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Velocity-sensorless tracking control and identification of switched-reluctance motors

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Abstract

We present a solution to the speed sensorless control problem for switched-reluctance motors under parametric uncertainty. Our main results guarantee velocity tracking control for velocity references with constant reference acceleration under the assumption that the load torque, the rotor inertia, the resistance and inductances are unknown. Under a persistency of excitation condition on a function which depends only on reference trajectories, we guarantee uniform global asymptotic stability therefore, we establish conditions for the identification of the physical parameters of the system. Our theoretical findings are supported by illustrative simulation results.

1 Introduction

The fact that switched-reluctance motors are highly reliable and cheap to construct [20] make them very attractive for a number of applications. However, from a modelling viewpoint they are complex machines: *e.g.*, as they are doubly-salient brushless motors [8], it is not possible to apply well-established AC motor rotating field theory; also, the nonuniform nature of electromechanical torque leads to torque ripple. This and their commonly practised operation in magnetic saturation to maximise the torque, make control, modelling and identification of switched-reluctance motors a challenging active research area among various communities.

There exist a large number of efficient heuristically-based and experimentally-validated identification and control approaches for switched-reluctance machines with different control objectives: torque and flux control –[1], velocity control –[5], state estimation –[6], parameters identification –[3], *etc.* Of particular interest is to reduce the number of mechanical sensors (position and velocity) in the loop as this has a direct impact in the cost of production. For instance, in [5] the authors propose an efficient inductance model-based sensorless control scheme to run the SRM from stand-still to high speed.

Articles on control of switched-reluctance drives that include a rigorous stability analysis, especially in a sensorless context (let alone under parametric uncertainty), are rare. For instance, in [17] there is reported a proportional-derivative-based controller but which relies on the knowledge of the load torque and full-state feedback. The main result in [4] establishes global asymptotic stability for a passivity-based controller in the case of unknown load however, it uses both angular velocity and position measurements. In [13] the authors present an adaptive position-feedback controller which uses an approximate-differentiation filter instead of velocity measurements and it is guaranteed (under a high-gain condition) that the tracking errors converge to zero. However, the model used in [13] comprises only the rotor dynamics and it is assumed that the currents are valid physical control inputs. Although the latter is fully justified, for instance in view of the torque-sharing approach –see [17], the analysis of the stator control loop is not included in the former reference. It is worth remarking that the lack of analysis in a sensorless context is certainly not the case for other electrical machines such as induction motors – see [12, 7], or permanent-magnet synchronous machines –[18, 16]. In [2] a nonlinear controller using a similar scheme is presented however, the authors suppose the mechanical load-torque τ_L known and the model includes viscous friction, which eases considerably the stability analysis.

We present a robust adaptive speed sensorless controller for switched-reluctance motors with uncertainty in all physical parameters. The control scheme is composed of an outer rotor control loop of proportional-integral-derivative type in which the stator currents are considered as virtual control inputs and is robust to the uncertainty in the inertia. Then, an inner adaptive tracking control loop for the stator currents is incorporated. We establish uniform global exponential stability in the case that the stator parameters are known and for the overall adaptively controlled system we establish uniform global asymptotic stability; notably, we establish necessary and sufficient conditions for the convergence of the physical parameter estimates. As far as we know, similar results have not been reported in the literature.

The rest of the paper is organised as follows. In the following section we describe the motor model; in Section 3 we describe the control approach. The formal statements and proofs are provided in Section 4 and some simulations that illustrate our theoretical findings are presented in Section 5, before concluding with some remarks.

2 The motor model

Switched-reluctance motors operation is based on the phenomenon that can be observed when two displaced opposite polarised magnets are aligned. The basic structure of a machine with 6 poles in the stator and 4 poles in the rotor

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is depicted in Figure 1. Each of the symmetrical pole-pairs are simultaneously energised (using a single coil and only positive currents) producing a magnetic field that aligns the winding-free rotor. Mechanical force is produced by exciting, in a suitable way (usually sequentially), the different stator phases.

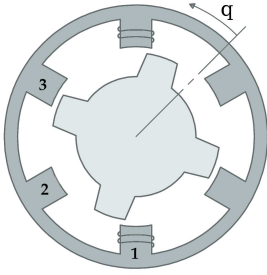


Fig. 1. Diagram of a switched-reluctance motor

After experimental evidence, the three stator phases of a switched-reluctance motor may be assumed to be magnetically decoupled *i.e.*, the mutual inductance among stator phases is negligible. Under such hypothesis an experimentally-validated three-phases dynamic model is proposed in [8]. However, although the model used in the latter reference accounts for magnetic saturation, it defines a nonlinear non-invertible map currents-torque. Therefore, for control and *analysis* purposes we assume that the flux is linear in the currents that is, the model of the stator dynamics is given, for each phase j , by

$$u_j = L_j(q)\dot{x}_j + K_j(q)\omega x_j + R x_j$$

where all variables are scalar, u_j denotes the voltage applied to the stator terminals, q is the angular rotor position, ω is the angular velocity, x_j denotes the stator current, R denotes the stator winding resistance,

$$L_j(q) = \ell_0 - \ell_1 c_j, \quad c_j := \cos\left(N_r q - (j-1)\frac{2\pi}{3}\right)$$

with $\ell_0 > \ell_1 > 0$, denotes the phase inductance, N_r is the number of poles and

$$K_j(q) = \frac{\partial L_j}{\partial q} = N_r \ell_1 s_j, \quad s_j := \sin\left(N_r q - (j-1)\frac{2\pi}{3}\right),$$

corresponds to the phase-inductance variation relative to the rotor angular position. Hence, in matrix form, we have

$$L(q)\dot{x} + K(q)\omega x + R x = u \quad (1)$$

where $L := \text{diag}\{L_j\}$, $K := \text{diag}\{K_j\}$ and $x = [x_1, x_2, x_3]^\top$. For further development, we remark that there exist constants ℓ_m , ℓ_M and k_M such that for all² $q \in [-\pi, \pi]$ and $j \in \{1, 2, 3\}$,

