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Global Adaptive Linear Control of the Permanent-Magnet Synchronous Motor†

Antonio Loria[a], Gerardo Espinosa–Pérez[b*] and Sofía Avila–Becerril[b]

[a] *CNRS, at LSS-Supelec, 3 Rue Joliot Curie, Gif sur Yvette France. E-mail:* loria@lss.supelec.fr
[b] *FI – UNAM, A.P. 70-256, 04510 México D.F., MEXICO.* gerardoe@unam.mx, soavbec@gmail.com

SUMMARY

We contribute with a linear time-varying controller for the permanent magnet synchronous motor. We solve the open problem of speed-tracking control by measuring only stator currents and the rotor angular positions, under parametric uncertainty. Integral action is used to compensate for the effects of the unknown load-torque and adaptation is employed to estimate the unknown parameters. In the case that parameters are known (except for the load) we show that the origin of the closed-loop system is uniformly globally exponentially stable. For the case of unknown parameters we prove uniform global asymptotic stability hence, we establish parametric convergence. In contrast to other adaptive control schemes for electrical machines, we use a reduced-order adaptive controller. Indeed, adaptation is used only for the electrical dynamics equations. Moreover, not surprisingly, the closed-loop system has a structure well-studied in adaptive-control literature. Performance is illustrated in a numerical setting. Copyright © 2012 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Control of electrical machines has been deeply studied both from a practical and from a theoretical perspective for a long time –see [15], [22]. On one hand, field-oriented control is the preferred scheme in industrial applications, due to its structure based on nested proportional-integral loops –[11]. On the other hand, academic contributions which have better captured the attention of practitioners, are those obtained from a passivity-based perspective [17] and those using feedback linearization –[5]. In [7] the authors present an injection-and-damping-assignment controller for the permanent-magnet synchronous motor in Hamiltonian coordinates; the control design is carried out following a procedure designed for general Hamiltonian systems to which integral action is added. In [25] the authors use a feedback linearizing controller and then, design a Luenberger-type observer and apply a certainty-equivalence controller; (local) asymptotic stability is established via Lyapunov's first method. In [19] it has been shown that there exists a downward compatibility between a passivity-based control for induction motors and its corresponding field-oriented control. Some proportional-integral control tuning rules for field-oriented control of induction motors have been proposed in [4] by exploiting its passivity properties, and in [14] a feedback linearization controller was proposed based on the stability properties of field-oriented control of induction motors. The article [24] presents a locally exponentially stabilizing controller without velocity nor

*Correspondence to: FI – UNAM, A.P. 70-256, 04510 México D.F., MEXICO. gerardoe@unam.mx.
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position measurements however, it relies on parametric knowledge (except for the load torque) and internal viscous friction.

Control of the permanent-magnet synchronous motor under parametric uncertainty and partial state feedback has also been addressed via a range of complex nonlinear control approaches – see [23, 3] for sensorless schemes and [2, 8] for adaptive controllers. The authors of [2] solved a problem of adaptive control without velocity measurements and established asymptotic convergence of tracking errors (for induction motors). In [8] an adaptive controller for permanent-magnet synchronous motors is proposed using measurement of currents and positions only. Although in [8] exponential stability is claimed, it relies on a condition of persistency of excitation along the closed-loop trajectories which is (clearly) impossible to verify. Furthermore, as the main proof in [8] relies on tools for *linear* time-varying systems, it is implicitly required that the closed-loop trajectories are bounded. Although this is proved and not assumed, exponential stability can be established only on compact subsets of the state-space that is, it may be established that the origin is globally asymptotically stable and locally exponentially stable. Since the analysis does not establish that stability and convergence are uniform in the initial times robustness cannot be guaranteed[†].

In this note we solve the speed-tracking problem without velocity measurement and under parametric uncertainty, for permanent-magnet synchronous motors via (adaptive) PID control. Our controller is composed of two parts conceived separately: on one hand, a PI^{2}D controller –see [16], for the rotor dynamics and on the other, a linear time-varying "tracking" for the stator current dynamics. The PI^{2}D controller consists in a proportional (to the position error) term, a derivative term in which velocities are replaced by approximate differentiation and integral feedback of the position errors and the approximate derivative. The integral action, and not adaptive control, compensates for the effects of the unknown constant torque-load.

The application of PI^{2}D control for the permanent-magnet synchronous motor relies on the structural properties of the machine –mainly passivity. Indeed, the PI^{2}D controller forms an outer control loop which acts on the rotor (the mechanical part of the machine). This control law enters as a *virtual* input through a reference trajectory purposely designed for one of the stator currents. An inner loop is composed of control laws to drive the stator currents to the appropriate operating point.

We address two cases: first, with known parameters but unknown constant load-torque and second, with all unknown parameters. In the first case we establish *uniform* global exponential stability. In the second we establish *uniform* global asymptotic stability under a persistency of excitation condition on (a function of) the *reference* trajectories and for sufficiently large control gains (independent of the initial conditions). The technical tools to establish our main results are tailored for nonlinear time-varying systems –see [20]; in particular, we establish that the unknown parameters are asymptotically estimated if and only if the aforementioned persistency-of-excitation condition holds.

We emphasize that for non-autonomous systems the uniform stability properties that we establish guarantee local input to state stability hence, our main results supersede others in the literature by guaranteeing robustness to bounded additive disturbances. Furthermore, using converse Lyapunov functions, our main results may be extended to address the sensorless problem via certainty-equivalence output feedback. Last but not least, our controllers (PID control for the mechanical variables and linear time-varying for the electrical part) are comparably much simpler than others in the literature.

The rest of the paper is organized as follows. In the following section we formulate the problem and discuss its solution, under the assumption that the parameters (except load torque) are known; a formal statement is made in Section 3. Then, this result is extended to the case of unknown parameters in Section 4. Numerical simulations are presented in Section 5. We conclude with some remarks in Section 6.

[†]This is a drawback encountered in numerous articles on Model Reference Adaptive Control (MRAC) where classic linear systems theory is inappropriately used to analyze nonlinear systems. The readers are invited to see [20] for further discussions.

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2. PROBLEM FORMULATION AND ITS SOLUTION

2.1. The model

Consider the well–known dq model of the non–salient permanent-magnet synchronous motor – [5, 21],

$$\begin{aligned} L\frac{di}{dt} &= -Ri - \omega\Phi\mathbf{J}\varrho - \omega\mathbf{J}Li + U \\ J\dot{\omega} &= n_p\Phi i_q - \tilde{\tau}_L \\ \dot{\theta} &= \omega \end{aligned}$$

where $i = [i_d,\, i_q]^\top$ and $U = [u_1,\, u_2]^\top$ are the stator currents and voltages respectively, θ and ω are the mechanical (position and speed) variables, $\tilde{\tau}_L$ is the load torque, L is the proper inductance of the stator windings, R corresponds to the stator resistance, Φ is the magnetic field, J is the moment of inertia and n_p is the number of pole pairs and

$$\mathbf{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix};\quad \varrho = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

For the purpose of control design and analysis we introduce the state $x \in \mathbb{R}^4$, $x = [x_1,\, x_2,\, x_3,\, x_4]$ with $[x_1,\, x_2] = i$, $x_3 = \omega$, $x_4 = \theta$ and the parameters

$$\tau_L = \frac{\tilde{\tau}_L}{J}, \qquad \sigma = \frac{n_p\Phi}{J}.$$

Then,

$$\begin{aligned} L\dot{x}_1 &= -Rx_1 + Lx_2x_3 + u_1 && \text{(1a)} \\ L\dot{x}_2 &= -Rx_2 - Lx_1x_3 - \Phi x_3 + u_2 && \text{(1b)} \\ \frac{\dot{x}_3}{\sigma} &= x_2 - \frac{\tau_L}{\sigma} && \text{(1c)} \\ \dot{x}_4 &= x_3 && \text{(1d)} \end{aligned}$$

Given any rotor angular velocity reference $x_3^* : \mathbb{R}_{\geq 0} \to \mathbb{R}$, twice differentiable, bounded and with bounded derivatives (almost everywhere), the control goal is to design a dynamic position-feedback controller for the system (1) with measurable states $[x_1, x_2, x_4]^\top$ and unknown (constant) perturbation τ_L, such that

$$\lim_{t\to\infty}(x_3 - x_3^*) = 0$$

and Lyapunov stability is ensured.

