

# Logical Outline

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Assuming sufficient equations, resolve all  $D$ -operators as per  $n^{th}$ -Differentials into variables  $f_i$ . Make use of backward substitution of all available equations/conditions. Once done, one will arrive at the form

$$y\phi^1(\vec{x})_i + y^2\phi^2(\vec{x})_i + .. + y^m\phi^n(\vec{x})_i \quad (1)$$

where we have made the substitutions  $(y^n, \dots, y) \rightarrow \left(\frac{\delta^n f}{\delta x^n}\right), \dots, \frac{\delta f}{\delta x}$ , or  $y \rightarrow f^1$  for a single variable  $f$  as resolved to.

The Proof is quite simple: Make the substitutions  $y_i^n \rightarrow \left(\frac{\delta f}{\delta x}\right)_i^n$  as above in the former sense, we then have the system:

$$G_1(y^n, \dots, y, \vec{x}) = 0, G_2(y^n, \dots, y, \vec{x}) = 0, \dots, G_i(y^n, \dots, y, \vec{x}) = 0 \quad (2)$$

Here, the coefficient functions  $\Phi_i(\vec{x})$  span over  $\vec{x}$  and  $y$  is what we aim to solve outside of this context.

After backward substitutions on the above series of sufficient equations, we define the maximal polynomial solution to be  $P(\Phi)$ :  $Max(\Phi_j) = 0 \forall i$  with  $Max(\Phi_j) := |y\phi^1(\vec{x})_i + y^2\phi^2(\vec{x})_i + .. + y^m\phi^n(\vec{x})_i|$  using  $y^n, \dots, y$ . This equation above will always have a non infinite solution for finite values for  $\phi_i(\vec{x})$  due to the F.T.A, and forcing an asymptotic solution at any point makes for discontinuity as pointed out. The argument pertaining to trend-lining the solution as an asymptotic one, has the effect of pointing out the mechanical failures that arise in the solution should the polynomial condition exist.

However to completely resolve the matter of continuity, one needs to show that solutions to the base polynomial from  $\vec{x} \rightarrow \overrightarrow{x+h}$  has a continuous differential change in  $y, y+p$ .

A key point to note here is that, for any single  $x$ , the metric  $ds$  in:

$$ds^2 = (y^n\phi_m(\vec{x}_k + h) + .. + \phi_0(\vec{x}_k + h))^2 - (y^n\phi_m(\vec{x}_j) + .. + \phi_0(\vec{x}_j))^2 \quad (3)$$

is unrestricted and can be made as small as desired. With the solution to the Lagrange-Minimal of  $ds^2$  as above; being the link to h solution, regardless of the changes in  $(\vec{x}, t)$

Analysis of the polynomial (1), with smooth continuous functions  $\Phi_i(\vec{x})$  for *coefficients*, the analysis of continuity and smoothness would follow along the following lines.

For every single  $h$ ,  $Max(\Phi_i)(\overrightarrow{x+h}) = 0$  will have a solution. Furthermore, the base is overall increasing over the next iteration or decreasing. Given this,

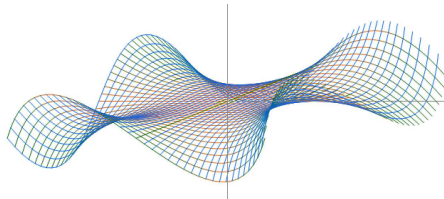
<sup>1</sup>With the former aimed at making the argument in the continuous sense.

$y$  increases or decreases, meaning that in the next two iterations we will have for instance  $y < y + p < y + 2p$ . One can vary  $h$  in order to make  $p$  as small as desired, and as there is no restriction here and as such continuity is implied. To see this more clearly refer to F.T.A proof by Gauss(Version 1) or, R.Iyer. Subtle changes to  $\Phi_i$  with  $h$  will vary the coefficients differentially(and thus the associated surfaces in the way of deformation from its previous iteration state), which will in turn vary  $p$  in a continuous manner as the surfaces will deform differentially with  $h$ . Additionally as the coefficients are continuous surfaces, the intersection solution points will additionally vary differentially. So as  $\Phi_i(\overrightarrow{x + \vec{h}})$  varies differentially, all  $r(\text{Cos}(\theta) + i\text{Sin}(\theta))_{ij}$  varies differentially, and thus the intersection points of:

