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Numerical predictions of the behaviour of soft clay with two anisotropic elastoplastic models Paulo J. Venda Oliveira^{1,*} and Luís J.L. Lemos^{2,*}

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ABSTRACT

The numerical predictions obtained with the Melanie and MIT-E3 models are compared. Firstly, the performance of the constitutive models is checked against undrained triaxial tests. The models are then used to replicate the behaviour of an embankment built on soft clay. The numerical results are compared with the field data in terms of settlements, lateral displacements and excess pore pressures. Additionally, the numerical predictions are also analysed in terms of yield area, contours of vertical effective stresses, horizontal effective stresses and shear stresses and in terms of effective stress paths.

Keywords: Mit-E3 model, Melanie model, embankment on soft clay, Boston Blue Clay, numerical modelling, anisotropy.

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1. Introduction

The wave of urban expansion currently being experienced in developed countries, especially those with fast growing economies, is often associated with the creation of extensive embankments. These are needed for the construction of residential and/or industrial areas and modern motorways and railways. The shortage of land on the outskirts of large cities has led to land being occupied in recent decades which had been overlooked because of its poor geotechnical characteristics, namely high compressibility and/or low strength.

To make sure that embankments on this type of geotechnical formation are constructed safely, mathematical models are used to help predict the behaviour of these building works. In most cases, the finite elements method is used, and soil behaviour is reproduced by means of constitutive models, which usually assume that soil is an isotropic material. However, with the exception of some overconsolidated soils, the generality of the soils show anisotropic behaviour, both in strength and in stiffness. Thus, the constitutive models based on isotropic conditions, like the Modified Cam Clay (MCC) model, are not very suitable to replicate the real behaviour of natural clays.

In order to improve the numerical predictions, several constitutive laws have been proposed to model the anisotropic behaviour of natural soils. Some authors have proposed anisotropic constitutive models based on critical state soil mechanics [1], essentially adjusting the MCC model for anisotropic conditions. The Melanie (MEL) model [2, 3] considers a non-associative flow rule. The S-Clay1 model introduces a rotational yield surface and a rotational hardening rule [4, 5]. The S-Clay1S adds on the previous model to take into account the de-structuring of the soil [6] while others consider a combination of isotropic and kinematic hardening [7]. Some constitutive models describe the clay behaviour with a bounding surface plasticity considering the initial and the induced anisotropy, such as the MIT-E3 model [8-10] and the model presented by Anandarajah and Dafalias [11]. Constitutive models based on the concept of scaled memory [12] and using multi-yield-surface plasticity [13] have also been proposed.

This paper compares the performance of two elastoplastic anisotropic models, the simpler MEL model, and the more complex and sophisticated MIT-E3 model, which can simulate several characteristics of natural clays.

Initially, the capabilities and limitations of the two models to replicate the response of undrained triaxial tests in terms of stress paths and stress-strain behaviour is analysed. The aim of this analysis is to show clearly, if the inclusion of important behaviour characteristics of the soils, in addition to strength anisotropy, contributes to the improvement of the numerical results. Some of these characteristics are included in the MIT-E3 model, namely [8]: (*i*) non-linear elastic behaviour for small strain levels; (*ii*) the generation of plastic strains in unload-reload cycle; (*iii*) a smooth transition between the normally consolidated state to an overconsolidated state.

The ability of the two models to replicate the behaviour of a constructed embankment is then analysed. The embankment studied, located on the I-95 motorway near Boston, was previously studied using two isotropic soil models [14]. Another 11m high embankment, with a different geotechnical profile of the soil foundation on the same motorway has also been analysed with MCC and MIT-E3 models [15].

In order to show the diferences between the MEL and MIT-E3 models in the numerical predictions of the behaviour of an embankment, the settlements, the horizontal displacements and the excess pore pressures obtained from the two models are compared with each other and with the field data. Special attention is paid to the evolution of the maximum horizontal displacements with the settlement and to the variation of excess pore pressures with the height of the embankment. The analyses of the yield domain and the contours of the effective vertical and horizontal stress

increments is particularly important, as this helps the numerical results obtained from the two elastoplastic models to be understood.

This work is carried out using a two-dimensional finite element (FE) code, upgraded in the University of Coimbra with several constitutive models [16], enabling elastoplastic analyses with coupled consolidation to be made.

2. Main features of the constitutive models

The MEL and MIT-E3 are elastoplastic models, thus their behaviour can be separated into elastic and plastic components. The elastic component is calculated by means of the generalized Hooke's law, while the plastic component is found by using the incremental theory of plasticity. This requires the definition of a yield function, hardening laws, a failure criterion and a function of plastic potential.

2.1. Melanie model

In order to contemplate the anisotropic behaviour, the Melanie model considers an ellipse oriented in the direction of the K_0^{nc} line as yield criterion (Figure 1). This yield criterion, which generates a significantly different behaviour in triaxial compression and extension, is represented on the p'-q plane by [2]:

$$F(p',q,K) = A^2 \left(p'.\cos\theta + q.\sin\theta - \frac{K}{AC} \right)^2 + B^2 \left(q.\cos\theta - p'.\sin\theta \right)^2 - \frac{K^2}{C^2} = 0$$
(1)

with:

$$\theta = \operatorname{arctg}\left[\frac{3\left(1 - K_0^{\operatorname{nc}}\right)}{1 + 2K_0^{\operatorname{nc}}}\right]$$
(2)

$$A = 2\left(\sin\theta + \cos\theta\right) \tag{3}$$

$$B^{2} = \frac{A^{2} \cos \theta}{C \sin^{2} \theta} \cdot \left(\frac{2}{A} - C \sin \theta\right)$$
(4)

where C is a constant with value 0.6, and the hardening parameter, K, is given by:

$$K = \frac{\mathbf{p'_c}}{2} \mathbf{A.C.} (\cos\theta + \mathrm{tg}\theta.\sin\theta)$$
(5)

The MEL model uses a non-associated flow rule, expressed by the orientation of the plastic strain vector (\boldsymbol{e}^{p}) with the bisector direction between the normal to the ellipse and the straight line that joins the origin and the plastification point (Figure 1). This flow rule was established in order to take into account the experimental findings obtained with soft soils [2, 3].