$$0 < \ell_m \leq \|L_j(q)\| \leq \ell_M, \quad \|K_j(q)\| \leq k_M. \quad (2)$$

The rotor dynamics is given by a simple double integrator of the load torque τ_L and the mechanical torque of electrical origin, τ_e , which depends on the angular rotor position and the stator currents that is,

$$J\dot{\omega} = \tau_e(q, x) - \tau_L \quad (3a)$$

$$\dot{q} = \omega \quad (3b)$$

where and J is the total rotor inertia. Considering that the behaviour of the stator windings is decoupled, the mechanical torque of electrical origin corresponds to the sum of torques produced by each of the three phases,

$$\tau_e(q, x) = \sum_{j=1}^3 \frac{1}{2} K_j(q) x_j^2.$$

3 Control problem and its solution

For the system (1), (3) the control problem is to design a dynamic controller whose output $u = [u_1 \ u_2 \ u_3]^\top$ depends on the stator currents and rotor angular positions, such that $\omega(t)$ tracks a reference trajectory $\omega^*(t)$ under the following standing hypotheses.

Assumption 1

- The function ω^* is bounded, differentiable and $\dot{\omega}^*$ is (piecewise) constant;
- the inertia J is unknown and belongs to an interval of known limits J_m, J_M *i.e.*, $J \in [J_m, J_M]$;
- the load torque τ_L is piece-wise-constant and unknown.

We emphasise that from a control-theory perspective this problem is open.

The control approach consists in applying two control loops: an outer loop to robustly stabilise the rotor dynamics via the virtual control input τ_e and an inner loop to stabilise the stator dynamics using currents measurement to guarantee the tracking control goal $\tau_e \rightarrow \tau_e^*$.

² Throughout the paper we denote by $\|\cdot\|$, the Euclidean norm of vectors and the induced norm of matrices.

3.1 Robust control of rotor velocity

Since the rotor equation (3a) is linear with a (piecewise) constant perturbation τ_L/J and a vanishing disturbance, we chose to use proportional-integral-derivative (PID) control. Since the velocities ω are not measured we use the PI²D controller, introduced in [14] for robot manipulators. Its name comes from the fact that it corresponds to a modified PID controller; it consists in a correction term proportional to the tracking errors e_q , a ‘derivative’ term proportional to filtered velocities ϑ and a double integral action, both on e_q and ϑ . Besides its mathematical simplicity, the choice of this controller is motivated by the fact that it conserves the passivity properties of Lagrangian systems and ensures asymptotic stability, provided a property of detectability holds –see [15]. Moreover, since it is a PID controller it is model-free, which is fundamental in the present context in which we assume that the physical parameters are unknown. The PI²D *tracking* controller for the rotor dynamics is defined by

$$\tau_d = -k_p e_q - k_d \vartheta + \nu + \dot{\omega}^* \quad (4a)$$

$$\dot{\nu} = -k_i (e_q - \vartheta) \quad (4b)$$

$$\dot{q}_c = -a(q_c + b e_q) \quad (4c)$$

$$\vartheta = q_c + b e_q \quad (4d)$$

where k_p, k_i, k_d, a, b are positive constants and $e_q = q - q^*$ with

$$q^*(t) = \int_0^t q^*(s) ds, \quad q^*(0) = q_0^* \in [-\pi, \pi].$$

Since the variable to be controlled is ω , the initial value $q^*(0)$ is innocuous. Equations (4c) and (4d) correspond to the widely-used ‘‘approximate differentiation’’ filter

$$\vartheta = \frac{b}{p+a} e_\omega$$

where p is the Laplace variable and $e_\omega := \omega - \omega^*$. That is, ϑ is not an estimate of the velocity e_ω but a filtered version of it; in the limit case, when the pole is at $-\infty$, $\vartheta = e_\omega$ modulo the DC gain b/a .

The following is a preliminary result on the stability of the solutions of (3a) driven by the PI²D controller.

Proposition 1 *Consider the PI²D controller (4), let $k'_p := k_p - (k_i/\varepsilon)$ where $\varepsilon \in (0, 1)$, $k_i \leq \varepsilon$ and let*

$$\tau_e^* = \eta \tau_d, \quad \eta > 0. \quad (5)$$

Then, the solution

$$\left\{ [q \ \omega \ \nu]^\top = [q^* \ \omega^* \ \nu^*] \right\}, \quad \nu^* := \frac{\tau_L}{\eta} + \left(\frac{J}{\eta} - 1 \right) \dot{\omega}^* \quad (6)$$

of the system (3a), (3b) with $\tau_e = \tau_e^$ and under Assumption 1, is uniformly globally exponentially stable, provided that*

$$A(\eta) := \begin{bmatrix} 0 & 1 & 0 & 0 \\ -(\eta/J)k'_p & 0 & -(\eta/J)k_d & (\eta/J) \\ 0 & b & -a & 0 \\ -k_i & -k_i/\varepsilon & k_i & 0 \end{bmatrix} \quad (7)$$

is Hurwitz. Consequently, if $\tau_e \neq \tau_e^$ the system is input-to-state stable with input $\tilde{\tau}_e := \tau_e - \tau_e^*$.*

Remark 1 The parameter η is introduced to underline the robustness of the PI²D controller. Indeed, the closed-loop equilibrium for the rotor dynamics depends on the (possibly unknown) parameter η but not stability of that equilibrium. Based on [10], standard but somewhat lengthy computations, show that the matrix $A(\eta)$ in (9) is Hurwitz for any $J \in [J_m, J_M]$ and $\eta \in [\eta_m, \eta_M]$ if

$$b \geq \left[\frac{\eta_M}{J_m} + 1 \right] a + 1, \quad k'_p > k_d.$$

Hence, the eigenvalues of A may be made negative by properly choosing the control gains k_p, k_d and k_i without the knowledge of η and J .

