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**1 Saturated hydraulic conductivity determined by on**  
**2 ground mono-offset Ground-Penetrating Radar inside**  
**3 a single ring infiltrometer**

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4 **Abstract.** In this study we show how to use GPR data acquired along  
5 the infiltration of water inside a single ring infiltrometer to inverse the sat-  
6 urated hydraulic conductivity. We used Hydrus-1D to simulate the water in-  
7 filtration. We generated water content profiles at each time step of infiltra-  
8 tion, based on a particular value of the saturated hydraulic conductivity, know-  
9 ing the other van Genuchten parameters. Water content profiles were con-  
10 verted to dielectric permittivity profiles using the Complex Refractive In-  
11 dex Method relation. We then used the GprMax suite of programs to gen-  
12 erate radargrams and to follow the wetting front using arrival time of elec-  
13 tromagnetic waves recorded by a Ground-Penetrating Radar (GPR). The-  
14 oretically, the 1D time convolution between reflectivity and GPR signal at  
15 any infiltration time step is related to the peak of the reflected amplitude  
16 recorded in the corresponding trace in the radargram. We used this relation-  
17 ship to invert the saturated hydraulic conductivity for constant and falling  
18 head infiltrations. We present our method on synthetic examples and on two  
19 experiments carried out on sand soil. We further discuss on the uncertain-  
20 ties on the retrieved saturated hydraulic conductivity computed by our al-  
21 gorithm from the van Genuchten parameters.

## 1. Introduction

22 Soil hydraulic properties, represented by the soil water retention  $\theta(h)$  and hydraulic  
23 conductivity  $K(h)$  functions, dictate water flow in the vadose zone, as well as partitioning  
24 between infiltration and runoff. Their evaluation has important implications for mod-  
25 eling available water resources and for flood forecasting. It is also crucial in evaluating  
26 the dynamics of chemical pollutants in soil and in assessing the potential of groundwater  
27 pollution.

28 Soil hydraulic functions can be described by several mathematical expression [*Kosugi*  
29 *et al.*, 2002], among them the van Genuchten function [*van Genuchten*, 1980]. The deter-  
30 mination of the parameters defining the van Genuchten soil water retention function [*van*  
31 *Genuchten*, 1980] is usually done using laboratory experiments, such as the water hanging  
32 column [*Dane and Hopmans*, 2002].

33 The hydraulic conductivity function can be estimated either in the laboratory, or in  
34 situ using infiltration tests. Among the large number of existing infiltration tests [*Angulo-*  
35 *Jaramillo et al.*, 2000], the single [*Muntz et al.*, 1905] or double ring infiltrometers [*Boivin*  
36 *et al.*, 1987] provide the field saturated hydraulic conductivity by applying a positive  
37 water pressure on the soil surface, while the disk infiltrometer [*Perroux and White*, 1988;  
38 *Clothier and White*, 1981] allows to reconstruct the hydraulic conductivity curve, by  
39 applying different water pressures smaller than or equal to zero. For infiltration tests, the  
40 volume of infiltrated water versus time is fitted to infer the soil hydraulic conductivity at or  
41 close to saturation. These tests are time-consuming and difficult to apply to landscape-  
42 scale forecasting of infiltration. Furthermore, their analysis involve various simplifying

43 assumptions, partly due to the ignorance of the shape of the infiltration bulb. This lack of  
44 knowledge on the form of the infiltration bulb has to be filled to get accurate informations  
45 on the soil water retention  $\theta(h)$  function and consequently on hydraulic conductivity  $K(h)$   
46 function. This can be done by water content sensing.

47 Vereecken [*Vereecken et al.*, 2008] and Evett and Parkin [*Evett and Parkin*, 2005] give  
48 a state of the art on the different techniques available for soil moisture measurements.  
49 Among the large panel presented, geophysical methods take an important part, mainly  
50 because they are contact free and/or easy to use. The most commonly used hydro-  
51 geophysical methods are electrical resistivity measurements [*Goyal et al.*, 2006; *Zhou et al.*,  
52 2001] and electromagnetic methods [*Sheets and Hendrickx*, 1995; *Akbar et al.*, 2005]. This  
53 paper focuses on the use of Ground-Penetrating Radar (GPR) as a tool for monitoring  
54 water infiltration in soil.

55 For few decades GPR has been known as an accurate method to highlight water vari-  
56 ation in soils [*Huisman et al.*, 2003; *Annan*, 2005]. Different techniques are available in  
57 the literature for monitoring water content in soils using GPR. Tomography imaging be-  
58 tween boreholes during an infiltration has been done by Binley [*Binley et al.*, 2001] and  
59 Kowalsky [*Kowalsky et al.*, 2005] among others. Many advances were done during the last  
60 years on Off-Ground GPR using full waveform inversion, for instance to invert soil hy-  
61 draulic properties (Lambot [*Lambot et al.*, 2006, 2009] and Jadoon [*Jadoon et al.*, 2012]).  
62 Grote [*Grote et al.*, 2002] and Lunt [*Lunt et al.*, 2005] used two-way travel time variations  
63 from a reflector at a known depth to monitor water content variation with time. Finally,  
64 multi-offset GPR survey techniques, i.e. CMP<sup>1</sup> or WARR<sup>2</sup>, were carried out during infil-

65 tration processes in the works of Greaves [*Greaves et al.*, 1996] or Mangel [*Mangel et al.*,  
66 2012].

