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Abstract— We propose a control strategy for switched-reluctance motors with unknown load, which consists in two separate control loops, for the rotor (mechanical) dynamics and the stator (electrical) dynamics. The novelty of the approach resides in using an alternative rotor model which corresponds to that of an harmonic oscillator hence, it is linear in the rotation coordinates. The control law is of proportional-integral-derivative type and it is implemented through a virtual control input, generated via the mechanical torque of electrical origin. A second control loop is closed around the stator dynamics via a current tracking controller. As far as we know, we establish for the first time global exponential stability considering that the load torque is unknown.

I. INTRODUCTION

Switched-reluctance motors (SRM) are highly reliable, technologically simple and relatively cheaper than other electrical machines. However, technological simplicity comes at the cost of model mathematical complexity: the map flux-current is highly nonlinear and the mechanical torque is a nonlinear function of the stator currents and angular positions. These difficulties are responsible for the lack of theoretical foundation in the control of switched-reluctance machines, in spite of a number of articles which provide efficient *ad hoc* control methods, either via state-feedback as in [1], [2] or partial state-feedback –[3], [4].

A natural approach in control of electromechanical systems is to control separately the mechanical (slow) dynamics separately from the electrical (fast) dynamics. This methodology naturally leads to what is called cascaded-based control –see the seminal paper [5]. In the case of switched-reluctance motors the first control loop is used to drive the rotor variables (velocity and position) to a desired reference and a second loop is closed around the stator dynamics via current feedback, in order to to steer the currents to a regime such that the current drives the rotor velocities to the desired reference.

Although appealing, the realization of this method is obstructed by the fact that the two closed-loop subsystems are *feedback* interconnected via nonlinear functions of the

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states: currents enter nonlinearly in the mechanics equations and viceversa. One interconnection stymies the implementation of rotor control, while the second poses difficulties in sensorless control. To overcome the difficulty of control implementation, the *torque sharing* technique is adopted –see [6], [7] which exploits the physical properties of the machine by 'allocating' the control action through one phase at a time. The second difficulty is overcome using an alternative model in which the rotor is regarded as a linear oscillator, this idea comes from [8] although in this reference the context is fundamentally different; that of rotor angle estimation for permanent-magnet synchronous motors hence, was not used for tracking control purposes.

In this paper we address the problem of velocity/position control of switched reluctance motors. We assume that angular velocities and angular positions as well as currents, are measured. It is also assumed that the load torque is unknown but constant. The control approach follows the rationale of [5] however, we use a small-gain argument rather than a "cascades" argument to conclude stability. The rotor-dynamics controller is of PID-type and is implemented using a tracking nonlinear controller for the stator loop. We provide proofs of Lyapunov global exponential stability in closed loop. As far as we know our control approach is novel and global exponential stability has not been established for this type of machines with unknown payload.

The rest of the paper is organized as follows. In the next section we present the dynamic model, we assume that the inductances are functions of the rotor angular positions only. For clarity of exposition, in Section III we describe the first control loop: for the rotor dynamics; in Section IV we present the stator-currents controller and in Section V we present our main results.

II. PROBLEM STATEMENT AND SOLUTION

Considering the experimentally-validated fact that the mutual inductance among stator phases is negligible the low–current three–phase SRM dynamic model is given by

$$u_j = \mathbf{L}_j(\theta)\frac{di_j}{dt} + \mathbf{K}_j(\theta)\omega i_j + Ri_j \tag{1a}$$

$$J\dot{\omega} = \mathbf{T}_e(\theta, i) - T_L(\theta, \omega) \tag{1b}$$

$$\dot{\theta} = \omega \tag{1c}$$

for each $j \in \{1, 2, 3\}$, hence we make the standing assumption that there exists β_x such that $|i| \leq \beta_x$.

For each phase j, u_j is the voltage applied to the stator terminals, i_j is the stator current and ψ_j is the flux linkage.

R represents the stator winding resistance, J is the total rotor inertia, $L_j(\theta)$ corresponds to the phase inductances, given by

$$\mathbf{L}_j(\theta) = l_0 - l_1 \cos\left(N_r\theta - (j-1)\frac{2\pi}{3}\right)$$

where $l_0 > l_1 > 0$ and $\mathbf{K}_j(\theta) = \frac{\partial \mathbf{L}_j}{\partial \theta}$ corresponds to the phase-inductance variation relative to the rotor angular position. Based on the assumption that inductances are decoupled, the total mechanical torque $\mathbf{T}_e$ corresponds to the sum of the torques produced by each phase that is,

$$\mathbf{T}_e = \frac{1}{2}\sum_{j=1}^{3}\mathbf{K}_j(\theta)i_j^2.$$

Following ideas of [8] an alternative representation of the motor model can be obtained if the periodic functions that appear on it are obtained as solutions of the differential equation