Proof of Proposition 1. In view of (4a) and (5), Equation (3a) is equivalent to

$$\dot{\omega} = -\frac{\eta}{J} \left[k'_p e_q + \frac{k_i}{\varepsilon} e_q + k_d \vartheta \right] + \frac{\eta}{J} (\nu + \dot{\omega}^*) - \frac{\tau_L}{J} + \frac{\tilde{\tau}_e}{J}.$$

Next, let

$$z := \nu - \nu^* - \frac{k_i}{\varepsilon} e_q, \quad (8)$$

where ν^* is defined in (6) then,

$$\dot{e}_w = -\frac{\eta}{J} \left[k'_p e_q + k_d \vartheta - z \right] + \frac{\tilde{\tau}_e}{J}.$$

On the other hand, Equations (4b), (4c) are equivalent to

$$\dot{\vartheta} = -a\vartheta + be_\omega$$

so differentiating on both sides of (8), using (4b) and rearranging terms we obtain the rotor closed-loop dynamics³

$$\dot{\xi} = A\xi + B\tilde{\tau}_e, \quad \xi := [e_q \ e_\omega \ \vartheta \ z]^\top \quad (9)$$

where $B = [0 \ (1/J) \ 0 \ 0]^\top$. Now, the Hurwitz property of A is equivalent to the existence of $P = P^\top > 0$ and $\gamma_1 > \|PB\|$ such that $-(A^\top P + PA) = Q$ for a given positive definite matrix Q , therefore

$$V_1(\xi) = \frac{1}{2} \xi^\top P \xi$$

satisfies

$$\dot{V}_1(\xi) \leq -\frac{1}{2} \xi^\top Q \xi + \frac{\gamma_1}{J} \|\xi\| |\tilde{\tau}_e| \quad \text{a.e.} \quad (10)$$

that is, V_1 is an ISS-Lyapunov function and $\tilde{\tau}_e = 0$ implies global exponential stability of $\{\xi = 0\}$. The result follows observing that $\xi = 0$ implies that $[q \ \omega \ \nu]^\top = [q^* \ \omega^* \ \nu^*]^\top$. \blacksquare

3.2 Control of the stator dynamics

Equation (9) is valid if and only if $\tau_e^* = \eta\tau_d$ for a given τ_d ; this is accomplished by solving $\tau_e^*(q, x^*) = \eta\tau_d$ for x^* . To that end, we employ the so-called torque sharing technique introduced in [17]. To that end consider the commutation smooth functions⁴ $q \mapsto m_j$ such that $\sum_{j=1}^3 m_j(q) = 1$ then, in order to satisfy the resulting equation

$$\frac{1}{2} \left[x_1^{*2} K_1(q) + x_2^{*2} K_2(q) + x_3^{*2} K_3(q) \right] = \eta\tau_d [m_1(q) + m_2(q) + m_3(q)]$$

we define

$$x_j^* = \left[\frac{2\eta\tau_d m_j}{N_r \ell_1 s_j(q)} \right]^{1/2} \quad \forall j \in \{1, 2, 3\}$$

where $s_j(q)$ may be equal to zero and the quotient above may be negative. For the x_j^* to be well-posed we exploit the physics of the reluctance machine as in [8, 17] and introduce a current-switching policy as follows. Let the sets

$$\Theta_j^+ = \{q \in [-\pi, \pi] : s_j(q) \geq 0\}$$

$$\Theta_j^- = \{q \in [-\pi, \pi] : s_j(q) < 0\}$$

and let

$$m_j(q) = \begin{cases} m_j^+(q) & \text{if } \tau_d \geq 0 \\ m_j^-(q) & \text{if } \tau_d < 0 \end{cases}$$

where

$$m_j^+(q) > 0 \ \forall q \in \Theta^+, \quad m_j^+(q) = 0 \ \forall q \in \Theta^-,$$

$$m_j^-(q) > 0 \ \forall q \in \Theta^-, \quad m_j^-(q) = 0 \ \forall q \in \Theta^+.$$

Because the functions s_j are sinusoids out of phase by $2\pi/3$, for each q and τ_d there always exists (at least) one $j \in \{1, 2, 3\}$ such that

$$\frac{\tau_d m_j(q)}{s_j(q)} \in (0, \infty).$$

Furthermore, to make the phase transitions smooth we introduce hysteresis around the switching condition $s_j = 0$ *i.e.*,

$$x_j^* := \begin{cases} \left[\frac{2\eta}{N_r \ell_1} \right]^{1/2} \left[\frac{\tau_d m_j(q)}{s_j(q)} \right]^{1/2} & \text{if } |s_j(q)| > \delta_K \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

³ Strictly speaking, (9) is defined for almost all t , except for the countable number of points where $\ddot{\omega}^* \neq 0$ and $\dot{\tau}_L \neq 0$ however, the (Carathéodory) solutions are defined for all t .

⁴ Smoothness is assumed for simplicity, piecewise continuity is enough.

where δ_K is the hysteresis design parameter. Under these conditions we have $\tau_e^*(q, x^*) = \eta\tau_d$ so to ensure that $\tau_e \rightarrow \eta\tau_d$ we must solve the new tracking control problem $x \rightarrow x^*$.

The rationale behind the design of the tracking controller for the stator dynamics builds on the observation that under the action of the tracking control law

$$u = L(q)\dot{x}^* + \omega^* K(q)x + Rx^* - k_{px}e_x, \quad e_x = x - x^*, \quad (12)$$

the origin of the closed-loop equation

$$L(q)\dot{e}_x + [R + k_{px}]e_x = -K(q)x e_\omega \quad (13)$$

with zero input ($e_\omega \equiv 0$) is globally exponentially stable, provided that $k_{px} > 0$. This is clear from the fact that (13) is reminiscent of a linear system with stable drift and $L(q)$ is positive definite uniformly in q hence exponential stability of the origin of

$$L(q)\dot{e}_x = -[R + k_{px}]e_x$$

is equivalent to that of the origin of

$$\dot{e}_x = -[R + k_{px}]e_x.$$

Furthermore, the system may be rendered input to state stable from the input e_ω provided that the gain k_{px} dominates over the input ‘‘gain’’ function $K(q)x$; note that this is feasible since q and x are measured states. Under these conditions one may invoke a small-gain argument to establish global exponential stability of the origin of the closed-loop system (9), (13).

