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To cite this version:

Thomas Mathä, Olivier Pierrard. Search in the product market and the real business cycle. Journal of Economic Dynamics and Control, 2011, 10.1016/j.jedc.2011.03.001 . hal-00828982

HAL Id: hal-00828982

https://hal.science/hal-00828982v1

Submitted on 1 Jun 2013

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PII: S0165-1889(11)00046-7
DOI: doi:10.1016/j.jedc.2011.03.001
Reference: DYNCON 2547

To appear in: *Journal of Economic Dynamics & Control*

Received date: 2 July 2009
Revised date: 13 January 2011
Accepted date: 11 February 2011

Cite this article as: Thomas Mathä and Olivier Pierrard, Search in the product market and the real business cycle, *Journal of Economic Dynamics & Control*, doi:10.1016/j.jedc.2011.03.001

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Search in the Product Market and the Real Business Cycle

Thomas Mathä[a], Olivier Pierrard[a,b,*]

[a] *Economics and Research Department, Central Bank of Luxembourg, 2 bd. Royal, L-2983 Luxembourg, Luxembourg*
[b] *Department of Economics, Catholic University of Louvain, 3 place Montesquieu, B-1348 Louvain-la-Neuve, Belgium*

Abstract

Empirical evidence suggests that most firms operate in imperfectly competitive markets. We develop a search-matching model between wholesalers and retailers. Firms face search costs and form long-term relationships. Price bargain results in both wholesaler and retailer markups, which depend on firms' relative bargaining power. We simulate the general equilibrium model and explore the role of product market search frictions for business cycles. We conclude from the simulation exercise that incorporating product market search *structure* and *shocks* improve the standard real business cycle model to reproduce US business cycle fluctuations.

Keywords: Business cycle, Frictions, Search, Product market, Price bargain
JEL: E10, E31, E32

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1. Introduction

Empirical evidence suggests that most firms operate in imperfectly competitive markets where they have some power of setting prices themselves and form long-term relationships with their customers, which are predominantly other firms. These relationships are typically governed by implicit or explicit contracts. Available evidence also shows that firms produce substantial effort, typically in form of advertising or marketing, to find new customers and sell their products to. For instance, in the US total advertising expenditures have been close to 2.5% of GDP over the last 10 years. This effort may also cause an economic chain reaction by increasing sales, consumption and employment. In the standard Walrasian real business cycle model (hereafter RBC, see for instance King and Rebelo (1999) for an in depth exposition), the product market is perfectly competitive

*Corresponding author. Tel.: +32 4774 4449; fax: +32 4774 4920.
Email addresses: `thomas.mathae@bcl.lu` (Thomas Mathä), `olivier.pierrard@bcl.lu` (Olivier Pierrard)

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and adjustments occur without frictions. Given the above stylised facts, this paper aims to provide a more realistic story of business relationships and price formation mechanisms than in the RBC model and explores whether this can play a significant role in the explanation of economic fluctuations.

More precisely, we replace the Walrasian product market of the standard business cycle model with a product market with frictions by following Pissarides (2000) and the associated search-matching literature. In our model, downstream producers act as wholesalers and have long-term relationships with upstream retailers who in turn sell to final consumers. Retailers act as intermediaries between producers and consumer; they alleviate the search costs for final consumers. We believe that this is fair characterization of most product markets in industrialized economies. Only in very special markets do producers sell directly to consumers without intermediaries. Wholesalers produce effort (e.g. advertising or marketing) to find retailers to sell their products to. Retailers produce effort (e.g. by employing purchasing managers) to find wholesalers to buy their products from in order to refill their stores and enlarge their selection. The amount of products exchanged therefore depends on their respective search efforts. Moreover, every buyer-seller contact generates a surplus over which the wholesaler and the retailer bargain. We therefore provide a story how wholesalers and retailers meet in the market and for the subsequent price formation mechanism between them. Still, our model makes use of a simple representation and remains very close to the standard RBC model.[1]

The role of marketing frictions (usually consumer search frictions) have been analyzed to explain industry and firm dynamics (Fishman and Rob (2003)) and in the international business cycle literature to explain price differences between countries (Alessandria (2004, 2009)) or the behaviour of imports, exports and the terms of trade (Drozd and Nosal (2011)). In standard (closed economy) RBC models, most papers introducing imperfections in the product market assume monopolistic wholesalers, as in Blanchard and Giavazzi (2003), Messina (2006) or the recent DSGE literature (see for instance Christiano et al. (2005) or Smets and Wouters (2003)). Alternatively, in Fagnart et al. (2007), wholesalers experience privately observed and uninsured idiosyncratic shocks, which gen-

[1] We acknowledge that the model in this paper provides a too simplistic view of what in reality can be considered a rather complex relationship between buyer and seller. Clearly, contracting parties do not only bargain prices, but also special discounts, after sales services, etc... However, this is beyond the scope of this paper.

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erates a sub-optimal equilibrium. But none of these approaches allow for bilateral relationships and negotiations between wholesalers and retailers. Very recent related approaches that aim at providing better descriptions of customer-firm relationships are those by Hall (2008), Arseneau and Chugh (2007) and Kleshchelski and Vincent (2007). Hall (2008) explores customer search and seller recruiting by adapting principles of the labour market search and matching models to the product market. In his model, producers invest heavily in attracting final customers, as they receive a large share of the surplus. Hall's approach is concerned with retail markets. He incorporates frictions but no bargaining between customers and sellers. Arseneau and Chugh (2007) extend Hall's model and explore the effects of different bargaining assumptions. They specifically set out to analyse how the distributive role of prices through the notion of fairness affects price dynamics. In Kleshchelski and Vincent (2007), customers incur switching costs. Customers and firms form long-term relationships and idiosyncratic marginal shocks are only incompletely passed through to prices. However, all three approaches are concerned with the relationship between retail firms and final consumers, whereas our model provides a story of firms' relationships and of the price formation process. Moreover, we provide an in depth exploration of the RBC properties and compare simulation results to real US data.

Our key findings are as follows: First, the price bargain results in two markups, one for the wholesalers and another for the retailers. The respective markup depends on the relative bargaining power of the wholesaler and the retailer. Markups are procyclical following a productivity shock and countercyclical following a product market shock. Second, we explore whether the search structure improves business cycle statistics and impulse response functions when they are driven by a productivity shock. We show that a crucial element is the way search costs are modelled. In particular, convex search costs are able to produce hump-shaped dynamics for all variables and increase the persistence of output. Alternative approaches with sunk costs or durable search through marketing capital also generate interesting results.[2] Third, we construct historical productivity and product market shocks using the solution to the model and quarterly US data for some key variables. We show that the product market shock helps to explain the observed US business cycle fluctuations. Fourth, we provide some extensions and we introduce price rigidities

[2]The labour market literature also recently stressed these kind of results (role of vacancy opening costs for the volatility of labour market variables), see for instance Yashiv (2006) or Fujita and Ramey (2007).

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and adjustments along the extensive margin. However, these alterations do not really improve the fit of the model.

In the business cycle literature, search frictions are most commonly incorporated in the labour market following the contributions of Merz (1995) and Andolfatto (1996). From our simulations, we believe that the consideration of product market search *structure* and *shocks* may be important in this respect and merit further exploration and future research.

Section 2 provides selective evidence on the product market functioning and further motivation of this paper. Sections 3 and 4 develop and discuss the search-matching model with frictions in the product market and price bargain. Sections 5 and 6 present the calibration and some numerical simulations for US data. Section 7 discusses several extensions. Section 8 concludes.

2. On firms' business environment

It is widely accepted that most firms operate in markets which are governed by imperfections or frictions. By providing search effort firms try to overcome market imperfections. This is motivated by recent evidence, in particular from recent firm surveys. Following the lead by Blinder et al. (1998) firm surveys have been conducted for several industrial countries/areas over the last decade, which improved our understanding of firms' business environment and price setting practices.[3] According to those surveys, it seems that a fair characterisation of a typical or representative firm's business environment is that it operates an imperfectly competitive market and uses some form of mark up pricing above marginal cost as its predominant form of price-setting practice, thereby implying that it is able to exert some market power.[4] This firm engages in business-to-business (B2B) rather than business-to-consumer (B2C) relationships, where it typically does most business with repeat customers and forms long-term relationships with them. A long-term relationship, which can also be regarded as one form of an implicit contract based on principles of trust and fairness, is an effective way to reduce search cost, which the firm otherwise would have to bear. Together, these surveys provide a strikingly coherent set of empirical results and a chal-

[3]See Amirault et al. (2004) for Canada, Fabiani et al. (2006, 2007) for 9 euro area (EA) countries, Nakagawa et al. (2000) for Japan, Apel et al. (2005) for Sweden and Hall et al. (2000) for the UK.

[4]For example, 54% of euro area firms answered that they use a mark up pricing strategy. 73% of euro area firms said that their main market is the domestic market. In Canada, the corresponding figure is 81%.

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lenge to many modelling assumptions usually employed in standard Walrasian macroeconomic models.