2.2. Ideal state-feedback control

The control design method is reminiscent of backstepping control; it also exploits the structure of the model (1) and the natural properties of the motor, *passivity* in particular. Firstly, we regard x_2 as a virtual control input to Equation (1c) and introduce the PI^{2}D controller for the mechanical dynamics (1c), (1d). The PI^{2}D controller in closed loop with the rotor dynamics defines a passive map. Then, we design a tracking controller $(u_1,\, u_2)$ for the electrical dynamics *i.e.*, Equations (1a) and (1b).

For clarity of exposition we depart from an *ideal* control scheme that could be used if all the parameters were known and the velocities were available from measurement. Although these are restrictive conditions which *we do not assume* to hold in our main statements, by describing such scenario we identify the difficulties that arise in the context of angular-position feedback control and parametric uncertainty.

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Consider Equations (1c), (1d) and let us regard x_2 as a virtual control input. Let

$$x_2^* = \frac{\tau_L}{\sigma} + \frac{\dot{x}_3^*}{\sigma} + v_3 \tag{2a}$$
$$v_3 = -k_p e_4 - k_d e_3, \quad k_p,\ k_d > 0 \tag{2b}$$

where x_3^* is the velocity reference, hence $\dot{x}_4^* = x_3^*$; let the tracking errors be defined as $e_i = x_i - x_i^*$. Using $x_2 = e_2 + x_2^*$ in (1c) and subtracting $\dot{x}_4^* = x_3^*$ from (1d) we obtain

$$\frac{1}{\sigma}\dot{e}_3 = -k_p e_4 - k_d e_3 + e_2 \tag{3a}$$
$$\dot{e}_4 = e_3 \tag{3b}$$

whose origin is globally exponentially stable for any positive k_p and k_d, provided that $e_2 = 0$. Moreover, this system is input to state stable from the input e_2 and defines an output-strictly-passive map $e_2 \mapsto e_3$. In other words, x_2^* stabilizes (1c), (1d) at the equilibrium[‡] $x_4 = x_4^*$ and $x_3 = x_3^*$.

With this in mind, we proceed to design the second part of the controller, in order to steer $e_2 \to 0$ asymptotically. Under ideal conditions (parameter knowledge and state measurement feedback) this is achieved via the control laws

$$u_1^* = Rx_1^* + L\dot{x}_1^* - Lx_2x_3^* - k_1' e_1, \quad k_1' > 0 \tag{4a}$$
$$u_2^* = \Phi x_3^* + Lx_1x_3^* + Rx_2^* + L\dot{x}_2^* - k_2' e_2, \quad k_2' > 0 \tag{4b}$$

with x_2^* as in (2a) and x_1^* being a continuously differentiable function, given as reference for x_1; typically, x_1^* is chosen constant. Under these conditions the closed-loop electrical system is

$$L\dot{e}_1 - Lx_2e_3 + Re_1 + k_1' e_1 = 0 \tag{5a}$$
$$L\dot{e}_2 + Lx_1e_3 + \Phi e_3 + Re_2 + k_2' e_2 = 0. \tag{5b}$$

The interest of these equations relies in their structure; notice that, defining $e_{12} := [e_1\ e_2]^\top$, the previous equations have the form

$$\dot{e}_{12} = Ae_{12} + B(e_3)e_{12} + Ce_3 \tag{6}$$

where the poles of A depend on the control gains k_1' and k_2' hence it may be easily rendered Hurwitz, $B(e_3)$ is a skew-symmetric matrix such that $B(0) = 0$ and C depends on x_1^* and x_2^*. Therefore, using the quadratic Lyapunov function $V = |e_{12}|^2$ one may easily establish global exponential stability for

$$\dot{e}_{12} = Ae_{12} + B(e_3)e_{12}.$$

Furthermore, provided that C in (6) is bounded, exponential stability of $e_{12} = 0$ is conserved if $e_3 \to 0$ exponentially. The latter may be established invoking output-injection arguments –see [1]. In addition to this, input-to-state stability of (6) with input e_3 may be established using Lyapunov theory. Thus, stability of the origin of the overall closed-loop equations (3), (5) follows invoking small-gain arguments. The proof of these claims follows as a corollary of our results, presented in Section 3 below.

Besides the stability properties that it ensures, the controller (2)–(4) is attractive due to its simplicity; note that the right-hand side of (2) corresponds to a simple Proportional Derivative controller with load compensation and a feedforward term, while the control laws (4) are linear time-varying and ensure tracking control for the stator current dynamics with reference x_2^*. However, simplicity comes at the price of conservatism; on one hand, for the mechanical part (2) the controller requires the exact value of load-torque as well as velocity measurements. On the other hand, to compute u_2^* in (4a) one requires the time derivative of x_2^* in (2a). In the next section we relax these conservative assumptions and present a modification of the state-feedback controller previously discussed. The controller is of PID type and the velocities are replaced with approximate derivatives.

[‡]Note that the definition of x_4^* is of little importance in regards to the control objective.

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2.3. *Position–feedback control with unknown load–torque*

Inspired by the ideal structure (2) we redefine the virtual rotor controller as

$$x_2^* := \nu + \frac{\dot{x}_3^*}{\sigma} + v_3 \tag{7a}$$

$$v_3 = -k_p e_4 - k_d \vartheta, \quad k_p,\, k_d > 0. \tag{7b}$$

As before, v_3 is of Proportional "Derivative" type except for the fact that the unavailable velocity x_3 is replaced by the variable ϑ which is the output of the approximate-differentiation filter,

$$\dot{q}_c = -a(q_c + be_4) \tag{8a}$$

$$\vartheta = q_c + be_4, \quad a,\, b > 0 \tag{8b}$$

whose output ϑ satisfies

$$\dot{\vartheta} = -a\vartheta + be_3\,. \tag{9}$$

The effect of the load-torque, that is the constant perturbation τ_L/σ in (1c), is compensated for by integral action that is, ν in (7a) is defined as

$$\dot{\nu} = -k_i(e_4 - \vartheta) \tag{10}$$

where k_i is a constant positive gain. Thus, the uncertainty in the load-torque can be coped with by modifying the current reference x_2^*.

With the redefinition of the reference x_2^* the stator control laws (4) would remain unchanged if not because the new definition of x_2^*, in (7a), implies that

$$\dot{x}_2^* = [\dot{\tilde{\nu}} + \frac{\ddot{x}_3^*}{\sigma} + ak_d\vartheta] - (k_p + bk_d)e_3 \tag{11}$$

where (recalling that the load-torque is assumed constant) we introduced $\tilde{\nu} = \nu - \tau_L/\sigma$. From here, it is clear that $\dot{x}_2^*$ may not be part of u_2^* since it depends on the unmeasurable velocity errors e_3. Therefore, in the control implementation we shall only use the first three terms in brackets on the right-hand-side of (11), that is, let

$$u_2^* = \Phi x_3^* + Lx_1x_3^* + Rx_2^* + v_2 + \rho - k_2' e_2 \tag{12a}$$

$$\rho = L[\dot{\tilde{\nu}} + \frac{\ddot{x}_3^*}{\sigma} + ak_d\vartheta] \tag{12b}$$

$$v_2 = -\varepsilon(e_4 - \vartheta)\,. \tag{12c}$$

The role of v_2 shall become clear from the proof; it corresponds to a Lyapunov-redesign term –see [10] which is added to enhance negative semi-definiteness of the time derivative of a Lyapunov function candidate –see next section.

