence, which can be used to identify an elastoplastic damageable constitutive
132 equation from which the cohesive law can be extracted. The correspon-
133 dence is first explained and illustrated on two academic bulk behaviors: 1/
134 the damageable elasticity and 2/ the standard damageable elastoplasticity.
135 From these academic examples, we derive the generality of the correspon-
136 dence which thus can be employed without any assumption on the bulk
137 behavior. The experimental uniaxial response of any damageable material is
138 then converted onto a bulk behavior with only hardening (no damage) and a
139 cohesive zone model incorporating all the softening effects. Such a cohesive
140 law is hence suitable for describing damaging up to cracking. The shape
141 of the identified CZMs changes with the material ductility and appears to
142 be in close agreement with some standard CZM formulations used in the
143 computational codes.

144 **2. Experimental procedures**

145 *2.1. Experimental setup and correlation technique*

146 The experimental setup shown in Figure 2 involves a uniaxial testing
147 machine equipped with a 100 kN load cell and a CCD camera. Its optical
148 axis is set perpendicular to the specimen surface and remains fixed during
149 the test.

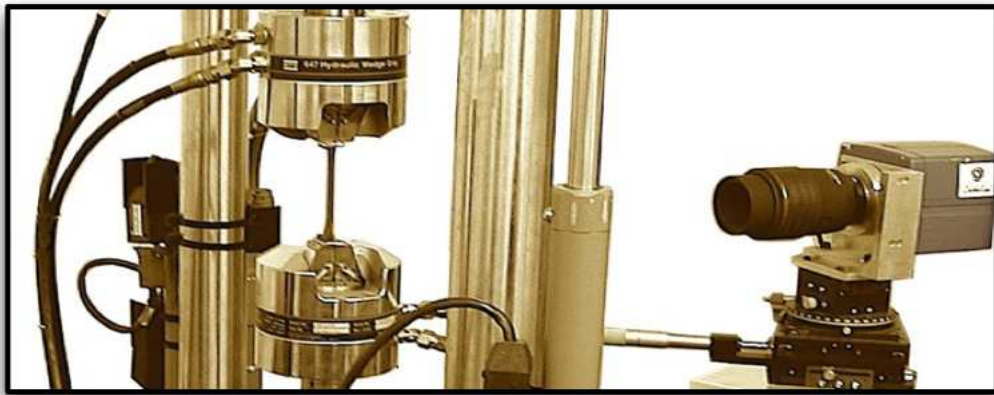


Figure 2: A general view of the experimental setup.

The speckle of the digitized image represents local optical signatures, which are used to track the material surface elements. The in-plane displacement vectors are obtained by a direct digital image correlation method. A normalized discrete correlation function is computed at selected pixels (e.g. initially positioned on a regular grid) of consecutive images. For this study, we used *KelKins* image processing software (Bornert et al., 2009) developed for the computation of kinematical fields. The surface of the specimen observed by the visible CCD camera is speckled with black and white paint in order to obtain a random pattern defining the local optical signature of each

material surface element. A classical digital image correlation algorithm allowed us to determine the in-plane components of the displacement field on a regular rectangular grid. **To determine the displacement at each point of the grid, we choose to perform a direct correlation computation and use a normalized intercorrelation function $\mathcal{C}(\phi)$ (see Latourte et al. (2008) for details):**

$$\mathcal{C}(\phi) = \langle I_1, I_2^\phi \rangle / \sqrt{\langle I_1, I_1 \rangle \langle I_2^\phi, I_2^\phi \rangle}$$

150 **where I_1 and I_2 are the intensity functions of two images separated**
 151 **by a small strain increment, $I_2^\phi(x) = I_2(x + \phi(x))$ for any point x**
 152 **and $\langle a, b \rangle$ is the scalar product of any light intensities a and b on**
 153 **the correlation subset Z : $\langle a, b \rangle = \int_Z a(x) \cdot b(x) dx$. With a zero order**
 154 **function ϕ and a computation of the intercorrelation function for**
 155 **multiple pixel shifts, the displacements are estimated with a 1-pixel**
 156 **resolution. To achieve higher resolutions, the discrete intercorre-**
 157 **lation function is interpolated in the neighborhood of its discrete**
 158 **maximum by a quadratic polynomial (Wattrisse et al., 2001a). The**
 159 **in-plane velocity and strain components were derived from the displacement**
 160 **data by a numerical differentiation method based on a local polynomial ap-**
 161 **proximation of the displacement field. Local time fitting of displacement**
 162 **fields often involves a second order polynomial while local space fitting is**
 163 **associated with a coupled first order polynomial of the two in-plane coordi-**
 164 **nates. Recent works show that in most situations the error on the**
 165 **displacement field is the combination of two contributions (Bornert**
 166 **et al., 2009): 1/ the model error related to the mismatch between**
 167 **the real transformation and one used as the shape function on the**

168 displacement field, and 2/ the ultimate error linked to the prop-
169 agation of the image noise on the displacement and to the gray
170 levels interpolation bias. In order to minimize both contributions
171 in situations involving strongly localized strain fields, we chose to
172 use an incremental strain computation (the correlation grid is reg-
173 ularly upgraded every 5% macroscopic deformation increments),
174 and we used bi-linear shape functions on the displacement fields.

175 The image processing has been widely presented in previous works (Wattrisse
176 et al., 2001a). Readers interested in its application can also refer to Huon
177 et al. (2007); Latourte et al. (2008); Wattrisse et al. (2001b,c) or Chryso-
178 choos et al. (2008) where the image processing was applied to different types
179 of materials, i.e. metals, shape memory alloys, polymers, ceramics, reinforced
180 concretes. In the case of brittle materials (e.g. ceramics and concretes), nat-
181 ural speckle was advantageously used. Indeed the cracks that occur at the
182 specimen surface may induce a degradation of the local optical signatures
183 and bad image correlations can be performed. For ductile materials, recent
184 work (Lopez-Crespo et al., 2009) showed that it is possible to still use natu-
185 ral speckle to analyse elastic-plastic behaviour around a crack tip. However,
186 in the present application, the polished surface of specimens forced us to
187 use black and ~~with~~white paint. To limit and check the possible correla-
188 tion problems, an incremental image processing was done verifying at each
189 computational step the quality of the correlation factor.

190 *2.2. Materials and test procedure*

191 Monotonic tensile tests were performed on flat specimens at room tem-
192 perature (about 293 K). A slow crosshead velocity (2.5 mm/min) was kept

Table 1: Measured properties of the tested materials: Young’s modulus (E), tensile yield stress (σ_y), maximal conventional stress (σ^{peak}), (mean Hencky) failure strain ($\varepsilon_{11}^{\text{max}}$), and initial cross section (S_0).

Material	Abbr.	E (GPa)	σ_y (MPa)	σ^{peak} (MPa)	$\varepsilon_{11}^{\text{max}}$	S_0 (mm \times mm)
High-strength steel	Steel 1	150	360	630	0.22	13.5×2.5
Ductile steel (Lüders bands)	Steel 2	210	380	450	0.31	10×2.5
2024 T3 aluminium alloy	Dural	75	310	480	0.15	10×3
Copper (99%)	Cu	100	200	290	0.23	13.5×2

193 constant throughout all tests. During the tensile test, 12-bit grey level im-
194 ages were captured at a frame rate of 2 Hz. The optical lens used allowed
195 us to obtain a spatial resolution of about $40 \mu\text{m}\cdot\text{pixel}^{-1}$. Force signal \mathcal{F} ,
196 crosshead displacement ΔL and time t were also recorded and stored in each
197 acquired image file. As shown in Figure 3, subscripts 1, 2 and 3 respectively
198 correspond to the length, width and depth of the specimen, with DIC giving
199 the in-plane components u_1 and u_2 of the displacement during tensile loading
200 in direction 1.

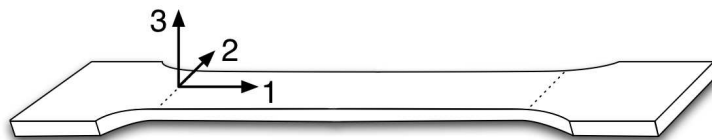


Figure 3: Schematic representation of the sample and frame of reference.

201 The main mechanical characteristics of the tested specimens are summa-
202 rized in Table1. Figure 4a shows the conventional stress $\sigma_c = \mathcal{F}/S_0$ (where
203 S_0 is the initial cross section of the specimen gauge part) as a function of the

204 mean Hencky strain $\bar{\varepsilon}_{11}$ (computed throughout the sample gauge part), in
 205 the loading direction for each specimen. These materials are ductile but show
 206 different modes of failure. In particular, Steel 1 and Dural specimens failed
 207 suddenly while the fracture of Steel 2 and Cu specimens was more progres-
 208 sive. Steel 2 presented a plastic plateau associated with the displacement
 209 throughout the specimen gauge part of a high strain rate zone associated
 210 with Lüders band propagation.