To substantiate the above points, consider that in the US, 85% of firms surveyed by Blinder et al. (1998) indicate that they mainly engage in long-term relationships with their customers. 77% of their main customers are other firms. The corresponding figures for other industrialised economies are of similar magnitudes (EA 70% and Sweden 86% for the share of long-term customers and 75 and 70% for the share of other firms as main customers). Furthermore, these long-term relationships are mainly governed by contracts. 50% of US firms responded that they have 60% or more of their sales covered by explicit or written contracts, which according to Blinder et al. (1998) is estimated to correspond to 38% of US GDP. The contracts typically last one year (both the median and the mode are 12 months). Furthermore, explicit contracts are of more importance for firms that are primarily engaged in B2B relationships. Surveys for other industrialised countries generally corroborate these findings (see Amirault et al. (2004), Apel et al. (2005) and Fabiani et al. (2006, 2007)).

Since the product market is imperfect, (i) firms' price setting policies make use of some form of mark up pricing over marginal cost and (ii) there is need for search of customers, and thus advertising and marketing effort, and a need for search for suppliers. With regard to first point, empirical estimates of the mark up of prices over marginal costs vary widely. Hall (1988) for example estimated U.S manufacturing to exhibit sizeable returns to scale and price cost mark ups. However, as shown by Felipe and Mc Combie (2002), the coefficient estimates based on value terms cannot unambiguously be interpreted as a mark up due to omitted variable bias and large estimates of increasing return of scale being flawed. Other studies using gross output data, such as Domowitz et al. (1988), report smaller mark ups in U.S. manufacturing data. Basu (1996) reports that neither increasing returns to scale nor imperfect competition appear to be a source of procyclical productivity and concludes that is rather a combination of variable factor utilisation and productivity shocks that drives cyclical productivity. Similarly, Basu and Fernald (1997) report returns to scale to be small and the mark up of price over marginal costs being at around 1.08.

With regard to the second point, search and thus advertising, marketing, promotions, public relations, and sales managers provided almost 600,000 jobs in 2006. Similarly, roughly an equally

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large number of people were engaged in purchasing and buying occupations (Bureau of Labor Statistics, 2007). This represents almost 0.5% of total US employment for each. And still in the US, total annual expenditures in advertising in all medias represented on average 2.4% of GDP over the last decades. Put differently, advertising expenditures amounted to 271 billions US dollars in 2005. Figure 1 also presents the cyclical properties of advertising expenditures; they are positively correlated with GDP, have a higher volatility than GDP, especially over the last years and are very persistent. This figure includes spending for advertising in newspapers, magazines, radio, television, direct mail, billboards and displays, Internet, and other forms, and thus also includes the advertising that is directly targeted towards final consumers, such as car manufacturers' or pharmaceuticals' television adverts. Although producers may directly target consumers, consumers buy via intermediaries and hence advertising towards consumers indirectly affects retailers.

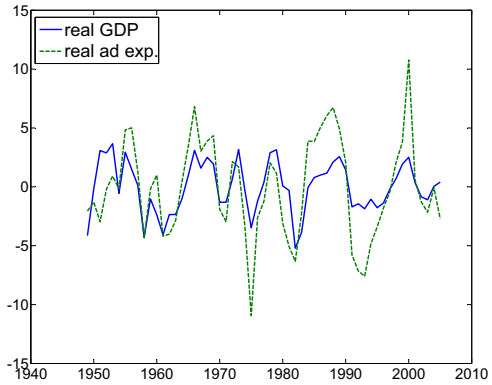

Fig. 1. Cyclical fluctuations of real advertising expenditures (% deviations from HP-trend). *Notes*: Data from www.galbithink.org/ad-spending.htm and Bureau of Economic Analysis. The yearly nominal series are GDP-deflated, logged and HP-filtered ($\lambda = 100$) to extract the business cycle components.

3. Model

There are three types of agents in the economy: households, wholesale firms and retail firms. Goods are produced by wholesale firms and consumed by households. However, conversely to the standard real business cycle literature, we do not assume that products are directly exchanged between producers and consumers; instead we introduce retailers as intermediaries. Retailers buy

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from producers, who act as wholesalers, and sell to households. Trade frictions are present in the product market between wholesalers and retailers, and we provide an explicit theory of price determination since every wholesaler-retailer cont(r)act generates a surplus over which firms bargain. More precisely, the product market consists of a two-sided search market between sellers (wholesale firms) and buyers (retail firms). Let T_t be the number of contracts between wholesale-retail pairs at period t, a contract meaning that both parties agree to exchange one unit of output. These contracts terminate and the pairs separate at the exogenous rate $0 < \chi < 1$. The contract duration is, thus, on average given by $1/\chi$. This results in a continuous depletion of the stock of contracts, and thus trade volume, and consequently a need to refill it. In order to do so, wholesale firms provide a search effort S_t (marketing or advertising expenditures) to find new buyers; and retail firms provide a search effort D_t (by purchasing employees) to find new suppliers. The number of new matches between sellers and buyers is increasing and concave in the search efforts and assumed to be generated by a standard Cobb-Douglas matching function:

$$M_t = \bar{m} S_t^{\gamma} D_t^{1-\gamma}, \tag{1}$$

where $\bar{m} > 0$ and $0 < \gamma < 1$. In analogy to the labour market, the relationship between the search effort of wholesalers and retailers can be regarded as a product market equivalent of the "Beveridge curve". It has search effort of wholesalers on the vertical axis and search effort of retailers on the horizontal axis. The curve slopes downwards as wholesalers produce higher effort (advertising) when retailers are reluctant to buy goods. Downward and upward shifts in this curve would signify structural improvements and deteriorations in the efficiency of the matching process, respectively. Movements along the curve, in contrast, imply a cyclical adjustment without alteration of the matching efficiency.

The trade volume evolves according to:

$$T_t = (1-\chi) T_{t-1} + M_t. \tag{2}$$

3.1. Households

The economy is populated by a large number of infinitively lived households. Their time endowment is normalized to 1 and split between work N_t and leisure $1 - N_t$. Their current utility is defined as:

$$\mathcal{U}(C_t, 1-N_t) = \log(C_t) + \frac{\theta}{1-\eta}\left((1-N_t)^{1-\eta} - 1\right), \tag{3}$$

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where C_t represents consumption. Utility is assumed to be concave in its arguments and specified as in King and Rebelo (1999): $\theta \geq 0$ and $\eta \geq 0$ is the parameter governing the labour supply elasticity. Households receive their income from lending capital to wholesale firms at interest rate $r_t + \delta$ and from working at their wage rate w_t. In each period, they choose the size of the capital investment I_t and labour supply N_t in order to maximize the present discounted value of their life-time utility:

$$W_t^H = \max_{I_t, N_t} \left\{ \mathcal{U}(C_t, 1 - N_t) + \beta E_t \left[W_{t+1}^H \right] \right\}, \tag{4}$$

subject to the constraints:

$$C_t + I_t = w_t N_t + (r_t + \delta) K_{t-1} + \Pi_t, \tag{5}$$

$$I_t = K_t - (1 - \delta) K_{t-1}, \tag{6}$$

where β denotes the discount factor. Equation (5) is the budget constraint. Households own both wholesale and retail firms and ultimately receive their profits Π_t. Equation (6) is the capital accumulation equation and δ denotes the exogenous capital destruction rate. The first order conditions are:

$$\frac{1}{C_t} = \beta E_t \left[(1 + r_{t+1}) \frac{1}{C_{t+1}} \right], \tag{7}$$

$$\frac{\theta}{(1 - N_t)^\eta} = \frac{w_t}{C_t}. \tag{8}$$

3.2. Wholesale firms

The economy is composed of a continuum of identical wholesale firms using capital K_{t-1} and labour N_t to produce tradable products T_t through a Cobb-Douglas production function:

$$T_t = \epsilon_t K_{t-1}^\alpha N_t^{1-\alpha}, \tag{9}$$

where ϵ_t is a productivity shock and $0 < \alpha < 1$. Given the sale price P_t, the firms choose their optimal search effort, i.e. the level of advertising expenditures S_t to find new buyers, as well as the optimal capital-labour ratio to produce the output level T_t. They take as given q_t^S, the rate at which every effort leads to a new match. The rate is defined as:

$$q_t^S = M_t / S_t. \tag{10}$$

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Hence, each wholesaler faces the following dynamic programming problem:

$$W_t^W = \max_{S_t, N_t} \left\{ P_t T_t - w_t N_t - (r_t + \delta) K_{t-1} - \kappa \frac{S_t^\mu}{\mu} + E_t \left[\beta_t W_{t+1}^W \right] \right\}, \tag{11}$$

subject to the constraints (2), (9) and (10). w_t and $r_t + \delta$ are respectively the labour and the capital costs. We have $\kappa \geq 0$ and assume a convex search cost $\mu \geq 1$.[5] The discount factor β_t is compatible with the pricing kernel of the consumers-shareholders:

$$\beta_t = \beta \frac{C_t}{C_{t+1}}. \tag{12}$$

The first order condition for the search intensity is:

$$\frac{\kappa S_t^{\mu-1}}{q_t^S} = P_t - \Lambda_t + E_t \left[\beta_t (1 - \chi) \frac{\kappa S_{t+1}^{\mu-1}}{q_{t+1}^S} \right], \tag{13}$$

where Λ_t is the real marginal cost and given by:

$$\Lambda_t = \frac{1}{\epsilon_t} \left(\frac{w_t}{1 - \alpha} \right)^{1-\alpha} \left(\frac{r_t + \delta}{\alpha} \right)^{\alpha}. \tag{14}$$

The optimal capital-labour ratio is:

$$\frac{K_{t-1}}{N_t} = \frac{\alpha}{1 - \alpha} \frac{w_t}{r_t + \delta}. \tag{15}$$

3.3. Retail firms

The economy is also composed of a continuum of identical retail firms buying tradable products T_t and selling them to households. At the given purchase price P_t, the firms choose their optimal search effort D_t, i.e. by setting aside the necessary number of purchasing and buying employees to find and bargain with new wholesalers. They take as given q_t^D, the rate at which every effort leads to a new match. The rate is defined as:

$$q_t^D = M_t / D_t. \tag{16}$$

Hence each retailer faces the following dynamic programming problem:

$$W_t^R = \max_{D_t} \left\{ T_t - P_t T_t - \kappa \frac{D_t^\mu}{\mu} + E_t \left[\beta_t W_{t+1}^R \right] \right\}, \tag{17}$$

[5] See section 5 for a discussion.