Remark 2.1

It is important to stress that contrary to other works in the literature, we do not use adaptive control to estimate the constant load-torque. The controller relies on the much simpler approach, commonly used in control practice, of compensating the perturbation via integral action. The fact that velocities are replaced by dirty derivatives, requires that the Proportional Integral controller involves integration of angular positions e_4 and filter outputs ϑ. For simplicity, we use a unique gain for both integrators. See [18] for an analysis of the PI^{2}D controller from a passivity viewpoint.

3. PI^{2}D CONTROL WITH KNOWN PARAMETERS

Proposition 3.1

Consider the system (1) in closed loop with $u_1 = u_1^*$, $u_2 = u_2^*$, given by Equations (4a), (12), (7), (8) and (10). The origin of the closed-loop system with state $\varsigma := [e_1\, e_2\, e_3\, e_4\, \vartheta\, \tilde{\nu}]^\top$, is uniformly globally exponentially stable, for sufficiently large gains. □

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The proof is established in three essential steps. Firstly, we derive the closed-loop equations then, we introduce a Lyapunov function and finally, we evaluate uniform integrability conditions of the trajectories, which lead to the conclusion of exponential stability. More precisely, according to [20, Lemma 3] uniform global exponential stability is equivalent to the existence of constants $c_1,\ c_2 > 0$, $p \in [1, \infty)$ such that for all $t_\circ \in \mathbb{R}_{\geq 0}$, $x_\circ \in \mathbb{R}^n$, all solutions $x(\cdot, t_\circ, x_\circ)$ satisfy the

$$\text{uniform } \mathcal{L}_\infty \text{ bound:} \qquad \sup_{t\geq t_\circ} |x(t)| \leq c_1|x_\circ|; \tag{13a}$$

$$\text{uniform } \mathcal{L}_p \text{ bound:} \qquad \left(\lim_{t\to\infty} \int_{t_\circ}^{t} |x(t)|^p\right)^{1/p} \leq c_2|x_\circ|. \tag{13b}$$

Note that the previous conditions are reminiscent of boundedness and p-integrability, commonly used in adaptive control to establish (using Barbalăt's lemma) *convergence* to zero. The bounds (13) are more conservative since they are uniform in the initial times and have linear growth in the normalized initial states however, they imply *uniform global exponential stability* which clearly is a much stronger property than convergence. The essence of the proof consists in verifying the Inequalities (13) for $\varsigma(t)$.

3.1. The closed-loop equations

From (11) we see that $\rho = L\,[\dot{x}_2^* + (k_p + bk_d)e_3]$. Using this in (12a) and u_2^* (as defined in the latter) in (1b) we obtain

$$\begin{aligned} L\dot{e}_1 &= -k_1 e_1 + Lx_2 e_3; \quad k_1 = k_1' + R && (14)\\ L\dot{e}_2 &= -k_2 e_2 + \Delta e_3 + v_2; \quad k_2 = k_2' + R && (15)\\ \Delta &= L(k_p + bk_d) - (\Phi + Lx_1). && (16) \end{aligned}$$

The introduction of Δ is motivated by the possibility of writing the equations as a nominal e_{12}–dynamics perturbed by the 'input' e_3, as done in Section 2.2.

On the other hand, the dynamics equation for the rotor speed can be written as

$$\frac{1}{\sigma}\left(\dot{x}_3 - \dot{x}_3^*\right) = x_2 - \frac{\tau_L}{\sigma} - \frac{\dot{x}_3^*}{\sigma} + x_2^* - x_2^*$$

which is equivalent to

$$\frac{1}{\sigma}\dot{e}_3 = e_2 + \tilde{\nu} - k_p e_4 - k_d\vartheta\,. \tag{17}$$

Furthermore, we define

$$\begin{aligned} z &= \tilde{\nu} - \frac{k_i}{\varepsilon}e_4, \quad 0 < k_i < \varepsilon \ll 1 && (18)\\ k_p' &= k_p - \frac{k_i}{\varepsilon} && (19) \end{aligned}$$

so that

$$\begin{aligned} \frac{1}{\sigma}\dot{e}_3 &= e_2 - k_p' e_4 - k_d\vartheta + z && (20)\\ \dot{z} &= -k_i(e_4 - \vartheta) - \frac{k_i}{\varepsilon}e_3. && (21) \end{aligned}$$

Let $x = [e_1,\ e_2,\ e_3,\ e_4,\ \vartheta,\ z]^\top$; note that stability of $\{x = 0\}$ is equivalent to that of $\{\varsigma = 0\}$. Thus, in the sequel we seek to establish (13) for the trajectories generated by Equations (14), (15), (20), (21) and $\dot{e}_4 = e_3$.

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3.2. *Stability analysis*

Consider the following functions which are proposed after [16],

$$V_1 = \frac{L}{2}(e_1^2 + e_2^2) \tag{22a}$$

$$V_2 = \frac{1}{2}\left(\frac{e_3^2}{\sigma} + k_p' e_4^2 + \frac{k_d}{b}\vartheta^2 + \frac{\varepsilon}{k_i} z^2\right) \tag{22b}$$

$$V_3 = \frac{1}{\sigma}\varepsilon e_3(e_4 - \vartheta), \qquad \varepsilon \ll 1 \tag{22c}$$

$$V := V_1 + V_2 + V_3\,. \tag{22d}$$

We establish that V is positive definite and its derivative is negative semidefinite.

3.2.1. Positivity of V. Using the triangle inequality we see that

$$-\frac{1}{2\sigma}\varepsilon\left(e_3^2 + e_4^2 + \vartheta^2\right) \le V_3 \le \frac{1}{2\sigma}\varepsilon\left(e_3^2 + e_4^2 + \vartheta^2\right) \tag{23}$$

therefore, given any control gains, there always exists a constant $1 \gg \varepsilon > 0$ and for such ε, there exist positive reals α_1, α_2 such that the function V satisfies

$$\alpha_1 |x|^2 \le V(x) \le \alpha_2 |x|^2 \quad \forall\, x \in \mathbb{R}^6. \tag{24}$$

3.2.2. Negativity of $\dot{V}$. The total time derivative of V_1 along the trajectories of (14), (15) yields

$$\dot{V}_1 = -k_1 e_1^2 + L x_2 e_1 e_3 - k_2 e_2^2 + \Delta e_2 e_3 + v_2 e_2; \tag{25}$$

the derivative of V_2 is given by

$$\dot{V}_2 = e_2 e_3 - \frac{k_d a}{b}\vartheta^2 - \varepsilon z(e_4 - \vartheta) \tag{26}$$

where we used $\dot{e}_4 = e_3$. Finally, the derivative of V_3 satisfies

$$\begin{aligned}\dot{V}_3 = {} & \frac{k_d a}{2b}\vartheta^2 - \frac{\varepsilon b'}{2\sigma}e_3^2 - \frac{\varepsilon k_p'}{2}e_4^2 + \varepsilon(e_4 - \vartheta)(z + e_2) \\ & -\frac{1}{2}\begin{bmatrix} e_3 \\ e_4 \\ \vartheta \end{bmatrix}^\top \begin{bmatrix} \frac{\varepsilon b'}{\sigma} & 0 & -\frac{\varepsilon a}{\sigma} \\ 0 & \varepsilon k_p' & \varepsilon(k_p' - k_d) \\ -\frac{\varepsilon a}{\sigma} & \varepsilon(k_p' - k_d) & k_d\left(\frac{a}{b} - 2\varepsilon\right) \end{bmatrix} \begin{bmatrix} e_3 \\ e_4 \\ \vartheta \end{bmatrix}\end{aligned}$$

where we used $b' := b - 1$. The matrix above is positive semidefinite if

$$\frac{k_d a}{b} \ge \varepsilon\left[2k_d + \frac{a^2}{\sigma b'} + \frac{(k_p' - k_d)^2}{k_p'}\right] \tag{27}$$

which holds for sufficiently small values of ε. Note, from (18), that this restricts the choice of k_i but not of the other control gains. Next, we use (12c) to see that, under the condition (27), the total time derivative of V satisfies

$$\begin{aligned}\dot{V} \le {} & -\frac{1}{2}\left[k_1 e_1^2 + k_2 e_2^2 + \frac{k_d a}{b}\vartheta^2 + \frac{\varepsilon b'}{2\sigma}e_3^2 + \varepsilon k_p' e_4^2\right] \\ & -\frac{1}{2}\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}^\top \begin{bmatrix} k_1 & 0 & L x_2 \\ 0 & k_2 & (\Delta + 1) \\ L x_2 & (\Delta + 1) & \frac{\varepsilon b'}{\sigma} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}.\end{aligned} \tag{28}$$