$$\dot{\rho} = \omega\mathbb{J}\rho, \quad \rho(0) = \rho_\circ \in \mathbb{R}, \tag{2}$$

where $\rho = [\rho_1, \rho_2]^\top$ and

$$\mathbb{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

That is, the variable θ is replaced by the two variables ρ_1, ρ_2 which are constrained to the circumference $\mathbb{S}^{\rho_\circ}$ of radius $\rho_\circ$, centered at the origin, given by $\rho_1(t) = (\rho_\circ/\sqrt{2})\cos(\theta(t) + \pi/4)$ and $\rho_2(t) = (\rho_\circ/\sqrt{2})\sin(\theta(t) + \pi/4)$ which satisfy $\rho(t) \in \mathbb{S}^{\rho_\circ}$ for all $t \geq 0$.

In the new coordinates the rotor dynamics (1b) takes the form

$$J\dot{\omega} = T_e(\rho, x) - T_L \tag{3}$$

in which the mechanical torque is now expressed as a function of ρ. That is, although T_e and $\mathbf{T}_e$ are different functions $T_e(\rho, x)$ and $\mathbf{T}_e(\theta, i)$ represent the same quantity. The definition of T_e is explained below. The arguments of T_L are dropped for the sequel and it is assumed (unless explicitly specified otherwise) that T_L is constant.

Furthermore, defining $x := i$ to avoid a cumbersome notation, the stator equation becomes

$$L(\rho)\dot{x} + K(\rho)\omega x + Rx = u \tag{4}$$

where $L(\rho) = \mathrm{diag}[L_1(\rho), L_2(\rho), L_3(\rho)]$ with

$$L_1(\rho) = l_0 - l_1\frac{a}{2}(\rho_2 + \rho_1) \tag{5}$$

$$L_2(\rho) = l_0 - l_1\left[\frac{ab}{4}(\rho_2 - \rho_1) - \frac{a}{4}(\rho_2 + \rho_1)\right] \tag{6}$$

$$L_3(\rho) = l_0 - l_1\left[-\frac{ab}{4}(\rho_2 - \rho_1) - \frac{a}{4}(\rho_2 + \rho_1)\right] \tag{7}$$

$$a = \sqrt{2}, \quad b = \sqrt{3} \tag{8}$$

and $K(\rho) = \mathrm{diag}[K_1(\rho), K_2(\rho), K_3(\rho)]$ with

$$K_1(\rho) = l_1\frac{a}{2}(\rho_2 - \rho_1) \tag{9}$$

$$K_2(\rho) = l_1\left[-\frac{ab}{4}(\rho_2 + \rho_1) - \frac{a}{4}(\rho_2 - \rho_1)\right] \tag{10}$$

$$K_3(\rho) = l_1\left[\frac{ab}{4}(\rho_2 + \rho_1) - \frac{a}{4}(\rho_2 - \rho_1)\right] \tag{11}$$

Thus, in the new coordinates and under the conditions described in Section II, the motor dynamics is defined by Equations , (3) and (4).

The **control problem** consists in driving the angular velocity ω to a set-point reference ω^*. We solve it by designing separately a set-point control law for the rotor dynamics (2), (3) and a tracking controller for the stator dynamics (4) as shown in Figure 1. The rotor dynamics is forced to *synchronize* with a reference oscillator model (in polar coordinates); this is accomplished via a virtual control law of proportional-integral-derivative type plus a feedforward term and it is implemented based on the torque-sharing approach of [9] hence x^* which is a virtual control input to (3) is fed as reference to the stator controller. The convergence of the actual current x to the desired reference x^*, is ensured by the control input u, which is implemented in the form of an input voltage. Each control loop is input to state stable so a small-gain argument applies to guarantee stability of the interconnected system
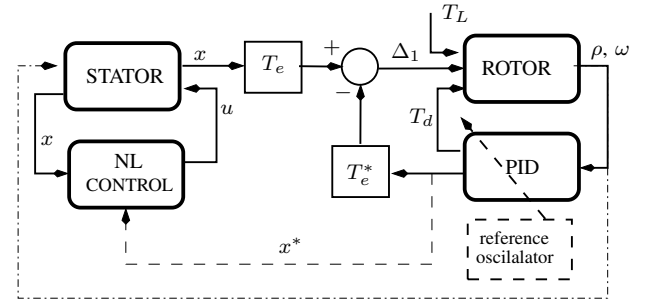

Fig. 1. Control block-diagram (for analysis purpose).