The combination of the Mohr Coulomb criterion (MC) and MEL model gives the latter a failure criterion that confirms experimental results [3]. Figure 1 shows the composite model with the two yield criteria. Inside the yield surfaces of the MEL/MC model the behaviour is linear elastic. The MC criterion is a boundary beyond which irreversible strains arise, thereby reducing the elastic domain. Thus, the elastic domain does not occupy the whole of the ellipse of the MEL model and is dependent on the shear strength characteristics of the soil, c' and ϕ' [17]. The MC criterion does not show hardening and considers an associated flow rule. This type of flow rule leads to dilatancy which is unrealistic for high plasticity soils but may be present in silty and soft sandy soils.



Fig. 1. Representation of Melanie model associated with Mohr-Coulomb criterion in p'-q and ε_v - ε_q planes [2].

2.2. MIT-E3 model

The MIT-E3 model is an anisotropic model appropriate for predicting the behaviour of clays with an overconsolidation ratio (OCR) of less than eight [9]. This model describes the behaviour of a normally consolidated soil by means of a yield surface with the form of an ellipsoid initially oriented in the consolidation direction (Figure 2), and written as:

$$F = -c^{2}p'(2\alpha'-p') + \sum_{i=1}^{5} (\{s_{i}\} - p'\{b_{i}\})^{2} = 0$$
(6)

where c is the ratio of the semi-axes of the ellipsoid, α ' controls the size of the yield surface, {s} is the deviatoric stress tensor and {b} is a second order tensor describing the orientation of the yield surface [8]. The stress state corresponding to the normally consolidated soil is represented by the point V in Figure 2.

The failure criterion coincides with the critical state criterion, and is defined by a conical surface whose vertex is located at the origin (Figure 2). This criterion is defined by the friction angles obtained in triaxial compression and extension tests (ϕ'_{TC} , ϕ'_{TE}) for the critical state conditions, which means large strain conditions. The MIT-E3 model

uses a non-associated flow rule that allows the K_0 and critical state conditions to be satisfied, so that when the stress state approaches the failure cone the energy dissipated is completely consumed by the shear strain, with null volumetric strain.



Fig. 2. MIT-E3 model. Yield, failure and loading surfaces (based on [8]).

The evolution of the yield surface with the plastic flow is controlled by two hardening rules, expressed in terms of plastic volumetric strain, which control the change in the size and the orientation of the yield surface. The rotational hardening is based on the assumption that the principal axes of anisotropy rotate towards the principal stress axes, therefore controlling the rate of change of anisotropy of the soil [8].

The undrained shear strength and the postpeak strain softening behaviour can also be controlled by the MIT-E3 model [8, 9].

In order to predict the behaviour of overconsolidated clays, the MIT-E3 model includes a hysteretic model and a bounding surface plasticity model. The hysteretic model simulates the elastic non-linear behaviour of an overconsolidated clay in an unloading-reloading cycle by smooth stiffness variation (Figure 3). When formulating this model, the load reversal point must be identified, calculating the stiffness as a

function of the distance between the current stress state and the reversal point [18]. The evolution of the tangential bulk modulus is described by the following function:

$$K = \frac{1+e}{\kappa_o (1+\delta)} p'$$
⁽⁷⁾

with the elastic non-linearity being found from the change of δ in accordance with expression:

$$\delta = \text{C.n.}(\ln_e \xi + \xi_s)^{n-1} \tag{8}$$

where *e* is the void ratio, κ_o the initial slope of the unloading/reloading phase in e-log_e p' space (Figure 3), *C* and *n* are material parameters of the MIT-E3 model characterised by the non-linearity at small strain levels, $\xi e \xi_s$ are variables which measure the stress amplitude, respectively in terms for volumetric and shear stresses, depending on the distance between the current stress state and the reversal point [8].



Fig. 3. MIT-E3 model. Variation of κ parameter during the reloading (based on [9]).

With the inclusion of the bounding surface plasticity model, there are two surfaces (Figure 2). The internal surface that passes through the current stress state R, called load surface f_0 , and the external surface designated by bounding surface plasticity f, described by the yield function of the normally consolidated clay (equation 6). The

load surface is homothetic to the bounding surface, with size ratio α_0'/α' [9]. For overconsolidated stress states, R, the plastic behaviour is linked to the behaviour of the image point I, located on the bounding surface. Therefore, the hardening of the bounding surface is controlled by pseudo-plastic strains defined from the stress state at the image point.

With the bounding surface plasticity model it is possible to predict the development of plastic strains in overconsolidated stress states (Figure 4). This allows coupling between the volumetric and distortional behaviour, and thus simulates the generation of excess pore pressures during undrained shearing and a smooth transition between the overconsolidated and the normally consolidated states.