67 The work presented here is based on mono-offset monitoring of infiltration with on-  
68 ground surface GPR as related by Haarder [*Haarder et al.*, 2011], Moysey [*Moysey*, 2010],  
69 Lai [*Lai et al.*, 2012], Dagenbach [*Dagenbach et al.*, 2013] and Saintenoy [*Saintenoy et al.*,  
70 2008]. Haarder [*Haarder et al.*, 2011] used a constant offset on-ground GPR coupled with  
71 dye tracing to exhibit preferential flows. They found that a GPR was able to map deep  
72 infiltration comparing to dye tracer, but they did not manage to resolve the infiltration  
73 patterns (by-pass flow, fingering...). Moysey [*Moysey*, 2010] studied the infiltration inside  
74 a sand box from the surface with on-ground GPR. He used the reflection from the wetting  
75 front as well as from the ground wave and the bottom of the box, to monitor the water  
76 content. He also modelled his experiment and estimated the van Genuchten parameters  
77 using semblance analysis. As Léger [*Léger et al.*, 2013], he found that the most poorly  
78 constrained parameter was  $n$ . Lai [*Lai et al.*, 2012] used a joint time frequency analysis  
79 coupled with grayscale imaging to measure infiltration and drainage in controlled con-  
80 ditions in laboratory. They were able to follow the peak frequency of the GPR wavelet  
81 associated with the wetting front using time frequency analysis and then determined the  
82 rate of water infiltration in unsaturated zone. Saintenoy [*Saintenoy et al.*, 2008] mon-  
83 itored the wetting bulb during an infiltration from a Porchet infiltrometer. They were  
84 able to identify the dimension of the bulb with time and good agreement was found with  
85 modelling.

86 On the continuity of those studies, we present a method for monitoring the wetting  
87 front during infiltration using on-ground GPR with fixed offset inside a ring infiltrometer.

88 The objectives of this paper were i) to check if the proposed method is accurate enough to  
 89 monitor wetting front during infiltration with different boundary conditions, ii) to invert  
 90 saturated hydraulic conductivity using the model of Mualem-van Genuchten [*Mualem*,  
 91 1976; *van Genuchten*, 1980] , and iii) to analyze the uncertainties using a simplified MC  
 92 uncertainty analysis. The method has been tested on synthetic examples and on two field  
 93 data sets.

## 2. Background

### 2.1. Unsaturated Flow Equation

94 In this study we consider one-dimensionnal vertical water flow in a soil, described by  
 95 the one-dimensional Richard's equation [*Richards*, 1931]. Its expression in term of water  
 96 content is

$$\frac{\partial \theta}{\partial t} = \frac{\partial K(\theta)}{\partial z} + \frac{\partial}{\partial z} \left[ D(\theta) \frac{\partial \theta}{\partial z} \right], \quad (1)$$

97 where  $K(\theta)$  is the hydraulic conductivity as a function of water content, and  $D(\theta)$  is  
 98 water diffusivity (Childs and Georges-Collis [*Childs and Collis-George*, 1950]), expressed  
 99 in terms of water content as  $D(\theta) = K(\theta) \frac{\partial h}{\partial \theta}$ .

### 2.2. Hydraulic Properties Functions

100 Several mathematical functions exist to model the hydraulic properties of porous me-  
 101 dia [*Kosugi et al.*, 2002]. We chose the van Genuchten model [*van Genuchten*, 1980]  
 102 with the relation of Mualem [*Mualem*, 1976], giving the following expression for the water  
 103 retention curve:

$$\theta(h) = \theta_r + (\theta_s - \theta_r) \left( 1 + (\alpha h)^n \right)^{\frac{1}{n} - 1}, \quad (2)$$

104 where  $\theta_s$  is the saturated water content,  $\theta_r$ , the residual water content, and  $\alpha$  and  $n$ , two  
 105 fitting parameters which are respectively linked to the matric potential and the slope of  
 106 the water retention curve at the inflexion point. The hydraulic conductivity function is  
 107 described by

$$K(\theta) = K_s \Theta^\lambda \left[ 1 - \left[ 1 - \Theta^{\frac{n}{n-1}} \right]^{\frac{n}{n-1}} \right]^2, \quad (3)$$

108 with  $K_s$  the saturated hydraulic conductivity,  $\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r}$  the effective saturation and  $\lambda$  a  
 109 factor that accounts for pore tortuosity. The  $\lambda$  parameter has an influence on the shape  
 110 of the hydraulic conductivity function. However in this study we concentrated on the  
 111 inversion of only one parameter, the saturated hydraulic conductivity. We fixed  $\lambda$  equal  
 112 to 0.5 as reported in [Mualem, 1976].

### 2.3. Petrophysical Relationships

113 Several empirical and conceptual relationships exist to convert soil dielectric permit-  
 114 tivity to volumetric water content. Using the fact that the experiments presented here  
 115 have been made in a quarry of Fontainebleau sand, considered as pure silica, we used the  
 116 CRIM relation [Birchak *et al.*, 1974; Roth *et al.*, 1990], which relates the relative dielectric  
 117 permittivity of bulk media,  $\varepsilon_b$ , to the volumetric summation of each components of it.  
 118 Thus for a tri-phasic medium comprising water, air and silicium, we obtain

$$\sqrt{\varepsilon_b} = \theta \sqrt{\varepsilon_w} + (1 - \phi) \sqrt{\varepsilon_s} + (\phi - \theta), \quad (4)$$

119 where  $\varepsilon_w = 80.1$ ,  $\varepsilon_s = 2.5$  are respectively the relative dielectric permittivity of water  
 120 and silica,  $\phi$  the porosity and  $\theta$  the volumetric water content.



## 2.4. Dielectric Permittivity Versus Electromagnetic Wave Velocity

121 Surface GPR consists in a transmitting antenna, being a dipole, positioned on the  
 122 surface, that emits short pulses of spherical electromagnetic (EM) wave in response to  
 123 an excitation current source, and a receiving antenna, also located at the surface, which  
 124 converts the incoming EM fields to an electrical signal source to be treated. Following the  
 125 works of Annan [*Annan, 1999*], the velocity of electromagnetic waves is

$$v = \frac{c}{\sqrt{\varepsilon' \mu_r \frac{1 + \sqrt{1 + \tan^2 \delta}}{2}}}, \quad (5)$$

126 where  $\delta$  is the loss factor as a function of the dielectric permittivity, frequency and elec-  
 127 trical conductivity,  $\varepsilon'$  is the real part of the relative dielectric permittivity,  $\mu_r$  the relative  
 128 magnetic permeability and  $c$  is the velocity of EM waves in air equal to  $0.3 \text{ m/ns}$ . Con-  
 129 sidering the case of non magnetic soil with low conductivity, in the range of 10 MHz  
 130 to 1 GHz, the real part dominates the imaginary part of the dielectric permittivity and  
 131 neglecting Debye [*Debye, 1929*] effect, equation (5) reduces to:

$$v = \frac{c}{\sqrt{\varepsilon'}}. \quad (6)$$

132 We used this equation to compute the travelling time of an EM wave through a layer of  
 133 soil of known thickness with a given dielectric permittivity.