However, the control law (12) is not implementable since \dot{x}^* depends on the unmeasured velocity ω indeed,

$$\dot{x}_j^* = \begin{cases} \alpha_j [\rho_j + \delta_j e_\omega] & \text{if } |s_j(q)| > \delta_K \\ 0 & \text{otherwise} \end{cases}$$

where, if $|s_j(q)| > \delta_K$,

$$\begin{aligned} \alpha_j &= \frac{1}{2} \left[\frac{2\eta}{N_r \ell_1} \right]^{1/2} \left[\frac{m_j \tau_d}{s_j} \right]^{-1/2} \\ \rho_j &= \frac{m_j}{s_j} \left[(k_d a + k_i) \vartheta - k_i e_q + \frac{\partial m_j}{\partial q} \frac{\tau_d \omega^*}{m_j} \right] - \frac{m_j N_r c_j}{s_j^2} \tau_d \omega^* \\ \delta_j &= \frac{1}{s_j} \left[-m_j (k_p + k_d b) + \frac{\partial m_j}{\partial q} \tau_d \right] - \frac{m_j N_r c_j}{s_j^2} \tau_d \end{aligned} \quad (14)$$

otherwise, $\alpha_j = \rho_j = \delta_j = 0$.

Therefore, we introduce the following control law which is reminiscent of u defined in (12) except that we drop the term $\alpha_j \delta_j e_\omega$ in the definition of $\dot{x}^* = [\dot{x}_1^*, \dot{x}_2^*, \dot{x}_3^*]^\top$ that is, let

$$u = L(q)\alpha\rho + \omega^* K(q)x + Rx^* - k_{px}e_x, \quad (15)$$

where $\alpha = \text{diag}\{\alpha_1, \alpha_2, \alpha_3\}$, $\rho = [\rho_1 \ \rho_2 \ \rho_3]^\top$ and $\delta = [\delta_1 \ \delta_2 \ \delta_3]^\top$. Therefore, (15) is equivalent to

$$u = L(q)\dot{x}^* + \omega^* K(q)x + Rx^* - k_{px}e_x - L(q)\alpha\delta e_\omega. \quad (16)$$

The closed-loop equation (1) with (16) yields

$$L(q)\dot{e}_x = -[R + k_{px}]e_x - [K(q)x + L(q)\alpha\delta]e_\omega \quad (17)$$

which is also reminiscent of a perturbed linear system with stable drift; in this case the input gain is given by

$$g(t, x, y) := [K(q(t))x + L(q(t))\alpha(t, y)\delta(t, y)]$$

in which we underline the dependence of α and δ on $\omega^*(t)$, $q(t)$ and the measurable output $y := [e_q \ \vartheta \ \nu]^\top$. Moreover, since L , K , m_j , $\frac{\partial m_j}{\partial q}$ and ω^* are uniformly bounded, there exists a non-decreasing function $\gamma_2 : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ such that

$$\|g(t, x, y)\| \leq \gamma_2(\|y\|, \|x\|).$$

Note that γ_2 depends on $\eta_M \geq \eta$ but not on the possibly unknown constant η . Thus, it may be established that (17) is input-to-state stable with respect to the input e_ω , for an appropriate choice of the gain k_{px} depending on γ_2 , hence on $\|y\|$ and $\|x\|$.

4 Main results

From the previous developments we see that the closed-loop system (9), (17) consists in the interconnection of two input-to-state stable systems for which the feedback gains may be adjusted to ensure global exponential stability. The proof of this claim constitutes the first of our main contributions.

4.1 Robust rotor control

Proposition 2 Consider the system (1), (3) in closed loop with the PI^2D controller (4) under the conditions of Proposition 1 and the control law (15) where $x^* = [x_1^* \ x_2^* \ x_3^*]^\top$ is defined via (11) with $\eta := \hat{J} \in [J_m, J_M]$. Let the control gains and \hat{J} , which is a constant estimate of J , be such that the matrix $A(\hat{J})$ in (7) is Hurwitz for any $J \in [J_m, J_M]$ –see Remark 1. Assume that the stator parameters ℓ_0, ℓ_1 and R are known. Then, there exist a real number $k'_{px} > 0$ and a non-decreasing function $k''_{px} : \mathbb{R}_{\geq 0}^3 \rightarrow \mathbb{R}_{\geq 0}$ such that defining

$$k_{px} = k'_{px} + k''_{px}(\|y\|, \|x\|, \|x^*\|),$$

with $e_x := x - x^*$, the origin of the closed-loop system $\{[\xi, e_x] = [0, 0]\}$ is uniformly globally exponentially stable.

Proof. In view of the definition of x^* we have $\tau_e^* = \hat{J}\tau_d$ so the rotor closed-loop dynamics yields (9) with $A = A(\hat{J})$. In view of Proposition 1 the equilibrium defined by (6) is uniformly globally exponentially stable if $\tilde{\tau}_e = 0$. Let $q_m > 0$, $\varepsilon \in (0, 1)$ and $Q = Q^\top > 0$ be such that $Q = Q_1 + Q_2$ where $Q_1 = Q_1^\top$ is positive definite and $Q_2 = Q_2^\top$ is positive semidefinite such that $q_m\|\xi\|^2 \leq \frac{1}{2}\xi^\top Q_1 \xi$ and $\xi^\top Q_2 \xi \geq (J/\eta)\varepsilon(b-1)e_\omega^2$. Let the Hurwitz assumption generate via [9, Theorem 4.2], a matrix P and positive reals γ_1, p_m and p_M such that $Q = -(A^\top P + PA)$, $\|PB\| \leq \gamma_1$ and

$$p_m\|\xi\|^2 \leq V_1(\xi) \leq p_M\|\xi\|^2.$$

Then, the derivative of V_1 satisfies (10) with Q as defined above and since τ_e and τ_e^* are quadratic functions uniformly bounded in $q, 2|\tilde{\tau}_e| \leq k_M|e_x^\top(x+x^*)|$ so

$$\dot{V}_1(\xi) \leq -q_m\|\xi\|^2 + \frac{k_M\gamma_1}{2}\|\xi\||e_x^\top(x+x^*)| - (J/\eta)\varepsilon(b-1)e_\omega^2 \quad \text{a.e.}$$