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subject to the constraints (2) and (16). We impose the same search cost and convexity as for wholesale firms.[6] The discount factor β_t is still defined by (12). The first order condition for the search intensity is:

$$\frac{\kappa D_t^{\mu-1}}{q_t^D} = 1 - P_t + E_t \left[\beta_t (1-\chi) \frac{\kappa D_{t+1}^{\mu-1}}{q_{t+1}^D} \right]. \tag{18}$$

3.4. Price formation and markups

Each product market match yields pure economic rents equal to the expected search costs for wholesalers and retailers (including foregone profits). The agreed price is such that these rents are shared and in addition each party is compensated for its incurred costs of forming the match. We follow the labour market literature (see for instance Pissarides (2000)) and assume that the rent sharing is a solution to a Nash (1950) bargaining problem. More precisely, prices are (re-)negotiated between wholesalers and retailers at the beginning of every period through a Nash bargain over the surplus resulting from the match.[7] Because all firms are identical, there is one single price in the economy:

$$\max_{P_t} \left(\frac{\partial W_t^W}{\partial T_t} \right)^{1-\lambda} \left(\frac{\partial W_t^R}{\partial T_t} \right)^{\lambda}, \tag{19}$$

where $0 \leq \lambda \leq 1$ is the retailer bargaining power. This gives:

$$\lambda \, \frac{\partial W_t^W}{\partial T_t} \; = \; (1-\lambda) \, \frac{\partial W_t^R}{\partial T_t}, \tag{20}$$

that simplifies to:

$$P_t \; = \; \lambda \Lambda_t + (1-\lambda). \tag{21}$$

Prices are therefore a weighted average of the marginal cost and 1 and increasing (resp. decreasing) in the bargaining power of the wholesalers (resp. retailers). If wholesalers have no bargaining power, the price is equal to its marginal cost. If retailers have no bargaining power , the price is

[6] See the calibration and simulation sections for a discussion.

[7] Note that P_t represents a *real* producer price (price of final/consumer products are still normalized to 1 as in a standard real business cycle model). The marginal mark up is equal to the average mark up because all wages are (re-)negotiated every period.

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equal to 1.

Wholesaler and retailer markups

The model generates two markups. The wholesaler markup is the markup over the marginal cost and the retailer markup is the markup over the producer price. They are respectively defined as:

$$\varphi_t^W = P_t - \Lambda_t = (1-\lambda)(1-\Lambda_t), \tag{22}$$

$$\varphi_t^R = 1 - P_t = \lambda(1-\Lambda_t). \tag{23}$$

The wholesaler markup is therefore the fraction $1-\lambda$ (wholesaler bargaining power) of the total surplus $1-\Lambda_t$ of the match while the retailer markup is the fraction λ (retailer bargaining power) of the total surplus of the match.

It is straightforward to see that the two markups are negatively correlated with marginal costs. A productivity shock increases output, reduces marginal costs and produces procyclical markups. We obtain a similar correlation with a government shock (higher output and lower marginal costs due to lower wages) and a preference shock for leisure (lower output and higher marginal cost due to higher wages). On the other hand, more frictions in the product market (negative shock on $\bar{m}$ or positive shock on κ) decrease the marginal cost and output, and therefore imply countercyclical markups.

3.5. Equilibrium definition

Given the initial conditions on T_{t-1} and K_{t-1}, an equilibrium of this economy is a sequence of prices $\{\mathcal{P}_t^r\}_{t=1}^{\infty} = \{r_t, w_t, P_t\}_{t=1}^{\infty}$ and a sequence of quantities $\{\mathcal{Q}_t\}_{t=1}^{\infty} = \{C_t, N_t, I_t, S_t, D_t\}_{t=1}^{\infty}$ such that:

- given the sequence of prices $\{\mathcal{P}_t^r\}_{t=1}^{\infty}$, $\{C_t\}_{t=1}^{\infty}$ is a solution to the household first order condition (7)

- given the sequence of prices $\{\mathcal{P}_t^r\}_{t=1}^{\infty}$, $\{N_t, I_t\}_{t=1}^{\infty}$ is a solution to the product and household contraints (9) and (5)

- given the sequence of prices $\{\mathcal{P}_t^r\}_{t=1}^{\infty}$, $\{S_t, D_t\}_{t=1}^{\infty}$ is a solution to the wholesaler and the retailer first order conditions (13) and (18)

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- given the sequence of quantities $\{\mathcal{Q}_t\}_{t=1}^{\infty}$, $\{r_t, w_t\}_{t=1}^{\infty}$ clears the capital market (15) and the labour market (8)

- the price $\{P_t\}_{t=1}^{\infty}$ is set according to the Nash bargain solution (20)

Finally, GDP is defined as:

$$Y_t = C_t + I_t = T_t - \kappa \left(\frac{S_t^{\mu}}{\mu} + \frac{D_t^{\mu}}{\mu} \right), \tag{24}$$

and T_t and K_t evolve according to the law of motion (2) and (6).

4. Inefficiency sources

The economy we describe is characterised by two sources of inefficiency. The first source arises from search costs that induce an inefficiently low level of output. Proposition 1 shows that when search costs disappear, the steady state tends to the Walrasian steady state: labour and capital are priced at their respective marginal productivity and firms make no profits while output is maximised.[8]

Proposition 1. *(Search costs and Walrasian output)*
When $\kappa \longrightarrow 0$ (no search costs), the steady state solution tends to the Walrasian one.

Proof. When $\kappa \longrightarrow 0$, the first order conditions (13) and (18) simplify to $P = \Lambda$ and $P = 1$. Combining $\Lambda = 1$ with equations (14) and (15), we obtain $w = (1-\alpha)\epsilon(K/N)^{\alpha}$ and $r + \delta = \alpha\epsilon(K/N)^{\alpha-1}$. This means that prices are normalized to 1, wages are equal to the marginal productivity of labour and interest rates (incl. depreciation) are equal to the marginal productivity of capital. Moreover, by replacing P, w, $r + \delta$ in equations (11) and (17) and using (9), we see that profits of wholesalers and retailers are equal to zero. This solution is therefore equivalent to the Walrasian one. ■

The second source of inefficiency results from search externality. In a decentralized economy, a search process exhibits externality and, in most cases, the decentralized equilibrium is different from the social planner's equilibrium. With search frictions in the labour market, Hosios (1990), in

[8] Although the steady state tends to the Walrasian one, still the dynamics are different from the dynamics of a standard Walrasian real business cycle.

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a static environment, and Merz (1995), in a dynamic environment, show that an efficiency condition (workers' bargaining power equal to the elasticity of matching with respect to unemployment) exists such that the externality is internalized and the decentralized outcome is strictly equivalent to the social planner's outcome. In proposition 2, we show that a similar condition exists when search frictions exist in the product market.

Proposition 2. *(Externality in the decentralized economy)*
When $\lambda = 1 - \gamma$, the decentralized equilibrium is strictly equivalent to the social planner's problem.

Proof. The social planner's problem is solved in Appendix A and the equivalence between the two solutions is proved. ■

5. Calibration

The technology shock is the exogenous driving force and is assumed to be AR(1):

$$\epsilon_t = \epsilon_{t-1}^{\rho} \exp\left(u_t^{\epsilon}\right), \tag{25}$$

where ρ is the autoregressive parameter and $u_t^{\epsilon} \sim N(0, \sigma_{\epsilon}^2)$.

We consider two versions of the model. We first remove all the frictions in the product market to obtain a standard Walrasian real business cycle model, where labour and capital are priced at their respective marginal productivity and prices are normalized to 1 (the standard Walrasian real business cycle is presented in Appendix B, "RBC" hereafter). We then add frictions. In this case, labour and capital are priced below their respective marginal productivity, prices are endogenous and lower than unity and firms make profits.

We calibrate our RBC model parameters $\{\beta,\ \delta,\ \alpha,\ \theta,\ \eta,\ \rho,\ \sigma_{\epsilon}\}$ on US quarterly data to reproduce some stylized facts for the US economy. We fully follow King and Rebelo (1999). The discount factor is calibrated to yield an average return to capital of 6.5% per annum: $\beta = 1/(1 + 0.065/4)$. To match a capital-output ratio of 8, capital depreciation rate is set to $\delta = 0.025$. We set $\alpha = 1/3$, which is the standard value for the long run capital income share. We assume that $\eta = 1$ in order to obtain a double log utility function: $\mathcal{U}(C_t, 1 - N_t) = \log(C_t) + \theta \log(1 - N_t)$. We choose $\theta = 3.3$ to match $N = 0.20$, which means that 20% of total available time is used for work. Finally, we set $\rho = 0.979$ and $\sigma_{\epsilon} = 0.0072$ to obtain a realistic standard deviation and autocorrelation of output.