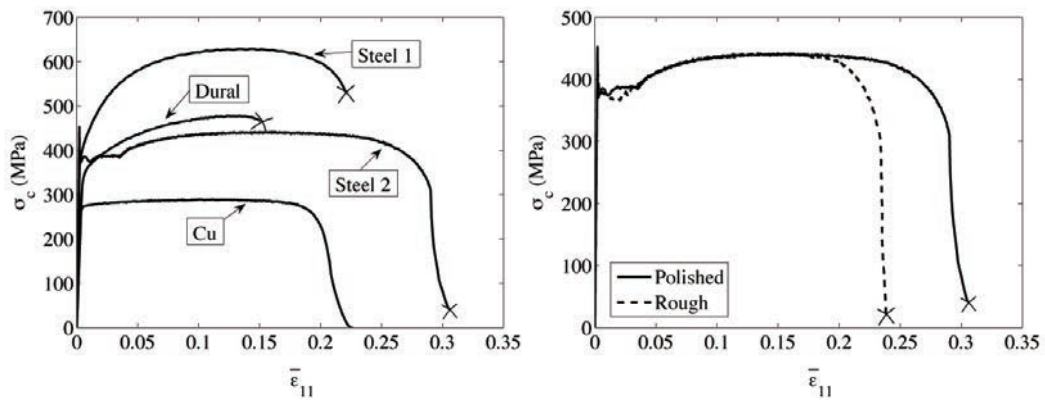


Figure 4: Uniaxial conventional stress σ_c vs. mean Hencky strain $\bar{\varepsilon}_{11}$ diagrams: (a) tested materials. (b) Example of polishing influence for the Steel 2 specimens. Crosses are associated with the specimen rupture.

211 Even slight geometrical imperfections of the specimen surface can give
 212 rise to early crack inception. We then performed comparative tensile tests
 213 on polished and unpolished specimens to highlight the influence of geomet-
 214 rical surface defects on the overall specimen response. Figure 4b shows the
 215 two responses in terms of σ_c vs. $\bar{\varepsilon}_{11}$ diagrams. As expected, we observed
 216 no significant difference before the maximum load, but the curves diverged
 217 significantly during the specimen softening, i.e. greater strains were reached
 218 with polished specimens. As the onset of localization phenomena is crucial

219 for CZM identification, the required tests were systematically performed with
220 specimens polished on all faces.

221 **3. Experimental observations**

222 During tensile tests, the gauge part of the specimen is generally consid-
223 ered as a material volume uniformly stressed and strained. This convenient
224 hypothesis allows the ~~experimenter~~experimentalist to estimate stress and
225 strain from load cell and extensometer signals. However, several years ago,
226 based on our concern about obtaining reliable full-field strain measurements,
227 we began to question the homogeneity of these ‘simple’ tensile tests and to
228 consider the gauge part of specimens as a possibly non-uniformly strained
229 structure (Wattrisse et al., 2001b). The next sub-section shows some aspects
230 of the early and gradual development of strain localization. We arbitrarily
231 chose kinematical results obtained with Cu samples to illustrate the field
232 properties that will be used to detect and assess damage in the sequel.

233 *3.1. Localization zone*

234 The early and gradual development of localization can be observed in
235 Figure 5 which represents the temporal evolution of longitudinal Eulerian
236 strain rate profiles $D_{11}(X_1, X_2 = 0, t)$, where X_1 and X_2 denote the in-plane
237 Lagrangian coordinates. This longitudinal profile, using contour plots, was
238 captured at each time t in the middle of the gauge part of the specimen (i.e.
239 $X_2 = 0$). Contour plots were chosen to underline the progressive narrowing
240 of the localization zone. The load signal was also superimposed to indicate
241 the overall sample response. **Note that the apparent slope of the load-**
242 **time curves (Figure 5 and the following) can be slightly different in**

243 comparison to the slope of the load-strain curves (Figure 4) since
 244 the overall constant velocity can lead to a non constant average
 245 strain rate on the specimen gauge length.

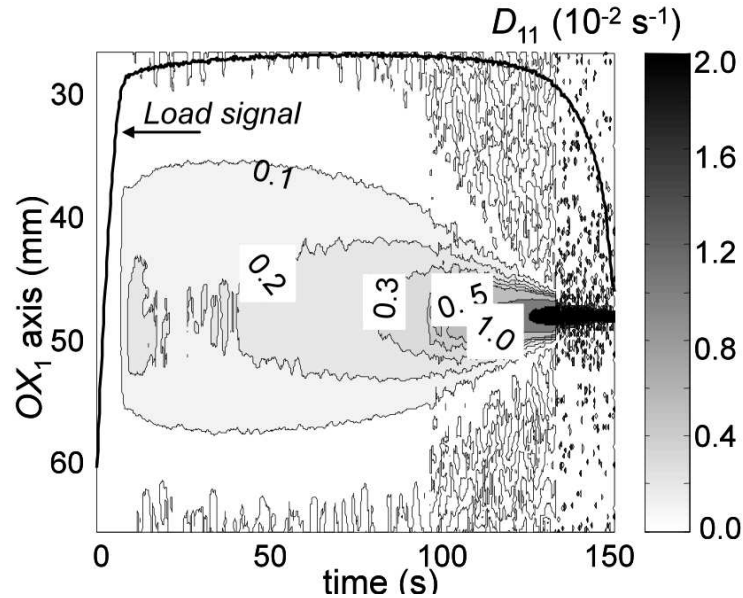


Figure 5: Copper specimen. Time course of the longitudinal components $D_{11}(X_1, X_2 = 0, t)$ of the Eulerian strain rate tensor. Profiles were captured at $X_2 = 0$, i.e. along the OX_1 symmetry axis of the sample.

246 Another illustration of the localization precocity can be given in terms of
 247 acceleration. Generally, for quasi-static loadings, the acceleration amplitude
 248 remains negligible as compared with the gravity acceleration, this latest being
 249 often ignored in equilibrium equations. Nevertheless, the high performances
 250 of current DIC techniques allow reliable assessments of very low acceleration
 251 intensity and the analysis of acceleration fields appears to be of some interest
 252 for early detection of material flow heterogeneity. Using the same graphical
 253 format as the one used in Figure 5, the longitudinal profile of the acceleration
 254 component $\gamma_1 = \partial^2 u_1 / \partial t^2$, where u_1 is the longitudinal displacement, is plot-

255 ted in Figure 6a. At the very beginning of the test, acceleration data were
 256 not plotted until a ‘significant’ signal was reached in order to avoid parasitic
 257 effects of the finite stiffness of the testing machine. At the end of the test,
 258 acceleration data were no longer plotted once the displacement gradients be-
 259 came too high for the spatial and temporal resolutions of the CCD camera,
 260 thus inducing an awkward fitting (inconsistent smoothing) of the accelera-
 261 tion data. The vertical dashed lines correspond to the profiles plotted in
 262 Figure 6b while the horizontal dashed line (placed at $X_1 \simeq 48$ mm) roughly
 263 separates regions with positive and negative accelerations. This line then cor-
 264 responds to a specimen cross-section characterized by a zero-acceleration, or
 265 in other words to the center of the strain localization zone shown in Figure 5.
 266 This has already been observed in stroke-controlled tests (Wattrisse et al.,
 267 2001b), the strain localization zone corresponding to the inflection point of
 268 the longitudinal velocity profile. We then estimated that the damage pref-
 269 erentially developed in such zone where the crack finally occurs. Figure 6b
 270 shows examples of longitudinal acceleration profiles $\gamma_1(X_1, X_2 = 0, t)$ cap-
 271 tured at $t = 40, 80$ and 120 s, respectively. Note that the reconstruction of
 272 acceleration profiles in the Lagrange configuration is necessary to make the
 273 profiles comparable. It can be verified that the encircled point located at
 274 $X_1 \simeq 48$ mm is the sole point keeping a zero-acceleration throughout this
 275 velocity-controlled test.

276 In Figure 7a-d (left), profiles of the longitudinal acceleration $\gamma_1(X_1, X_2 =$
 277 $0, t)$ obtained for the 4 studied materials are shown. To facilitate the com-
 278 parison of material responses we used normalized scales of time and space,
 279 with $t^* = 1$ corresponding to the instant of specimen breaking and X_1^* corre-

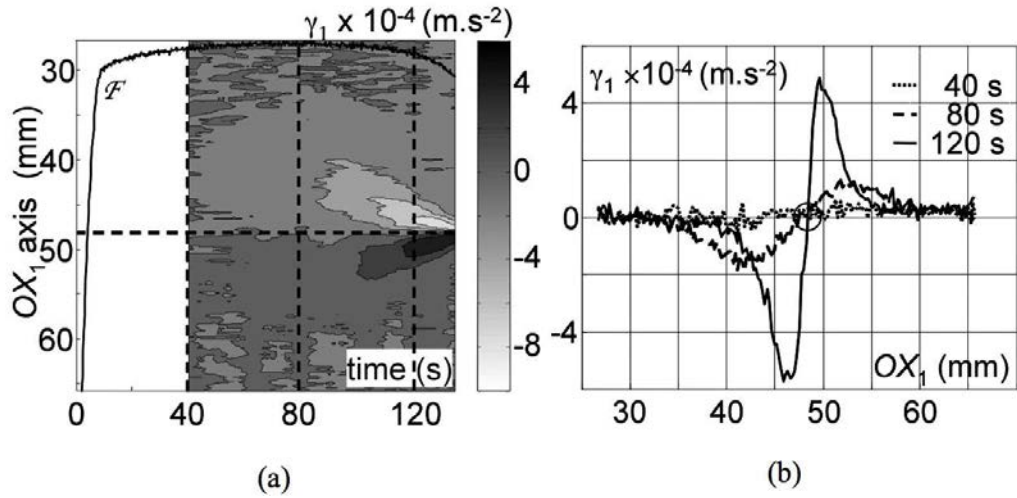


Figure 6: Copper specimen. (a) Lagrangian representation of the time course of the acceleration profile $\gamma_1(X_1, X_2 = 0, t)$. (b) Acceleration distributions along the OX_1 axis at 3 given times. The intersection point of the 3 profiles corresponds to the zero-acceleration point.