On the other hand, substituting u from (15) in (1) we obtain the stator closed-loop equation

$$L(q)\dot{e}_x = -[R + k_{px}]e_x - g(t, x, y)e_\omega. \quad (18)$$

Consider the function $V_2 : \mathbb{R}_{\geq 0} \times \mathbb{R}^3 \rightarrow \mathbb{R}_{\geq 0}$ defined by

$$V_2(t, e_x) = \frac{1}{2}e_x^\top L(q(t))e_x$$

which is positive definite and radially unbounded since $\|L(q)\|$ is uniformly bounded and positive definite actually, in view of (2),

$$\frac{\ell_m}{2}\|e_x\|^2 \leq V_2(t, e_x) \leq \ell_M\|e_x\|^2.$$

The total derivative of V_2 along the trajectories of (18) satisfies

$$\dot{V}_2(t, e_x) \leq -e_x^\top \left[(k_{px} - \gamma_2^2\lambda_1 - \frac{k_M}{2}\omega^*)I - \frac{\lambda_3}{2}e_x^\top K(q(t))e_x K(q(t)) \right] e_x + \left[\frac{1}{\lambda_1} + \frac{1}{2\lambda_3} \right] e_\omega^2$$

for which we used $\frac{d}{dt}L(q(t)) = K(q(t))[e_\omega + \omega^*]$, $e_x^\top K(q)e_x e_\omega \leq (1/\lambda_3)e_\omega^2 + \lambda_3[e_x^\top K(q)e_x]^2$ and $e_x^\top g e_\omega \leq \lambda_1\gamma_2^2\|e_x\|^2 + (1/\lambda_1)e_\omega^2$. Thus, the total derivative of the Lyapunov function $V_3 := V_1 + V_2$ along the closed-loop trajectories satisfies

$$\begin{aligned} \dot{V}_3 \leq & - \left[(J_m/\eta_M)\varepsilon(b-1) - \frac{1}{\lambda_1} - \frac{1}{2\lambda_3} \right] e_\omega^2 - \left[q_m - \frac{\lambda_2 k_M \gamma_1}{2} \right] \|\xi\|^2 \\ & - \left[k_{px} - \lambda_1\gamma_2^2 - \frac{k_M}{2}(\omega^* + \lambda_3 k_M\|e_x\|^2 + \frac{\gamma_1}{\lambda_2}\|x+x^*\|^2) \right] \|e_x\|^2 \quad \text{a.e.} \end{aligned}$$

where we used $|e_x^\top(x+x^*)|\|\xi\| \leq (1/\lambda_2)\|x+x^*\|^2 + \lambda_2\|\xi\|^2$. Therefore, given q_m and b , for any J, η and ε , there exist positive numbers $\lambda_1, \lambda_2, \lambda_3$ and c such that, defining $k'_{px} > 0$ and

$$k''_{px} := \frac{k_M}{2}(\omega^* + \lambda_3 k_M\|e_x\|^2 + \frac{\gamma_1}{\lambda_2}\|x+x^*\|^2) + \lambda_1\gamma_2(\|y\|, \|x\|)^2,$$

V_3 satisfies

$$\dot{V}_3(t, \xi, e_x) \leq -c \left[\|\xi\|^2 + \|e_x\|^2 \right] \quad \text{a.e.}$$

Note that k''_{px} is a non-decreasing function of $\|y\|$, $\|x\|$ and $\|x^*\|$ since $\|x^*\|^2 = \mathcal{O}(\tau_d)$ where τ_d is linear in y . The result follows. \blacksquare

4.2 Control under full parametric uncertainty

Now we assume that in addition to J and τ_L , the physical parameters ℓ_0 , ℓ_1 and R are also unknown. Let $C(q) := \text{diag}\{c_j(q)\}$, $S(q) := \text{diag}\{s_j(q)\}$ then, $L(q) = \ell_0 I - \ell_1 C(q)$ and $K(q) = \ell_1 N_r S(q)$. We introduce the *constant* estimate of ℓ_1 , $\hat{\ell}_{1o} \in [\ell_m, \ell_M]$ to redefine η in (11) as

$$\eta := \frac{\hat{J}\ell_1}{\hat{\ell}_{1o}} \quad (19)$$

so α in (14) and x^* in (11) depend only on known quantities. With this notation, the control law (15) may be written as

$$u = \ell_0 \alpha \rho - \ell_1 C(q) \alpha \rho + \omega^* \ell_1 N_r S(q) x + R x^* - k_{px} e_x$$

which is linear in the physical parameters ℓ_0 , ℓ_1 and R therefore,

$$\begin{aligned} u &= \Psi(t, y, e_x)^\top \Theta - k_{px} e_x, \\ \Psi(t, y, e_x)^\top &:= [\alpha \rho \quad \omega^* N_r S(q) x - C(q) \alpha \rho \quad x^*], \\ \Theta &:= [\ell_0 \quad \ell_1 \quad R]^\top. \end{aligned} \quad (20)$$

We stress that Ψ is a function of t , the closed-loop states e_x and the measured outputs $y = [e_q \vartheta \nu]^\top$; indeed, one should read $q(t)$ and $x = e_x + x^*$ in place of q and x while α , ρ and x^* are functions of t , y and known constants.