A detailed description of the MIT-E3 model can be found in some technical papers [8-10, 16, 19].



Fig. 4. Modelling an unloading-reloading cycle with the MIT-E3

model (based on [8]).

3. Soil parameters

Considering that one of the parts of this work is to analyse the behaviour of an embankment constructed over a deposit of Boston Blue Clay (BBC), it was decided to

use in all analyses the parameters of this soil. Taking into account that the BBC parameters given in the literature [8, 15, 20] exhibit some variability, particularly in terms of compressibility and strength characteristics, it was decided to use, as base, the parameters considered by Borja et al. [14], which were taken from experimental research [21-23]. In this way, the MEL parameters are obtained (Table 1).

To guarantee some consistency between the parameters of MEL and MIT-E3 models, allowing the direct confrontation of the results, the evaluation of parameters e_{100} , λ , ν , ϕ'_{TC} and ϕ'_{TE} of the MIT-E3 model was made from the values considered by Borja et al. [14]. The values reported by Whittle [8] were adopted for the other parameters of the MIT-E3 model, irrespective of whether they came from experimental studies (K_0^{nc} , κ_0) or from parametric analysis (c, ψ_0 , C, n, w, S_t, γ , h). Table 2 gives all 15 parameters of the MIT-E3 model used for BBC.

4. Numerical prediction of soil behaviour

Before simulating the embankment behaviour, the ability of the two constitutive models to replicate the response of BBC under CK_0U triaxial tests is investigated. Normally consolidated and overconsolidated (OCR=4) samples are used to match the ground conditions. Compression and extension stress paths are considered in order to assess the loading conditions observed under and on the sides of the embankment. Table 3 summarizes the initial conditions used. Axisymmetric FE analyses are carried out to simulate the conditions imposed for the triaxial tests.

The Figures 5 and 6 compare the numerical predictions obtained with the two soil models for the normally consolidated and the overconsolidated samples.



b) OCR = 4

Fig. 5. Computed stress-strain for CK₀U triaxial tests using MEL/MC and MIT-E3 models.



Fig. 6. Computed stress paths for CK₀U triaxial tests using MEL/MC and MIT-E3 models.

As expected, the stress-strain response (Figure 5) shows for, both soil models, a marked strength anisotropy, achieving values of undrained strength in the compression paths (c_{uTC}) higher than those achieved in extension paths (c_{uTE}). The ratio $\frac{c_{uTC}}{c_{uTE}}$ predicted by the MIT-E3 model is 1.8 and 2.3, respectively for normally consolidated samples (OCR=1) and overconsolidated samples (OCR=4). However, with the MEL/MC model this ratio increases, fundamentally due to a distinct stress-strain response in the extension path, provided that the c_{uTC} is very similar. For normally consolidated samples in extension path, the MEL/MC model originates an unrealistic stress-strain response with a c_{uTE} close to zero.

The greater differences between the numerical predictions of the two models are observed for small strains. Thus, the MIT-E3 model generates a non-linear stress-strain response with stiffness anisotropy, providing a more rigid behaviour in compression than in extension paths. On the other hand, with the MEL/MC model the stress-strain response is linear showing the same stiffness in compression and extension paths. As expected, both models indicate stress paths (Figure 6) that develop throughout the "elliptical" surfaces, with the line K_0 as the main axis, and these generate strong strength anisotropy.

In compression, the stress paths predicted with the MEL/MC model are controlled by the MEL model inside the envelope of MC. When the stress paths reach the MC criterion, this criterion controls the stress path evolution, not allowing its crossing. Thus, for normally consolidated samples, the stress paths remain unchanged when they contact the MC criterion, while for overconsolidated samples the stress paths move along the MC criterion.

In overconsolidated states and in extension stress paths of normally consolidated samples the shape of the yield surface of the MEL model leads to part of the behaviour evolving within the yield surface in a linear elastic regime, represented on the graph by p'-q vertical stress paths. After contact with the yield surface the behaviour reflects the plasticity effect, showing slight hardening of the yield surface. This fact is responsible for the c_{uTE} approximately zero obtained with normally consolidated samples, taking into account that the yield surface of MEL model crosses the abscissa's axis in the area where the effective stress path reaches the ellipse of yield surface.

On the other hand, the MIT-E3 model infers a non-linear stress-strain behaviour for small strain levels, giving rise to a non vertical effective stress paths, which induces a gentle evolution of the stress-strain response. This behaviour is naturally different from that predicted by the MEL/MC model, which finds there is elastic linear behaviour within the yield surface, which leads to an abrupt transition between the elastic and plastic behaviour (Figure 5).

The numerical results predicted by the MIT-E3 model show the excellent ability of this model to predict some characteristics of the soil behaviour - notably strength anisotropy and non-linearity of the stress-strain behaviour - for the full range of strains, regardless of the OCR value.

5. Characteristics and numerical modelling of the embankment

Having established some of the potentials and limitations of the MEL/MC and MIT-E3 constitutive models for the prediction of soil behaviour, they are used to predict the behaviour of the I-95 test embankment. This embankment has already been studied with isotropic models, i.e. the MCC model on its own, or associated with the Von Mises model [14]. Another embankment of the I-95 motorway, but with different geotechnical profile, was analysed with MCC and MIT-E3 models [15].