280 sponding to the initial gage length of the specimen. The conventional stress
 281 was as usual superimposed to give a landmark of the specimen response.

282 Except for Steel 2 (Figure 7b), where the passage of the Lüders band at
 283 the very beginning of strain hardening induced a deceleration-acceleration
 284 propagating wave, a steady concentration of level curves of positive and neg-
 285 ative acceleration was clearly observed on both sides of the necking zones.
 286 The section where the fracture eventually took place was then characterized
 287 by zero acceleration. This cross section can be clearly predetermined on the
 288 basis of Figure 7a-d (right). Acceleration profiles captured at 3 times were
 289 plotted as a function of a normalized Lagrangian coordinate X_1^* . Whatever
 290 the studied material, we observed that the profiles clearly intersect at a sin-
 291 gular point corresponding to the cross section where the crack finally occurs.

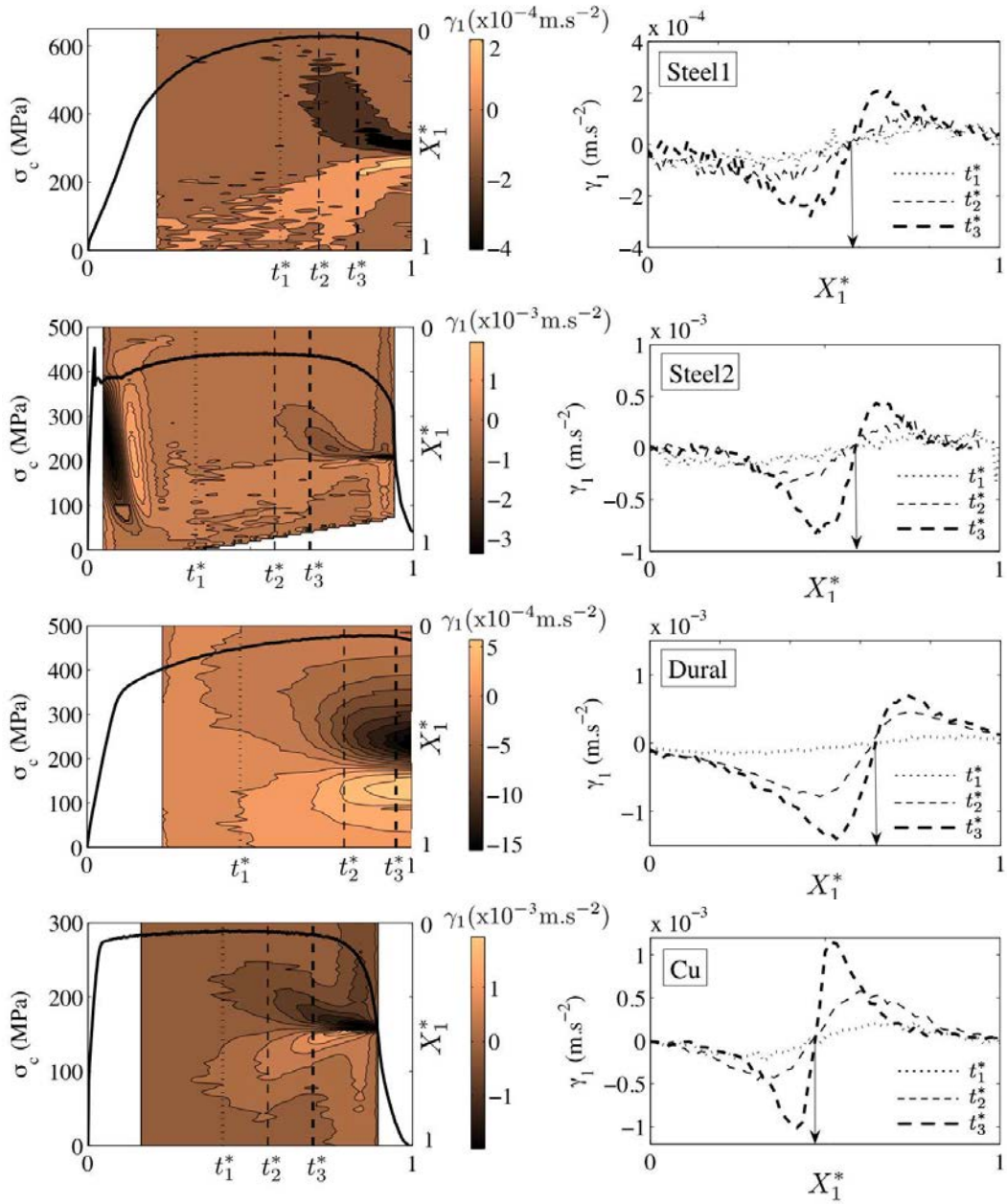


Figure 7: Longitudinal acceleration profiles: (a) Steel 1; (b) Steel 2; (c) Dural; (d) Cu. (left) Time course of the conventional stress (solid line) and spatiotemporal representation of accelerations (contour plots); (right) selected acceleration profiles at 3 different instants ($t_1^* < t_2^* < t_3^*$) marked by vertical lines on the left plots.

292 The coordinate of this cross section did not systematically correspond to the
293 centre ($X_1 = L_0/2$) of the gauge length despite the considerable care exer-
294 cised concerning accurate specimen polishing and positioning. This result is
295 however not surprising if we think about the heterogeneous distribution of
296 microstructural defects within the sample gauge part that may rapidly give
297 rise to heterogeneous mesoscopic responses of the material.

298 *3.2. Material rotations*

299 A 1D analysis of CZM using non-homogeneous tensile tests remains con-
300 sistent as long as irrotational transformation is observed. Using the full-field
301 displacement fields, maps of material rotations can be computed from the
302 polar decomposition of the transformation gradient tensor $F = RU$, where
303 R is an orthogonal tensor while U is a symmetric definite positive tensor.
304 In order to visualize these rotations, we plotted in Figure 8 the distribution
305 over the specimen gauge part of angles $\alpha = |\cos^{-1} R_{11}|$ computed at different
306 times for Cu specimens. At the beginning of the test, the greatest rotation
307 angles occurred in the connection zone of the specimen where slight shear
308 stresses developed (Figure 8 (left)). When necking occurred, the greatest ro-
309 tations appeared on both sides of the necking shoulders (Figure 8 (middle)).
310 However, the rotation angles α were less than 0.25 degree ($\simeq 5.10^{-3}$ rad.)
311 and remained negligible (Figure 8 (right)). Until the crack occurred, the
312 maximal logarithmic shear strain ε_{12} , which was inside the specimen gauge
313 part for the 4 materials, remained much lower than $10^{-2} \times \varepsilon_{11}$ throughout
314 the test, with ε_{11} corresponding to the associated tensile strain. The eigen
315 directions of the strain tensor then remained fixed and parallel to directions
316 $i = 1, 2, 3$ already defined.

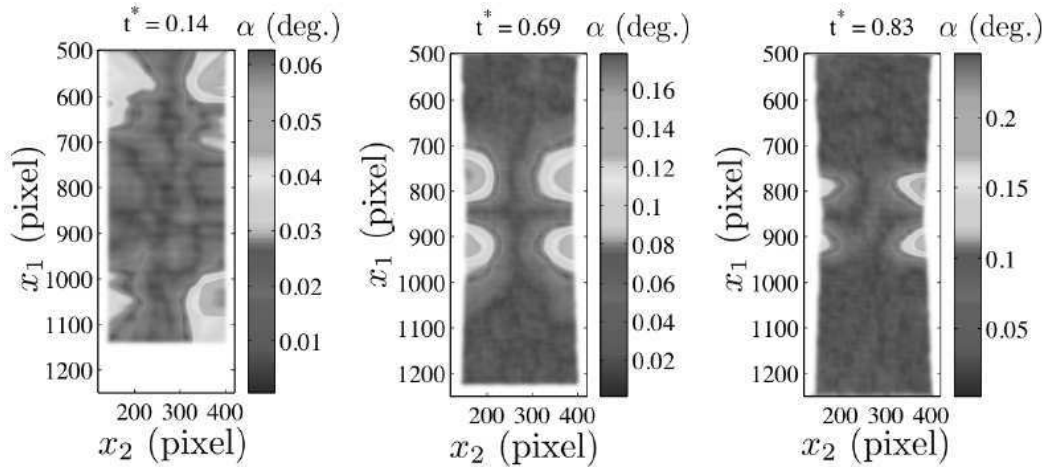


Figure 8: Material rotations (degrees) at 3 different times for Cu specimen.

317 The simple 1D mechanical analysis of the mechanical behavior presented
 318 hereafter can then be legitimated by the absence of significant rotations and
 319 the lowness of shear strains. In particular, an additive partition of elasto-
 320 plastic strain is possible for a classical multiplicative decomposition of the
 321 transformation gradient (Lee, 1969). Indeed, with $F = F_e F_p$, it arises when
 322 there are no longer any rotations:

$$F \simeq U = U_e U_p, \quad (1)$$

$$\varepsilon \simeq \log U = \log U_e + \log U_p = \varepsilon_e + \varepsilon_p, \quad (2)$$

323 where U_e and U_p are elastic and plastic dilatation tensors respectively asso-
 324 ciated with the polar decomposition of F_e and F_p .

325 3.3. Damage field

326 Plasticity mechanisms are generally supposed to be isochoric. With elas-
 327 tic strain remaining small, volume variations are then associated with poros-
 328 ity induced by microvoids or opening of microcracks (Lemaitre, 1992). The

329 kinematical data were then used to estimate volume variations associated
 330 with micro-void development and then to quantify the material volumetric
 331 damage. This measurement is the first step in identifying an 'equivalent
 332 surface damage' corresponding to a cohesive law.