Proposition 3 Consider the system (1), (3) in closed loop with the PI^2D controller (4) under the conditions of Proposition 1 and

$$u = \Psi(t, y, e_x)^\top \hat{\Theta} - k_{px} e_x, \quad (21a)$$

$$\dot{\hat{\Theta}} = -k_\theta \Psi(t, y, e_x) e_x, \quad k_\theta > 0. \quad (21b)$$

Let the control gains and $\hat{J} \in [J_m, J_M]$ be such that the matrix $A(\eta)$, with η as in (19), is Hurwitz for any $J \in [J_m, J_M]$. Define $\tilde{\Theta} := \hat{\Theta} - \Theta$. Then, the origin of the closed-loop system, $\{[\xi, e_x, \tilde{\Theta}] = [0, 0, 0]\}$ is uniformly globally stable (i.e., the origin is uniformly stable and the solutions are uniformly globally bounded) and the tracking errors ξ and e_x satisfy

$$\lim_{t \rightarrow \infty} \|\xi(t)\| = 0 \quad \lim_{t \rightarrow \infty} \|e_x(t)\| = 0.$$

Proof. A direct computation using

$$u = \Psi(t, y, e_x)^\top \Theta - k_{px} e_x + \Psi(t, y, e_x)^\top \tilde{\Theta},$$

leads to the closed-loop equations

$$L(q) \dot{e}_x = [R + k_{px}] e_x - g(t, x, y) e_\omega + \Psi(t, y, e_x)^\top \tilde{\Theta} \quad (22a)$$

$$\dot{\tilde{\Theta}} = -k_\theta \Psi(t, y, e_x) e_x. \quad (22b)$$

Next, consider the Lyapunov function $V : \mathbb{R}_{\geq 0} \times \mathbb{R}^4 \times \mathbb{R}^3 \times \mathbb{R}^3 \rightarrow \mathbb{R}_{\geq 0}$ defined by

$$V(t, \xi, e_x, \tilde{\Theta}) := V_3(t, \xi, e_x) + \frac{1}{2k_\theta} \|\tilde{\Theta}\|^2$$

which is positive definite and radially unbounded. The Hurwitz property of A implies by Proposition 2 that the total derivative of V along the closed-loop trajectories generated by (22) satisfies

$$\dot{V}(t, \xi, e_x, \tilde{\Theta}) \leq -c \left[\|\xi\|^2 + \|e_x\|^2 \right] \leq 0 \quad \text{a.e.} \quad (23)$$

Uniform global stability of the origin follows integrating $\dot{V} \leq 0$ along the closed-loop trajectories hence $\xi \in \mathcal{L}_\infty$, $e_x \in \mathcal{L}_\infty$ and, in view of (2), a simple inspection at the closed-loop equations show that $\dot{\xi} \in \mathcal{L}_\infty$ and $\dot{e}_x \in \mathcal{L}_\infty$. Furthermore, the first inequality in (23) implies that $\xi \in \mathcal{L}_2$, $e_x \in \mathcal{L}_2$. The result follows from Barbalat's lemma. \blacksquare

Under an additional condition of persistency of excitation, one may establish the uniform convergence of the parameter estimation errors.

Proposition 4 Consider the system (1), (3) in closed loop with the PI^2D controller (4) under the conditions of Proposition 1 and (21) under the conditions of Proposition 3. Then, the origin of the closed loop system is uniformly globally asymptotically stable if and only if $\Psi_0(t) := \Psi(t, 0, 0)$ is persistently exciting that is if there exist $\mu > 0$ and $T > 0$ such that

$$M(t) := \int_t^{t+T} \Psi_0(s)\Psi_0(s)^\top ds \geq \mu I \quad \forall t \geq 0. \quad (24)$$

Proof. The proof of Proposition 4 follows by applying [11, Theorem 1]. The first condition of the latter is that the origin is uniformly globally stable, this is established in Proposition 3. From the proof of the latter, we have $\dot{V} \leq Y_0(\xi, e_x)$ a.e. with $Y_0(\xi, e_x) = -c[\|\xi\|^2 + \|e_x\|^2] \leq 0$. It is left to find differentiable locally bounded auxiliary functions with the property that their derivatives are negative semi-definite on certain sets. The first auxiliary function must be negative semi-definite on $\{Y_0 = 0\}$ hence, we look for a function $W_1 : \mathbb{R}_{\geq 0} \times \mathbb{R}^4 \times \mathbb{R}^3 \times \mathbb{R}^3 \rightarrow \mathbb{R}$ such that

$$[\xi, e_x] = [0, 0] \implies \dot{W}_1 \leq 0.$$

We propose

$$W_1(t, \xi, e_x, \tilde{\Theta}) = -e_x^\top L(q(t))^\top \Psi_0(t)^\top \tilde{\Theta}$$

whose total derivative along the closed-loop trajectories generated by (22) yields

$$\begin{aligned} \dot{W}_1(t, \xi, e_x, \tilde{\Theta}) &= -e_x^\top (k_{px} + R) \Psi_0(t)^\top \tilde{\Theta} - g(t, x, \xi)^\top \Psi_0(t)^\top \tilde{\Theta} e_\omega - \tilde{\Theta}^\top \Psi(t, y, e_x) \Psi_0(t)^\top \tilde{\Theta} \\ &\quad - e_x^\top \dot{L}(q(t))^\top \Psi_0(t)^\top \tilde{\Theta} - e_x^\top L(q(t))^\top \dot{\Psi}_0(t) \tilde{\Theta} + k_\theta e_x^\top L(q(t))^\top \Psi_0(t)^\top \Psi(t, y, e_x) e_x. \end{aligned}$$