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The matrix in the expression above is positive semidefinite for positive values of all the control gains and if $k_1k_2\varepsilon b' \geq 2\sigma(\Delta+1)^2k_1 + 2\sigma L^2k_2x_2^2$, which holds if

$$k_2\varepsilon b' \geq 4\sigma(\Delta+1)^2 \tag{29a}$$
$$k_1\varepsilon b' \geq 4\sigma L^2x_2^2. \tag{29b}$$

Note that the expressions above impose that k_1 and k_2 depend on measurable states hence, these may be chosen as strictly positive functions of x_2 and x_1 respectively.

We conclude that there exists a constant $\alpha_3 > 0$ such that, defining $y^\top := [e_1,\, e_2,\, e_3,\, e_4,\, \vartheta]$,

$$\dot{V}(x) \leq -\alpha_3|y|^2 \leq 0 \quad \forall\, x \in \mathbb{R}^6. \tag{30}$$

That is, V is negative semidefinite and the origin of the system, $\{x = 0\}$, is uniformly globally stable *i.e.*, the solutions are uniformly globally bounded and the origin is Lyapunov stable –see [9]. It is left to show uniform global exponential convergence of the error trajectories.

Remark 3.1
Note that $\dot{V}$ is negative semidefinite. One may not invoke Lasalle's principle because the system is non-autonomous. Now, (30) implies that $y \in \mathcal{L}_2$ and all signals are bounded. From this, it follows from Barbalăt's lemma that $y(t) \to 0$. However, on one hand, this argumentation does not lead to the convergence of z hence of the integrator variable $\tilde{\nu}$. On the other, from Babrbalăt's lemma, the convergence may not be guaranteed to be uniform in the initial conditions hence our purpose to verify the uniform bounds in (13).

3.2.3. Uniform global exponential stability. To complete the proof we show that the conditions (13) hold. From (30) we see that

$$\dot{V}(x(t)) \leq -\alpha_3|y(t,t_\circ,x_\circ)|^2 \leq 0 \quad \forall\, t_\circ \in \mathbb{R}_{\geq 0},\ x_\circ \in \mathbb{R}^6$$

which is equivalent to

$$V(x(t)) - V(x(t_\circ)) \leq -\alpha_3 \int_{t_\circ}^{t} |y(s,t_\circ,x_\circ)|^2 ds \quad x(t_\circ) = x_\circ$$

hence, in view of the positivity and boundedness of V –see (24), we have

$$\alpha_1|x(t)|^2 \leq V(x(t)) \leq V(x(t_\circ)) \leq \alpha_2|x_\circ|^2. \tag{31}$$

It follows that for all $t \geq t_\circ \geq 0$ and all $x_\circ \in \mathbb{R}^6$

$$\int_{t_\circ}^{t} |y(s,t_\circ,x_\circ)|^2 ds \leq \frac{\alpha_2}{\alpha_3}|x_\circ|^2\,; \tag{32}$$

$$|x(t)| \leq c_1|x_\circ| \qquad c_1 := \sqrt{\frac{\alpha_2}{\alpha_1}} \tag{33}$$

so (13a) holds. It is left to find a uniform $\mathcal{L}_2$ bound on $z(t)$. For this, consider the function

$$V_4 = \frac{1}{\sigma}e_3 z\,. \tag{34}$$

Its total time derivative along the closed-loop trajectories yields

$$\dot{V}_4 = -z^2 - z(e_2 - k_d\vartheta - k_p'e_4) - \frac{1}{\sigma}e_3(-k_i(e_4 - \vartheta) - \frac{k_i}{\varepsilon}e_3) \tag{35}$$

which after the triangle inequality, satisfies

$$\dot{V}_4 \leq -z^2 + \frac{1}{2}\left[z^2 + \delta|y|^2\right] \tag{36}$$

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for an appropriate but innocuous choice of δ. Integrating on both sides of (36) along the trajectories, from $t_\circ$ to t we obtain

$$2e_3(t)z(t) - 2e_3(t_\circ)z(t_\circ) \leq -\sigma \int_{t_\circ}^{t} z(s)^2 ds + \sigma \int_{t_\circ}^{t} \delta |y(s)|^2 ds\,. \tag{37}$$

Now, even though they are of indefinite sign, the terms on the left hand side of the inequality are bounded by $2|x_\circ|^2$ hence, using (32) we obtain

$$\int_{t_\circ}^{t} z(s)^2 ds \leq \left[\frac{\delta\alpha_2}{\alpha_3} + 2\sigma\right] |x_\circ|^2 \quad \forall\, t \geq t_\circ \geq 0. \tag{38}$$

Uniform global exponential stability follows from (32), (33) and (38) since the conditions (13) are satisfied.

Remark 3.2
It may be showed that the function $V + \varepsilon_2 V_4$ is a strict Lyapunov function for the closed-loop system with a negative definite derivative (for $\varepsilon_2 \ll 1$). However, this may be established under further restrictions on the control gains hence, it is beyond interest in this paper.

Remark 3.3 (Robustness)
The property of uniform global exponential stability cannot be overestimated. It implies (local) Input to State Stability that is, under the presence of relatively small additive disturbances, one recovers asymptotic stability of a residual compact set whose "size" depends on the disturbances' magnitudes. In particular, consider the case of slowly-varying load torques; say $\tau_L = \tau_L^* + \Delta\tau_L(t)$ where τ_L^* is constant and $\Delta\tau_L$ is bounded with continuous and bounded first derivative. Then, a simple inspection shows that the closed-loop system takes the form

$$\dot{x} = F(t, x) + d(t) \tag{39}$$

where $\dot{x} = F(t, x)$ corresponds to the closed-loop dynamics (14), (15), (20), (21) and (9), and the disturbance $d(t)$ consists in terms $\Delta\tau_L(t)$ and $\dot{\Delta}\tau_L(t)$ which enter in equations (15), (20) and (21). From Proposition 3.1 and converse Lyapunov theory we deduce that (39) is input to state stable with input d.

4. CONTROL UNDER PARAMETRIC UNCERTAINTY

Now we relax the assumption that R, L, Φ and $\sigma(J, n_p)$ are known and we introduce an adaptive control law to estimate online, R, L and Φ. We establish a necessary and sufficient condition for these parameters to be estimated. In addition, we show that the controller is robust with respect to the uncertainty on σ. In the case that the reference acceleration $\dot{x}_3^*$ is piece-wise constant, we establish uniform global asymptotic stability for the overall closed-loop system. The proof follows from the analysis in Section 3 and using familiar arguments based on persistency of excitation, to conclude parametric convergence. However, we stress the importance of using technical tools tailored for *nonlinear* systems –see [20], as opposed to classical adaptive control systems theory for linear systems.