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Table 1 summarises the calibration (line "RBC"; a similar calibration is found in King and Rebelo (1999), table 2, p. 955).

Table 1
Parameter values

	β	δ	α	θ	η	ρ	σ_ϵ
RBC	0.984	0.025	0.33	3.34	1	0.979	0.0072
$\mu = 1$	0.984	0.023	0.33	3.30	1	0.979	0.0072
$\mu = 2$	0.984	0.021	0.33	3.06	1	0.979	0.0072

	λ	γ	χ	κ	$\bar{m}$
$\mu = 1$	0.5	0.5	0.25	0.05	0.52
$\mu = 2$	0.5	0.5	0.25	0.35	0.51

We also have the RBC parameters in the model with frictions. However, because of these frictions, giving the same values to the RBC parameters changes the steady state. In particular, the capital-output ratio and the employment level decrease (see proposition 1). To keep a similar steady state, we re-calibrate some of these parameters. More precisely, we change δ and θ to keep a capital-output ratio of 8 and an employment level of 0.2. The TFP process is similar in the RBC model and the model with frictions (same ρ and σ_ϵ). We check in the next section whether or not a similar TFP process introduced in different models (RBC *vs.* frictions) changes the output volatility and persistence. The other parameters $\{\lambda,\ \gamma\ \ \chi,\ \kappa,\ \bar{m},\ \mu\}$ are specific to the model with search frictions.

The "Hosios' parameters"

If retailers (wholesalers) have no bargaining power, *i.e.* no market power, the wholesaler (retailer) appropriates all rents from the contractual relationship. In reality, the bargaining power of retailers and wholesalers is between these two extreme cases. The bargaining power may also vary across markets and depend for instance on the relative size of buyers and sellers (and for example

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whether firm-specific investments have to be undertaken, which gives rise to the classic hold up problem). Since we do not have any priors and data on "economy-wide" bargaining power, we assume that wholesalers and retailers have the same bargaining power: $\lambda = 1 - \lambda = 0.5$, that is the total markup in the economy is equally split between them. In order to have a Pareto optimal outcome (see proposition 2), we impose $\gamma = 1 - \lambda = 0.5$. The Pareto optimality assumption is fairly standard in the labour matching literature.[9]

Search costs and total markup

In our model, prices can be adjusted every period and in any case χ governs the average length of an actual wholesaler-retailer relationship (not a price contract). Empirical evidence (see section 2) shows that the length of business relationships is on average one year (or alternatively the fraction of newly formed business relationships is about 25%) and we therefore set $\chi = 0.25$. According to US data available (see section 2) we set advertising expenditures equal to 2.4% of output. We have no information for retailers' search costs and assume they are equivalent to wholesalers' costs. Total search costs are therefore:

$$\frac{costs}{T} = \frac{\kappa}{\mu}\left(S^{\mu} + D^{\mu}\right) = 4.8\%.$$

Another important parameter is the convexity μ of search costs. We assume linear costs, that is $\mu = 1$, as usually in the literature with search unemployment, as Mortensen and Pissarides (1994), Merz (1995) or Andolfatto (1996). From equations (13) and (18), we have:

$$\frac{1-\Lambda}{1-\beta(1-\chi)} = \frac{\mu}{\chi}\frac{costs}{T}, \tag{26}$$

which implies a total mark up $1 - \Lambda$ (for wholesalers and retailers) of 5%. On the one hand, this is slightly lower than the mark ups reported in previous studies (see section 2). Moreover, mark ups reported in these studies are usually for wholesalers. On the other hand, this lower value might be justified as our mark up refers to an economy-wide mark up of price over marginal cost. This value is expected to be lower than those based on sector data alone since the public sector (where we can assume a negligible mark up) represents substantial part of the total economy data. Given these uncertainties, we alter the functional form of the search cost representation and include quadratic

[9] See for instance Merz (1995) and Andolfatto (1996) for such a calibration.

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search costs, that is $\mu = 2$. This implies a total mark up of 10% which is more in line with the recent empirical literature. Chari et al. (2000) and Edmond and Veldkamp (2009) who based on the work of Basu and Fernald (1997) use for example an economy-wide mark up of close to 10%.

The idea that search costs are convex implies diminishing returns in the matching function and is, we believe, quite intuitive. To show the diminishing returns, let us define a net matching function as $t(S_t, D_t) = M(S_t, D_t) - \kappa S_t^{\mu}/\mu$. When $\mu = 1$, $t(aS_t, aD_t) = a\,t(S_t, D_t)$ and we have constant returns to scale. When $\mu > 1$, $t(aS_t, aD_t) < a\,t(S_t, D_t)$ and we have diminishing returns to scale. The intuition is that in high demand states many agents want to advertise, thus there is more competition for customers than in low demand states. It is therefore conceivable that one unit of search effort, *i.e.* one ad in a newspaper or one advertising time slot on television has a lower effectiveness than in low demand states. As it turns out convex search costs are important for creating hump-shaped impulse response functions for some key variables.

Other parameters

A result of the calibration exercise is that the value of κ is unimportant. Indeed, at given total costs and at given μ, a higher κ will only imply lower steady state values for S and D, and a higher implied value for $\bar{m}$ (matching efficiency parameter), leaving unchanged $\kappa S/q^S$ and $\kappa D/q^D$ and hence the log-linearised equations.[10] We simply choose κ to have matching probabilities close to 0.5. Another direct implication is that assuming different κ's for wholesalers and retailers would not change our simulation results.

The parameters for each version of the model with frictions (linear *vs.* quadratic search costs) are displayed in table 1 (lines "$\mu = 1$" and "$\mu = 2$"). In the next section, we provide simulations for both assumptions. Moreover, given the uncertainties around the calibration of the bargaining power and the estimation of total search costs, we conduct a sensitivity analysis on both variables.

[10] As already explained, this is because parameters are calibrated to match some steady state values. In simulation exercises, changes in κ have obviously steady state and dynamic effects.

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6. Simulations

First, we investigate how the search *structure* modifies the transmission of a productivity shock to the economy. Second, we explore whether or not search *shocks* can help to improve the reproduction of important US data time series.

6.1. RBC properties

We use the autoregressive productivity shock and simulate three different models: (i) the standard Walrasian real business cycle model (RBC), (ii) the model with frictions in the product market presented in section 3 and linear search costs ($\mu = 1$), and (iii) the same model but with quadratic search costs ($\mu = 2$). We compare results to the business cycle characteristics of US data (see Appendix C). The simulation results as well as the US statistics are reported in table 2 and figure 2 (similar results for the RBC version are found in King and Rebelo (1999), table 3 p. 957 and figure 10 p. 968).

It is well known that although the Walrasian RBC model does a good job in reproducing consumption and investment behaviour, it suffers from some weaknesses: (i) employment is not volatile enough and wages are too procyclical, (ii) the autocorrelation of output is too weak although the autocorrelation of the shock is close to one (not enough endogenous persistence in the model), and (iii) there are no smooth impulse responses (except for consumption and wages).

The results with linear search frictions are very close to the RBC results. The only improvement comes from the - slightly - smoother reaction of GDP and investment due to the search and matching process on the product market. We also see that frictions dampen the absolute volatility of output (the numbers are provided between brackets in table 2). Looking at variables specific to the model with search frictions, we see that the advertising expenditures are procyclical and highly volatile as in the real data. However, the reaction is very short-lived (autocorrelation close to zero). The short-lived costs combined with $\mu = 1$ results in a benign increase in surplus and therefore markups (see equation (26) in section 5), which in turn implies an almost negligible fall in the producer price (relative standard deviation of 0.01 with respect to 0.59 in data).

The story is completely different once we introduce quadratic search costs. Although the initial reaction of advertising expenditures is similar, the persistence is much higher and explains why

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Table 2
Steady state and cyclical properties

	relative standard deviation			
	US data	RBC	$\mu = 1$	$\mu = 2$
P_t	0.59		0.01	0.24
ad_t	2.57		2.89	4.53
Y_t	(1.56) 1.00	(1.39) 1.00	(1.34) 1.00	(0.98) 1.00
C_t	0.81	0.44	0.44	0.50
I_t	3.41	3.38	3.47	3.62
N_t	1.00	0.49	0.51	0.49
w_t	0.64	0.54	0.55	0.56
r_t	0.31	0.03	0.03	0.02

	first-order autocorrelation				contemporaneous correlation with output			
	US data	RBC	$\mu = 1$	$\mu = 2$	US data	RBC	$\mu = 1$	$\mu = 2$
P_t	0.80		0.71	0.34	-0.17		-0.94	-0.37
ad_t	0.88		0.02	0.43	0.77		0.58	0.55
Y_t	0.88	0.72	0.78	0.90	1.00	1.00	1.00	1.00
C_t	0.88	0.79	0.79	0.76	0.86	0.94	0.95	0.92
I_t	0.92	0.71	0.79	0.88	0.94	0.99	0.99	0.97
N_t	0.94	0.71	0.70	0.58	0.83	0.97	0.96	0.69
w_t	0.81	0.76	0.76	0.86	0.24	0.98	0.98	0.98
r_t	0.55	0.71	0.71	0.87	-0.06	0.96	0.84	0.94

Notes: All variables have been logged (with the exception of the real interest rate) and detrended with the HP filter. US data: see Appendix C; RBC: standard Walrasian real business cycle model *à la* King and Rebelo (1999) presented in Appendix B; $\mu = 1$ and $\mu = 2$: models with frictions in the product market presented in section 3. Numbers between brackets: absolute standard deviations. P_t : real producer price, $ad_t = \kappa S_t^{\mu}/\mu$: advertising expenditures, Y_t : gross domestic product, C_t : consumption, I_t : investment, N_t : employment, w_t : wage, r_t : interest rate.