By construction, the ~~displacement~~ **transformation** gradient F ~~of the transformation~~ is such that:

$$\det F = \frac{dv}{dV} \geq 0, \quad (3)$$

where dv and dV are respectively volumes occupied by an infinitesimal amount of matter in the current and reference configurations. The volume dilatation can be classically related to the Hencky strain tensor by:

$$\frac{dv}{dV} = e^{\text{tr} \varepsilon}, \quad (4)$$

where operator 'tr' symbolizes the trace operator. Standard DIC technique gives us access only to in-plane components u_1 and u_2 of the displacement and consequently to ε_1 and ε_2 . The out-of-plane component u_3 is not available. Nevertheless, analysis of the cross-section shape close to the necking regions led us to assume in a first approximation that $\varepsilon_2 = \varepsilon_3$. For example, the isotropic contraction of cross sections has been quite well verified experimentally for the grade of steel shown in Wattrisse et al. (2001a). With such a transverse isotropy assumption, the local relative volume variation $dv/dV - 1$ can be expressed only with the strain assessments:

$$\frac{dv}{dV} = e^{(\varepsilon_1 + 2\varepsilon_2)}, \quad (5)$$

Since a pure plastic transformation of the material (i.e. excluding microvoids and the very slight elastic dilatation) remains isochoric, the relative

volume variation can be regarded as a microvoid fraction within dV , with elastic effects being negligible even at finite strain:

$$\frac{dv}{dV} - 1 = \frac{dv_v}{dV}, \quad (6)$$

where dv_v is the microvoid volume. Since these microvoids reduce the surface that could withstand stress, a local scalar variable can be classically introduced to describe isotropic damage. This variable represents the *surface proportion of microvoids* (Chaboche, 1988; Ju, 1989; Kachanov, 1958, 1980; Krajcinovic, 1989; Lemaitre, 1992):

$$D = \frac{ds_v}{dS}, \quad (7)$$

where ds_v is, in this work, the surface of microvoids associated with the elementary surface dS perpendicular to the straining direction. Geometric modeling of microvoids is required to calculate the value of D from volume changes. Different models already exist in the literature to take the damage by cavitation, decohesion of matrix particles, crazing, etc., into account. The main damage mechanisms concerning the metals under investigation are microcracks opening and cavitation. **Following Wu et al. (2011) and** for the sake of simplicity, we propose to assume a simple spherical void shape, distribution and growing: *(i)* a uniform microvoid distribution, *(ii)* with voids of the same spherical shape (radius a), *(iii)* uniformly distributed within a volume element, *(iv)* and of same isotropic growth kinetics. The void density per unit length η is thus the same for all directions. Considering a virgin specimen in the reference configuration, we get for an elementary cubic volume:

$$\frac{dv_v}{dV} = \frac{4}{3}\pi(a\eta)^3. \quad (8)$$

According to Eq. (7) and the geometric assumptions made on the microvoid network, the local estimate of the damage parameter is: $D = \pi(a\eta)^2$, and can be expressed as a function of the relative volume variations derived from the kinematic measurements:

$$D = \alpha_D \left(\frac{dv_v}{dV} \right)^{2/3}, \quad (9)$$

333 with $\alpha_D = (3/4)^{2/3}\pi^{1/3} \simeq 1.2$.

334 Figure 9 (left) shows the time course of the relative volume variation pro-
 335 file along the straining direction for the 4 studied materials. Figure 9 (right)
 336 shows the damage evolution for 3 cross sections: the first one is centered on
 337 the necking zone (bold line) while the two others were chosen on each side of
 338 the crack zone (dashed lines). As often observed at the scale of the sample
 339 gauge part, in all cases failure occurs when local damage parameter values are
 340 lower than 1. This is partly due to the current limitations of the CCD camera
 341 (spatial and temporal resolutions) and to the image processing (space-time
 342 fitting necessary to filter noisy and discrete data before derivation). It is
 343 also due to the academic character of the chosen porosity model, which be-
 344 comes unsound as soon as anisotropic growth and coalescence of microvoids
 345 occur. Fortunately, the assessments show small damage values at this level
 346 and surely have a slight influence on what happens next. Moreover, the am-
 347 plitude of the damage parameter in the zero acceleration necking zone (bold
 348 line) is larger than away from this zone (dashed lines): again, the zero accel-
 349 eration zone corresponds exactly to the cross section where the failure occurs.
 350 **The inverse problem to obtain some volumetric informations from**
 351 **the surface measurements is a complex problem. A more accurate**
 352 **estimate of the local volume variations using full-3D measurements**

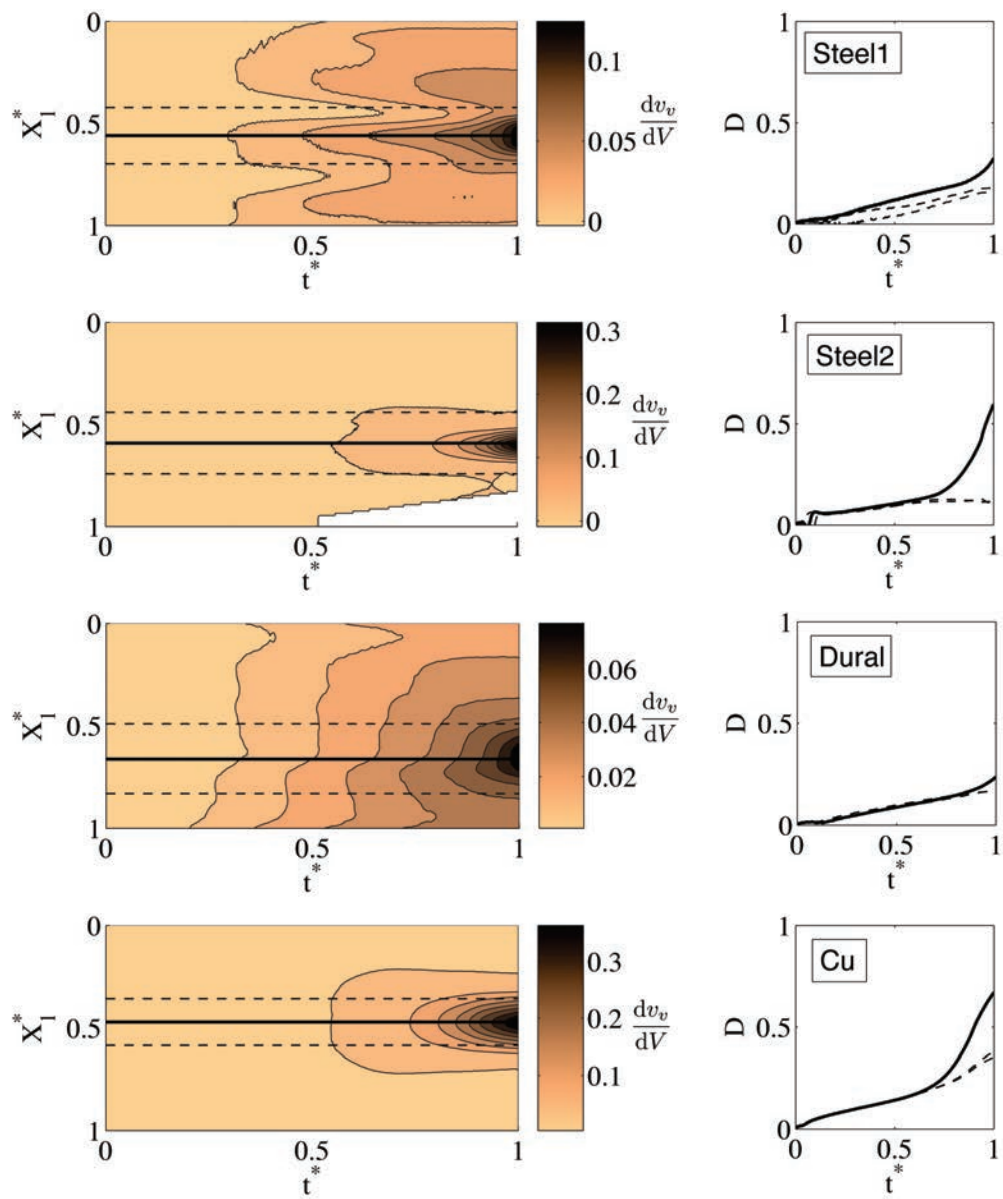


Figure 9: (Left) Time course representation of the Lagrangian longitudinal profile of the relative volume variations (isovalues). (Right) Kinetics of the damage parameter over 3 chosen cross sections located by the horizontal lines on the left figure.

353 will be investigated in a future work. However, the recent work of
 354 Wu et al. (2011) concerning the damage measurement by full-3D
 355 DIC reports no value of damage greater than 0.4 and the validity of
 356 such type of approach could be probably challenged for very large
 357 strains. The damage maps identified with our 2D measurements
 358 are shown in Figure 10 before and during the strain localization:
 359 no particular spatial inhomogeneities can be pointed out in the di-
 360 rection perpendicular to the tensile direction. In the sequel, a pure
 361 1D approach will thus be carried on.