## 2.5. Electromagnetic Modelling

134 Numerous techniques are available for simulating GPR data, e.g. ray-based methods  
 135 (e.g. Cai and McMechan [*Cai and McMechan, 1995*] or Sethian and Popovici [*Sethian  
 136 and Popovici, 1999*]), time-domain finite-difference full-waveform methods (e.g. Kunz  
 137 and Luebbers [*Kunz and Luebbers, 1996*] or Kowalsky [*Kowalsky et al., 2001*]), or finite  
 138 differences time domain (FDTD) (e.g. Irving and Knight [*Irving and Knight, 2006*]). We

139 used the GprMax 2D codes of Giannopoulos [*Giannopoulos, 2005*], which uses FDTD  
140 modelling to solve the maxwell equations in 2 dimensions.

### 3. Materials and Methods

#### 3.1. Experimental Set-up

141 We studied infiltration of a 5-cm thick water layer inside of a single ring infiltrometer  
142 in a sandy soil. The scheme of the apparatus is presented in Figure 1. The single ring  
143 infiltrometer was a 1-mm thick aluminum cylinder with a 60-cm diameter, approximately  
144 20-cm high, buried in the soil to a depth of 10 cm. GPR antennae (namely the transmitter  
145 T and the receiver R) were set up at a variable distance from the edge of the cylinder,  
146 noted  $X$ , in Figure 1. In all our field experiments, we used a Mala RAMAC system with  
147 antennae centered on 1600 MHz, shielded at the top. The inner part of the cylinder was  
148 covered with a plastic waterproof sheet. This allowed us to fill the cylinder with water  
149 and create an initial 5-cm thick water layer, while preventing infiltration into the sand  
150 before starting data acquisition. The beginning of the acquisition was launched by pulling  
151 away the plastic sheet to trigger water infiltration. The GPR system was set to acquire  
152 a trace every 10 s. With this apparatus, we performed two types of infiltration: i) a  
153 falling head infiltration consisting of pulling away the plastic sheet and leaving water to  
154 infiltrate into the sand freely with no additional refill, and ii) a constant head infiltration,  
155 when water was continuously added to the ring to maintain a 5-cm thick water layer  
156 during the infiltration experiment. In the following examples, we will show that GPR  
157 data acquired every 10 s during the infiltration experiment can be used to estimate the  
158 saturated soil hydraulic conductivity,  $K_s$ . In all GPR data presented below, we subtracted  
159 the average trace and applied an Automatic Gain Control (AGC) to the data in order

160 to make them clearer. The van Genuchten parameters,  $\alpha$ ,  $n$ ,  $\theta_r$ ,  $\theta_i$  of the sand have  
161 been determined in laboratory by several classical hanging water column experiments.  
162 We assumed arbitrarily a 5 % uncertainty for all the measured parameters. The sand was  
163 considered homogeneous. Its initial water content,  $\theta_i$ , and porosity,  $\phi$ , of the soil were  
164 determined using gravimetric measurements on field samples.

### 3.2. Modelling

165 Infiltration experiments were simulated by solving Richards equation (Eq. (1) ) using  
166 Hydrus-1D. The soil profile was 50 cm deep, assumed to be homogeneous, and divided into  
167 1001 layers. We used either an atmospheric boundary condition (BC) with no rain and  
168 no evaporation at the soil surface, for the falling head infiltration, or a constant pressure  
169 head of 5 cm to the top node, for the constant head infiltration, and for both case free  
170 drainage BC at the bottom. To simulate the 5-cm layer of water, the initial condition  
171 was set to a 5 cm pressure head in the top node. We simulated the first 10 minutes of  
172 the experiment with a time step of 10 s, i.e., with 60 water content snapshots. Using  
173 the CRIM relation (Eq. 4), each water content snapshot was converted to permittivity  
174 profiles (made of 1001 points), considering a three-phase media: sand (considered as pure  
175 silica), water, and air. Each one of these permittivity profiles were the input for the  
176 GprMax2D program [Giannopoulos, 2005]. GprMax2D gave simulated GPR monitoring  
177 of the infiltration process. We then picked the maximum amplitude of the signal to get  
178 the Two Way Travel (TWT) time of the wetting front reflection.

### 3.3. Inversion Algorithm

#### 179 3.3.1. Convolution Algorithm

180 Our inversion algorithm was based on the comparison between the arrival times of the  
 181 wetting front reflection observed in the radargrams acquired during the water infiltra-  
 182 tion experiment and the arrival times of these reflections computed from the theoretical  
 183 water content profiles modeled by Hydrus-1D. If a suitable relationship between water  
 184 content and dielectric permittivity is known, water content profiles, obtained by the reso-  
 185 lution of the Richards [*Richards, 1931*] equation (done by Hydrus1D in our case), can be  
 186 transformed to a 2D series of reflection coefficients:

$$R_{i,t} = \frac{\sqrt{\varepsilon_{i+1,t}} - \sqrt{\varepsilon_{i,t}}}{\sqrt{\varepsilon_{i+1,t}} + \sqrt{\varepsilon_{i,t}}}, \quad (7)$$

187 where  $\sqrt{\varepsilon_{i,t}}$  and  $\sqrt{\varepsilon_{i+1,t}}$  are the relative dielectric permittivity at the infiltration time  $t$   
 188 for two successive model cells centered at depth  $z_i$  and  $z_{i+1}$ . The effective depth where the  
 189 reflection coefficient is calculated is  $z_R = \frac{z_i + z_{i+1}}{2}$ . Knowing the dielectric permittivity of  
 190 each layer of the profile, the electromagnetic wave velocity (Eq. 6) and travel time can  
 191 be computed. The travel time is used to interpolate reflection coefficients to a constant  
 192 sampling interval. We used this depth to time conversion to compute a Ricker signal  
 193 in this time interval. The center frequency of the Ricker was set to 1000 MHz, central  
 194 frequency of the GPR signal recorded on the field. We derived it twice with respect to  
 195 time to simulate the transformation made by the emitter and the receiver in real antennae.  
 196 We then performed the convolution between this pseudo-GPR signal and the reflectivity  
 197 to obtain

$$O(t) = R(t) * \frac{\partial^2}{\partial t^2} I(t), \quad (8)$$

198 where  $O(t)$  is the output signal,  $R(t)$  is the reflectivity and  $I(t)$  is the input source of the  
 199 antenna.