Concerning the design of the *reference oscillator* which, as we shall see is crucial to our the control approach, for a given desired constant reference ω^*, we introduce θ^* as the angular position reference for θ that is, $\dot{\theta}^* = \omega^*$. Thus, the *reference coordinates* can be obtained as solutions of

$$\dot{\rho}^* = \omega^*\mathbb{J}\rho^*, \quad \rho^*(0) = \rho_\circ^* \in \mathbb{R} \tag{12}$$

with initial conditions $\rho_\circ^*$ left free as a design parameter, that is, with an abuse of notation, we denote the solutions to (12) as

$$\rho^*(t) := \frac{1}{\sqrt{2}}\begin{bmatrix} \rho_\circ^*\cos(\theta^*(t) + \pi/4) \\ \rho_\circ^*\sin(\theta^*(t) + \pi/4) \end{bmatrix}$$

where $\theta^*(t) = \omega^*t$ that is, we set $\theta^*(0) = 0$.

With the relationship between ρ^* and ω^* at hand, a virtual control law T_d is designed for the mechanical equation (3)

to steer $\omega \to \omega^*$ as if T_e were a control input. In this sense the reference mechanical torque is defined as

$$T_e^* = \frac{1}{2}\Big[K_1(\rho^*)x_1^{*2} + K_2(\rho^*)x_2^{*2} + K_3(\rho^*)x_3^{*2} \Big] \quad (13)$$

where x_j^* is a current reference trajectory for each phase which solves

$$\frac{T_e^*}{J} = T_d \quad (14)$$

for any given T_d. Hence, provided that $T_e = T_e^*$ the desired control torque T_d acts upon the mechanical equation to drive $\omega \to \omega^*$. By ensuring an accurate current tracking that is $x \to x^*$, it is guaranteed that $T_e \to T_e^*$ and consequently that $\omega \to \omega^*$. See Figure 1 and [5].

In order to solve (14) for x_j^* we exploit the physics of the *switched* reluctance motor. In view of the fact that the torque sign is only determined by the variation of the inductance and using ideas reported in [6] and [9] we introduce the following smooth current-switching policy. Define the sets

$$\Theta_j^+ = \Big\{ \rho^* \in \mathbb{S}^{\rho_\circ^*} : K_j(\rho^*) \geq 0 \Big\}$$

$$\Theta_j^- = \Big\{ \rho^* \in \mathbb{S}^{\rho_\circ^*} : K_j(\rho^*) < 0 \Big\}$$

where the superscripts $^+$ and $^-$ stand for required positive and negative torque respectively. Accordingly, let us introduce

$$m_j^+(\rho^*) > 0, \ \sum_{j=1}^{3} m_j^+(\rho^*) = 1 \quad \forall \rho^* \in \Theta^+,$$

$$m_j^-(\rho^*) > 0, \ \sum_{j=1}^{3} m_j^-(\rho^*) = 1 \quad \forall \rho^* \in \Theta^-$$

and given T_d, define

$$m_j(\rho^*) = \begin{cases} m_j^+(\rho^*) & \text{if} \quad T_d \geq 0, \\ m_j^-(\rho^*) & \text{if} \quad T_d < 0. \end{cases}$$

Then, the reference currents for $j \in \{1, 2, 3\}$ are given by

$$x_j^* = \begin{cases} \left[2J \dfrac{m_j(\rho^*)T_d}{K_j(\rho^*)} \right]^{1/2} & \text{if } K_j(\rho^*) \neq 0 \\ 0 & \text{otherwise,} \end{cases} \quad (15)$$

while, by construction. we have

$$T_d = m_1(\rho^*)T_d + m_2(\rho^*)T_d + m_3(\rho^*)T_d. \quad (16)$$

It is important to notice that the reference trajectory x^* depends on the current phase of the *reference* oscillator, in contrast to the usual practice where it is defined in terms of the actual position θ, see [6] and [7], requiring then full state feedback and known load torque. This may be considered as an advantageous modification of the classical torque-sharing approach.

In the rest of the paper we present the three integrating parts of the control approach. First, we describe the rotor dynamics control-loop then, we present the fast-dynamics control loop that is, the stator controller. In either case, we show that the system is input-to state stable. In the last part we establish stability of the complete controlled system by appealing to a small-gain argument.

III. ROTOR ROBUST STATE-FEEDBACK CONTROL

In this section the robust state-feedback control of the rotor dynamics is presented. First, we establish a result of practical stability with respect to the uncompensated constant disturbance induced by the load-torque, to later on, add an integrator to compensate for the load-torque and establish global exponential stability.