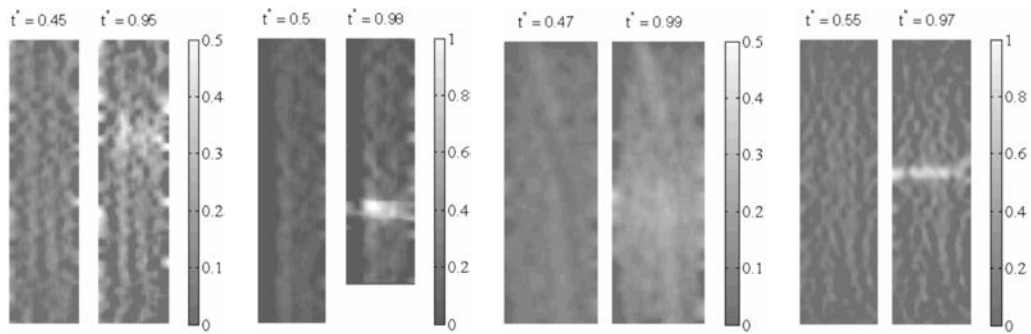


Figure 10: Damage maps before and during the strain localization (from left to right: Steel1, Steel2, Dural and Copper). Please note that the gray level bars were adapted to the range of the damage signal.

362 3.4. Local stress-strain responses

363 Since we have in mind to separate hardening bulk behaviors and softening
 364 surface ones, the local measurement of the damage field allows now to derive
 365 local stress-strain responses as well in the localization zone (where the crack
 366 will occur) as in the rest of the sample.

For standard tensile tests, it is generally assumed that the tensile Cauchy stress is uniformly distributed over the cross section of the specimen. This tensile stress is classically written as

$$\sigma(x_1, t) = \mathcal{F}(t)/S(x_1, t), \quad (10)$$

where $S(x_1, t)$ is the current cross section of the specimen at the longitudinal Eulerian coordinate x_1 and time t . For 1D data processing, while taking the mass balance and the transverse isotropy hypothesis into account, $S(x_1, t)$ can be related to the initial cross section by:

$$S(x_1, t) = S_0 e^{2\varepsilon_2(x_1, t)}. \quad (11)$$

According to the definition of the damage parameter (Lemaitre, 1992), the stress σ_{eff} relative to the effective surface is then:

$$\sigma_{\text{eff}}(x_1, t) = \frac{\sigma(x_1, t)}{1 - D(x_1, t)}. \quad (12)$$

367 Figure 11 shows the stress–strain diagrams at different loci along the
 368 gauge part of the Cu specimen. The influence of the gradual development
 369 of heterogeneity on the local stress-strain response is clearly shown by their
 370 successive divergence. At the beginning of the test, all the responses follow
 371 the same path but do not move on this path at a same speed. The more
 372 the cross section is close to the localization zone, the longer is the covered
 373 path. Divergence of local stress-strain responses appears when the specimen
 374 softens. We have already discussed in Wattrisse et al. (2001b) the different
 375 possible interpretations of such softening in terms of material and/or struc-
 376 ture effects. We particularly showed that strain softening properties strongly
 377 depend on the observation scale. In the framework of the present experiment,

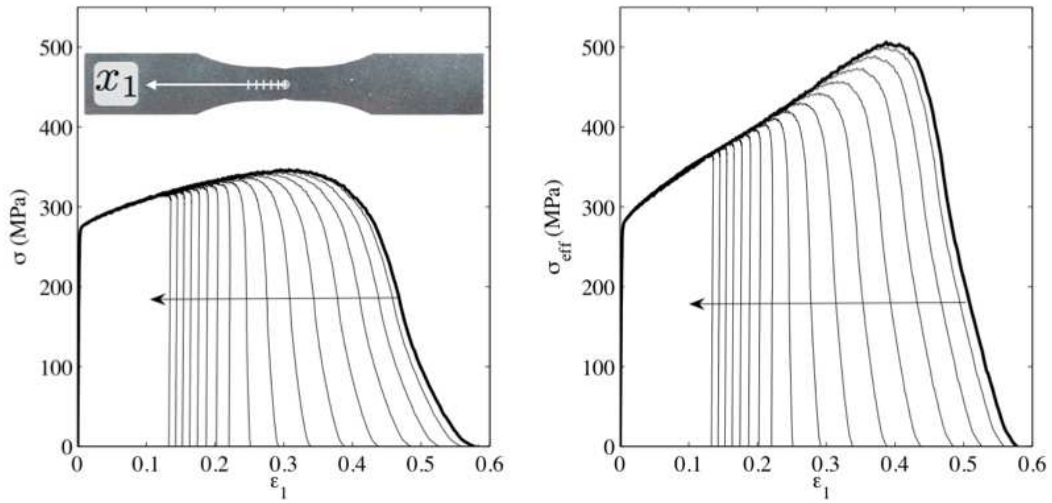


Figure 11: Local stress-strain diagrams (Cu). (a) Cauchy stress vs. Hencky strain; (b) Effective stress vs. Hencky strain. The horizontal arrow indicates a growing distance from the localization zone (bold line).

378 softening of the specimen is associated with a local elastic unloading for cross
 379 sections placed outside the necking zone. For cross sections placed inside the
 380 current necking zone, the load decrease is accompanied by a local softening,
 381 as soon as the localization zone size becomes smaller than the gauge length
 382 fixed by the optical system used to estimate the local strain, as it is shown in
 383 Figure 11a-b. This scenario ~~translates~~describes the competition between
 384 the load decrease and the local necking course. It naturally changes from one
 385 cross-section to another. The horizontal arrow in Figure 11 symbolizes the
 386 increasing distance to the necking region of the cross section concerned by
 387 the local response curve. A comparison of Figure 11a and Figure 11b shows
 388 the influence of damage on the local stress-strain response. The envelope
 389 curve (bold line) corresponds to the material points located in the rupture
 390 cross section. This particular stress-strain curve is considered to identify the

391 1D elastoplastic damageable response and extract the sought response of the
392 cohesive zone.

393 4. Towards a cohesive zone model identification

394 Before using these experimental data, we propose to remind the objective
395 of the CZM approach of fracture and to progressively introduce the identifi-
396 cation protocol of cohesive zone laws using several academic examples.

397 4.1. CZ response through heuristic 1D models

398 Whatever the damage mechanism, locations of defects (i.e. small regions
399 where stresses are not transmitted) are not known *a priori*. In CZM-based
400 simulations, failures only occur at predetermined locations, but a structure
401 may also be weakened without being broken. This is reflected by the ‘open-
402 ing’ of cohesive zones up to a value that corresponds to the maximum damage,
403 therefore leading to structural failure. By construction, the CZM approach
404 ‘concentrates’ the damage at the interfaces between adjacent volumetric el-
405 ements (FE meshes or FE meshed bodies) and the numerical ‘opening’ does
406 not represent a material displacement jump but instead summarizes at this
407 interface the overall kinematical effects induced by the damage at the vicinity
408 of this interface. From a strength standpoint now, CZM introduces cohesive
409 forces associated with the displacement jump so that the work related to
410 this couple represents the part of the volume deformation energy induced by
411 the damage mechanisms. From a rheological standpoint, the introduction of
412 cohesive zones leads to a clear separation of damage effects and implies a
413 series construction of deformation mechanisms: one rheological component
414 (the bulk part of the FE modeling) ~~translates~~describes the undamaged

415 material behavior such as elastoplasticity with hardening, while a second
416 component (the cohesive/surface part of the FE modeling) reflects the dam-
417 age effects. The relevance and efficiency of this separation of damage effects
418 are strongly related to the length scale at which the modeling is performed.

419 Usually, CZMs describe the material cohesion through a relationship be-
420 tween a cohesive stress R_C and a displacement jump $[u]$ between two contigu-
421 ous elements of the finite element mesh. From a more physical standpoint,
422 the resulting jump must be interpreted as the overall displacement induced
423 by damage (growth and coalescence of microvoids for example), and the on-
424 set and opening of a microcrack within a mesoscopic volume element. This
425 displacement jump $[u]$ surely cannot be interpreted as the gap between the
426 crack lips inasmuch as R_C would consequently vanish for any $[u] \neq 0$. To
427 construct the R_C vs. $[u]$ relationship, the first step is to split the overall
428 strain ε into a bulk strain ε_B that expresses the volumetric effects and a
429 ‘cohesive’ strain ε_C that summarizes the kinematic effects of damage effects.
430 Hereafter this *strain-formulated cohesive zone* will simply be referred to as
431 a *cohesive zone*.

432 The next subsection illustrates this additive strain decomposition through
433 heuristic, analytical 1D models. These rather academic exercises led us to
434 progressively construct a generic method to numerically perform this decom-
435 position for a set of experimental data.

436 *4.1.1. Damageable elasticity*

Let us consider the simple one-dimensional case of damageable elasticity. The constitutive equation classically reads:

$$\sigma = (1 - D)\sigma_{\text{eff}} = (1 - D)E\varepsilon, \quad (13)$$

where E is the Young modulus, and D is a damage parameter. For the sake of simplicity, we avoided using any subscript for the 1D stress and strain component. We only considered tensile loading ($\sigma \geq 0$). We also simply supposed that the damage rate is proportional to the positive part of the tensile strain rate expressed by:

$$\dot{D} = \langle \dot{\varepsilon} \rangle^+ / \varepsilon_R \quad \text{if} \quad \varepsilon = \varepsilon_M, \quad (14)$$

437 where ε_R is the strain level at which rupture occurs, $\varepsilon_M = \max\{\varepsilon(\tau), \tau \leq t\}$
 438 and $\langle x \rangle^+$ symbolizes the positive part of x . The damage then increases when
 439 the strain reaches the yield strain ε_M and as long as ε_M is less than ε_R .