Figure 7 shows the soil profile and the FE mesh used in the plane-strain analyses, consisting of 676 nodal points and 203 eight node isoparametric quadrilateral elements. The geometry and the characteristics of the mesh are similar to those used by Borja et al. [14]. Below the water table, the elements allow a coupled analysis of fluid flow and deformation to simulate the consolidation phenomenon in the soil foundation. Each of these elements has twenty nodal degrees of freedom, allowing the calculation of the displacements in eight nodes and the excess pore pressure in four corner nodes. These elements provide quadratic interpolation of displacements and linear interpolation of pore pressures.



Fig. 7. Soil profile and FE mesh for I-95 test embankment (based on [14]).

The boundary conditions applied to the mesh are such that the two vertical sides are restrained from moving in the horizontal direction, while the bottom boundary has zero displacements in both the vertical and horizontal directions.

In terms of hydraulic boundary conditions, the top boundary, corresponding to the water table, is permeable. The bottom boundary and the two vertical sides' boundaries are impermeable, so it is assumed that there is no flow of water through these boundaries. Above the water table, no water flow is considered in the numerical analyses.

In order to study the behaviour of the embankment, the soil foundation was instrumented with (Figure 7): (*i*) a vertical inclinometer tube under the slope of the embankment; (*ii*) a settlement plate; (*iii*) a piezometer.

The construction sequence used in the numerical analysis of the embankment is shown in Table 4. The elements used to simulate the embankment are generated sequentially in order to guarantee the top level of the embankment layers. The load applied by layers A and B of the embankment is divided into 5 sub-increments (Table 4), in order to avoid numerical instabilities. All embankment elements are initially stress-free.

6. Selection of the soil models and parameters for the numerical prediction of the embankment behaviour

The embankment itself is modelled by a linear elastic material, varying the Young's modulus (E') from the bottom to the top layer and during the sub-increments, in order to model the post construction confining stresses. Thus, for layer C (1 step) E' is equal to 5700 kPa, while for layers A and B the Young's modulus increases during the 5 steps of the application of the load. In this way, for layer B the Young's modulus

assumes the values 5000, 10000, 20000, 30000 to 40000 kPa, while for layer A the variation of E' is equal to 7800, 15000, 30000, 50000 and 78000 kPa.

The foundation of the embankment is composed of three types of soil: soft peat (layer 1), silty sand (layer 2) and BBC (layers 3 to 8). At depth, the overconsolidated ratio for the BBC layers decreases, changing from 4.5 (immediately below the silty sand) to 1.0, at a depth of 20 metres. This change is simulated by subdividing the clay into six layers (Figure 7), with the following OCR values [14]: 4.5, 3.0, 2.3, 1.8, 1.3, 1.0.

The parameters of the various models used for the numerical modelling of this embankment are given in Tables 1, 2 and 5, in accordance with the suggestions of Borja et al. [14] and Whittle [8]. Taking into account that, below the embankment and before its construction the peat was excavated and replaced by dense granular fill, the behaviour of this layer was modelled by a linear elastic material (Table 5).

The silty sand layer is simulated in all the analyses by the MCC model and the BBC behaviour is replicated by the MEL/MC and MIT-E3 models. With the MIT-E3 model, and in order to analyse the effect of the changes of the initial BBC stiffness, two analyses were carried out with values for κ_0 of 0.001 and 0.006. Figure 8 shows the effect of the change of the initial stiffness, indicated by parameter κ_0 , in an edometric reloading of a soil in an initially overconsolidated state. It can be seen that increased stiffness, which corresponds to a decreasing value of κ_0 , naturally gives rise to smaller strains, leading more quickly to yielding, and the length of path with non-linear elastic behaviour is reduced.

During the calculations, the coefficients of permeability of the silty sand and BBC layers vary with the void ratio, in accordance with the expression [27]:

$$\mathbf{k} = \mathbf{k}_0 \mathbf{x} \mathbf{10}^{\frac{\mathbf{e} - \mathbf{e}_0}{C_k}} \tag{9}$$

where $e_0 = 3.3$ represents the reference void ratio, $k_0 = 1.1 \times 10^{-3}$ m/day is the coefficient of permeability corresponding to e_0 , k is the corrected coefficient of permeability relative to the actual void ratio e, and C_k is a constant equal to 1.2 [28]. For all layers below the water table, a ratio between the horizontal and vertical permeability (k_h/k_v) of 4 is adopted [14].



Fig. 8. Influence of parameter κ_0 on an edometric recompression path.

Succinctly, this paper describes three numerical analyses, changing the soil model used to simulate the behaviour of BBC; one uses the MEL/MC model and other two use the MIT-E3 model, varying the κ_0 parameter. The vertical and horizontal displacements and the excess pore pressures are compared with the field data, thereby testing the reliability of the soil models to predict the behaviour of an embankment.

The models used in this paper do not take into account the creep phenomenon, which could be important in this case [14].

7. Vertical and horizontal displacements analysis

Figure 9 depicts the evolution over time of the settlements under the embankment regarding the settlement plate. In accordance with the numerical results of Zdravkovic et al. [29], it is noted that the MIT-E3 model gave rise to larger vertical

displacements contributing to a better approximation to the field data. This approximation was improved when κ_0 =0.006 is used. These results are the reflection of the greater complexity and more reliable behaviour of the MIT-E3 model, as was seen in Figures 5 and 6.



Fig. 9. Observed and computed evolution of the settlements with time on settlement plate.