A. Without load compensation

Consider the rotor dynamic model (2), (3) together with the state–feedback control law

$$T_d = -k_d e_\omega - k_p \rho^{*\top}\mathbb{J}^\top e_\rho + \nu + \dot\omega^* \quad (17)$$

where $e_\rho := \rho - \rho^*$, $e_\omega := \omega - \omega^*$ and ν is an integrating correction term added to compensate for the unknown constant payloads $\nu^* = \frac{T_L}{J}$. Note that T_d is of the proportional-integral-derivative type since the integral action ν is complemented by a proportional correction term $-k_p\rho^{*\top}\mathbb{J}^\top e_\rho$ projected into $\mathbb{R}$ and a derivative correction term $-k_d e_\omega$ proportional to angular velocity errors. In addition, the control law contains the feedforward term $\dot\omega^*$.

If it is defined $\tilde\nu := \nu - \nu^*$ and it is considered that $T_d - T_e^*/J = 0$, then the dynamic of e_ω can be written as

$$\dot e_\omega = -k_d e_\omega - k_p \rho^{*\top}\mathbb{J}^\top e_\rho + \tilde\nu + \Delta_1(t, e_x, e_\rho) \quad (18a)$$

$$\Delta_1 = \frac{1}{2}\left[x^\top K(e_\rho)x - x^{*\top}K(\rho^*)x^* + x^\top K(\rho^*)x \right]. \quad (18b)$$

On the other hand, the error dynamic equation for e_ρ is obtained subtracting (12) from (2) leading to the error equations

$$\dot e_\omega = -k_d e_\omega - k_p \rho^*(t)^\top \mathbb{J}^\top e_\rho + v \quad (19a)$$

$$\dot e_\rho = e_\omega \mathbb{J}\rho^*(t) + \omega\mathbb{J}e_\rho \quad (19b)$$

which are reminiscent of the thoroughly-studied error system in model-reference adaptive control –*cf.* [10]. Another interesting feature of these equations is that they may be regarded as a 'nominal' non-autonomous but periodic system, perturbed by the 'input' $v = \Delta_1 + \tilde\nu$. The interest of this observation relies on the following statement.

Proposition 3.1 (GES, robust to unknown payload): Let $k_p > 2|\rho_\circ^*|$ and let v be bounded then, the system (19) is input-to-state-stable with respect to the input v. Also, if $v \in \mathcal{L}_2$ the map $v \to e_\omega$ is output-strictly passive. In the case that $v \equiv 0$ (perfect load compensation and current tracking) the origin $(e_\rho, e_\omega) = (0, 0)$ of (19) is globally exponentially stable.

Proof: Consider the positive definite radially unbounded function V_{c1},

$$V_{c1}(e_\omega, e_\rho) = \frac{1}{2}\left(e_\omega^2 + k_p |e_\rho|^2 \right) \quad (20)$$

whose time derivative along the trajectories of (19) yields

$$\dot V_{c1}(e_\omega, e_\rho) = -k_d e_\omega^2 + e_\omega v. \quad (21)$$

Output strict passivity of the map $v \mapsto e_\omega$ follows by integrating on both sides of (21). The proof of global asymptotic stability under the condition $v \equiv 0$, follows invoking Krasovskiĭ-Lasalle's theorem for periodic systems –see *e.g.* [11, p. 179]: note that $e_w = 0$ implies that $\dot{V}_{c1} = 0$ and the only solution of $k_p \rho^*(t)^\top \mathbb{J}^\top e_\rho = 0$ for any t, is $e_\rho = 0$ that is, the origin is the largest invariant set contained in $\dot{V}_{c1} = 0$. Global exponential stability is established invoking standard results from adaptive control literature, observing that $\mathbb{J}\rho^*(t)$ is persistently exciting that is, there exist T_c and $\mu_c > 0$ such that

$$\int_t^{t+T} \mathbb{J}\rho^*(\tau)\rho^*(\tau)^\top \mathbb{J}^\top d\tau \geq \mu_c I. \tag{22}$$

As a matter of fact, it can be proved[1] that (22) holds with $T_c = \pi/\omega*$ and $\mu_c = |\rho_\circ^*|^2 T_c/2$. To establish input-to-state stability we construct an ISS-Lyapunov function for (22).

Lemma 3.1: There exists $\varepsilon_1 \in (0, 1)$ such that $\sum_{i=1}^3 V_{ci}$ with

$$V_{c2}(t, e_\omega, e_\rho) = \varepsilon_1 e_\omega \rho^*(t)^\top \mathbb{J}^\top e_\rho$$
$$V_{c3}(t, e_\rho) = -e_\rho^\top \int_t^\infty e^{(t-\tau)} \mathbb{J}\rho^*(\tau)\rho^*(\tau)^\top \mathbb{J}^\top d\tau e_\rho,$$

is an ISS Lyapunov function[2].