The time integration of Eq. (14) leads to:

$$D(t) = \int_0^t \langle \dot{\varepsilon}(\tau) \rangle^+ / \varepsilon_R d\tau = \frac{\varepsilon_M(t)}{\varepsilon_R}. \quad (15)$$

As proposed above, let us now assume an additive decomposition of the overall strain $\varepsilon = \varepsilon_B + \varepsilon_C$ which allowed us to consider a series model, associating elasticity with a first component and damage with a second one

~~—We then formulate:~~

$$\sigma = E\varepsilon_B = E(\varepsilon - \varepsilon_C) = R_C = h(\varepsilon_C), \quad (16)$$

440 where h is a function to be defined.

For a monotone loading until rupture, Eqs. (13-16) lead to the following relation:

$$\sigma = E \left\{ 1 - \frac{1}{\varepsilon_R} \left(\frac{\sigma}{E} + \varepsilon_C \right) \right\} \left(\frac{\sigma}{E} + \varepsilon_C \right). \quad (17)$$

Solving Eq. (17) gives a single solution compatible with the physical boundary values $\sigma(\varepsilon_C = 0) = 0$ and $\sigma(\varepsilon_C = \varepsilon_R) = 0$ and allows us to derive the function h expression. The response of the strain-formulated ‘cohesive zone’ is:

$$R_C = h(\varepsilon_C) = E (\sqrt{\varepsilon_R \varepsilon_C} - \varepsilon_C). \quad (18)$$

If unloading is considered at a given strain $\varepsilon = \varepsilon_M$, the damage stops at $D(t) = D_M = \varepsilon_M/\varepsilon_R$. Eqs. (13-16) allow us to derive the response of the cohesive zone during unloading:

$$R_C = h(\varepsilon_C) = E \left(\frac{1 - D_M}{D_M} \right) \varepsilon_C. \quad (19)$$

441 The unloading response is then a linear function of the cohesive strain,
 442 with the slope characterizing the straight line of the $\sigma - \varepsilon_C$ plane passing
 443 through the origin and the maximum damage state reached at $\varepsilon = \varepsilon_M$. The
 444 slope tends to infinity for small damage values as the derivative $d\sigma/d\varepsilon_C|_{\varepsilon_C=0}$
 445 of the cohesive zone response during the loading.

If ℓ_{fe} now represents a characteristic length associated with CZM, the cohesive law can then be written as:

$$R_C = \begin{cases} E \left(\frac{1-D_M}{D_M} \right) \frac{[u]}{\ell_{fe}} & \text{if } \begin{cases} ([u] < \ell_{fe} \varepsilon_R D_M^2) \\ \text{or} \\ ([u] = \ell_{fe} \varepsilon_R D_M^2 \text{ and } [\dot{u}] \leq 0) \end{cases} \\ E \left(\sqrt{\varepsilon_R \frac{[u]}{\ell_{fe}}} - \frac{[u]}{\ell_{fe}} \right) & \text{if } ([u] = \ell_{fe} \varepsilon_R D_M^2 \text{ and } [\dot{u}] > 0) \end{cases} \quad (20)$$

446 where $[u] = l_{fe}\epsilon_C$. Figure 12 illustrates the cohesive zone response associated
 447 with Eq. (20). Naturally, the validity of Eq. (20) is directly related to the
 448 consistency of the ‘physics’ included in the material constitutive Eqs. (13-14).

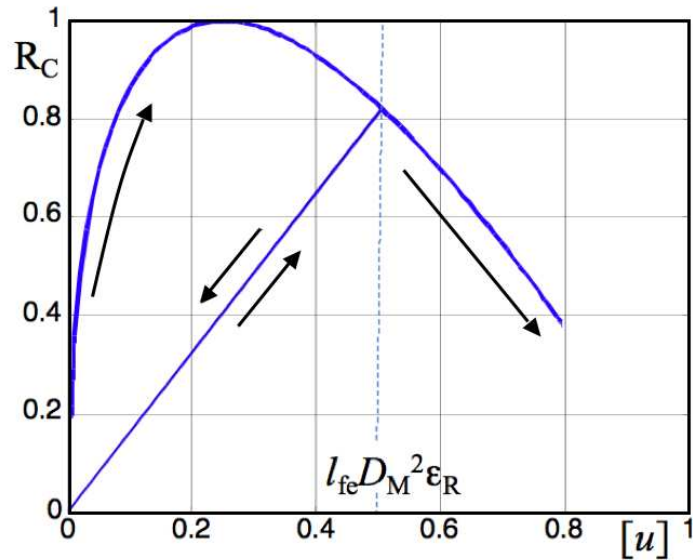


Figure 12: Example of normalized cohesive zone response for an elastic damageable material.

449 An extension of this damageable elasticity analysis to damageable elasto-
 450 plastic materials is proposed in the next paragraph.

451 4.1.2. Damageable elastoplastic model

452 Analytical expression of the cohesive zone constitutive equation cannot be
 453 obtained systematically. In what follows, we present a generic method to ex-
 454 tract the cohesive zone response in most complicated situations. The method
 455 is illustrated through the following damageable elastoplastic behavior. Once
 456 again, only tensile loadings ($\sigma \geq 0$) will be considered. We admitted the prin-
 457 ciple of strain equivalence proposed by [Chaboche and Lemaitre \(1978\)](#).

458 We chose a classical power law to describe the hardening, with the damage
 459 being introduced via the effective stress.

The elastic domain is defined by the yield function $f(\sigma, H) = \sigma - H - \sigma_y \leq 0$, where σ_y is the elastic limit while H stands for the power law hardening $H = K\varepsilon_p^n$, with K being the so-called hardening modulus and n the plastic exponent. The constitutive equations can be summarized by the following state and rate equations (**with effective stress replacing the stress in the yield function f**):

$$\left\{ \begin{array}{l} \text{State equation:} \\ \sigma_{\text{eff}} = E(\varepsilon - \varepsilon_p) \\ \text{Evolution equation:} \\ \dot{\varepsilon}_p = \begin{cases} 0 & \text{if } f < 0 \text{ or } (f = 0 \text{ and } \dot{f} < 0) \\ \left(\frac{H}{K}\right)^{\frac{1-n}{n}} \frac{1}{nK} \dot{\sigma}_{\text{eff}} & \text{if } f = 0 \text{ and } \dot{f} = 0 \end{cases} \end{array} \right. \quad (21)$$

The damage time course is, once more, modeled by a simplistic function of strain:

$$\dot{D} = \begin{cases} 0 & \text{if } \varepsilon < \varepsilon_M \\ m \left\langle \frac{\dot{\varepsilon}}{\varepsilon_R - \varepsilon_0} \right\rangle^+ \left(\frac{\varepsilon - \varepsilon_0}{\varepsilon_R - \varepsilon_0} \right)^{m-1} & \text{if } \varepsilon = \varepsilon_M \end{cases}, \quad (22)$$

460 where $\varepsilon_M(t) = \max(\varepsilon_0, \max\{\varepsilon(\tau), \tau \leq t\})$ with ε_0 the strain corresponding
 461 to the onset of damage ($D = 0$), ε_R is the rupture strain ($D = 1$), and $m > 0$
 462 is a constant parameter. The m value allows us to modify the ductility of
 463 the material response, with high m values inducing a loss of ductility.

For monotone tests, combining equations (21), (22) and the additive partition $\varepsilon = \varepsilon_B + \varepsilon_C$ associated with the hypothesis of a series model, the stress

reads:

$$\sigma = (1 - D)\sigma_{\text{eff}} = g(\varepsilon_B) = g(\varepsilon - \varepsilon_C) = R_C = h(\varepsilon_C), \quad (23)$$

464 where g and h are functions that respectively define the bulk and cohesive
 465 stress-strain responses (**h does not have same expression as in (16)**
 466 **since the overall behavior has changed**). The chosen elastoplastic bulk
 467 response is, by construction, described by Eq. (21). **Roughly speaking,**
 468 **this bulk response corresponds to the overall when no damage oc-**
 469 **curs.**

The following procedure is proposed to identify the :Ajoutercohesive re-
 sponse $h(\varepsilon_C)$. During monotone loading, the overall uniaxial response $\sigma - \varepsilon$
 is recorded, **as an increasing-decreasing function $\sigma = w(\varepsilon)$ (see Fig-**
ure 13). At each computational step k , it is then possible to determine ε_B
 as the solution of a nonlinear equation that we formally write as:

$$\varepsilon_B^k = (\varepsilon^k - \varepsilon_C^k) = g^{-1}(\sigma^k). \quad (24)$$

470 The non-regularity of the plastic behavior led us to develop a numerical
 471 computation of g^{-1} . **However, in this academic example, the func-**
 472 **tion g is piecewise given for monotone loading. First (hardening**
 473 **part), until the peak stress value σ^{peak} , i.e. $0 < \varepsilon < \varepsilon^{\text{peak}}$ with $\varepsilon^{\text{peak}}$**
 474 **such that $g(\varepsilon^{\text{peak}}) = \sigma^{\text{peak}}$, g is given by a time integration of (21)**
 475 **and g^{-1} appears as $\varepsilon_B^k = g^{-1}(\sigma^k) = ((\sigma^k - \sigma_y)/K)^{1/n} + \sigma^k/E$. Second**
 476 **(softening part), i.e. $\varepsilon > \varepsilon^{\text{peak}}$, the function g corresponds to an**
 477 **elastic discharge and g^{-1} reads $\varepsilon_B^k = g^{-1}(\sigma^k) = (\sigma^k - \sigma^{\text{peak}})/E + \varepsilon^{\text{peak}}$.**
 478 ~~With the couples~~ **The set $(\sigma^k, \varepsilon^k, \varepsilon_B^k)$ being known, the sought response**
 479 ~~$\sigma^k = R_C^k = h(\varepsilon_C^k)$~~ **cohesive function h is then immediately derived for each**