Let $\theta_1 = L$, $\theta_2 = R$, $\theta_3 = \Phi$, $\theta_4 = 1/\sigma$ and[§] $\theta = [\theta_1\ \theta_2\ \theta_3]^\top$. We denote by $\hat{\theta}$ the estimate of θ and the estimation errors by $\tilde{\theta} = \hat{\theta} - \theta$. Then, Equations (1) become

$$\theta_1 \dot{x}_1 = -\theta_2 x_1 + \theta_1 x_2 x_3 + u_1 \tag{40a}$$
$$\theta_1 \dot{x}_2 = -\theta_2 x_2 - \theta_1 x_1 x_3 - \theta_3 x_3 + u_2 \tag{40b}$$
$$\theta_4 \dot{x}_3 = x_2 - \frac{\tau_L}{\sigma} \tag{40c}$$
$$\dot{x}_4 = x_3 \tag{40d}$$

[§] θ_4 is not estimated online but is dealt with separately, hence the definition of θ and the explicit appearance of σ.

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and we introduce the certainty-equivalence adaptive controller

$$\begin{aligned} u_1 &= -\hat{\theta}_1 x_2 x_3^* + \hat{\theta}_2 x_1 + \hat{\theta}_1 \dot{x}_1^* - k_1 e_1 && \text{(41a)} \\ u_2 &= \hat{\theta}_3 x_3^* + \hat{\theta}_1 x_1 x_3^* + \hat{\theta}_2 x_2 - k_2 \hat{e}_2 + \hat{\theta}_1 \alpha + v_2 && \text{(41b)} \\ \dot{\hat{\theta}} &:= -\gamma \Psi e_{12}, \quad \gamma > 0 && \text{(41c)} \end{aligned}$$

where $\hat{e}_2 := x_2 - \hat{x}_2^*$, $e_{12} := [e_1,\ \hat{e}_2]^\top$,

$$\begin{aligned} \dot{\hat{x}}_2^* &:= \hat{\theta}_4 \dot{x}_3^* + \nu + v_3, \quad \dot{\hat{\theta}}_4 = 0 && \text{(42a)} \\ \alpha &:= -k_i(e_4 - \vartheta) + k_d a \vartheta && \text{(42b)} \\ \Psi^\top &:= \begin{bmatrix} \dot{x}_1^* - x_2 x_3^* & x_1 & 0 \\ \alpha + x_1 x_3^* & x_2 & x_3^* \end{bmatrix}. && \text{(42c)} \end{aligned}$$

Note that $\dot{\hat{x}}_2^* = \dot{\nu} + \dot{v}_3 = \alpha - (k_p + k_d b) e_3$ since $\dot{\hat{\theta}}_4 = 0$ and $\dot{x}_3^*$ is (piece-wise) constant.

Furthermore, we remark that in contrast to other adaptive control schemes for electrical machines, we use a reduced-order adaptive controller. Indeed, adaptation is used only for the electrical dynamics equations. To obtain the closed-loop equations corresponding to the electrical dynamics, let $\varphi = [L x_2 e_3 \ \ \Delta e_3 + v_2]^\top$ and $K_{12} :=$diag$[k_1\, k_2]$ where the latter are functions of the state, according to (29) and (16). Then, using (41) and (42) we obtain

$$\begin{bmatrix} L\dot{e}_{12} \\ \dot{\tilde{\theta}} \end{bmatrix} = \begin{bmatrix} -K_{12}(t,\xi_1) & \Psi(t,\xi_1)^\top \\ -\gamma \Psi(t,\xi_1) & 0 \end{bmatrix} \begin{bmatrix} e_{12} \\ \tilde{\theta} \end{bmatrix} + \begin{bmatrix} \varphi \\ 0 \end{bmatrix}. \tag{43}$$

Not surprisingly, for $\varphi = 0$, the previous system (43) has the familiar structure of model-reference-adaptive control systems. The analysis of Equation (43) with $\varphi = 0$ is standard routine under the following observations: if $\tilde{\theta} = 0$ the system reduces to that studied in the previous section otherwise, Barbalăt's lemma and standard signal-chasing arguments lead to the conclusion that $e_{12} \to 0$. Moreover, persistency of excitation of Ψ may be invoked to conclude that $\tilde{\theta} \to 0$. See for instance [2, 8]. Finally, a converse Lyapunov function for the system with $\varphi = 0$ might be used to analyze the perturbed dynamics (43).

The simplicity of such argumentation hides several technical fallacies which lead to wrong conclusions. Firstly, for notational simplicity Ψ is written in (42c) without arguments however, we emphasize that $\Psi : \mathbb{R}_{\geq 0} \times \mathbb{R}^6 \to \mathbb{R}^{3\times 2}$ *i.e.*, it is a function of time through the functions $t \mapsto x_1^*$, $t \mapsto x_3^*$ and their derivatives, as well as of the state $\xi_1 = [e_1\ \hat{e}_2\ e_3\ e_4\ \vartheta\ z]^\top$. More precisely, in place of x_2 one must read $\hat{e}_2 + \hat{x}_2^*$ where $\hat{x}_2^*$ is a function of the closed-loop state ξ_1. Correspondingly, one must read $e_1 + x_1^*(t)$ in place of x_1, *etc*. Therefore, invoking standard results as for *linear* systems –such as those in [1], requires to impose persistency of excitation on $\Psi(t, \xi_1(t))$ that is, along closed-loop trajectories –*cf.*[23, 2, 6]. Not only this is un-necessary but clearly impossible to verify for all t. Moreover, such reasoning may only lead to non-uniform attractivity which in turn, invalidates the invocation of converse Lyapunov theorems. To overcome these difficulties, we shall use the tools reported in [20], tailored for nonlinear-time-varying systems.

At this point we introduce the dynamics corresponding to the rotor variables; this may be computed as follows. We replace $x_2 = \hat{e}_2 + \hat{x}_2^*$ in (1c) and use (42a) to obtain

$$\theta_4 \dot{x}_3 = \hat{e}_2 + \hat{\theta}_4 \dot{x}_3^* + \nu + v_3 - \frac{\tau_L}{\sigma} \pm \theta_4 \dot{x}_3^*$$

and we redefine $\nu^* := \tau_L/\sigma - (\hat{\theta}_4 - \theta_4)\dot{x}_3^*$ where $\dot{x}_3^*$ is (piece-wise) constant hence, so is ν^*. Therefore,

$$\theta_4 \dot{e}_3 = \hat{e}_2 + \tilde{\nu} + v_3$$

which is exactly of the form (17) and therefore, leads to (20) with the only difference laying in the redefinition of ν^* hence, of z. In other words, the additional uncertainty $\tilde{\theta}_4 = \hat{\theta}_4 - \theta_4$ only changes

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the equilibrium of the z-dynamics part of the closed-loop system. The latter is

$$\begin{bmatrix} \theta_4 \dot{e}_3 \\ \dot{e}_4 \\ \dot{\vartheta} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} 0 & -k_p' & -k_d & 1 \\ 1 & 0 & 0 & 0 \\ b & 0 & -a & 0 \\ -k_i/\varepsilon & -k_i & k_i & 0 \end{bmatrix} \begin{bmatrix} e_3 \\ e_4 \\ \vartheta \\ z \end{bmatrix} + \begin{bmatrix} \hat{e}_2 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{44}$$

in which it is clear that the control gains may be chosen appropriately to ensure that the matrix above is Hurwitz.

The system (44) corresponds to the tracking control dynamics for the mechanical variables, under the action of the virtual control $\hat{x}_2^*$. The absence of a term involving parametric uncertainty in (44) (as it might be expected) comes on one hand from the fact that we chose to maintain $\hat{\theta}_4$ constant and $\dot{x}_3^*$ is assumed piece-wise constant and on the other hand, because ν^* is compensated for by integral action. Note that by replacing $\hat{e}_2$ with e_2, system (44) also corresponds to the closed-loop equations (20), (21) and (9) hence, in the case that $\hat{e}_2 = 0$, the origin is exponentially stable for suitable values of the control gains; also, the dynamics is input to state stable with input $\hat{e}_2$.