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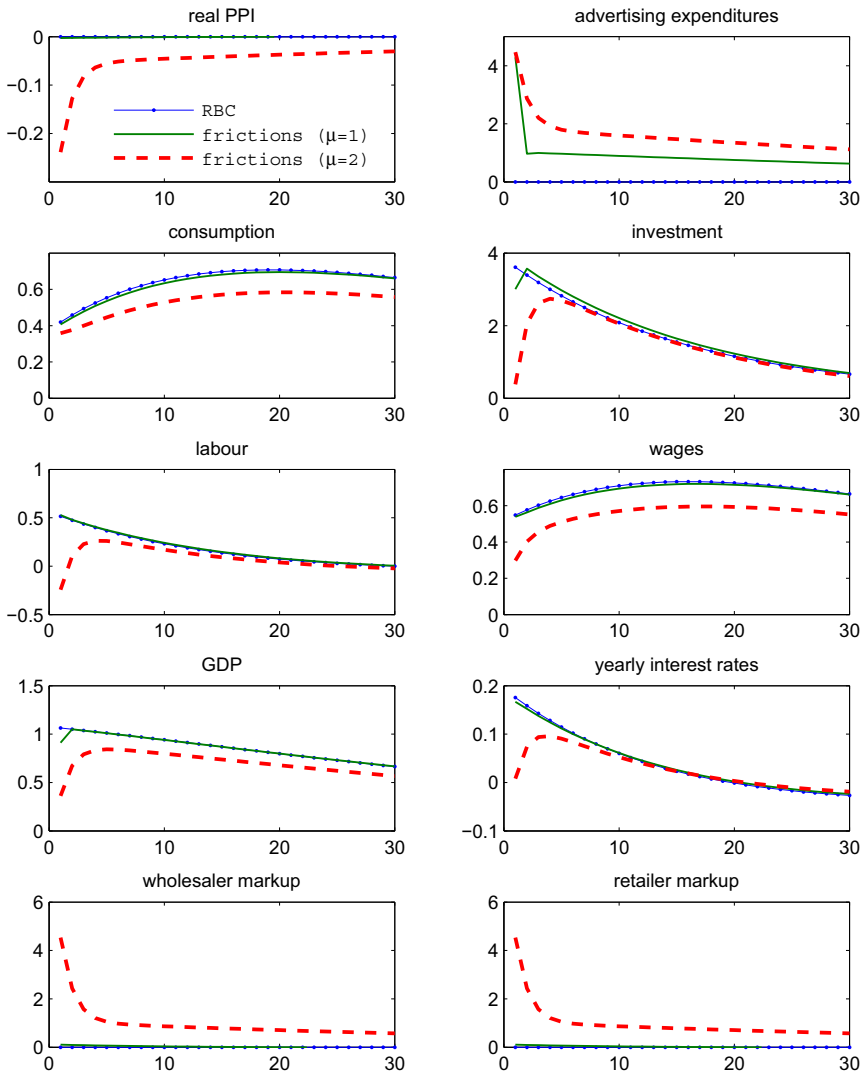

Fig. 2. Impulse response functions to a productivity shock (% deviations from steady-states)

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the surplus (markups) must strongly increase (see equation (26) in section 5). This in turn allows for a much stronger fall in producer prices (relative standard deviation increases from 0.01 with $\mu = 1$ to 0.24 with $\mu = 2$). As displayed in figure 2, quadratic search costs also quite remarkably produce hump-shaped reactions for all variables. Indeed, although linear and quadratic costs generate similar initial increases in *search costs*, the initial increases in the *search efforts* (S_t and D_t) are much smaller in case of a quadratic relationship between efforts and costs. As a result, only few new wholesaler-retailer relationships are initially created which in turn affects investment and labour. The hump-shaped reaction follows from the higher persistence in search costs and therefore in search effort. We see that the first-order autocorrelation of output strongly increases and is close to what is observed in the data. Obviously, this also further dampens the absolute volatility of output. Finally, we observe that search frictions do not tend to modify moments related to consumption, investment and the factor prices, with respect to the standard RBC model.

From the above analysis, we conclude that product market frictions *per se* do not really affect the transmission of shocks with respect to the standard RBC model. What is really important is the modelisation of search costs. On the one hand, convex search costs increase the persistence of output and generate hump-shaped reactions for most variables. Moreover, they require a strong reaction in markups which in turn allows for a high (and realistic) volatility of producer prices. On the other hand, they also reduce output volatility. Unless mentioned, we use the model with quadratic costs as benchmark in the remainder of the paper.

6.2. *Sensitivity analysis*

We now discuss the role of the bargaining power parameter and the calibration of search costs.

Bargaining power

The Hosios condition ($\gamma = 1 - \lambda = 0.5$) holds in the benchmark calibration. We look at the effects of changes in the retailers' bargaining power λ. A different λ does not change anything but the way the surplus is split between wholesalers and retailers. The higher the parameter λ, the higher is the surplus share the retailers receive, *i.e.* the lower is the producer price, the lower is the wholesaler markup and the higher is the retailer markup. As obvious from equation (21), the price volatility increases with λ. For instance, moving from $\lambda = 0.1$ to $\lambda = 0.9$ increases the relative standard deviation of P_t from 0.05 to 0.45 (0.59 in data). It is also worth noting that higher

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retailer (resp. lower wholesaler) markups allow retailers (resp. wholesalers) to bear higher (resp. lower) search costs. In other words, the calibration still assumes total search costs representing 4.8% of output, but these search costs are no longer evenly distributed.

Search costs

Total search costs themselves are uncertain. We associate search costs to advertising expenditures for wholesalers and assume similar costs for retailers. This is certainly arbitrary and maybe too restrictive. For instance, total marketing budgets (including all needed resources as labour, capital, etc.) could be a more realistic representation of wholesalers' search efforts. We let the total search costs vary from 2.5% to 7.5% of output. It implies that marginal costs move from 0.95 to 0.85 (see equation (26) in section 5), or equivalently that total surplus, or the total markup, moves from 5% to 15%. We can show that increasing search costs decrease their volatility and increase the volatility of producer prices (because of their lower steady state value). Output is also less volatile but more persistent. Moments for the other variables are also affected but only marginally.

In conclusion, higher frictions (search costs) combined with higher markups for retailers further improve the product market representation (higher price volatility and lower cost volatility) but this improvement is not really dramatic. Similarly, we could fine-tune the results by allowing for different μ's for wholesalers and retailers.

6.3. The product market shock

So far we looked at the effect of the search *structure* on business cycle fluctuations, including solely a productivity shock as source of fluctuation. But if we believe search to be important, we can expect that search related *shocks* could also help to explain economic fluctuations. In this section, we introduce a shock to the search cost parameter κ such that:

$$\kappa_t = \kappa^{1-\rho}\, \kappa_{t-1}^{\rho} \exp\left(u_t^{\kappa}\right), \tag{27}$$

where $\rho = 0.979$ is the autoregressive parameter (same autocorrelation than the productivity shock) and $u_t^{\kappa} \sim N(0, \sigma_{\kappa}^2)$, with $\sigma_{\kappa} = 0.072$. We first look at the impulse response functions of

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this product market shock. We then conduct a shock decomposition exercise.[11]

Impulse response functions

Figure 3 displays the IRF's of a product market shock, *i.e.* of a transitory increase in the search cost κ_t. For comparison purposes, we also show the IRF's of a positive productivity shock. We see that a product market shock increases advertising expenditures which forces wholesalers and retailers to augment their markups. As a result, there is a negative effect on the whole economy. As already mentioned in section 3, productivity shocks generate procyclical markups whereas product market shocks imply countercyclical markups.

Shock decomposition

The recursive solution of the log-linearised model has the form:

$$\mathbf{X}_t = A \ \left[\hat{K}_{t-1} \ \hat{T}_{t-1}\right]' + B \ \mathbf{E}_t \ , \tag{28}$$

where $\mathbf{X}_t$ is a vector of endogenous variables, $\mathbf{E}_t = [\hat{e}_t \ \hat{\kappa}_t]'$ is a vector including the two exogenous shocks, K_t and T_t are the two state variables of the model, and A and B are coefficient matrices. We represent log-linearised variables with a hat.