To see this note that $\sum_{i=1}^3 V_{ci}$ is positive definite if $k_p > 2|\rho_\circ^*|$ and ε_1 is sufficiently small. Furthermore, in view of (21) we have

$$\sum_{i=1}^3 \dot{V}_{ci} \leq -\bar{k}_{d3} e_\omega^2 - \varepsilon_1 \bar{k}_{p3} \left[\rho^{*\top}\mathbb{J}^\top e_\rho \right]^2$$
$$-\kappa(|\rho_\circ^*|) |e_\rho|^2 + \left[\varepsilon_1 \left[\rho^{*\top}\mathbb{J}^\top e_\rho \right] + e_\omega \right] v \tag{24}$$

where

$$\kappa(|\rho_\circ^*|) := \frac{\pi |\rho_\circ^*|}{2\omega^*} \left(|\rho_\circ^*| e^{-\pi/\omega^*} - \frac{1}{2} \right), \tag{25}$$

for appropriate values of ε_1. Then, it is enough to choose a constant α sufficiently small such that $|v| \leq \alpha |e_\omega, e_\rho|$ implies that $\sum_{i=1}^3 \dot{V}_c$ is negative definite. ∎

Proposition 3.1 establishes global exponential stability for the system without load torque. As a byproduct, the system is robust with respect to additive uncertainties for instance, to uncompensated for torque-load uncertainty (*i.e.*, $\tilde{\nu} = $ const.).

B. With compensation of unknown load

Now, we exploit the passivity properties of (19) to add a second control loop which we close with integral action to compensate for constant additive disturbances such as constant unknown payload. That is, let the variable ν in (17) be defined by

$$\dot{\nu} = -k_i \left(e_\omega + \varepsilon_1 \left[\rho^{*\top}\mathbb{J}^\top e_\rho \right] \right), \quad k_i > 0. \tag{26}$$

[1]Due to space constraints this proof is not included in the paper, but is available if requested to the authors.
[2]*ib idem*

Since the map $\left(e_\omega + \varepsilon_1 \left[\rho^{*\top}\mathbb{J}^\top e_\rho \right] \right) \mapsto \nu$ is passive (Eq. (26) is a simple integrator), the passivity and robustness properties of (19) are conserved.

Proposition 3.2 (GES with load compensation): The system (19) with $v = \tilde{\nu} + \Delta_1$ is input-to-state-stable with respect to bounded inputs Δ_1 and the map $\Delta_1 \mapsto \left(e_\omega + \varepsilon_1 \left[\rho^{*\top}\mathbb{J}^\top e_\rho \right] \right)$ is output-strictly passive with respect to inputs $v \in \mathcal{L}_2$. Moreover, if $\Delta_1 \equiv 0$ that is, if $v = \tilde{\nu}$, then the origin $(e_\rho, e_\omega, \tilde{\nu}) = (0, 0, 0)$ of (19) and

$$\dot{\tilde{\nu}} = -k_i \left(e_\omega + \varepsilon_1 \left[\rho^{*\top}\mathbb{J}^\top e_\rho \right] \right), \quad k_i > 0 \tag{27}$$

is globally exponentially stable for appropriate values of the gains k_p, k_d and k_i.

Proof: The total time derivative of $\sum_{i=1}^4 V_{ci}$, where

$$V_{c4}(\tilde{\nu}) := \frac{1}{2k_i} \tilde{\nu}^2, \tag{28}$$

along the trajectories of the closed-loop system (18), (19b) and (27), satisfies (24) with $v = \Delta_1$. As in the proof of Proposition 3.1 if $\Delta_1 \equiv 0$, global asymptotic stability follows invoking Lasalle's principle. Also, integrating the resulting expression of $\sum_{i=1}^4 \dot{V}_{ci}$ on both sides, we see that the map $\Delta_1 \mapsto \left(e_\omega + \varepsilon_1 \left[\rho^{*\top}\mathbb{J}^\top e_\rho \right] \right)$ is output-strictly passive.

Now we proceed to show that $V_c := \sum_{i=1}^5 V_{ci}$, with

$$V_{c5}(e_\rho, e_\omega, \tilde{\nu}) := -\varepsilon_3 \tilde{\nu} e_\omega - \frac{1}{2}\varepsilon_1 \varepsilon_3 k_i |e_\rho|^2, \tag{29}$$

qualifies as an ISS-Lyapunov function for (18), (19b) and (27) with input Δ_1. Note that V_c is positive definite for sufficiently small values of ε_1, ε_3 and $k_p > 2|\rho_\circ^*|^2$. The total time derivative of V_{c5} along the closed-loop trajectories yields

$$\dot{V}_{c5} = \varepsilon_3 k_i \left(e_\omega + \varepsilon_1 \left[\rho^{*\top}\mathbb{J}^\top e_\rho \right] \right) e_\omega$$
$$-\varepsilon_3 \tilde{\nu}^2 - \varepsilon_3 \tilde{\nu} \left(-k_d e_\omega - k_p \rho^{*\top}\mathbb{J}^\top e_\rho + \Delta_1 \right)$$
$$-\varepsilon_1 \varepsilon_3 k_i e_\rho^\top \left[\omega \mathbb{J} e_\rho + e_\omega \mathbb{J}\rho^* \right] \tag{30}$$