Let $\Omega \subset \mathbb{R}^{10}$ be a compact set; we see that for all $(t, [\xi, e_x, \tilde{\Theta}]) \in \mathbb{R}_{\geq 0} \times \Omega \cap \{Y_0 = 0\}$ we have

$$\begin{aligned} \dot{W}_1(t, \xi, e_x, \tilde{\Theta}) &= Y_1(t, \tilde{\Theta}) \leq 0, \\ Y_1(t, \tilde{\Theta}) &= -\tilde{\Theta}^\top \Psi_0(t) \Psi_0(t)^\top \tilde{\Theta}. \end{aligned}$$

Next, we look for a second auxiliary function W_2 such that its derivative is negative semi-definite on $\mathbb{R}_{\geq 0} \times \Omega \cap \{Y_0 = 0\} \cap \{Y_1 = 0\}$. Let $W_2 : \mathbb{R}_{\geq 0} \times \mathbb{R}^3 \rightarrow \mathbb{R}_{\geq 0}$ be defined by

$$W_2(t, \tilde{\Theta}) = - \int_t^\infty e^{t-\tau} \|\Phi(\tau, \tilde{\Theta})\|^2 d\tau$$

where $\Phi(t, \tilde{\Theta}) = \Psi_0(t)^\top \tilde{\Theta}$. Note that

$$\int_t^\infty e^{t-\tau} \|\Phi(\tau, \tilde{\Theta})\|^2 d\tau \geq \int_t^{t+T} e^{t-\tau} \|\Phi(\tau, \tilde{\Theta})\|^2 d\tau$$

and considering (24),

$$\int_t^{t+T} e^{t-(t+T)} \|\Phi(\tau, \tilde{\Theta})\|^2 d\tau \geq \mu e^{-T} \|\tilde{\Theta}\|^2$$

so we obtain

$$W_2(t, \tilde{\Theta}) \leq -\mu e^{-T} \|\tilde{\Theta}\|^2. \quad (25)$$

On the other hand, the total time derivative of W_2 along the closed-loop trajectories satisfies, after (22b) and (25),

$$\dot{W}_2(t, \tilde{\Theta}) \leq 2k_\theta \int_t^\infty e^{t-\tau} \tilde{\Theta}^\top \Psi_0(\tau) \Psi_0(\tau)^\top d\tau \Psi(t, y, e_x) e_x + \|\Phi(t, \tilde{\Theta})\|^2 - \mu e^{-T} \|\tilde{\Theta}\|^2.$$

Now, we have $\{Y_0 = 0\} \cap \{Y_1 = 0\} = \{[\xi, e_x, \Phi] = [0, 0, 0]\}$ hence, on this set and for all $t \in \mathbb{R}_{\geq 0}$,

$$\dot{W}_2(t, \tilde{\Theta}) \leq -\mu e^{-T} \|\tilde{\Theta}\|^2.$$

The functions \dot{V} , \dot{W}_1 and \dot{W}_2 are bounded from above by continuous functions which vanish simultaneously only at $[\xi, e_x, \Phi, \tilde{\Theta}] = [0, 0, 0, 0]$. The result follows invoking Theorem 1 of [11]. \blacksquare

5 Simulation Results

With aim at evaluating the controller of Proposition 4 we performed some numerical simulations in SIMULINKTM of MATLABTM, using the following parameters, taken from [8], $R = 0.3[\Omega]$, $\ell_0 = 24[\text{mH}]$, $\ell_1 = 19[\text{mH}]$ and $N_r = 25$, while the control gains are set to $a = 1500$, $b = 3200$, $k_p = 1050$, $k_i = 5e^{-4}$, $k_d = 1000$, $k'_{px} = 250$, $k''_{px} = 0$, $\eta = 13e^{-4}$ and the adaptive law gain to $k_\theta = \text{diag}[1e^{-4}, 1e^{-6}, 2.5e^{-5}]$. We stress that for implementation purpose, we use a constant value of k_{px} even though the *sufficient* condition previously established for *global* stability is that this gain depends on the measured states. The reason to fix $k''_{px} = 0$ is to avoid high values in the input voltages which drive the converters into saturation.

The simulation consists in imposing a realistic speed reference namely, the desired motor speed starts off from zero, accelerating at $36[\text{rad/s}^2]$ up to $90[\text{rad/s}]$. This value is kept constant for other 10s until $t = 12.5\text{s}$ when a new constant reference velocity, of $180[\text{rad/s}]$, is given to be reached in 0.5s. The new velocity reference is kept up to $t=17.5\text{s}$ when the motor is rapidly brought to a regime of inverse rotation at $90[\text{rad/s}]$. During the first 5s, the load torque equals to $1[Nm]$ and it is abruptly increased by 50% at $t = 5\text{s}$

The commutation functions $m_j : [0, 2\pi) \rightarrow \mathbb{R}_{\geq 0}$ which are illustrated with the currents in Figure 3, are defined as follows. Let

$$f(x) = \frac{10x^3}{(\pi/N_r)^3} - \frac{15x^4}{(\pi/N_r)^4} + \frac{6x^5}{(\pi/N_r)^5}$$

and $q_1 := \text{mod}(q, 2\pi/N_r)$, $q_2 := \text{mod}(q - 2\pi/3N_r, 2\pi/N_r)$, $q_3 := \text{mod}(q + 2\pi/3N_r, 2\pi/N_r)$ where the operator mod resets q that is, $q_j = \text{mod}(\beta_1, \beta_2)$ takes the initial value $q_j(0) = \beta_1$ and is reset to the latter when $q_j(t) = \beta_2$. Then,

$$m_j^+(q) = \begin{cases} f_j(q_j) & 0 < q_j \leq \frac{\pi}{3N_r} \\ 1 & \frac{\pi}{3N_r} < q_j \leq \frac{2\pi}{3N_r} \\ 1 - f_j(q_j) & \frac{2\pi}{3N_r} < q_j \leq \frac{\pi}{N_r} \\ 0 & \text{otherwise} \end{cases} \quad m_j^-(q) = \begin{cases} f_j(q_j) & \frac{\pi}{N_r} < q_j \leq \frac{4\pi}{3N_r} \\ 1 & \frac{4\pi}{3N_r} < q_j \leq \frac{5\pi}{3N_r} \\ 1 - f_j(q_j) & \frac{5\pi}{3N_r} < q_j \leq \frac{2\pi}{N_r} \\ 0 & \text{otherwise.} \end{cases}$$