480 **step k as: $\sigma^k = h(\varepsilon_C^k)$. The cohesive strain reads $\varepsilon_C^k = \varepsilon^k - \varepsilon_B^k =$**
 481 **$w^{-1}(\sigma^k) + g^{-1}(\sigma^k)$ and the cohesive function is defined formally by**
 482 **$h = [(w^{-1} + g^{-1})^{-1}]$.**

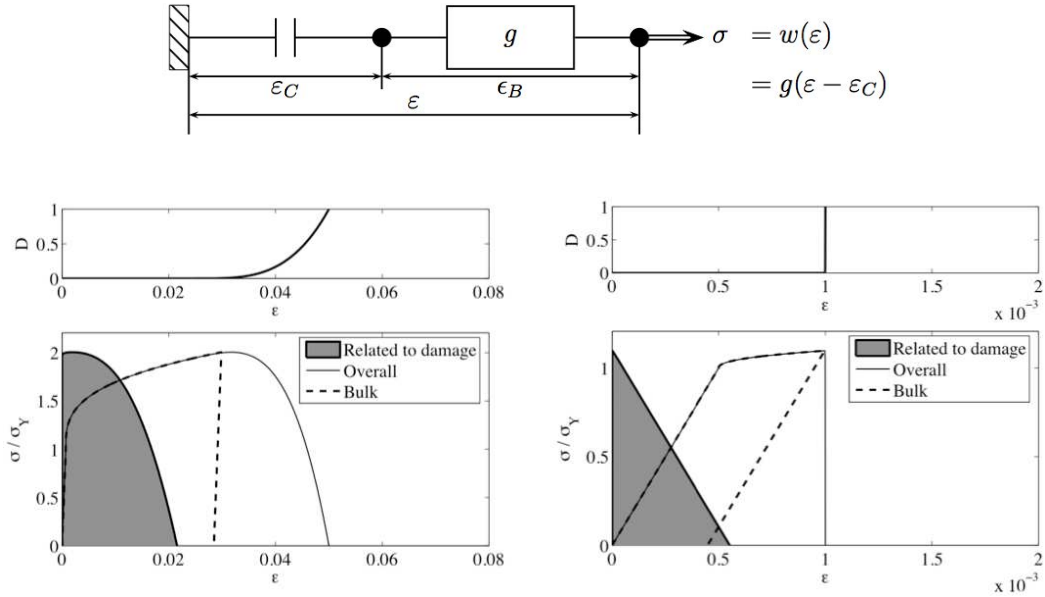


Figure 13: **(Top)** Series decomposition of a damageable elastoplastic response. **Damage (Middle) and stress (Bottom) vs strain:** (Left) Ductile material: $E = 200\text{GPa}$, $\sigma_y = 140\text{MPa}$, $\varepsilon_0 = 2.5\%$, $\varepsilon_R = 5\%$, $K = 440\text{GPa}$, $n = 0.32$, $m = 0.28$; (Right) Brittle material: $E = 300\text{GPa}$, $\sigma_y = 50\text{MPa}$, $\varepsilon_0 = 0.099\%$, $\varepsilon_R = 0.1\%$, $K = 400\text{MPa}$, $n = 0.43$, $m = 0.03$. **The shaded curves correspond to the cohesive zone response (σ/σ_y versus ε_C).**

483 Figure 13 shows the bulk response g and that related to damage h in the
 484 academic case of the damageable elasto-plastic model described above. Two
 485 sets of parameters corresponding to ductile and brittle behaviors were chosen.
 486 It is interesting to note that the identified responses of the cohesive zone have
 487 strong similarities with those of common (empirical) cohesive zone models
 488 described in the literature: trapezoidal shape (door-like) for ductile materials

489 and linear shape (triangle) for brittle materials Alfano (2006); Borst (2002);
490 Chandra et al. (2002).

491 Note that the identified response $h(\varepsilon_C^k)$ is not a regular function since
492 there is no initial compliance. The corresponding CZM (obtained by mul-
493 tiplying ε_C by a characteristic length) is thus categorized as an ‘extrinsic’
494 model (i.e. with an infinite interfacial stiffness). In this case, this feature
495 reflects the onset of damage from a stress threshold. The overall and bulk
496 stress–strain relations are the same before any damage occurs.

497 This academic analysis shows how a damageable elastoplastic behavior
498 can be locally splitted in the necking zone into a hardening volumetric be-
499 havior and a softening cohesive behavior. This approach is applied in the
500 following to experimental measurements. **Thus, in the sequel, the bulk**
501 **function g is not a priori given and will be implicitly deduced from**
502 **the damage measurement: no assumption is made on the form of**
503 **g or on the damage time course. In other words, the experimental**
504 **identification of the cohesive responses does *not* rest on simplistic**
505 **state and rate relations (21) and (22) that were used in the previous**
506 **academic example.**

507 4.2. *Experimental analysis of cohesive zone responses*

When identifying a CZ model associated with an incompressible bulk be-
havior, the sample has to be considered as a structure in which there is a
combination of structural and material effects. In this section, we propose
a protocol to derive the uniaxial cohesive zone response associated with the
loading direction during a monotonous tensile test. The objective is to deter-
mine the bulk part ε_B and the elasto-plastic part $\varepsilon_e + \varepsilon_p$ from the measured

tensile strain ε_1 . The cohesive zone strain is, by construction, the difference:

$$\varepsilon_C = \varepsilon_1 - \varepsilon_B. \quad (25)$$

The chosen bulk behavior corresponds to isochoric elastoplasticity. To extract this underlying elastoplastic response from the experimental data, the compressibility induced by the damage development must be taken into account. The virtual equivalent stress σ_{inc} associated with a given load \mathcal{F} must verify:

$$\mathcal{F} = \sigma_{\text{eff}} S_{\text{eff}} = \sigma_{\text{inc}} S_{\text{inc}}, \quad (26)$$

508 where $S_{\text{eff}} = S_0(1-D)e^{2\varepsilon_2}$ and $S_{\text{inc}} = S_0e^{-\varepsilon_1}$, with the latter being associated
 509 with a virtual isochoric deformation process. During the experiments, the
 510 computation of S_{eff} and S_{inc} at each loading step required knowledge of the
 511 strain components and of the damage variable derived from Eqs. (5) and (9).
 512 Figure 14 presents a basic sketch of the additive partition of the strain where
 513 the isochoric stress is adjusted in accordance with Eq. (26).

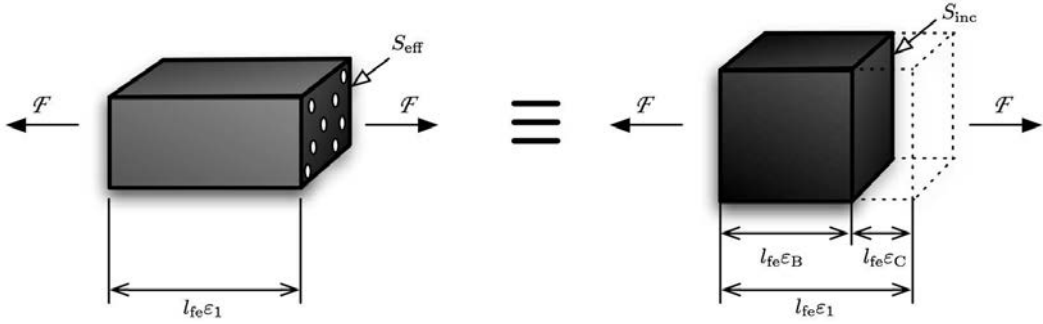


Figure 14: Basic sketch of the strain partition.

Regarding the effective behavior, the strain consists of an elastic and a plastic part:

$$\varepsilon_1 = \varepsilon_e + \varepsilon_p = \frac{\sigma_{\text{eff}}}{E} + \varepsilon_p, \quad (27)$$

514 including the kinematical effects of the microvoids presence.

On the other hand, the isochoric elastoplasticity in series with a cohesive zone leads to:

$$\varepsilon_1 = \underbrace{\varepsilon_e^* + \varepsilon_p^*}_{\varepsilon_B} + \varepsilon_C = \frac{\sigma_{\text{inc}}}{E} + \varepsilon_p^* + \varepsilon_C. \quad (28)$$

515 Note that standard linear elasticity was assumed for effective and isochoric
 516 responses. This hypothesis remained closely in line with our observations
 517 during unloading.