hence adding $\dot{V}_{c4}$ and the latter to (24) (in which we use $v = \tilde{\nu} + \Delta_1$) we obtain

$$\sum_{i=1}^5 \dot{V}_{ci} \leq -[\bar{k}_{d4} - \varepsilon_3 k_i] e_\omega^2 - \kappa(|\rho_\circ^*|) |e_\rho|^2$$
$$-\varepsilon_1 \bar{k}_{p4} \left[\rho^{*\top}\mathbb{J}^\top e_\rho \right]^2 - \frac{\varepsilon_3}{2} \tilde{\nu}^2 - \delta_2$$
$$+ \Delta_1 \left(\varepsilon_1 \left[\rho^{*\top}\mathbb{J}^\top e_\rho \right] + e_\omega - \varepsilon_3 \tilde{\nu} \right)$$

where

$$\delta_2 := \frac{1}{2} \begin{bmatrix} e_\omega \\ \rho^{*\top}\mathbb{J}^\top e_\rho \\ \tilde{\nu} \end{bmatrix} \begin{bmatrix} 2k_{d3} & 0 & \varepsilon_3 k_d \\ 0 & 2\varepsilon_1 k_{p3} & \varepsilon_3 k_p \\ \varepsilon_3 k_d & \varepsilon_3 k_p & \varepsilon_3 \end{bmatrix} \begin{bmatrix} e_\omega \\ \rho^{*\top}\mathbb{J}^\top e_\rho \\ \tilde{\nu} \end{bmatrix}.$$

Let ε_3 satisfy $\min \left\{ \frac{k_{d4}}{k_i}, \frac{\lambda_3}{k_d}, \frac{\varepsilon_1 \lambda_3}{k_p} \right\} \geq \varepsilon_3$ then, $\delta_2 \geq 0$ and $\sum_{i=1}^5 V_{ci}$ is an ISS-Lyapunov function for the system (18), (19b) and (27) with input Δ_1. Furthermore, if $\Delta_1 \equiv$

0, $\sum_{i=1}^{5} \dot{V}_{ci}$ is bounded by a quadratic negative definite function of the state and global exponential stability follows invoking standard Lyapunov theory. ∎

IV. STATOR ROBUST STATE-FEEDBACK CONTROL

The fast–dynamics control loop is introduced in this section. With this aim, consider (15) and define $x^* := [x_1^* \, x_2^* \, x_3^*]^\top$. For Equation (4) we introduce the control law

$$u^*(t,x) := L(\rho^*)\dot{x}^* + K(\rho^*)\omega^* x + Rx^* - k_{px}e_x \quad (31)$$

where $e_x := x - x^*$ and k_{px} is shorthand notation for $k_{px}(t, |e_x|)$ which is defined by a continuous function $k_{px} : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ such that $k_{px}(t, \cdot)$ is non-decreasing.

Now, defining $\sigma(\rho^*) := \dfrac{m_j(\rho^*)}{K_j(\rho^*)}$ we have after (15),

$$\dot{x}_j^* = \begin{cases} \left[\dfrac{J}{2\sigma(\rho^*)T_d}\right]^{1/2} \left[\dot{\sigma}T_d + \sigma\dot{T}_d\right] & \text{if } K_j(\rho^*) \neq 0 \\ 0 & \text{otherwise.} \end{cases} \quad (32)$$

Applying $u = u^*$ in (4) we see that

$$L(\rho)\dot{e}_x + [R + k_{px}]\,e_x = \tilde{L}(e_\rho)\dot{x}^* + K(e_\rho)\omega^* x - K(\rho)e_\omega x \quad (33)$$

where we defined $\tilde{L}(e_\rho) := L(\rho^*) - L(\rho)$ and we used the identity $K(e_\rho) = K(\rho) - K(\rho^*)$. Furthermore, we define

$$\Delta_2(t, e_\rho, e_x, \dot{x}^*) := \tilde{L}(e_\rho)\dot{x}^* - K(e_\rho)\omega^* x - K(\rho)e_\omega x$$

and, for further development we remark that there exist $c_1(l_0, l_1)$ and $c_2(l_0, l_1) > 0$ such that

$$|\Delta_2| \leq \left[c_1 \,|\dot{x}^*| + c_2 \,|x|\right]|e_\rho| + \left[c_3\rho_\circ \,|x|\right]|e_\omega|. \quad (34)$$

Proposition 4.1: Consider the control law (31) where

$$k_{px} := k_{px1} + \frac{1}{2}\left[c_1\,|\dot{x}^*| + c_2\,|x|\right]^2 + \frac{1}{2}\left[c_3\rho_\circ\,|x|\right]^2 \quad (35)$$

then, the closed-loop system (15)–(31) is input-to-state stable from the input $[e_\rho, e_\omega]^\top$. Moreover, in the case that $|\Delta_2| \equiv 0$, the origin $\{e_x = 0\}$ is globally exponentially stable with $k_{px} := k_{px1}$.