200 Some remarks have to be made about the comparison between 1D-temporal convolution  
201 and real electromagnetic signal. First of all, our inversion algorithm is based on the  
202 assumption that soil can be represented as a stack of homogeneous layers. The assumption  
203 of horizontal interfaces forces the reflection coefficient (equation (7)) to be expressed as  
204 a normal incidence case. Secondly, we considered that the 2-D plane waves computed by  
205 FDTD algorithm (modelling) and 3-D plane waves (experiments) could be treated as a 1-  
206 D temporal convolution. Third we neglect relaxation effects occurring when propagating  
207 an electromagnetic wave in water saturated sand.

### 208 **3.3.2. Inversion Procedure**

209 We used the TWT time obtained from the radargram (modelled or experimental) as  
210 data to be fitted to derive the saturated hydraulic conductivity, assuming the other 4  
211 van Genuchten parameters and initial water content were known. Using Hydrus-1D, we  
212 generated 60 water content snapshots using the saturated hydraulic conductivity in the  
213 range from 0.01 to 1 *cm/min*, with a step of 0.001 *cm/min*. For each value of  $K_s$ , we  
214 calculated the TWT time using our convolution algorithm and we computed the Root  
215 Mean Square Error (RMSE) between these times and the data as an objective function,  
216 to be computed as function of saturated hydraulic conductivity. The  $K_s$  value which  
217 corresponds to the minimum of the objective function was used as inverted value.

## **4. Falling Head Infiltration Experiment**

### **4.1. Numerical Example**

#### 218 **4.1.1. Forward modelling**

219 The set of hydrodynamical parameters used for this numerical example is presented in  
220 Table 1. The permittivity profiles, resulting from water content conversions from Hydrus-

221 1D to permittivity and which were used as input of GprMax2D program [*Giannopoulos*,  
 222 2005] are presented in Figure 2-a. The simulated GPR monitoring of the infiltration  
 223 process is shown in Figure 2-b. The horizontal axis is the number of traces simulated  
 224 with GprMax2D, two traces being separated by 10 seconds, as permittivity profiles are.  
 225 The vertical axis is the TWT time of the EM wave coming back to the receiver.

226 On the profile presented in Figure 2-b, we denote one particular reflection, labeled A. Its  
 227 arrival time is increasing as the wetting front moves deeper. This reflection is interpreted  
 228 as coming from the wetting front. The reflections labeled  $A'$  and  $A''$  are primary and  
 229 secondary multiples of reflection  $A$ . The reflection labeled  $B$  is the wave traveling in air  
 230 directly between the two antennae. After the 40<sup>th</sup> trace, the 5-cm layer of water has been  
 231 infiltrated, and drainage is starting. As a consequence, the permittivity of the upper part  
 232 of the medium decreases and the velocity increases (Eq. 6). The TWT time of reflection  
 233  $A$  increases more slowly, creating a change of slope in the reflection time curve (Fig. 2-b).  
 234 In Figure 2-c, we display two curves: the TWT time of the maximum peak of reflection  $A$   
 235 (obtained from Figure 2-b) and the TWT time calculated by the convolution Algorithm.

236 The result of the convolution algorithm is in good agreement with the GprMax2D  
 237 modelling.

#### 238 4.1.2. Inverse Modeling

239 We used the TWT time obtained from the radargram of Figure 2-b as data to be  
 240 fitted to derive the saturated hydraulic conductivity, assuming the other 4 van Genuchten  
 241 parameters and initial water content were known (see Table 1). The RMSE was minimized  
 242 for  $K_s = 0.121 \text{ cm/min}$ , which has to be compared with the value set as input, i.e.,

243  $K_s = 0.120 \text{ cm}/\text{min}$ . This result confirms the ability of our algorithm to invert saturated  
244 hydraulic conductivity.

## 4.2. Field experiment

### 245 4.2.1. Experimental Data and its Analysis

246 The experiment took place in a quarry of Fontainebleau sand in Cernay-La-Ville (Yve-  
247 lines, France). The middle of the antennae was positioned 11 cm away from the cylinder  
248 wall ( $X = 11 \text{ cm}$  in Fig. 1). The 5-cm water layer was fully infiltrated after about 10  
249 minutes, although in certain areas of the soil surface this time has been slightly shorter.  
250 The sand parameters measured by the hanging water column are given in Table 1 and  
251 initial volumetric water content is  $\theta_i = 0.09 \pm 0.01 \text{ cm}^3/\text{cm}^3$ . The recorded GPR data  
252 are shown in Fig. 3. In this profile, we denote three particular reflections. The one  
253 interpreted as coming from the infiltration front, labeled A, is visible during the first 30  
254 minutes of the acquisition, with an arrival time varying from 2 ns down to 9 ns. The other  
255 reflections come from the cylinder and are interpreted in [Léger and Saintenoy, 2012]. We  
256 determined the arrival time of the A reflection peak and inverted the saturated hydraulic  
257 conductivity using the same algorithm as for the synthetic case. We obtained the mini-  
258 mum of the objective function for  $K_s = 0.120 \text{ cm}/\text{min}$ . In parallel, we also carried out  
259 disk infiltrometer experiments, using the multi-potential method [Ankeny et al., 1991;  
260 Reynolds and Elrick, 1991]. We obtained a value of the saturated hydraulic conductivity  
261 of  $K_{Disk} = 0.108 \pm 0.01 \text{ cm}/\text{min}$ .