Thus, the overall closed-loop system (43), (44) may be regarded as the feedback interconnection of two input-to-state stable systems and uniform global asymptotic stability of the origin of (43), (44) may be inferred invoking a small-gain argument. This simple rationale hides the difficulty that φ in (43) depends on all of ξ_1 and not only on the "mechanical" variables; in turn, this imposes the challenge of actually constructing a strict Lyapunov function for the overall system –see [12, 13]. Instead, in order to show that the origin of the closed-loop system is uniformly globally asymptotically stable, we follow a direct but rigorous method of proof which relies on [20, Proposition 3]. The latter is an output-injection statement for nonlinear systems, analogous to the well-known output-injection lemma for linear adaptive systems –*cf.* [1]. In words, the output-injection lemma in [1] establishes that uniform complete observability is invariant under an output injection provided that the output is in $\mathcal{L}_2$. Its nonlinear "counterpart", [20, Proposition 3], establishes that uniform global asymptotic stability is invariant under (a uniformly $\mathcal{L}_2$) output injection.

Proposition 4.1
Let $t \mapsto x_1^*$ and $t \mapsto x_3^*$ be given bounded reference trajectories. Let x_1^* be twice continuously differentiable with bounded derivatives and let x_3^* be piece-wise continuous with $\dot{x}_3^*$ piece-wise constant. Consider the system (40) in closed loop with the adaptive controller (41), (42). Let σ_m, σ_M, L_M and Φ_M be known constants such that $\sigma \in [\sigma_m, \sigma_M]$, $L_M \geq L$ and $\Phi_M \geq \Phi$. Then, all tracking errors e_1, e_2, e_3, e_4, $\tilde{\nu}$ and ϑ converge to zero asymptotically provided the control gains (dependent on σ_m, σ_M, L_M and Φ_M) are sufficiently large. Furthermore, define $\tilde{\Psi} : \mathbb{R}_{\geq 0} \times \mathbb{R} \to \mathbb{R}^{3\times 2}$ as

$$\tilde{\Psi}(t,\delta)^\top = \begin{bmatrix} \dot{x}_1^*(t) - \delta x_3^*(t) & x_1^*(t) & 0 \\ x_1^*(t) x_3^*(t) & \delta & x_3^*(t) \end{bmatrix}$$

then, the origin of the closed loop system is uniformly globally asymptotically stable if *and only if* there exist $\mu > 0$ and $T > 0$ such that

$$M(t) := \int_t^{t+T} \tilde{\Psi}(s,1)\tilde{\Psi}(s,1)^\top ds \geq \mu \mathcal{I} \quad \forall\, t \geq 0. \tag{45}$$

Proof of Proposition 4.1: To apply [20, Proposition 3] we start by establishing uniform global stability hence, uniform boundedness of all trajectories $t \mapsto \xi$ where $\xi = [\xi_1^\top \ \tilde{\theta}^\top]^\top$. Following the developments of Section 3 we see that the total derivative of

$$\mathcal{V}(\xi) = V(\xi_1) + \frac{1}{2\gamma}|\tilde{\theta}|^2$$

along the trajectories of (43), (44) yields $\dot{\mathcal{V}}(\xi) = \dot{V}(\xi_1)$ where $\dot{V}(\xi_1)$ is upper-bounded by the right-hand side of (28) with e_2 replaced by $\hat{e}_2$. Define

$$\Delta_M = L_M(k_p + bk_d) + (\Phi_M + L_M|x_1|),$$

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and let (29) and (27) hold with Δ_M in place of Δ, with σ_m in (27), σ_M in (29) instead of σ and with L_M in place of L. Then, similarly to (30) we have

$$\dot{V}(\xi_1) \leq -\alpha_3|\hat{y}|^2 \leq 0, \quad \hat{y}^\top := [e_1\ \hat{e}_2\ e_3\ e_4\ \vartheta] \tag{46}$$

hence, uniform global stability of the origin as well as uniform square integrability of e_1, $\hat{e}_2$, e_3, e_4 and ϑ follow. Invoking standard signal-chasing arguments which involve Barbalăt's lemma, one obtains the convergence of $|\hat{y}(t)|$ to zero.

To establish uniform global asymptotic stability of the origin note that the analysis in Paragraph 3.2.3) holds, *mutatis mutandis* for the trajectories of (43)–(44) hence, from (32) and (46), there exists $c > 0$ such that

$$\int_{t_\circ}^{t} |\xi_1(s)|^2 ds \leq c|\xi_\circ|^2 \quad \forall\, t \geq t_\circ \geq 0. \tag{47}$$

Next, note that the inequality in (45) holds if and only if there exist T', $\mu' > 0$ such that

$$\int_{t}^{t+T} \tilde{\Psi}(s,\delta)\tilde{\Psi}(s,\delta)^\top ds \geq \mu\mathcal{I} \quad \forall\, t \geq 0, \forall \delta \in \mathbb{R}; \tag{48}$$

this follows from the fact that the choice of δ does not modify the rank of $M(t)$. In what follows we set $\delta = \hat{x}_2^*(0)$ and observe that $\tilde{\Psi}(t, \hat{x}_2^*(0)) = \Psi(t,0)$. Moreover, K_{12} is a diagonal matrix bounded from below, say by the constant diagonal matrix $K_{12}^m > 0$ then, let $K'_{12}(t,\xi_1) := K_{12}(t,\xi_1) - K_{12}^m$ so the equations (43) may be rewritten as

$$\begin{bmatrix} \dot{e}_{12} \\ \dot{\tilde{\theta}} \end{bmatrix} = \begin{bmatrix} -K_{12}^m/L & \tilde{\Psi}(t,\hat{x}_2^*(0))^\top/L \\ -\gamma\tilde{\Psi}(t,\hat{x}_2^*(0)) & 0 \end{bmatrix} \begin{bmatrix} e_{12} \\ \tilde{\theta} \end{bmatrix} + \mathcal{K}_1(t,\xi) \tag{49}$$

where the output-injection term

$$\mathcal{K}_1(t,\xi) := \begin{bmatrix} \varphi \\ 0 \end{bmatrix} + \begin{bmatrix} K'_{12}(t,\xi_1)/L & (\Psi(t,\xi_1)^\top - \Psi(t,0)^\top)/L \\ -\gamma(\Psi(t,\xi_1)^\top - \Psi(t,0)^\top) & 0 \end{bmatrix} \begin{bmatrix} e_{12} \\ \tilde{\theta} \end{bmatrix} \tag{50}$$

satisfies $\mathcal{K}_1 \equiv 0$ if $\xi_1 = 0$. Indeed, note that there exists $c > 0$ such that $|\Psi(t,\xi_1)^\top - \Psi(t,0)^\top| \leq c|\xi_1|$ while[¶] $|\varphi| \leq c|\xi_1|(c + |\xi_1|)$ therefore $|\mathcal{K}_1(t,\xi)| \leq c|\xi_1|(c + |\xi|)$. That is, $\mathcal{K}_1$ is a vanishing output injection. More precisely, in view of (46) and (47) we have

$$\int_{t_\circ}^{t} |\mathcal{K}_1(s,\xi_1(s))|^2 ds \leq \beta(|\xi_\circ|)|\xi_\circ|^2 \quad \forall\, t \geq t_\circ \geq 0$$

where $\beta : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is non-decreasing. To invoke [20, Proposition 3] it is only left to show uniform global asymptotic stability of the origin of (49), (44) under the condition that $\mathcal{K}_1 \equiv 0$ in (49) and $\hat{e}_2 \equiv 0$ in (44). Note that in this case, the two dynamics are decoupled. Uniform global asymptotic stability of the origin of (44) follows from the analysis in Section 3 while uniform global exponential stability of the origin of (49) with $\mathcal{K}_1 \equiv 0$ follows invoking standard theorems on linear time-varying adaptive control systems: observe that $K_{12}^m > 0$ is constant, $\tilde{\Psi}(t,\hat{x}_2^*(0))$ and $\dot{\tilde{\Psi}}(t,\hat{x}_2^*(0))$ are bounded a.e. and the PE condition (48) holds. Necessity of the PE condition also follows. See [1]. □

Remark 4.1

We stress that the origin is also uniformly (locally) exponentially stable; this follows from [20, Proposition 4] however, *global* exponential stability under the PI^{2}D controller seems out of reach, in view of the output-injection terms which may be seen as a nonlinear non-globally-Lipschitz vanishing perturbation. Nonetheless, the robustness properties described in Remark 3.3 still hold, notably for the case that $\ddot{x}_3^* \not\equiv 0$.