Proof: The total time derivative of

$$V_{c6}(e_x) := \frac{1}{2}\,|e_x|^2 \quad (36)$$

along the closed-loop trajectories yields

$$\dot{V}_{c6} \leq -[R + k_{px}]\,|e_x|^2 \\ + |e_x|\left(\left[c_1\,|\dot{x}^*| + c_2\,|x|\right]|e_\rho| + \left[c_3\rho_\circ\,|x|\right]|e_\omega|\right)$$

which, in view of (35), implies that

$$\dot{V}_{c6} \leq -[R + k_{px1}]\,|e_x|^2 + \frac{1}{2}\left(|e_\rho|^2 + |e_\omega|^2\right) \quad (37)$$

hence, V_{c6} is an ISS-Lyapunov function for the stator closed-loop system. The proof of the second statement follows directly observing that $|\Delta_2| \equiv 0$ implies that $\dot{V}_{c6} \leq -[R + k_{px}]\,|e_x|^2$. ∎

V. MAIN RESULT

We establish that the interconnection of the two control loops for the rotor dynamics and the stator dynamics, remains input-to state stable with respect to external inputs and that without inputs, (the origin of) the closed-loop system is globally exponentially stable.

Proposition 5.1: Consider the system (1) under the assumptions described in Section II in closed loop with the controller defined by (31), (32), (26), (17) and (15). Let k_{px} be given by (35) where

$$k_{px1} \geq \frac{3}{2}k_M^2\,|\rho_\circ^*|^2\left[|x^*| + |x|\right]^2 + k_{px2}, \quad k_{px2} > 0 \quad (38)$$

and let k_p, k_d and $\rho_\circ^*$ be such that (25) holds and

$$\min\left\{\frac{3\bar{k}_{p4}}{\varepsilon_1},\; 3\bar{k}_{d5},\; 2\kappa(|\rho_\circ^*|)\right\} \geq 3\beta_x^2 k_M + 1. \quad (39)$$

Then, the origin of the closed-loop system is globally exponentially stable.

Proof: The closed-loop system corresponds to (19), (27) and (33). The term Δ_1 in (18b) satisfies $2\Delta_1 = x^\top K(e_\rho)x + e_x^\top K(\rho^*)x^* + e_x^\top K(\rho^*)x$ hence, using $|K(\rho^*)| \leq k_M\,|\rho_\circ^*|$ we see that

$$|\Delta_1| \leq k_M\,|\rho_\circ^*|\,|e_x|\left[|x^*| + |x|\right] + k_M\,|e_\rho|\,|x|^2. \quad (40)$$

Then, the total time derivative of $V_c := \sum_{i=1}^{6} V_{ci}$ yields

$$\begin{aligned}
\dot{V}_c \leq\; & -[\bar{k}_{d5} - 0.5]e_\omega^2 - \varepsilon_1\bar{k}_{p4}\left[\rho^{*\top}\mathbb{J}^\top e_\rho\right]^2 - \frac{\varepsilon_3\tilde{\nu}^2}{2} \\
& + k_M\,|\rho_\circ^*|\,|e_x|\left[|x^*| + |x|\right]\left(\varepsilon_1\left[\rho^{*\top}\mathbb{J}^\top e_\rho\right] + e_\omega - \varepsilon_3\tilde{\nu}\right) \\
& + k_M\,|e_\rho|\,|x|^2\left(\varepsilon_1\left[\rho^{*\top}\mathbb{J}^\top e_\rho\right] + e_\omega - \varepsilon_3\tilde{\nu}\right) \\
& -[R + k_{px1}]\,|e_x|^2 - \kappa(|\rho_\circ^*|)\,|e_\rho|^2
\end{aligned} \quad (41)$$

which, in virtue of the triangle inequality, implies that

$$\begin{aligned}
\dot{V}_c \leq\; & -[\bar{k}_{d5} - 1 - \frac{1}{2}\beta_x^2 k_M]e_\omega^2 - \frac{\varepsilon_3}{4}(1 - 2\varepsilon_3\beta_x^2 k_M)\tilde{\nu}^2 \\
& -[R + k_{px2}]\,|e_x|^2 - \left[\kappa(|\rho_\circ^*|) - \frac{3}{2}\beta_x^2 k_M\right]|e_\rho|^2 \\
& -\varepsilon_1(\bar{k}_{p4} - 1 - \frac{\varepsilon_1}{2}\beta_x^2 k_M)\left[\rho^{*\top}\mathbb{J}^\top e_\rho\right]^2.
\end{aligned} \quad (42)$$