### 262 4.2.2. Uncertainty Analysis

263 We attempted to evaluate the uncertainty in the saturated hydraulic conductivity re-  
264 trieved from GPR data fitting by using a modified Monte Carlo method. We qualified this

method as “modified Monte Carlo” in the sense that it is nor the Tarantola method [Tarantola, 1987] and neither the adaptive method proposed by the Guide to the expression of uncertainty in measurement [BIPM et al., 2011] published by the Joint Committee for Guides in Metrology (JCGM). We consider five major uncertainty sources, four from the van Genuchten parameters,  $\alpha$ ,  $n$ ,  $\theta_r$ ,  $\theta_s$  and one from the initial water content  $\theta_i$ . We do assume that all uncertainties can be described by gaussian distribution probability function centered on the value found by several water hanging column experiments with a standard deviation of 5 % of this value. With this definition we obtained the following set of a priori density function for experimental case:  $\mathcal{N}_\alpha(\alpha^\mu = 0.023 \text{ cm}^{-1}, \alpha^\sigma = 0.001 \text{ cm}^{-1})$ ,  $\mathcal{N}_n(n^\mu = 6.7, n^\sigma = 0.3)$ ,  $\mathcal{N}_{\theta_r}(\theta_r^\mu = 0.062 \text{ cm}^3/\text{cm}^3, \theta_r^\sigma = 0.001 \text{ cm}^3/\text{cm}^3)$ ,  $\mathcal{N}_{\theta_s}(\theta_s^\mu = 0.39 \text{ cm}^3/\text{cm}^3, \theta_s^\sigma = 0.01 \text{ cm}^3/\text{cm}^3)$ , and  $\mathcal{N}_{\theta_i}(\theta_i^\mu = 0.09 \text{ cm}^3/\text{cm}^3, \theta_i^\sigma = 0.01 \text{ cm}^3/\text{cm}^3)$ , where the  $\mathcal{N}$  stands for the gaussian/normal probability density function and the  $\mu$  and  $\sigma$  represent the mean and standard deviation. We generate multiple sets of parameters by sampling each gaussian distribution,  $\{\alpha^i, n^i, \theta_r^i, \theta_s^i, \theta_i^i\}$ , where the subscript “ $i$ ” is the iteration number. For each set the value of  $K_s$  minimising the objective function was computed by our inversion procedure presented above. We generated enough sets of parameters such as the histogram of  $K_s$  values look like a gaussian function with a stabilized standard deviation. We used this standard deviation as uncertainty on  $K_s$ .

We did not consider the uncertainties on radargram picking, because we evaluated it has a very weak influence comparing to the other uncertainties considered.

Using our analysis, we found in the case of falling head infiltration that  $K_s$  was equal to  $0.12 \pm 0.01 \text{ cm}/\text{min}$ . This narrow range of possible values is in agreement with disk infiltrometer value, and clearly shows the accuracy of our method.



## 5. Constant Head Infiltration Experiment

### 5.1. Numerical Example

#### 5.1.1. Forward Modelling

In this second case, a water layer of 5 cm above the ground was kept constant during the entire experiment. Similarly as above, using the same van Genuchten parameters as in the first synthetic example (Table 1), we modeled infiltration of water inside a ring infiltrometer by applying a constant pressure head of 5 cm to the top node during 10 minutes. The permittivity profiles are presented in Fig. 4-a, with each curve plotted every 10 s as in the previous case. Fig. 4-b shows the radargram simulated with GprMax2D. As can be seen, the reflection labeled *A* describing the position of the infiltration front, is returning at increasing times, because infiltration is being constantly fed by the constant ponding depth, contrary to the previous falling head case. In Fig. 4-c, we computed the TWT time of the wetting front using the convolution algorithm and picking the *A* reflection from the radargram in Fig. 4-b.

#### 5.1.2. Inverse Modelling

We inverted for the saturated hydraulic conductivity by minimizing the differences between the arrival times of the wetting front reflection obtained by the convolution algorithm and the arrival times picked from the simulated radargram in Fig. 4-b. The objective function was minimized for  $K_s = 0.119 \text{ cm}/\text{min}$ , to be compared with the value used for simulating the data:  $K_s = 0.120 \text{ cm}/\text{min}$ .

## 5.2. Field Experiment

The experiment took place in the same quarry of Fontainebleau sand as the previous experiment. The middle of the antennae was positioned in the middle of the ring ( $X = 30$

308 cm in Fig. 1). The GPR data are shown in Fig. 5 and were recorded during 80 minutes  
 309 (only a part of the radargram is presented). We used the van Genuchten parameters  
 310 determined in the laboratory using the hanging column experiments (Table 1) and we  
 311 measured on sand core samples an initial volumetric water content of  $\theta_i = 0.07 \pm 0.02$ .

312 In the profile presented in Fig. 5, the arrival time of reflection  $A$  ranges from 0 at the  
 313 beginning of the experiment to about 6 ns after 10 min. We picked the arrival time of the  $A$   
 314 reflection peak and computed the objective function using the same procedure as described  
 315 before. We obtained the minimum of the objective function for  $K_s = 0.089 \text{ cm}/\text{min}$ .  
 316 Again, this value has to be compared with the one obtained by the disk infiltrometer  
 317 experiment,  $K_{Disk} = 0.108 \pm 0.01 \text{ cm}/\text{min}$ . Using the same procedure as presented in  
 318 the earlier field example, we found a range of possible values for the saturated hydraulic  
 319 conductivity,  $K_s = 0.089 \pm 0.005 \text{ cm}/\text{min}$ . Despite the fact that we are not in the same  
 320 range as the disk infiltrometer method the discrepancy is very small and allows us to  
 321 conclude on the good accuracy of our method.