The previous result may be compared with related literature, such as [2] which establishes convergence of the tracking errors but also [8] where the PE condition on the regressor is imposed on a regressor $\Gamma(t, x(t))$ that is, along reference trajectories.

[¶]Without loss of generality c denotes a generic constant whose value is unimportant.

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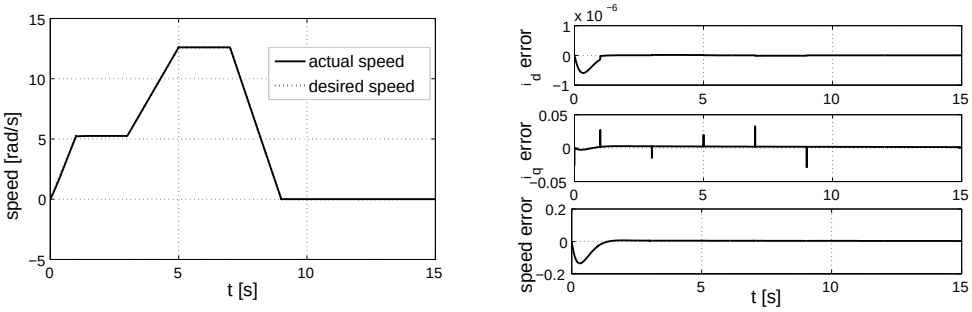
5. SIMULATION RESULTS

We have performed some numerical simulations using the parameters reported in [8] with the aim of evaluating the proposed controller and of comparing it with one of the existing reported schemes. We use $R = 3\Omega$, $L = 0.006H$, $\Phi = 0.33Wb$, $n_p = 6$ and $J = 0.01Kgm^2$. In order to work under a more stringent condition than in [8] the damping coefficient is set to zero. The evaluation consists in imposing a speed reference inspired by the signal profile proposed as a benchmark, by the French Working Group *Commande des Entraînements Electriques* – see `http://www2.irccyn.ec-nantes.fr/CE2/`. The desired motor speed starts at zero, increases with a slope of 5.25 $\frac{rad}{s^2}$ until it reaches the value $5.25\frac{rad}{s}$, at $t = 1s$. This value is kept constant until $t = 3s$ when it is increased again, with a slope of 3.675 $\frac{rad}{s^2}$, up to $12.6\frac{rad}{s}$ during 2 seconds. Then, the reference decreases (with a slope of 6.3 $\frac{rad}{s^2}$) and remains at zero for the rest of the simulation time. The applied load torque is constant and equals $1Nm$.

Following field-orientation control ideas, the desired value for x_1 is set to zero while the controller gains are set to $k_1 = 40$, $k_2 = 65$, $k_p = 5$, $k_d = 10$, $k_i = 0.005$, $a = 50$, $b = 50$, $\varepsilon = 0.02$ and $\gamma = 5$; the last one is used when parametric uncertainty is considered. It is assumed that the motor is at stand-still at the beginning of the simulation, *i.e.*, all the motor states are set to zero, and in a similar way the initial value of the derivative filter state q_c, the estimated load torque and the parameter estimates are considered equal to zero. The estimated value for the unknown parameter θ_4 was set to $\hat{\theta}_4 = 0.5\theta_4$. Such is the worst-case scenario with respect to the uncertainty, both for the load torque and the parameters.

Three different simulations were carried out. The first one illustrates the controller performance under the conditions stated in Proposition 3.1 *i.e.*, that all physical parameters except for the load torque are known. The second is related to Proposition 4.1, under parametric uncertainty. In this case, in addition to uncertainty on the model parameters, we added 400% uncertainty of the nominal value of τ_L *i.e.*, it was set to $5Nm$, from $t = 10s$ to $t = 15s$. One last simulation illustrates the robustness property mentioned in Remark 3.3 by adding a slowly time-varying sinusoidal perturbation of amplitude $2Nm$ and frequency $0.5Hz$.



(a) *Actual (continuous line) and desired (dashed line) speeds*

(b) *Currents and speed errors*

Figure 1. First scenario: with uncertainty in the load torque only and continuous speed reference

The simulation results for the first scenario (with known parameters) are showed in Figure 1. The reference speed profile together with the actual speed response in Figure 1(a), while the currents and speed error behaviors in Figure 1(b). It can be observed the remarkable performance of the PI^{2}D controller. The position error, e_4, is not showed to avoid graphical saturation and due to the fact that its behavior is unimportant for achieving the control objective, although it must be recognized that its rate of convergence to zero is considerably slower than that corresponding to the other errors.

In Figure 2(a) we show the required stator voltages. The noticeable spikes in the stator variables are due to sudden changes in the first and second derivatives of the reference speed. These spikes are avoided if a smooth reference is designed or a filter for the reference is included, but with the aim at evaluating the controller under stringer conditions, the discontinuities are not avoided during this

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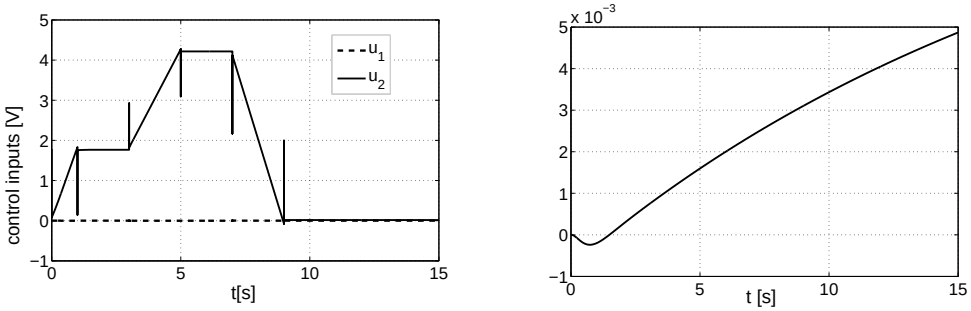
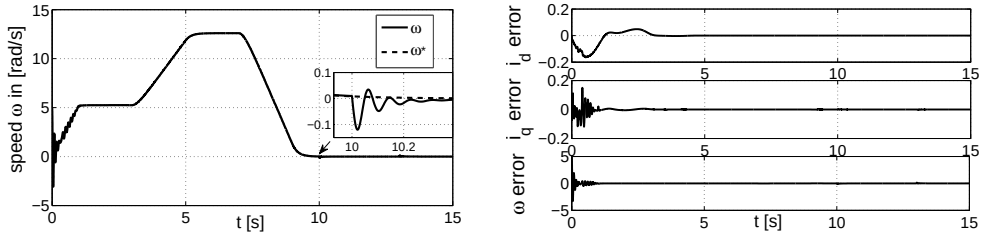


(a) *Stator voltages* (b) *Integral correction* ν

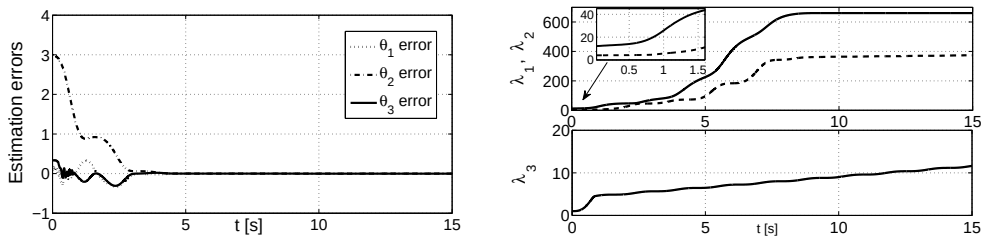
Figure 2. First scenario

simulation. In Figure 2(b) we depict the integral correction ν which compensates for the constant disturbance τ_L/σ. As expected, the convergence of this variable to its steady-state is very slow due to the small value of the integral gain k_i.