As already mentioned, the cohesive zone response was determined during monotone loadings on the rupture cross section to ensure continuous progress of the plasticity until rupture (no elastic unloading). For such loading, σ_{eff} represents the plastic flow stress as soon as σ_y is reached. The flow stress can be formally written:

$$\sigma_{\text{eff}} = \sigma_y + H(\varepsilon_p) = \sigma_y + H\left(\varepsilon_1 - \frac{\sigma_{\text{eff}}}{E}\right), \quad (29)$$

where H characterizes the strain hardening. **Young modulus E and yield stress σ_y are identified on the uniaxial and monotonic traction experiments. For any measured load \mathcal{F} , the principal components of the strain tensor, namely ε_1 and ε_2 (and $\varepsilon_3 = \varepsilon_2$ assuming again transverse isotropy) are computed with the DIC procedure and appropriate spatial differentiations. Combining (5), (6) and (9), the corresponding damage is computed as $D \simeq 1.2(e^{(\varepsilon_1+2\varepsilon_2)} - 1)^{2/3}$. With (10), (11) and (12), the associated effective stress is estimated as $\sigma_{\text{eff}} = \mathcal{F}/(S_0(1-D)e^{2\varepsilon_2})$ in the actual configuration. These data together with (29) allow the numerical computation of the strain**

hardening function H at any increment of elongation. In the same way, σ_{inc} associated with σ_{eff} (Eq. (26)) is the plastic flow stress developed by the incompressible bulk component. The principle of strain equivalence allows us to use the same hardening function H to numerically compute the isochoric plastic strain as a root of the nonlinear equation $H(\varepsilon_p^*) - (\sigma_{\text{inc}} - \sigma_y) = 0$. The cohesive strain can then be formally written:

$$\varepsilon_C = \varepsilon_1 - \underbrace{\frac{\sigma_{\text{inc}}}{E}}_{\varepsilon_e^*} - \underbrace{H^{-1}(\sigma_{\text{inc}} - \sigma_y)}_{\varepsilon_p^*}. \quad (30)$$

518 Since the hardening response H is experimentally known, finding ε_C for a
 519 given value of σ_{inc} leads to solve a non linear equation. **In other words,**
 520 **the procedure is identical to the academic case described in sec-**
 521 **tion 4.1.2 with the bulk function g (not a priori known in that case)**
 522 **now deduced from the overall response and the identification of the**
 523 **damage D .**

524 Figure 15 shows, for the four studied specimens, the apparent stress σ in
 525 the localization zone as a function of the overall and cohesive zone strains,
 526 respectively ε_1 and ε_C . The stress-strain relation σ_{inc} vs. ε_B associated with
 527 the virtual isochoric straining is also plotted on the same graphs. This rela-
 528 tion corresponds to the bulk behavior that should be resolved at the Gauss
 529 points in a FE method for each material, and therefore it should be used for
 530 determining the bulk parameters of constitutive equations. The relation σ vs.
 531 ε_C provides access to the cohesive zone response and corresponds to the CZM,
 532 which can be directly used in CZM-based simulations through incorporation
 533 of the length ℓ_{fe} corresponding to the link between the displacement jump $[u]$
 534 and the cohesive strain ε_C , as mentioned in **Section 4(20)**. The Figure 15 can

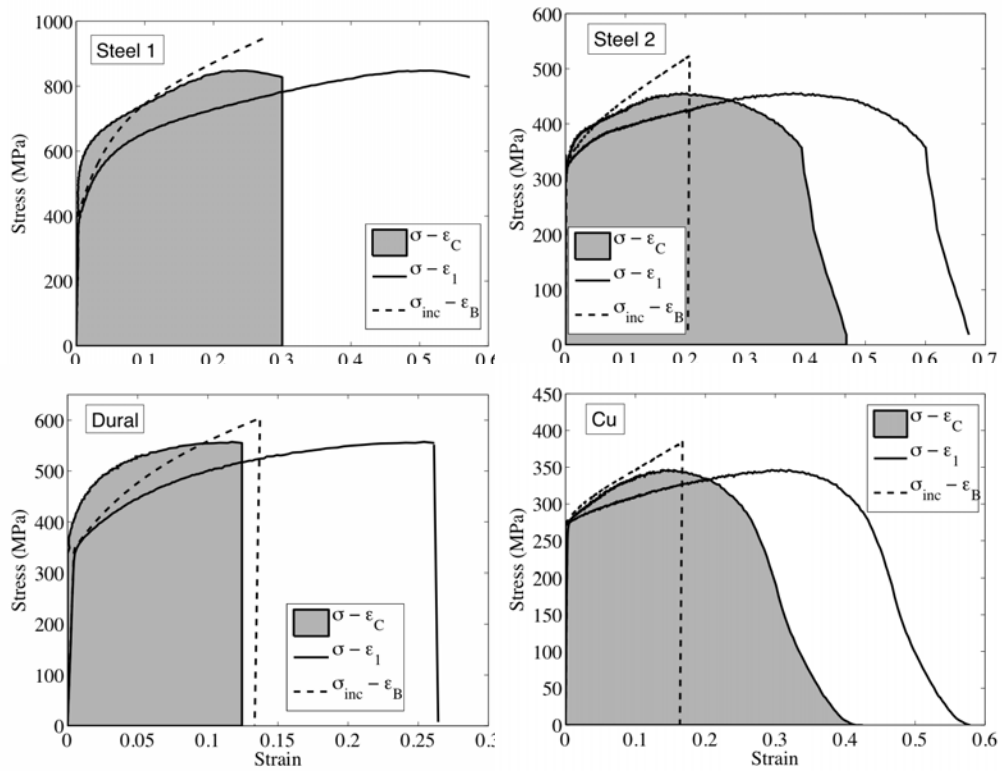


Figure 15: Local (within the localization zone) stress-strain diagrams for the studied materials. Measured σ vs. ϵ_{11} relation (solid line); Identified σ_{inc} vs. ϵ_C relation (dashed line); Identified cohesive zone response (solid line with grey area).

535 be considered as the central result of this study: the cohesive-volumetric de-
536 composition arises naturally from the uniaxial stress-strain response. While
537 the apparent behavior (dark lines) is elastoplastic and damageable, the lo-
538 cal bulk behavior (dashed lines) is elastoplastic, i.e. without any softening
539 behavior, and the entire softening process is incorporated in the cohesive re-
540 sponse (dark line with gray area). The local bulk responses exhibit higher
541 hardening than the apparent responses and the maximal cohesive stress cor-
542 responds to the ultimate tensile stress of the apparent local behavior. Since
543 the cohesive strain tends to increase only when the overall strain is high
544 enough to induce some damage, the difference between the cohesive strain
545 and the apparent strain at failure corresponds to the elastoplastic strain that
546 the material can tolerate without any onset of microdamage.

547 This experimental protocol thus validates the cohesive-volumetric decom-
548 position. However, at this stage of the study the final traction-separation
549 relationship of the cohesive law cannot be obtained since the cohesive strain
550 ε_C has to be related to the displacement jump $[u]$ identifying the cohesive
551 length ℓ_{fe} . The exact physical or numerical meaning of this length ℓ_{fe} will
552 be investigated in a forthcoming work. This length has indeed nothing to do
553 with the length-scale used for the identification which is related to the CCD
554 sensor characteristics, the optical lens, and the image processing. The rele-
555 vance of CZM-based computational predictions depends, as usual, *(i)* on the
556 mechanical relevance of the chosen constitutive equation, *(ii)* on the qual-
557 ity of the model identification and *(iii)* on the **finenessrefinement** of the
558 discretization chosen in association with the estimated damage distribution
559 heterogeneity level.

560 In summary, the main elementary bricks of the proposed protocol are the
561 following:

- 562 1. Use of the early character of the zone where the crack will finally occur
563 to monitor the damage kinetics
- 564 2. Assessment of the local volume change distribution during the test
565 using digital image correlation techniques
- 566 3. Correlation of this volume change to surface damage using a microme-
567 chanical model
- 568 4. Cohesive-volumetric decomposition using a dedicated strain partition-
569 ing.

570 Furthermore, we underline the fact that the second point has to be experi-
571 mentally improved using, for example, stereo-correlations to avoid the trans-
572 verse isotropy assumption, and that the third point can be replaced by any
573 convenient micromechanical model or energy balance. However, the final re-
574 sult on the cohesive-volumetric decomposition is obtained here without loss
575 of generality.

576 **5. Concluding comments**

577 **In this paper, an experimental analysis on standard traction**
578 **tests was proposed in order to identify cohesive zone models with-**
579 **out any assumption on the shape of the cohesive law nor on the final**
580 **crack path (like in debond tests).** This experimental analysis allowed
581 us to better grasp the physical sense of the so-called cohesive or interface law
582 and displacement jump. We proposed an experimental approach to estimate

583 local relative volume variations that correspond to microvoid fraction in the
584 case of elastoplasticity (with negligible elastic volume changes). The damage
585 parameter associated with these microvoids could be found through a voids
586 growth model. Although the model used in this paper is very simple, it may
587 be implemented to draw up an identification protocol of cohesive zone models
588 using kinematical full-field measurements.

589 We identified cohesive zone responses for four different ductile materials
590 and noted numerous similarities with the cohesive zone models commonly
591 used in cohesive-volumetric finite element simulations. The three main re-
592 sults of this work are the following:

- 593 • a cohesive zone model can be experimentally identified for each given
594 bulk behavior,
- 595 • such cohesive zone models belong to the class of so-called ‘extrinsic’
596 CZM,
- 597 • the classical empirical choices of door-like CZM for ductile materials
598 and bilinear CZM for brittle materials are experimentally and theoret-
599 ically confirmed.

600 In the extension of this work, it will be essential to perform numerical
601 validations by comparing numerical and experimental results for different
602 structures with complex loading paths in mode I. Pure shear tests should
603 now be considered to complete the identification of cohesive zone models.
604 It would also be of interest to extend the identification method to the brit-
605 tle behavior corresponding to a wide range of materials in dynamic loading

606 conditions. Moreover, the proposed strategy for CZM identifica-
607 tion will be improved in a forthcoming work: 1/ the accuracy of
608 the 2D approach has to be checked by complementary 3D mea-
609 surements, 2/ the assumption of an underlying volumetric damage
610 model has to be further analyzed (for example with the help of
611 loading-unloading experiments), 3/ the cohesive length — as the
612 key-parameter of CZM — has to be identified by comparison with
613 numerical simulations.

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