## 6. Discussion

322 The results presented above indicate clearly that a commercial surface GPR can be used  
 323 as a tool for monitoring the wetting front. Although the use of surface-based GPR data  
 324 to estimate the parameters of unsaturated flow models is not new [*Moysey, 2010*], our  
 325 method gives accurate values of the saturated hydraulic conductivity with uncertainties  
 326 comparable or smaller than those obtained with disk infiltrometer measurements. A dis-  
 327 tinct advantage of our approach is the simplicity of the algorithm and its rapidity to con-  
 328 verge, which is very encouraging for more complicated models ( stack of non-homogeneous  
 329 layers).

330 The discrepancy between saturated hydraulic conductivity determined by disk infil-  
331 trometry and that obtained with our GPR algorithm comes from different phenomena.  
332 First of all, the van Genuchten parameters determined from the water hanging column  
333 experiment are obtained with saturation coming from the bottom of the soil samples,  
334 whereas in our case, the infiltration is a ponded one, thus coming from the top.

335 Despite the fact that we upgraded the single ring infiltrometer by the use of GPR to  
336 monitor the wetting front, we still suffer from the problem of entrapped-air, which causes  
337 reduction of saturated water content and hydraulic conductivity. This issue cannot be  
338 fixed with ponded infiltration. Disk infiltrometer measurement monitoring may cause less  
339 problems, working with negative matric potentials [*Ankeny et al.*, 1991; *Reynolds and*  
340 *Elrick*, 1991].

341 During our modeling, we considered our soil as an homogeneous and isotropic one. Real  
342 soils exhibit heterogeneities, triggering preferential flows. Even in the case of our quarry  
343 of Fontainebleau sand, differences in packing and compaction could lead to creation of  
344 preferential flow paths.

345 One of the way to solve this issue could be to use a dual porosity model [*Gerke and van*  
346 *Genuchten*, 1993] and a Monte Carlo procedure to generate a high number of soil models  
347 with different parameters, as we did with the single porosity model in Hydrus-1D, and  
348 performed statistical analysis on the saturated hydraulic conductivity obtained.

349 An other source of error, already discussed above, comes from the assumption that a  
350 3D infiltration monitored by 3D electromagnetic waves can be treated as a 1-D temporal  
351 convolution. This limitation will be studied in future works, using Hydrus 2D/3D to  
352 simulate 2D axisymmetrical infiltration and 2D infiltration.

353 The results represent a promising step toward application of multi-parameters inver-  
354 sions. A first study in that direction was presented in Léger [*Léger et al.*, 2013].

## 7. Summary

355 This research investigated the use of on-ground surface GPR to monitor the wetting  
356 front during infiltration inside a ring infiltrometer. We showed by modeling and exper-  
357 iments that a standard GPR device was able to monitor the displacement of the water  
358 front in the soil. We tested in synthetic cases the ability of our algorithm to invert the  
359 saturated hydraulic conductivity, knowing the other van Genuchten parameters and the  
360 initial water content. Two infiltration experiments were performed, falling head infiltra-  
361 tion and constant head infiltration, in a quarry of Fontainebleau sand. The retrieved  
362 saturated hydraulic conductivity was comparable to that obtained with disk infiltrometer  
363 experiments. Uncertainty analysis accounting for all the van Genuchten parameters, was  
364 performed using a modified Monte Carlo method, and proved the robustness of our al-  
365 gorithm. Although results retrieved with GPR were in agreement with disk infiltrometry  
366 tests, we stress that further research is needed to improve our algorithm so as to determine  
367 the whole set of soil hydrodynamic parameters.

## Notes

- 368 1. Common MidPoints
2. Wide-Angle Reflection- Refraction

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Falling Head Infiltration							
	$\theta_i$	$\theta_r$	$\theta_s$	$\alpha$ ( $\text{cm}^{-1}$ )	$n$	$K_s$ ( $\text{cm}/\text{min}$ )	Retrieved $K_s$ ( $\text{cm}/\text{min}$ )
Numerical	0.17	0.07	0.43	0.019	8.67	0.120	0.121
Field	$0.09 \pm 0.01$	$0.062 \pm 0.003$	$0.39 \pm 0.01$	$0.023 \pm 0.001$	$6.7 \pm 0.3$	$0.108 \pm 0.01^*$	$0.120 \pm 0.013$
Constant Head Infiltration							
	$\theta_i$	$\theta_r$	$\theta_s$	$\alpha$ ( $\text{cm}^{-1}$ )	$n$	$K_s$ ( $\text{cm}/\text{min}$ )	Retrieved $K_s$ ( $\text{cm}/\text{min}$ )
Numerical	0.17	0.07	0.43	0.019	8.67	0.120	0.119
Field	$0.07 \pm 0.02$	$0.062 \pm 0.003$	$0.39 \pm 0.01$	$0.023 \pm 0.001$	$6.7 \pm 0.3$	$0.108 \pm 0.01^*$	$0.089 \pm 0.009$

**Table 1.** Hydrodynamic parameters for the numerical and field experiment. The \* indicates values of  $K_s$  measured from disk infiltrometer experiments.