The numerical predictions obtained with both MIT-E3 simulations shows that the influence of the initial stiffness is important to predict the behaviour of the embankment, basically due to the fact that the layers of BBC nearer the surface are initially overconsolidated. The high initial stiffness, at κ_0 =0.001, explains the smaller displacements of the initial instants, so that the settlement-time response is less curvilinear. As the initial stiffness of BBC increases (reduction of κ_0) the "rate" of approximation to the yield surface also grows. This is because there is less hardening, so that from this point the behaviour starts to evolve fundamentally in a plastic regime, which induces a substantial growth in vertical displacements, especially after more than 1000 days. When the initial stiffness is lower ($\kappa_0 = 0.006$), the path in the non-linear elastic regime is more "extensive" (Figure 8), corresponding to the slowing of the deformational rate in the plastic regime, which is expressed in a curvilinear settlement-time response.

The diagram in Figure 10 illustrates the evolution at depth of the horizontal displacements under the slope of the embankment for 2120 days.



Fig. 10. Observed and computed horizontal displacements on inclinometer for 2120 days.

The figure shows that the analyses with the MEL/MC and MIT-E3 models lead to diagrams that do not replicate the field data. This disparity decreases when the MIT-E3 model is considered.

The analysis with the MIT-E3 model, for κ_0 =0.001, give rise to a better simulation of *in situ* behaviour at the top of the BBC layer, even though in the central zone there is a very significant inflection in the horizontal displacements, which is not observed *in situ*. This inflection occurs in the upper layers of the BBC, which are initially overconsolidated, and is caused by the change in stiffness between the surface layers of the foundation soil and the BBC layers. This explanation is borne out by the fact that the inflection grows with increasing initial stiffness of the BBC layers, in other words, as the difference in stiffness between the BBC deposit and the layers of the foundation soil nearer the surface increases.

The diagram that relates the numerical predictions of the maximum horizontal displacement to the maximum settlement, (Figure 11), clearly shows that the horizontal displacements occur mainly during the loading phases, which are normally associated with pronounced gradients of $y_{max} / \Delta H_{max}$. This is clearly illustrated in Figure 12, which shows the change in $y_{max} / \Delta H_{max}$ over time, where a significant increase in the $y_{max} / \Delta H_{max}$ relation is noticed with the construction of layer B of the embankment, which decreases during the consolidation phases. On the whole, both models simulate this variation, even though the values from the MIT-E3 model are more expressive, since larger displacements are associated with it. However, the values obtained during loading fall short of those observed for embankments in general [30]: $y_{max} = (0.91 \pm 0.2) \Delta H$.

Figure 11 shows that the displacements on both sides of the embankment develop in the same way, despite the increased thickness of the BBC on the left side tending to raise the value of y_{max} . This fact reflects the compensating effect of the lateral berm. In the initial instants (Figure 12) it can be seen that the $y_{max}/\Delta H_{max}$ relation grows perceptibly on the right side, compared with the left. This situation is caused by layer A of the embankment which is much wider than the others layers (Figure 7), so that the zone under analysis on the left side is under the mid zone of layer A of the embankment, and consequently there are no significant horizontal displacements. With the placement of the next layers (B and C) it can be seen that the $y_{max}/\Delta H_{max}$ relation tends to have a similar value on both sides of the embankment.



Fig. 11. Numerical predictions of the maximum horizontal displacement (y_{max}) in both extremities of the embankment with maximum settlement (ΔH_{max}) .



Fig. 12. Numerical predictions of the ratio between the maximum horizontal displacement and maximum settlement over time, in both extremities of the embankment.

Figure 13 depicts the evolution over time of the ratio between the volume of soil vertically displaced along the base of the embankment (ΔV_{vert}) and the total volume of soil horizontally displaced on both vertical extremities of the embankment (ΔV_{hor}). This figure aims to show if the resultant settlements from the movement of the soil to the

sides is related to no volume change. Since, if there is no volume change, then the vertical deformation has to be compensated by an equal lateral expansion, i.e. $\Delta V_{vert} = \Delta V_{hor}$.

In the initial instants, up to 150 days, MIT-E3 model leads to a higher ratio $\Delta V_{vert}/\Delta V_{hor}$, which is fundamentally due to lesser ratio $y_{max}/\Delta H_{max}$, as was seen in Figure 12. The ratio $\Delta V_{vert}/\Delta V_{hor}$ has the lowest value during the placement of the layer B of the embankment. This fact corresponds to the movement of the soil mass from the bottom of the embankment to the sides, with deformation under almost zero volumetric strain. With time, the ratio $\Delta V_{vert}/\Delta V_{hor}$ tends to increase due to the consolidation of the foundation soil, corresponding to a lower ratio $y_{max}/\Delta H_{max}$.



Fig. 13. Numerical Predictions of ratio $\Delta V_{vert} / \Delta V_{hor}$ of the embankment.

8. Excess pore pressures analysis

Figure 14 illustrates the evolution of excess pore pressure head at the piezometer. At first, the behaviour is similar in all the numerical analyses, though there is a slight increase in the pore pressures generated by the MIT-E3 model, which could suggest a coupling effect between the volumetric and distortional components.

However, as time passes, the two MIT-E3 analyses show very similar behaviour, and for a period of over 1000 days they can almost be superimposed on the field data. However, the dissipation of the pore pressure obtained with the MIT-E3 model is less than predicted by the MEL/MC model. This is related to the increased settlements induced by the MIT-E3 model, given that this increase implies the need to expel a greater volume of water from the soil, thereby decreasing the rate of pore pressure dissipation.



Fig. 14. Observed and computed excess pore pressure head with time on piezometer.