(a) *Speed behavior under constant disturbance appearing at* $t = 10s$ (b) *State errors under constant disturbance*

Figure 3. Second scenario: full parametric uncertainty and under the effect of an additional disturbance appearing at $t = 10$s



(a) *Parameter estimation errors under the influence of a constant disturbance* (b) *Eigen-values of* $M(t)$ *defined in (45)*

Figure 4. Second scenario: the unknown parameters are estimated as expected, in view of the persistency of excitation.

In the second and third scenarios we use the adaptive controller (41) by filtering the reference x_3^* using a second order filter of the form

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where $\omega_n = 100$ while $\zeta = 20$. The numerical results obtained under uncertainty in both, the load torque and the motor parameters, are showed in Figures 3 and 4. It can be observed in Figure 3(a)

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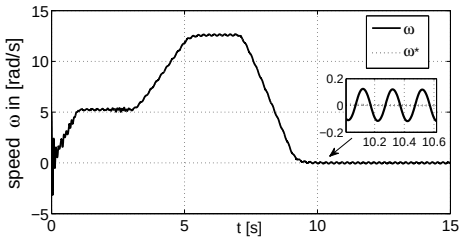
the capability of the control scheme to achieve the control objective even when, at $t = 10s$, the constant unknown disturbance in the load-torque is applied. As expected, the transient response at the beginning of the simulation exhibits some oscillations due to the error in the parameter values. This behavior is attenuated by using better values as initial conditions instead of zero. In Figure 3(b) is depicted the convergence of the state errors to zero, in spite of the perturbation. Figure 4(a) shows that the estimation errors converge to zero asymptotically and for completeness, in Figure 4(b) we depict the eigenvalues of $M(t)$ in (45). To induce the necessary excitation we have used $x_1^*(t) := \sin(2\pi t)$. Finally, Figure 5(a) shows the speed behavior under the aforementioned time–varying load torque disturbance. Input-to-state stability may be appreciated.



(a) *Speed behavior under time–varying disturbance*

Figure 5. Third scenario: the system works under the influence of a time-varying disturbance. A steady state error is appreciated, the controlled system is robust in the input-to-state sense

6. CONCLUDING REMARKS

We showed that a position-feedback controller of PID type solves the speed tracking control problem for permanent-magnet synchronous motors even in the case of unknown parameters. From a practical perspective, besides the remarkable dynamic performance achieved by the closed–loop system, the proposed design avoids the use of (noisy) speed sensor and does not rely on the knowledge of the load torque. The controller's robustness established analytically is also evident from the numerical simulations.

REFERENCES

1. B. D. O. Anderson, R.R. Bitmead, C.R. Johnson, Jr., P.V. Kokotović, R.L. Kosut, I. Mareels, L. Praly, and B.D. Riedle. *Stability of adaptive systems*. MIT Press, Cambridge, MA, USA, 1986.
2. P. Aquino, M. Feemster, D. Dawson, and A. Behal. An adaptive partial state feedback control of the induction motor: Elimination of rotor flux and rotor velocity measurements. In *IEEE Proceedings of the Conference on Decision and Control*, pages 977–982, Tampa, FL, USA, 1998.
3. A. Behal, M. Feemster, D. Dawson, and A. Mangal. Sensorless rotor velocity tracking control of the permanent magnet stepper motor. In *IEEE Conf. Control Appl.*, pages 150–155, Anchorage, Alaska, 2000.
4. G. W. Chang, G. Espinosa-Pérez, R. Ortega, and E. Mendes. Tuning rules for the PI gains of field–oriented controllers of induction motors. *IEEE Transactions on Industrial Electronics*, 47(3):592–602, 2000.
5. J. Chiasson. *Modeling and High Performance Control of AC Drives*. Wiley, 2005.
6. Jun Hu D. Dawson and T. Burg. *Nonlinear control of electric machinery*. Marcel Dekker Inc, 1998.
7. A. Donaire and S. Junco. On the addition of integral action to port-controlled Hamiltonian systems. *Automatica*, 45(4):1910–1916, 2009.
8. S. Di Gennaro. Adaptive output feedback control of synchronous motors. *International Journal of Control*, 73:1475–1490, 2000.
9. W. Hahn. *Stability of motion*. Springer–Verlag, Berlin, 1967.
10. H. Khalil. *Nonlinear systems*. Prentice Hall, 3rd ed., New York, 2002.
11. W. Leonhard. *Control of Electrical Drives*. Springer, 2001.
12. A. Loría R. Kelly, and A. Teel. Uniform parametric convergence in the adaptive control of mechanical systems. *European J. of Contr.*, 11(2):87–100, 2005.
13. M. Malisoff and F. Mazenc. *Constructions of Strict Lyapunov Functions*. Springer, 2009.
14. R. Marino, S. Peresada, and P. Tomei. Adaptive input-output linearizing control of induction motors. *IEEE Transactions on Automatic Control*, 38(2):208–221, 1993.

DOI: 10.1002/acs

15. J. Meisel. *Principles of Electromechanical Energy Conversion*. Mc. Graw Hill, 1961.
16. R. Ortega, A. Loría, and R. Kelly. A semiglobally stable output feedback PI^2D regulator for robot manipulators. *IEEE Transactions on Automatic Control*, 40(8):1432–1436, 1995.
17. R. Ortega, A. Loría P. J. Nicklasson, and H. Sira-Ramírez. *Passivity–based Control of Euler–Lagrange Systems*. Springer-Verlag, London, 1998.
18. R. Ortega, A. Loría P. J. Nicklasson, and H. Sira-Ramírez. *Passivity-based Control of Euler-Lagrange Systems: Mechanical, Electrical and Electromechanical Applications*. Series Comunications and Control Engineering. Springer Verlag, London, 1998. ISBN 1-85233-016-3.
19. R. Ortega, P. J. Nicklasson, and G. Espinosa-Pérez. On speed control of induction motors. *Automatica*, 32(3):455–460, 1996.
20. E. Panteley, A. Loría, and A. Teel. Relaxed persistency of excitation for uniform asymptotic stability. *IEEE Transactions on Automatic Control*, 46(12):1874–1886, 2001.
21. C. J. Retter. *Matrix and space-phasor theory of electrical machines*. Akadémiai Kiadó, Budapest, 1987.
22. D. Taylor. Nonlinear control of electric machines: An overview. *Control Systems Magazine*, 14(6):41–51, 1994.
23. P. Tomei and C. M. Verrelli. A nonlinear adaptive speed tracking control for sensorless permanent magnet step motors with unknown load torque. *Int. J. Adapt. Control Signal Process.*, 22:266–288, 2008.
24. P. Tomei and C. M. Verrelli. Observer-based speed tracking control for sensorless permanent magnet synchronous motors with unknown load torque. *IEEE Trans. on Automat. Contr.*, 56(6):1484–1488, 2011.
25. G. Zhu, L.-A. Dessaint, O. Akhrif, and A. Kaddouri. Speed tracking control of a permanent-magnet synchronous motor with state and load torque observer. *IEEE Trans. on Indus. Electronics*, 47(2):346–355, 2000.

DOI: 10.1002/acs