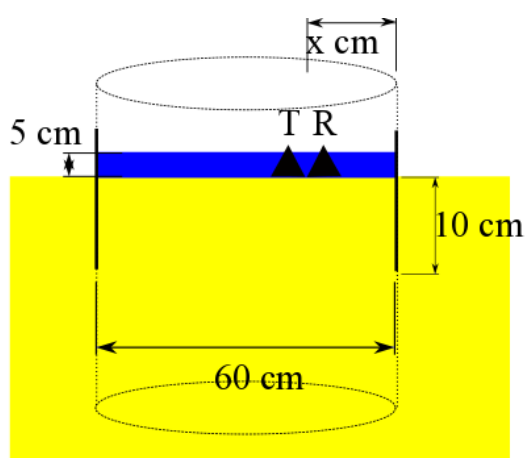
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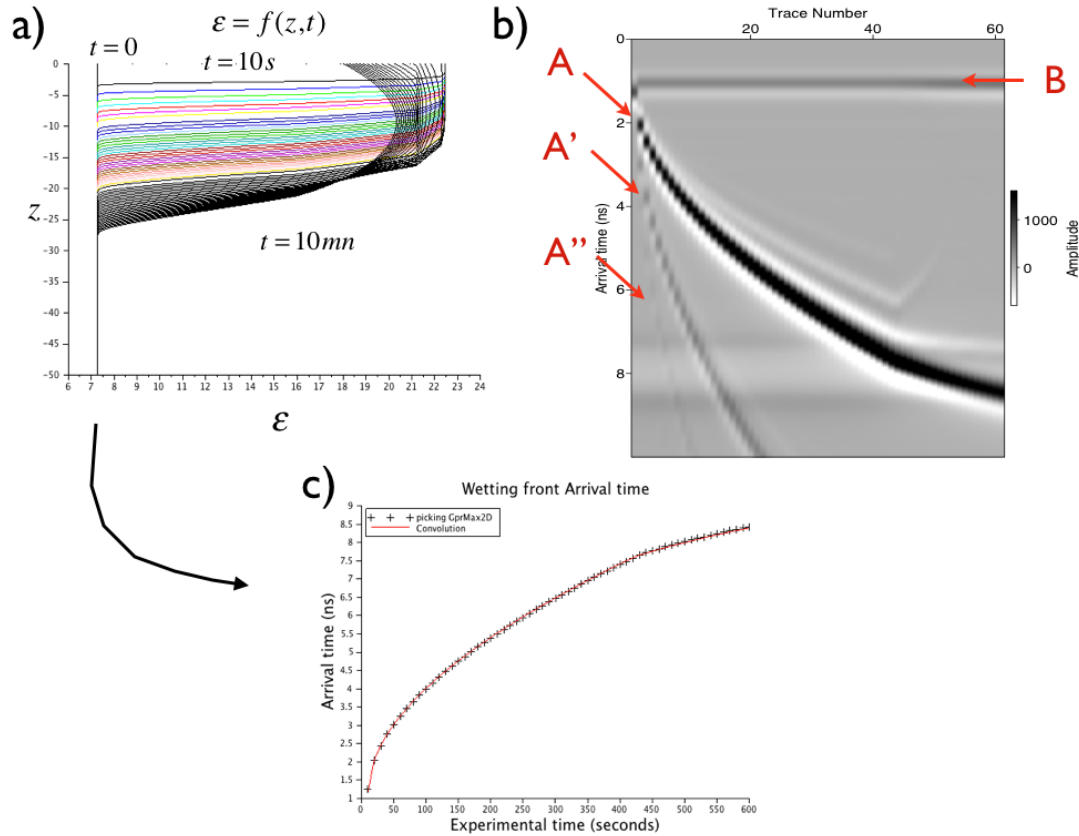
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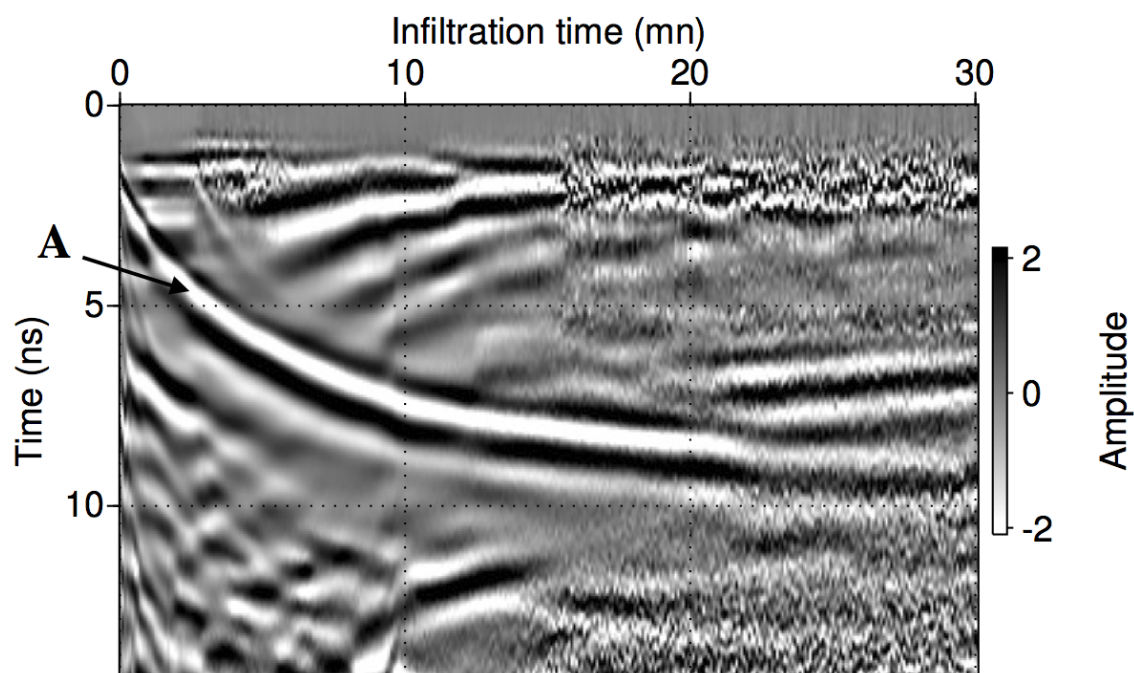
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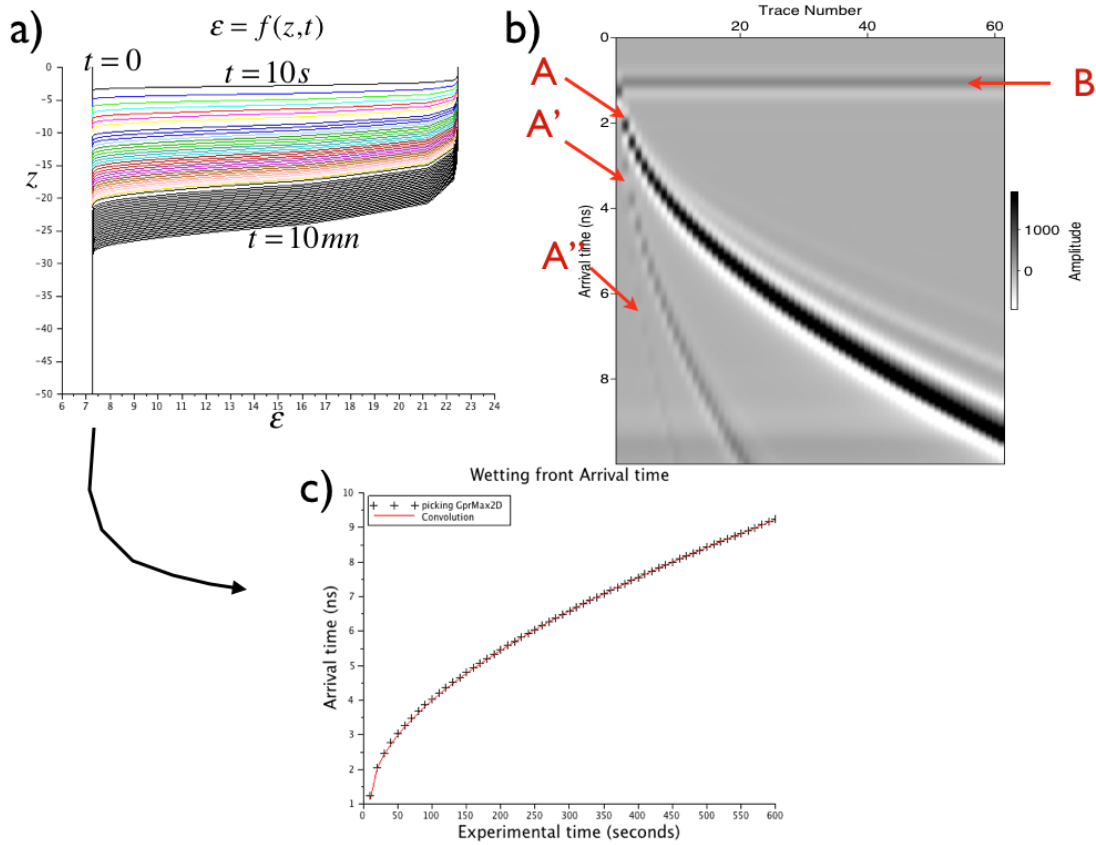
**Figure 1.** Experimental set up at its initial state.