During the placement of layers B and C of the embankment, all the numerical analyses predict excess pore pressures lower than 60% of measured value, which is of the same order of magnitude as the increment of vertical load applied by the embankment (14.4 kPa), reflecting an undrained response. This fact is probably due to the heterogeneity of the soil medium in terms of permeability, which in the numerical analyses was assumed to be homogeneous. In reality, there will be local variations of

the permeability of the soil, which cannot be taken into account in the numerical analysis. Otherwise, in the numerical analyses there is some dissipation of excess pore pressure during the construction of the embankment.

The evolution of excess pore pressure head with embankment height (Figure 15) shows that during the placing of the first two layers of the embankment the excess pore pressures increase approximately the same as the increment of total vertical stresses, i.e., $\overline{B} (\Delta u / \Delta \sigma_v) \cong 1,0$. This reveals a quasi undrained behaviour, which is related to a normally consolidated state [30]. Thus, the construction of the layers A and B of the embankment is sufficient to impose a normally consolidated state in some areas of BBC deposit, which were initially overconsolidated. It is found that the value of \overline{B} obtained in the loading phases is close to that observed for embankments in general [30]: $\overline{B} = 1,05 \pm 0,15$. As a consequence of higher dissipation of the excess pore pressure during the construction of the embankment, as seen in Figure 14, the numerical results reveal a behaviour that tends to deviate progressively from the real behaviour.



Fig. 15. Observed and computed excess pore pressure versus height of embankment.

9. State of stress analysis

Figures 16 and 17 show the extension of the yielding areas, the diagrams with the contours of the vertical $(\Delta\sigma'_y)$ and horizontal $(\Delta\sigma'_x)$ effective stress increments and the shear stresses increments $(\Delta\tau_{xy})$, predicted by MEL/MC and MIT-E3 (with κ_0 =0.006) models for t=2120 days.



Fig. 16. Numerical predictions obtained with MEL/MC model for t = 2120 days. Yield area, contours of vertical and horizontal effective stress increments and contours of shear stress increments.



Fig. 17. Numerical predictions obtained with MIT-E3 model (κ_0 =0.006) for t = 2120 days. Yield area, contours of vertical and horizontal effective stress increments and contours of shear stress increments.

The yield areas predicted by the two constitutive models (Figures 16a and 17a) mostly affect deeper layers, which are normally consolidated or lightly overconsolidated. Yielding continues to the layers nearest the surface, namely under the centre of the embankment, corresponding to the greatest increments of vertical stresses.

The MEL/MC model induces more extensive yield areas up to the superficial layers, with a simultaneous yielding of MEL and MC models. On the other hand, with the MIT-E3 model the yielding areas are concentrated in deeper layers associated with a greater irregularity. These facts could lead to the assumption that the settlements would

be larger with the consideration of the MEL/MC model. However, the opposite is found to be true. It is supposed that this behaviour is the reflection of the non-linear elastic law of the MIT-E3 model.

Analysis of the diagrams for the vertical effective stresses (Figures 16b and 17b), determines more regularity and uniformity in the distribution of $\Delta\sigma'_{y}$ through the various foundation soil layers with the MEL/MC model, as opposed to the MIT-E3 model predictions, which show a greater concentration of $\Delta\sigma'_{y}$ in the surface layers, due to the greater stiffness of the soil.

Comparing the diagrams of $\Delta \sigma'_x$ (Figures 16c and 17c), it can be seen that the MCC model indicates a better distribution of the contours throughout the foundation soil than is given by the MIT-E3 model. The MIT-E3 model indicates a progressive concentration of increments in horizontal stresses in the vicinity of the foot of the slope, which is related to the inflection in the diagram of the horizontal displacements (Figure 10).

Analysis of the $\Delta \tau_{xy}$ isograms (Figures 16d and 17d) shows a similar qualitative evolution for the two models, though in quantitative terms the MIT-E3 model gives higher values (by around 50%). As a result of the coupling between the volumetric and distortional components, the MIT-E3 model allows that the development of the pore pressures will depend on the variation of tangential stresses, too. This fact explains, partially, the greater increments in pore pressure predicted by the MIT-E3 model, as illustrated in Figure 14.

Figure 18 shows the computed effective stress paths at some points of the foundation soil (described in Figure 7), predicted by MEL/MC and MIT-E3 (with κ_0 =0.006) models. Beneath the embankment's axis (points B₁ and B₂) the paths are naturally in compression and both models give similar stress paths. This fact reflects the behaviour observed in Figure 6, where small differences between the effective stress paths predicted by both models in compression are observed. In point B₂ the influence

of the failure criterion of the two models is clearly seen, which truncates the progress of the deviatoric stress.



Fig. 18. Numerical predictions of the effective stress paths in points A₁, A₂, B₁, B₂, C₁ and C₂ of the soil foundation.

On the sides of the embankment (points A_1 , A_2 , C_1 and C_2) the stress paths are naturally in extension, with p' increasing at the points nearer the surface (A_1 and C_1), and decreasing slightly at the deeper points (A_2 and C_2). In all these points, the MEL/MC model predicts a reversal path immediately after the start of the consolidation (t=320 days), with the increasing of the deviatoric stress (q), while the effective stress path induced with the MIT-E3 model remains almost unchangeable after this time.

The predictions with the MEL/MC model at points A_1 and C_1 , where the yielding was not reached (Figures 16a and 17a), show longer effective stress paths than with the MIT-E3 model. Reflecting this fact the greater widening of the yield surface of

the MEL model induced by the OCR. In deeper points (A2 and C2), where the soil is normally consolidated, the stress paths originates by the MIT-E3 model are longer, because the yield surface of this model for OCR=1 is wider. These results are consistent with the behaviour seen in the simulation of triaxial tests (Figure 6).