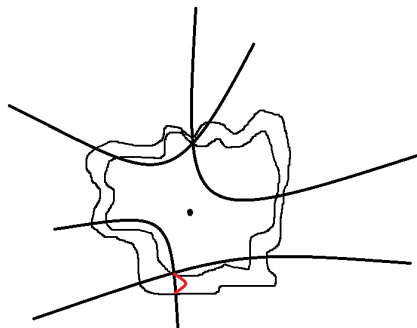
$$\begin{aligned} \text{Re}([\lambda]I(\Phi_i(\overrightarrow{x + \vec{h}}))) &= \text{Re}([\lambda]Q(\Phi_i(\overrightarrow{x + \vec{h}}))) \\ \text{Im}([\lambda]I(\Phi_i(\overrightarrow{x + \vec{h}}))) &= \text{Im}([\lambda]Q(\Phi_i(\overrightarrow{x + \vec{h}}))) \end{aligned}$$

respectively(with variables  $[\lambda, \dots, \lambda^n]$ ), and so will the  $(r, \theta)$  pairs that solve the above simultaneously. With little effort one can see how this works upon analysis of this intersection, which varies differentially for each differential change in the coefficients, should these be differentiable and continuous, and thus  $h$  can be made as small as desired in the way of making  $p$  as small as desired. There are no restrictions on the limit.

The continuous changes to the coefficients changes differentially the form of:



This will in turn vary the intersection points differentially and continuously.



Applying the **Sequence criterion for the Derivative** argument will aide in proving the differentiability of the variation of the intersection point.

With differential changes in  $x + dx, y + dy, z + dz$  (as expressed by  $\frac{\delta}{\delta t}(Re, \Phi_i) +$

$\nabla(\text{Re}, \Phi_i)$

It all ultimately comes down to a single polynomial type D-operator equation that needs solving with  $\Phi_i(x(t), y(t), z(t), \dots)$  as coefficients at various  $x, y, z, \dots$ . Naturally where,  $\Phi_i$  differentiable, the following arguments can be made: Note the aim here is to build a platform capable of continuity arguments of the form; if,  $\Phi_i(x + f, y + g, z + h) - \Phi_i(x, y, z)$  can be made arbitrarily small, so too will  $(y_2 - y_1)$  in relation.

If  $\Phi_i$  is continuously partially differentiable so too is  $\Phi_i(x, y, z) \rightarrow \rho(r, \theta)$ , and true in addition for fixed  $\theta$ . Here I would argue that this differentiability extends to the variation of the simultaneous intersection as described previously.

Many cases including those N.S, lead to a single linear D-Operator. I.e., polynomial of degree 1. In such cases the same technique can be applied, with the intersection visualized as by Gauss' proof V1.

The *path* along which the intersection point varies, does so with change in  $\Phi_i(x, y, z, t)$  and function domain type. The *path* can be expected to exhibit predictable behaviour with regards to actual types<sup>2</sup> of  $\Phi_i$ .

Studying the geometry of the  $\Phi_i$  will be of great value.

$$\frac{\partial \Phi_i}{\partial x_j} \tag{4}$$

A simple means of achieving this and a means of analysing the movement of solution points over the  $r, \theta$  Plane. As and when necessary one can represent  $x, y, z, \dots$  as functions of  $r, \theta$ .

Let all positive terms of (1) as  $+U$ , and the rest as  $-U$ . Let us denote each term in (1) as  $E_i$ . To honestly save on time expressing these thoughts, and for simplicity sake, let us denote  $E_1$  simply as 1, etc.

$$1_E = \frac{1}{2 + 3 + 4 + 5 + \dots + i} \tag{5}$$

and in general:

$$i_E = \frac{i}{2 + 3 + 4 + 5 + \dots} \tag{6}$$

These expressions tell us the influence exhibited by each term in the equation by the influence it has on the shapes of  $\text{Re}(1)$  and  $\text{Im}(1)$ .

Aside from the 'constant', i.e., the first term in (1), the other coefficient expressions  $\Phi_i$  can only affect/deform the shape of  $\text{Re}, \text{Im}$  outside of  $\theta$  coinciding with  $\pi, \pi/2n$ . We want to see the solution points of (1) behave somehow erratically, and the conditions necessary for this to occur, are, over some  $r, \theta$  connected region around a solution point for fixed  $\Phi_i$ , is if the shapes of  $\text{Im}, \text{Re}$  deform upward and thereafter downward for a few cycles. These graphs can't deform sideways :).

Sideways expansion can only occur with changes in the 'constant' expression.

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<sup>2</sup>Exponential, Transcendental...

This will only occur at regions where  $\Phi_i$  are dominant right at the point of intersection. We are not in anyway interested in exactly where this is, instead, enforcing the deformation necessary at this juncture.