To construct productivity and product market shocks, we follow Chari et al. (2007), Benk et al. (2005) or Nolan and Thoenissen (2009). We assign values to some endogenous variables in $\mathbf{X}_t$ using US quarterly data, and compute the shock values $\mathbf{E}_t$ back from the recursive equilibrium of our model. We choose endogenous variables (also called observable) for which we have reliable data and, when possible, related to the product market. The result is a four variables vector $\mathbf{X}_t = [\hat{Y}_t \ \hat{C}_t \ \hat{I}_t \ \hat{P}_t]'$ including GDP, consumption, investment and real producer prices. For each variable we use the US data values from 1971:1 to 2006:4 and represent them as percentage deviations from their HP-trend. Next, we need to construct the vector of two state variables of the model. We construct K_t from the capital accumulation equation (6) and US data on investment. We construct

[11] We rely on κ as product market shock, as this is the parameter we use to identify search frictions in section 4. Alternatively, we could use the matching efficiency parameter $\bar{m}$ and obtain similar results.

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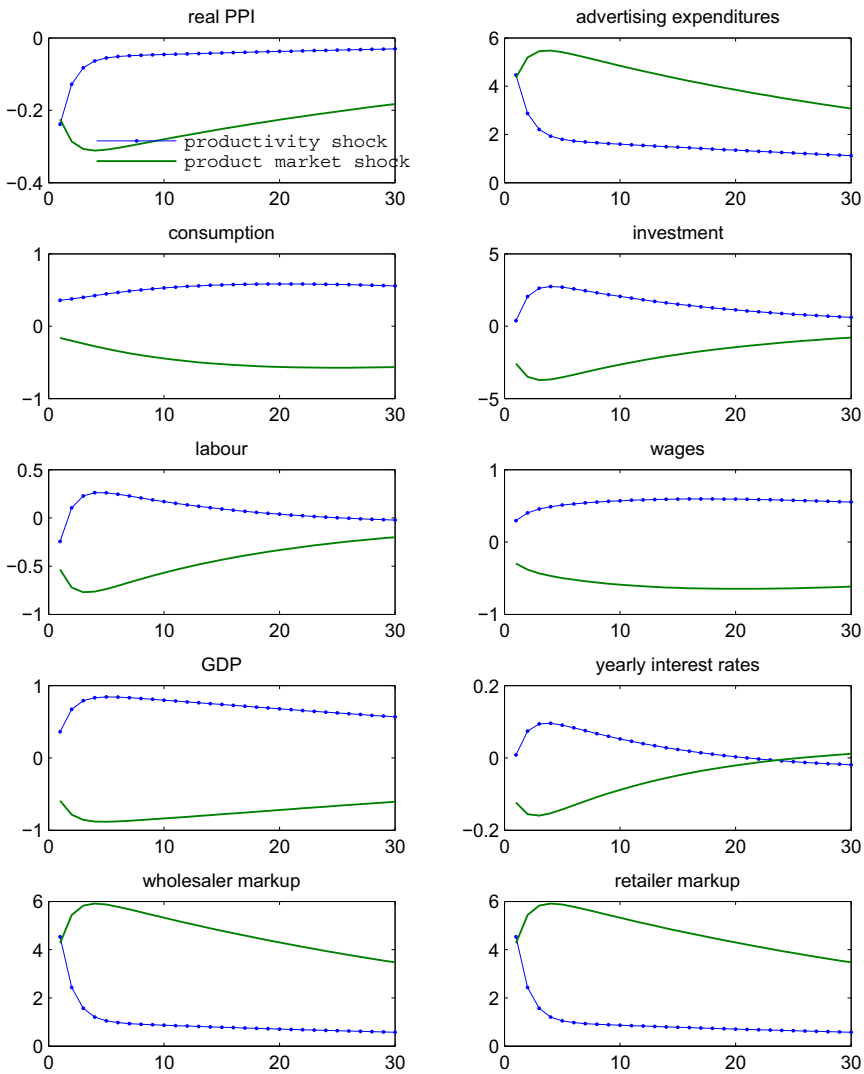

Fig. 3. Impulse response functions to a productivity shock *vs.* a product market shock (% deviations from steady-states)

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T_t from the output definition equation (24) and US data on GDP and advertising expenditures.[12]

Having four equations for two unknown shocks gives an overidentification of the shocks. Overidentification still allows for a unique determination of the two shocks through an OLS estimation procedure:

$$\tilde{\mathbf{E}}_t = (B'B)^{-1}B'\left(\mathbf{X}_t - A\left[\hat{K}_{t-1}\ \hat{T}_{t-1}\right]'\right). \tag{29}$$

Finally, we estimate the autocorrelations of the two shocks through a system of seemingly unrelated regressions. We then substitute them back into (28) and estimate the shock process again until the two autocorrelation parameters converge.[13] After iteration, the autocorrelations are respectively $\rho_\epsilon = 0.84$ and $\rho_\kappa = 0.67$, and the correlation between the two shocks is $corr(\epsilon_t, \kappa_t) = 0.73$. Figure 4 plots the estimated shocks, along with the main US recession periods. As we can see, the search cost shock is procyclical, meaning that as the economy expands search effort will become more costly. Increasing search costs can for example emanate from limited advertising space in the newspapers or television, which is treated as an inventory. In low demand states this space is sold off cheaper than in high demand states, thus increasing the cost in expansionary periods. Figure 5 also shows that, although we do not have a full identification, the two shocks reproduce quite well the four observable, the only weakness being the lack of volatility for consumption.[14]

To have a quantitative idea of how the real data Y^d differ from the similar series Y^m generated by the model, we define:

$$distance = \frac{\sum_{t=1}^{T}\left(Y_t^d - Y_t^m\right)^2}{T}, \tag{30}$$

where T represents the number of periods. The first line ("data variance") in Table 3 simply gives the variance for each observable, which is equivalent to the distance as defined in (30), when $Y_t^m = 0\ \forall t$. The second line ("productivity shock") reports the distance between real and simulated data when we only assume a productivity shock, that is when conduct the iteration exercise described

[12]The construction of T_t is admittedly imprecise. The advertising expenditure data ad_t are for wholesalers, and we assume that retailers have identical expenditures. We therefore construct $T_t = Y_t - 2 \times ad_t$. We describe the US data in Appendix C.

[13]The standard deviations of the shocks also differ from the initial guesses, but this is unimportant since we we use a first order approximation to solve the model.

[14]As a robustness check, we also compute the shocks with combinations of three observable out of the four in the original sample. This gives broadly similar results. Smaller constructions with two observable would imply exact identifications.

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above with $\mathbf{E}_t = [\hat{\epsilon}_t]$. We see that a productivity shock may partially reproduce the fluctuations of output, consumption and producer prices, but at the expense of investment. The third line ("product market shock") gives the distance when we only assume a product market shock, that is when conduct the iteration exercise described above with $\mathbf{E}_t = [\hat{\kappa}_t]$. Interestingly, we see that the product market shock improves the reproduction of output - and investment - fluctuations with respect to the productivity shock, underlining the role of search shocks to explain US business cycles. However, the fit for consumption and producer prices is weaker. Finally, the fourth line ("productivity + product market shocks") assumes both shocks simultaneously as described above and, obviously, this improves the fit for all variables.

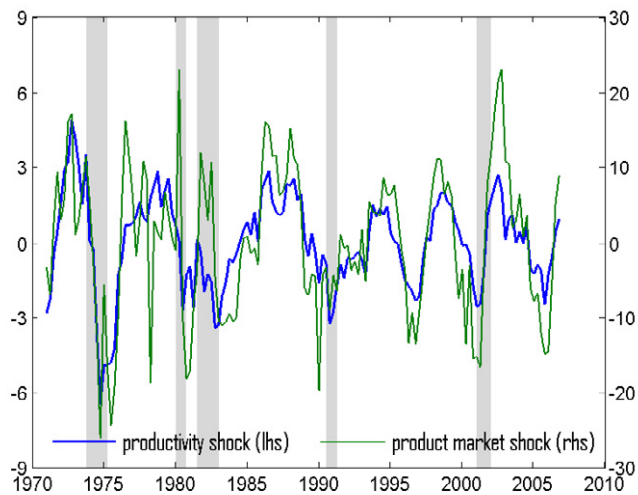

Fig. 4. Evolution of productivity and product market shocks (% deviations from trend). *Notes*: Shaded areas represent the US main recessions, according to The National Bureau of Economic Research. Source: www.nber.org.

7. Extensions

With flexible prices, static search and adjustment along the extensive margin, the model we develop remains quite simple. We hereafter discuss possible related extensions.

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Table 3
Data variance and distance between US data and simulated data with different combinations for shocks

	Y_t	C_t	I_t	P_t
data variance	2.40	1.57	28.07	0.83
productivity shock	1.78	1.05	45.40	0.37
product market shock	1.65	1.52	20.07	0.70
productivity + product market shocks	0.66	0.99	17.02	0.06

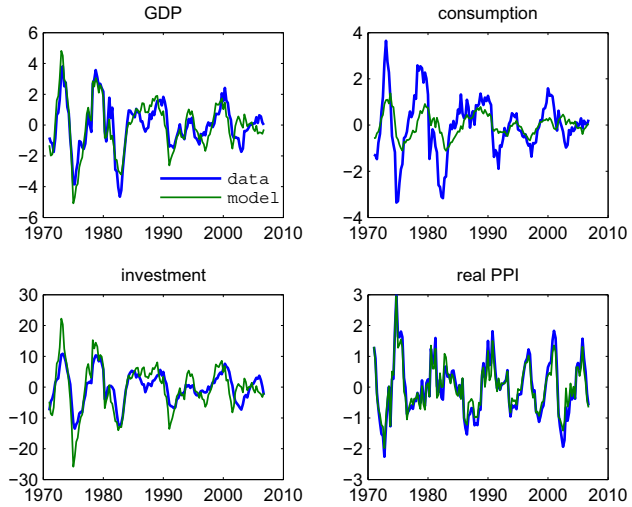

Fig. 5. Time series comparison: data vs. model (% deviations from HP-trend)

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7.1. Price rigidities

In the model, prices are bargained every period without any cost. However, prices may be subject to convex adjustment costs (see for instance Rotemberg (1982, 1983)) or staggered contracts (see for instance Taylor (1999) or Calvo (1983)). In the extreme case (constant prices), the volatility of the total markup would be unchanged but completely due to the volatility of the wholesaler markup (the retailer markup remaining unchanged). As a result, the volatility of advertising expenditures also strongly increases. In other words, price rigidities destroy (with respect to the data) the cyclical properties of the product market. However, since price rigidities only change the split of the surplus between wholesalers and retailers, other cyclical properties remain unchanged.