10. Conclusions

This work has compared the numerical results obtained by two constitutive models based on anisotropic conditions: the Melanie model associated with the Mohr Coulomb criterion (MEL/MC) and the MIT-E3 model. The latter includes important characteristics observed in the behaviour of clays, in particular [8]: (*i*) an elastoplastic model for normally consolidated clay with anisotropic strength; (*ii*) a hysteretic behaviour with a non-linear elastic behaviour for small strain levels; (*iii*) a bounding surface which allows the generation of plastic strains in an unload-reload cycle.

Initially the ability of the two soil models to replicate the response of BBC under triaxial loading was investigated. Compression and extension stress paths were considered for normally and overconsolidated (OCR = 4) samples. Some conclusions can be drawn from this study:

- The MIT-E3 model is naturally found to be more suitable than the MEL/MC model to predict the behaviour of BBC, since it incorporates important behaviour characteristics.
- For reduced deformation levels the MIT-E3 model predicts stiffer non-linear stressstrain behaviour than that obtained with the MEL/MC model.
- The differences between the two models are more visible under extension, with distinct stress paths, reflecting the effect of the shape of each model's yield surfaces.

Finally, the MEL/MC and MIT-E3 constitutive models were used to predict the behaviour of an embankment built over a BBC deposit. Some numerical results were compared with the field data, namely settlements, horizontal displacements and excess pore pressures. Several conclusions can be drawn from the embankment study:

- The MIT-E3 model predicts the embankment behaviour more accurately than the MEL/MC model. The results show the importance of the constitutive model in the numerical predictions, with a better-developed model that incorporates important soil behaviour characteristics naturally providing better results.
- The two numerical analyses performed with MIT-E3 (κ_0 =0.001 and κ_0 =0.006) show the huge influence of initial stiffness on the development of displacements.
- The MIT-E3 model predicts lower dissipation of the excess pore pressures than the MEL/MC model. This is essentially because the MIT-E3 model generates larger settlements, which are associated with the need to expel a greater volume of water from the zone beneath the embankment (with greater excess pore pressure) towards the sides where pore pressures are lower.
- Both models originate coherent effective stress paths, in compression and in extension, below and on the sides of the embankment, respectively.
- In compression paths, both models generate similar stress paths, being truncated at the same level, when failure is reached (point B₂).
- In extension (points A₁, C₁, A₂ and C₂), the stress paths depend on the shape of the yield surface and its widening originated by the OCR. The influence of the shape is shown at deeper points (A₂ and C₂), where the soil is normally consolidated. At the points nearer the surface, the stress paths depend fundamentally on the widening related to the increase of the OCR, this aspect being more significant with the MEL model.

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Notation

The following symbols are used in this paper:

$\overline{\mathbf{B}}$	=	parameter of pore pressure ($\Delta u/\Delta \sigma_v$);
CLS	=	critical state line;
c _{uTC}	=	undrained strength in compression paths;
c _{uTE}	=	undrained strength in extension paths;
eo	=	initial void ratio;
$\mathbf{k}_{\mathbf{h}}$	=	horizontal permeability coefficient;
K ₀ ^{nc}	=	coefficient of earth pressure at rest for a normally consolidated state;
к ₀ ^{ос}	=	coefficient of earth pressure at rest for an overconsolidated state;
k _v	=	vertical permeability coefficient;
OCR	=	overconsolidation ratio;
VCL	=	virgin consolidation line;
y _{max}	=	maximum horizontal displacement;
Δ	=	increment, variation;
ΔH	=	settlement;
ΔV_{vert}	=	volume of soil vertically displaced along the base of the embankment;
ΔV_{hor}	=	total volume of soil horizontally displaced on both vertical extremities of the
		embankment;
γ	=	unit weight;
σ' _x	=	effective horizontal stress;
σ' _y	=	effective vertical stress;

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Parameter	Description	Value
$e_{\lambda 0}$	Void ratio for a K ₀ normally consolidated sample at p'=1 kPa	3.56
λ	Slope of the virgin compression line in $e - \ln p$ ' space	0.147
κ	Slope of the swelling line in $e - \ln p$ ' space	0.06
K_0^{nc}	Coefficient of earth pressure at rest for normally consolidated clay	0.50
ν	Poisson's ratio	0.30
c'	Cohesion	0.0
φ'	Angle of shearing resistance	26.5 ^(*)

Table 1. MEL/MC parameters used for BBC (based on [14]).

(*) Establish from [14] for compression stress paths with the equation [24]: $M = \frac{6 \sin \phi'}{3 - \sin \phi'}$.