In view of (39), this expression is negative definite for sufficiently small ε_1 and ε_3. Global exponential stability follows invoking standard Lyapunov theorems. ∎

VI. SIMULATION RESULTS

We have tested our main result in simulations using SIMULINK™of MATLAB™. The parameters of the motors are $R = 5$, $l_0 = 0.030H$, $J = 0.01kg - m^2$ and $N_r = 4$. The desired motor speed set-point is fixed to 50rad/s. The applied load-torque is constant and equals 0.1[Nm]. The control gains are fixed to $k_p = 1000$, $k_d = 2000$ and $k_{px} = 100$. Two different implementaion scenarios are presented. In the first (Figs. 2 and 3) we use a sharp step of 50rad as reference; note the short transient at the expense of multiple peaks in the
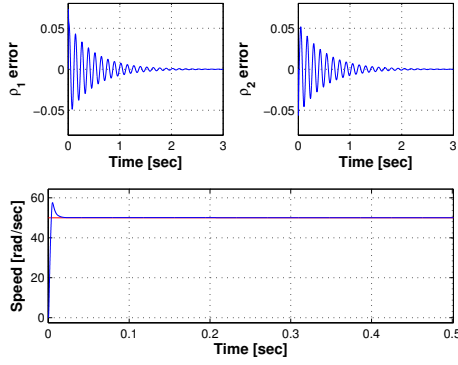
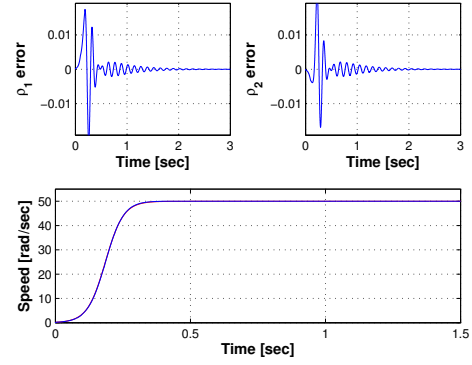Fig. 2. Position errors and tracking using a sharp step reference

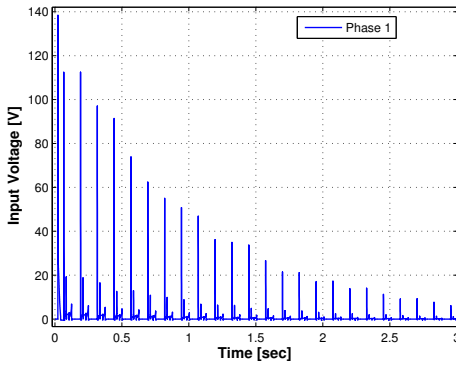Fig. 4. Position errors and tracking using a smooth step reference

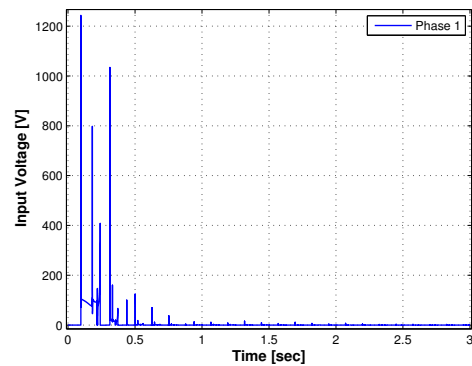Fig. 3. Input voltage for the phase 1 using a step reference $w*$

Fig. 5. Input voltage for the phase 1 using a smooth reference $w*$

input voltage due to the switching-based control implementation via the torque-sharing approach. In the second scenario, the control set-point is also 50 rad/s however, the reference is implemented as a smooth step using a reference signal generated by

$$\omega(t)^* = \omega_0^* + \frac{(\omega_f^* - \omega_0^*)}{2}\left(1 + \tanh(t - T)\right) \qquad (43)$$

where $\tanh(t - T) = \dfrac{1 - e^{-\gamma(t-T)}}{1 + e^{-\gamma(t-T)}}$ The simulation results are depicted in Figs 4 and 5.

Note that in both cases the errors $e_\rho(t)$ converge to zero asymptotically that is, the rotor synchronizes with the virtual rotor, generated by the reference oscillator.

VII. CONCLUSIONS

We have presented a PID-based controller for switched-reluctance motors based on a model-reference approach and we established global exponential stability of the closed-loop system. Our controller is tailored to go hand in hand with a state observer and to be implemented using certainty equivalence hence, this is a preliminary stage towards the solution of full-sensorless control that is, without measurement of rotor angular positions and velocities. This is under current investigation.

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