Let

$$F = \frac{+E}{|-E|} \quad (7)$$

The solution points will be erratic for a few cycles iff  $F$  fluctuates over  $0 < m \leq 1, n > 1$  in  $\{m < F < n|m, n \in \mathbb{R}\}$  at/around  $(r, \theta)$  values forming the region  $R$  where the solution is found. I.e., vary the values of  $\Phi_i$  alone, around this solution region, and the deformation caused will vary the solution point around  $R$ . Noting the dominant term within  $\phi_i$  at or around  $R$  will let you know the solution influencing term in  $\Phi_i$  via the potential to deform, again one can see that  $\Phi_{I,E} I$  will have to be erratic to produce an erratic solution movement. This way of thinking will take a bit of bravery due to the many complex moving parts.

An interesting thing to note, and a trivial result of this calculus, is that the coefficient function parameters in N.S too, will need be oscillatory/erratic to effect an erratic/turbulent solution/result. Therefore N.S is likely stable for stable coefficient parameters.

## 1 Some final remarks and a Lemma

$D$ -Operators are expressed as the variable associated with the polynomial being studied and is in polar  $r, \theta, \rho$ .  $\Phi_i$  coordinate system is agnostic.

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} = 0 \quad (8)$$

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial[\rho u_i u_j]}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + \rho f_i \quad (9)$$

$$\frac{\partial(\rho e)}{\partial t} + (\rho e + p) \frac{\partial u_i}{\partial x_i} = \frac{\partial(\tau_{ij} u_j)}{\partial x_i} + \rho f_i u_i + \frac{\partial(\dot{q}_i)}{\partial x_i} + r \quad (10)$$

The Einstein summation convention dictates that: When a sub-index (here  $i$  or  $j$ ) is twice or more repeated in the same equation, one sums across the n-dimensions. This means, in the context of Navier-Stokes in 3 spacial dimensions, that one repeats the term 3 times, each time changing the indice for one representing the corresponding dimension (i.e., 1, 2, 3 or  $x, y, z$ ). Equation 1 is therefore a shorthand representation of:  $\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_1)}{\partial x_1} + \frac{\partial(\rho u_2)}{\partial x_2} + \frac{\partial(\rho u_3)}{\partial x_3} = 0$ . Equation 2 is actually a superposition of 3 separable equations which could be written in a 3-line form: one line equation for each  $i$  in each of which one sums the three terms for the  $j$  sub-index.

### 1.1 Classic $\rightarrow, \otimes, \nabla$ notation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0 \quad (11)$$

$$\frac{\partial(\rho \vec{u})}{\partial t} + \vec{\nabla} \cdot [\overline{\rho u \otimes u}] = -\vec{\nabla} p + \vec{\nabla} \cdot \vec{\tau} + \rho \vec{f} \quad (12)$$

$$\frac{\partial(\rho e)}{\partial t} + \vec{\nabla} \cdot ((\rho e + p) \vec{u}) = \vec{\nabla} \cdot (\vec{\tau} \cdot \vec{u}) + \rho \vec{f} \cdot \vec{u} + \vec{\nabla} \cdot (\vec{q}) + r \quad (13)$$

## 2 $n^{Th}$ -Differentials

We start with:

$$f''_{\Delta}(x) = \frac{f(x+2h) - 2f(x+h) + f(x)}{\Delta h^2} = \frac{\frac{f(x+2h)-f(x+h)}{h} - \frac{f(x+h)-f(x)}{h}}{\Delta h} \quad (14)$$

Considering  $f(x+2h), 2f(x+h), f(x)$  as separate variables  $f_2, 2f_1, f_0$  respectively, we have:

$$f''_{\Delta}(x) = \frac{f_2 - 2f_1 + f_0}{\Delta h^2} \quad (15)$$

As every subsequent derivative makes use of the change  $x, x+h$ , using the above; we can generalize to form:

$$\sum \frac{f_{n,ih}}{\Delta h^{2n}} = \frac{f(x+nh) + .. + (m - (k-1))(-1)^{k-1} f(x + (n - (k-1))h) + ..}{\Delta h^{2n}} \quad (16)$$

all the way to  $(-1)^{k-1} f(x)$ , the final term. Where in the above  $m, k$  are the number of terms and the  $k$ -th term from the left respectively.

Using this beautiful formula, we can replace the  $n$ -th differential with:

$$\frac{f_n + (m-1)f_{m-1} + .. + (m - (k-1))(-1)^{k-1} f_{m-(k-1)} + ..}{\Delta h^{2n}} \quad (17)$$