Param	eter	Description					
e ₁₀₀	tests)	Void ratio for a K_0 normally consolidated sample at the reference stress ($\sigma'_v = 100$ kPa)	2.94				
λ	intal 1 [14])	Slope of the virgin compression line in e -ln p ' space	0.147 ^(a)				
κ_0	srime et al.	Initial slope of swelling line in <i>e</i> -ln <i>p</i> ' space	0.001				
K_0^{nc}	m expe Borja (Coefficient of earth pressure at rest for normally consolidated clay	0.50				
ν	ed fro ed on	Poisson's ratio	0.30				
ϕ_{TC}	/aluate (base	Critical-state angle of shearing resistance in triaxial compression	26.5 ^(b)				
ϕ_{TE}	Ē	Critical-state angle of shearing resistance in triaxial extension	39.5 ^{(b),(c)}				
С		Parameter affecting the non-linear volumetric swelling behaviour (hysteretic elasticity)	22				
n	tudies	Parameter affecting the non-linear volumetric swelling behaviour (hysteretic elasticity)	1.60				
w	metric s 8])	Parameter affecting the non-linearity at small strains in undrained shear (hysteretic elasticity)	0.07				
h	para ttle [Parameter affecting the irrecoverable plastic strain	0.2				
c	ed from (Whi	Parameter affecting the undrained shear strength (geometry of bounding surface)	0.86				
\mathbf{S}_{t}	aluate	Parameter affecting the degree of strain softening	4.5				
γ	Eva	Parameter affecting the generation of pore pressures induced by shear in overconsolidated clay	0.5				
Ψ_0		Parameter affecting rotation of bounding surface	100				
(a)	Whittle [8] indicates $\lambda = 0.184$, while Ladd et al. [15] report values for λ between 0.084 and 0.4.						
(b)	Whitte [8] and Ladd et al. [15] indicate values of $\phi_{TC} = 33.4^{\circ} \phi_{TE} = 45.9^{\circ}$. Ladd et al. [15] report						
	other data with $\phi_{TC} = \phi_{TE}$.						

Table 2. MIT-E3 parameters used for BBC (based on [8, 14, 15]).

(c) Others data suggests $\phi_{TE} \approx 40^{\circ}$ [8] and $\phi_{TE} \approx 35^{\circ}$ [25].

Table 3. Initial state of stress used in the triaxial test predictions.

OCR	K ₀	e_0	σ' _{v0} (kPa)	σ' _{h0} (kPa
1	0.50	2.943	100.0	50.0
4	1.077*	2.999	25.0	26.9

(*) Calculated with the equation [26]: $K_0^{\text{oc}} = K_0^{\text{nc}} \cdot OCR^{\sin\phi_{\text{TC}}}$

Time (days)	No. of steps	Description
0 - 75	5	Layer A, $\Delta H = 1.22 \text{ m}$
75 - 175	5	Layer B, $\Delta H = 6.40$ m
175 - 295	6	Consolidation.
295 - 320	1	Layer C, $\Delta H = 1.22 \text{ m}$
320 - 2120	11	Consolidation.

Table 4. Embankment construction steps (based on [14]).

	Initial state			Elastic param.		MCC plastic parameters				Permeability
Material	K_0	γ (kN/m ³)	OCR	E' (MPa)	ν'	λ	ĸ	$e_{\lambda 0}$	М	k_h/k_v
Embankment		19.0		5.7 - 78	0.3					
Granular fill ⁽¹⁾	0.5	15.0		10	0.3					
Silty sand	0.4	15.0	1.0	(*)	0.3	0.025	0.005	2.50	1.50	4.0
BBC	1.15-0.5 ⁽²⁾	15.0	4.5-1.0	(*)	0.3	Se	ee Table	s 1 and 2	2.	4.0

Table 5. MCC soil foundation parameters used in numerical embankment predictions.

⁽¹⁾Below the embankment and before its construction the soft peat was excavated and replaced by granular fill. Warying with the OCR value. ^(*) Calculated by expression [24]: $E' = \frac{3(1 + e_0)(1 - 2\nu')}{\kappa} p'_c$

⁽²⁾Varying with the OCR value.

CAPTIONS

Fig. 1. Representation of Melanie model associated with Mohr-Coulomb criterion in p'-q and ε_v - ε_q planes [2].

Fig. 2. MIT-E3 model. Yield, failure and loading surfaces (based on [8]).

Fig. 3. MIT-E3 model. Variation of κ parameter during the reloading (based on [9]).

- Fig. 4. Modelling an unloading-reloading cycle with the MIT-E3 model (based on [8]).
- Fig. 5. Computed stress-strain for CK_0U triaxial tests using MEL/MC and MIT-E3 models. a) OCR = 1. b) OCR = 4
- Fig. 6. Computed stress paths for CK₀U triaxial tests using MEL/MC and MIT-E3 models.
- Fig. 7. Soil profile and FE mesh for I-95 test embankment (based on [14]).
- Fig. 8. Influence of parameter κ_0 on an edometric recompression path.
- Fig. 9. Observed and computed evolution of the settlements with time on settlement plate.
- Fig. 10. Observed and computed horizontal displacements on inclinometer for 2120 days.
- Fig. 11. Numerical predictions of the maximum horizontal displacement (y_{max}) in both extremities of the embankment with maximum settlement (ΔH_{max}) .
- Fig. 12. Numerical predictions of the ratio between the maximum horizontal displacement and maximum settlement time, in both extremities of the embankment.
- Fig. 13. Numerical Predictions of ratio $\Delta V_{vert} / \Delta V_{hor}$ of the embankment.

Fig. 14. Observed and computed excess pore pressure head with time on piezometer.

- Fig. 15. Observed and computed excess pore pressure versus height of embankment.
- Fig. 16. Numerical predictions obtained with MEL/MC model for t = 2120 days. Yield area, contours of vertical and horizontal effective stress increments and contours of shear stress increments.
- Fig. 17. Numerical predictions obtained with MIT-E3 model (κ_0 =0.006) for t = 2120 days. Yield area, contours of vertical and horizontal effective stress increments and contours of shear stress increments.
- Fig. 18. Numerical predictions of the effective stress paths in points A_1 , A_2 , B_1 , B_2 , C_1 and C_2 of the soil foundation.