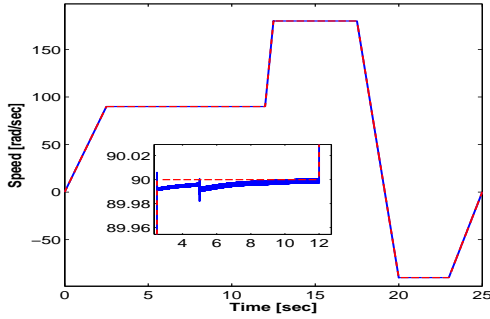


Fig. 2. Velocity Tracking

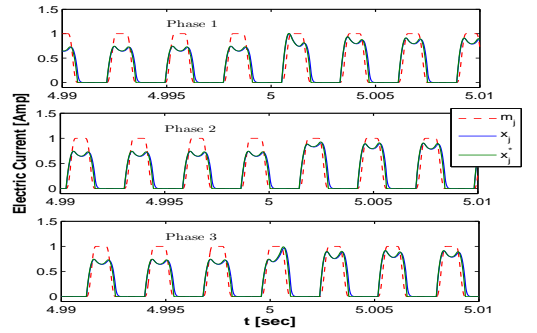


Fig. 3. Electric current around $t = 5\text{s}$ when the load torque changes. The dashed lines correspond to the commutation signals m_j .

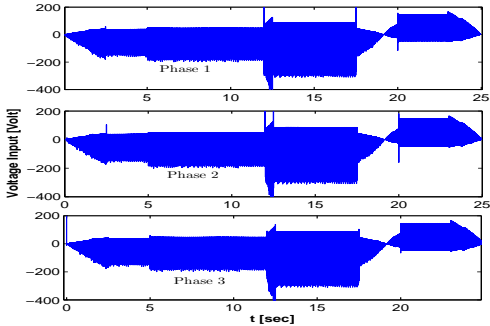


Fig. 4. Voltage control inputs u_j

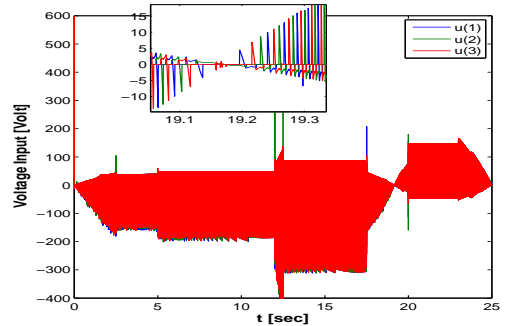


Fig. 5. Voltage control inputs u_j

The velocity response, which is the variable of interest, is depicted in Figure 2; one may appreciate a perfect tracking of the angular velocity and good performance. The tracking performance for the three currents is illustrated in Figure 3. The latter depicts, on a zoomed window, the three stator currents and their references, driven by the commutation signals $m_j(q(t))$; one may appreciate the phase difference of the commutation signals which alternately activate each phase current to inject the virtual control τ_d into the mechanical subsystem. Notice also the effect of the sudden load-torque increase at $t = 5\text{s}$ which results in a change of current reference *i.e.*, the motor requires more electrical current in order to remain at the required velocity set-point –*cf.* zoomed window in Figure 2. Back to Figure 3 one may appreciate that the reference current is tracked by the actual currents without delay, in spite of the step. The input voltages (physical control inputs) are depicted in Figures 4 and 5.

The last two figures illustrate the performance of the adaptation law. In Figure 6 are illustrated the responses of the estimated inductance parameters ℓ_0 , ℓ_1 and the estimated resistance R , which converge to their true values. Finally, Figure 7 depicts the eigenvalues of $M(t)$ in (24) which are clearly positive on the simulation window, this illustrates that the condition of persistency of excitation is satisfied.

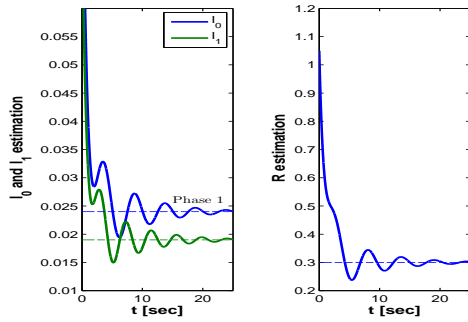


Fig. 6. Estimated parameters converging to the true values

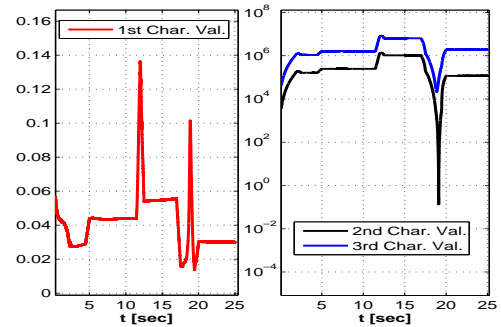


Fig. 7. Verification of the condition of persistency of excitation: eigenvalues of $M(t)$ in (24) on the window of simulation.

6 Concluding Remarks

We presented a robust adaptive controller for the switched-reluctance motor, considering both the stator and rotor dynamics. We employ fairly simple control laws (PID, approximate differentiation), widely used in control practise. Although our results guarantee robustness with respect to bounded disturbances (possibly unmodelled dynamics) it is of utmost importance in motor control, to design controllers with guaranteed performance. In that regard, an interesting avenue of research is to exploit more modern adaptation techniques which palliate side-effects such as sensitivity to noise by introducing the so-called σ -modification. The recent paper [19] presents promising results for model-reference-adaptive control whose applicability (modulo necessary modifications) to motor control, is interesting to analyse. Further ongoing research focuses on the sensorless control problem that is, avoiding the use of position measurements.

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