7.2. The search process

So far, we have assumed that search has only current effects ("static search"). In other words, if no match is created, search effort is lost and firms must start over from scratch in the next period. Alternatively, we could assume that search today has lasting effects ("durable search"). Below, we propose two different versions of durable search. In the first, a sunk cost must be paid to enter the product market and there is no cost afterwards. In the second, search is a flow variable allowing to build a marketing capital stock.

Sunk - or entry - cost

Fujita and Ramey (2007) propose a model with sunk costs to enter into the labour market. We hereafter transpose their approach to the product market. If a wholesaler wants to find a new buyer, (s)he needs first to pay a sunk - or entry - cost to enter the search market. The sunk cost SC may differ across wholesalers and is drawn from a probability distribution f. The total mass S_t of wholesalers searching for buyers is therefore composed of newcomers n_t, paying a sunk cost, and old stayers who haven't yet matched:

$$S_t = (1 - q_t^S)S_{t-1} + n_t. \tag{31}$$

The Lagrangian for the wholesaler's maximization problem is:

$$\mathcal{L} = E\left[\sum_{t=0}^{\infty} \beta^t \frac{C_0}{C_t}\left\{P_t T_t - \Lambda_t T_t + \phi_t\left((1-\chi)T_{t-1} + q_t^S S_t - T_t\right)\right\}\right]. \tag{32}$$

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The first order condition is $\partial\mathcal{L}/\partial T_t = 0$ and we determine the amount of newcomers as:

$$n_t = \int_0^{\mathcal{L}_{S_t}} f(SC)\, dSC, \tag{33}$$

where $\mathcal{L}_{S_t} = \partial\mathcal{L}/\partial S_t$. For simplicity reasons, we keep the original formulation for the retailers (static search) with $\mu = 1$.[15] We assume a uniform probability distribution function f and calibrate it to keep the same steady state. We then implement a productivity shock and compare this sunk cost model with the benchmark model (static search for both wholesalers and retailers with $\mu = 2$).

Figure 6 shows that this durable search framework may generate hump-shaped IRF's and persistent output reactions as static search with convex costs can do. Moreover, it also allows to amplify the effect of the shock on output. Indeed, this formulation transforms search from a flow to a stock variable, and therefore enhances its reaction and persistence.

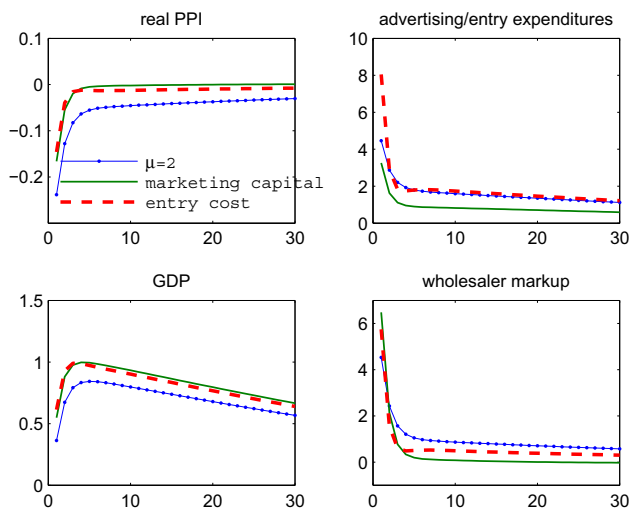

Fig. 6. Search costs: alternative specifications (% deviations from steady-states)

Marketing capital

Following Drozd and Nosal (2011), we assume that wholesalers spend S_t to build marketing cap-

[15] In section 6, we see that convex costs ($\mu = 2$) are important to generate realistic IRF's. Here, we purposely keep $\mu = 1$ for the retailers to better understand if sunk costs may provide an alternative explanation.

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ital m_t according to:

$$m_t = (1-\tau)m_{t-1} + S_t - \frac{\varphi}{m_{t-1}}(S_t - \delta m_{t-1})^2. \tag{34}$$

This formulation implies that marketing capital depreciates at the rate τ and that there is an adjustment cost. The matching rate in turn depends on the amount of marketing capital and the Lagrangian for the wholesaler's maximization problem is:

$$\begin{aligned}\mathcal{L} &= E\left[\sum_{t=0}^{\infty}\beta^t\frac{C_0}{C_t}\left\{P_tT_t - \Lambda_tT_t - \kappa S_t + \phi_t^1\left((1-\chi)T_{t-1} + q_t^S m_t - T_t\right)\right.\right.\\ &\quad \left.\left. +\phi_t^2\left((1-\tau)m_{t-1} + S_t - \frac{\varphi}{m_{t-1}}(S_t - \delta m_{t-1})^2 - m_t\right)\right\}\right]. \end{aligned} \tag{35}$$

The first order conditions are $\partial\mathcal{L}/\partial T_t = 0$, $\partial\mathcal{L}/\partial S_t = 0$ and $\partial\mathcal{L}/\partial m_t = 0$. Again, we assume that the costs are linear in search and, for simplicity reasons, we keep the original formulation for the retailers (static search) with $\mu = 1$ (see footnote 15). We set $\tau = 0.20$ as in Drozd and Nosal (2011) and $\varphi = 1$ to generate realistic volatility for advertising expenditures.

Figure 6 shows that this marketing capital framework may generate hump-shaped IRF's and persistent output reactions as static search with convex costs can do. It also amplifies the effect of the shock on output. Again, the reason is that marketing capital is a stock rather than a flow variable.

7.3. *Intensive margin on the product market*

In the previous models, every match exchanges one unit of goods every period, *i.e.* there is only adjustment along the extensive margin. We now assume that firms can also choose the exchanged quantity, *i.e.* they also adjust along the intensive margin. Practically, we replace equation (9) with:

$$h_t^{1+v}T_t = \epsilon_t K_{t-1}^{\alpha} N_t^{1-\alpha}, \tag{36}$$

where T_t is the number of matches and h_t is the exchanged quantity per match. We impose $v > 0$ which implies that we have decreasing returns to scale in the production function. The Lagrangian for the wholesaler's maximization problem is:

$$\mathcal{L}^W = E\left[\sum_{t=0}^{\infty}\beta^t\frac{C_0}{C_t}\left\{P_th_tT_t - \Lambda_th_t^{1+v}T_t - \kappa\frac{S_t^{\mu}}{\mu} + \phi_t^W\left((1-\chi)T_{t-1} + q_t^S S_t - T_t\right)\right\}\right]. \tag{37}$$

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The first order conditions are $\partial\mathcal{L}^W/\partial T_t = 0$ and $\partial\mathcal{L}^W/\partial S_t = 0$. The Lagrangian for the retailer's maximization problem is:

$$\mathcal{L}^R = E\left[\sum_{t=0}^{\infty}\beta^t\frac{C_0}{C_t}\left\{h_tT_t - P_th_tT_t - \kappa\frac{D_t^\mu}{\mu} + \phi_t^R\left((1-\chi)T_{t-1} + q_t^D D_t - T_t\right)\right\}\right]. \tag{38}$$

The first order conditions are $\partial\mathcal{L}^R/\partial T_t = 0$ and $\partial\mathcal{L}^R/\partial D_t = 0$. We successively examine a right to manage and an efficient bargaining determination of hours. These two approaches are standard in the general equilibrium literature, see for instance de Walque et al. (2009).

Right to manage

In this setup, the price is first bargained and then wholesalers are free to choose the quantity exchanged at the bargained price so as to maximise profits. This gives $(1+v)h_t^v = P_t/\Lambda_t$ and $P_t = (1-\lambda) + \lambda/(1+v)$. The exchanged quantity depends positively on the wholesaler's markup.

Efficient bargaining

In this setup, the exchanged quantity and prices are simultaneously bargained over to maximise the joint surplus of wholesalers and retailers. This gives $(1+v)h_t^v = 1/\Lambda_t$ and $P_t = \lambda h_t^v\Lambda_t + (1-\lambda)$. The exchanged quantity depends positively on the total markup of the economy. It is worth noting that in both the right to manage ("RTM") and the efficient bargaining ("EB") alternatives, producer prices are constant and given by the same expression.