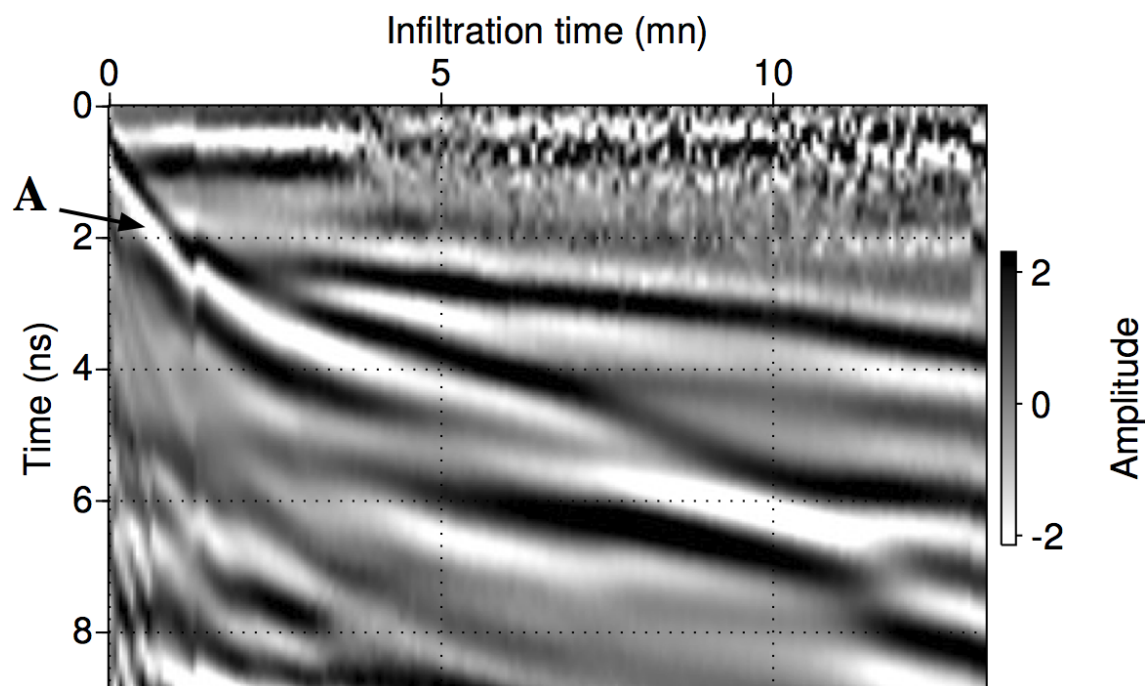
**Figure 2.** Falling head infiltration from a 5-cm thick water layer. a) Permittivity profiles: each curve is plotted every 10 s. b) Radargram simulated with GprMax2D; reflection A is coming from the wetting front, B is the direct wave, A' and A'' are multiples of reflection A. c) TWT time computed by the convolution algorithm from the permittivity profiles (plain red line) and TWT time obtained by picking of A peak in fig b).



**Figure 3.** Experimental GPR data acquired during the falling head infiltration (using a 5-cm initial water layer). Reflection A is the reflection coming from the wetting front



**Figure 4.** Constant head infiltration with 5 cm of water. a) Permittivity profiles, each curve is plotted every 10 s. b) Radargram simulated with GprMax2D, reflection A is the wetting front, B is the direct wave, A' and A'' are multiples. c) Two Way Travel Time computed with our convolution algorithm from the simulated permittivity profiles.



**Figure 5.** GPR data acquired during a constant head (5 cm) infiltration. Reflection A is the reflection coming from the wetting front.