Simulations

We calibrate v to keep $h = 1$ at the steady state as in the benchmark model ("$\mu = 2$"). Figure 7 displays the IRF's. RTM and EB have very close dynamics. After a positive shock, adjustment along the extensive margin is less important (lower expenditures) because firms can immediately adjust along the intensive margin. This increases the absolute volatility of output and makes the model more realistic. However, the intensive margin also removes the hump-shaped reaction of output and cannot explain price fluctuations anymore.

7.4. Comparison with other approaches

One success of the benchmark model (frictions with quadratic search costs) is to generate hump-shaped reactions for all variables. There are of course other existing explanations for such

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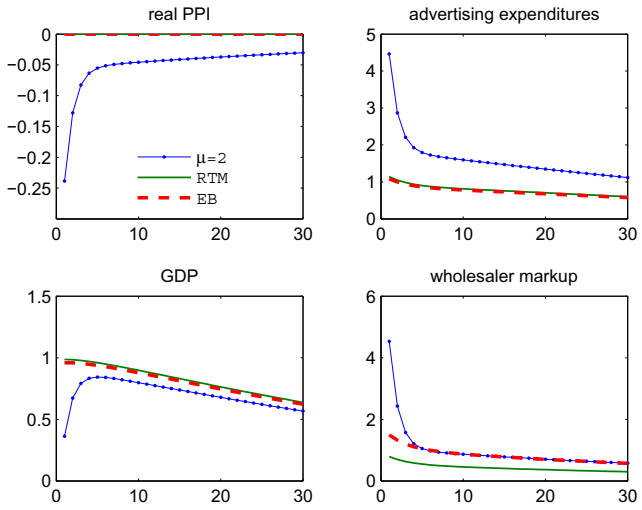

Fig. 7. Trade on the product market: alternative specifications (% deviations from steady-states)

dynamics. One the one hand, labour market search as in Merz (1995) or Andolfatto (1996) will lead to employment being hump-shaped, but nor investment neither output. On the other hand and obviously, capital adjustment costs produces hump-shaped investment but no hump-shaped labour and output. Combining the two as for instance de Walque et al. (2009) may therefore lead to a dynamics similar to ours. Multi-sector models also have interesting properties. Benhabib et al. (2006) show that a three-sector model has a strong propagation mechanism under conventional parameterizations and lead to hump-shaped consumption, output and investment. This however comes at the expenses of a too high labour volatility. We instead generate hump-shaped reactions with only search in the product market without relying upon any extra features. Moreover, we show that alternative search processes, as sunk cost or marketing capital, also allow amplifying shocks, as is the case in the multi-sector model.

8. Conclusion

This paper develops a theoretical model, where both wholesale and retail firms provide search effort (*i.e* through advertising expenditures and employment of sales and purchasing managers) to meet their customers in a product market with search frictions. Firms form long-term contrac-

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tual relationships and downstream producers or wholesalers bargain over prices with upstream retailers, who in turn sell to the final consumers. We show that the way search costs are modelled is crucial for the results. In particular, convex search costs generate hump-shaped reactions for all variables and provide nice statistical properties (compared to real data). We also discuss several extensions.

At least two less straightforward extensions could be interesting for future research. First, we could introduce a monetary dimension and compare with the standard monopolistic competition New Keynesian set up (see Smets and Wouters (2003) or Christiano et al. (2005)) how frictions affect monetary transmission and inflation persistence. Second, this paper explores the business cycle properties of search in the product market and so far, nothing is said on the welfare implications of our findings. For instance, given the adverse effects of search frictions on the level of output, policies aiming at reducing these imperfections (lower entry barriers, role of subsidies, taxation, trade associations, ...) might prove powerful. We could also use this setup to discriminate between product market and labour market regulations (see for instance Messina (2006) or Fang and Rogerson (2011) for models with monopolistic competition).

Acknowledgments

For helpful comments, we thank an anonymous referee, the editor Michel Juillard, and seminar participants at the Central Bank of Luxembourg, the European Central Bank and the EEA Meeting 2008. The views expressed in this paper are personal views of the authors and do not necessarily reflect those of the Banque centrale du Luxembourg or the Eurosystem.

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Appendix A. The social planner's problem

Social planner

The social planner's maximization problem is:

$$W_t = \max_{C_t, S_t, D_t} \left\{ \log(C_t) + \frac{\theta}{1-\eta} \left((1-N_t)^{1-\eta} - 1 \right) + \beta \, E_t \left[W_{t+1} \right] \right\},$$

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subject to the constraints:

$$\begin{aligned} T_t &= C_t + K_t - (1-\delta)K_{t-1} + \kappa\frac{S_t^\mu}{\mu} + \kappa\frac{D_t^\mu}{\mu}, \\ T_t &= F(K_{t-1}, N_t) \;=\; \epsilon_t K_{t-1}^\alpha N_t^{1-\alpha}, \\ T_t &= (1-\chi)T_{t-1} + M(S_t, D_t). \end{aligned}$$

The three first order conditions are:

$$\frac{1}{C_t} = \beta E_t\left[\frac{1-\delta}{C_{t+1}} + \frac{\theta}{(1-N_{t+1})^\eta}\,\frac{\alpha}{1-\alpha}\,\frac{N_{t+1}}{K_t}\right], \tag{P1}$$

$$\frac{\kappa S_t^{\mu-1}}{C_t}\,\frac{1}{M_{S_t}} = \frac{1}{C_t} - \frac{\theta}{(1-N_t)^\eta}\,\frac{1}{F_{N_t}} + \beta(1-\chi)\;E_t\left[\frac{\kappa S_{t+1}^{\mu-1}}{C_{t+1}}\,\frac{1}{M_{S_{t+1}}}\right], \tag{P2}$$

$$\frac{\kappa D_t^{\mu-1}}{C_t}\,\frac{1}{M_{D_t}} = \frac{1}{C_t} - \frac{\theta}{(1-N_t)^\eta}\,\frac{1}{F_{N_t}} + \beta(1-\chi)\;E_t\left[\frac{\kappa D_{t+1}^{\mu-1}}{C_{t+1}}\,\frac{1}{M_{D_{t+1}}}\right]. \tag{P3}$$

Decentralised equilibrium

In section 3, the three similar first order conditions are:

$$\frac{1}{C_t} = \beta E_t\left[(1+r_{t+1})\frac{1}{C_{t+1}}\right], \tag{D1}$$

$$\frac{\kappa S_t^{\mu-1}}{q_t^S} = P_t - \frac{w_t}{F_{N_t}} + E_t\left[\beta_t(1-\chi)\frac{\kappa S_{t+1}^{\mu-1}}{q_{t+1}^S}\right], \tag{D2}$$

$$\frac{\kappa D_t^{\mu-1}}{q_t^D} = 1 - P_t + E_t\left[\beta_t(1-\chi)\frac{\kappa D_{t+1}^{\mu-1}}{q_{t+1}^D}\right]. \tag{D3}$$

Using equations (8) and (15), we can rewrite equation (D1) as:

$$\frac{1}{C_t} = \beta E_t\left[\frac{1-\delta}{C_{t+1}} + \frac{\theta}{(1-N_{t+1})^\eta}\,\frac{\alpha}{1-\alpha}\,\frac{N_{t+1}}{K_t}\right]. \tag{D4}$$

Using equations (10), (12), (8) and (21), we can rewrite equation (D2) as:

$$\frac{\kappa S_t^{\mu-1}\gamma}{M_{S_t}} = (1-\lambda)\left(1 - \frac{C_t\theta}{(1-N_t)^\eta}\,\frac{1}{F_{N_t}}\right) + \beta E_t\left[\frac{C_t}{C_{t+1}}(1-\chi)\,\frac{\kappa S_{t+1}^{\mu-1}\gamma}{M_{S_{t+1}}}\right]. \tag{D5}$$

Using equations (16), (12), (8) and (21), we can rewrite equation (D3) as:

$$\frac{\kappa D_t^{\mu-1}(1-\gamma)}{M_{D_t}} = \lambda\left(1 - \frac{C_t\theta}{(1-N_t)^\eta}\,\frac{1}{F_{N_t}}\right) + \beta E_t\left[\frac{C_t}{C_{t+1}}(1-\chi)\,\frac{\kappa D_{t+1}^{\mu-1}(1-\gamma)}{M_{D_{t+1}}}\right]. \tag{D6}$$

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Equivalence

The social planner's equilibrium is equivalent to the decentralized equilibrium if and only if the first order conditions (P1)-(P2)-(P3) are equivalent to the first order conditions (D4)-(D5)-(D6). We see that equations (P1) and (D4) are always identical. We also see that $\gamma = 1 - \lambda$ is a sufficient and necessary condition to ensure that the system of equations (P2)-(P3) is equivalent to the system of equations (D5)-(D6).

Appendix B. The standard Walrasian real business cycle model

The equations of the standard Walrasian real business cycle model are:

$$\begin{aligned}
\frac{1}{C_t} &= \beta E_t \left[(1 + r_{t+1}) \frac{1}{C_{t+1}} \right], \\
\frac{\theta}{(1 - N_t)^{\eta}} &= \frac{w_t}{C_t}, \\
\epsilon_t K_{t-1}^{\alpha} N_t^{1-\alpha} &= C_t + K_t - (1 - \delta) K_{t-1}, \\
w_t &= \epsilon_t (1 - \alpha) \left(\frac{K_{t-1}}{N_t} \right)^{\alpha}, \\
r_t + \delta &= \epsilon_t \alpha \left(\frac{K_{t-1}}{N_t} \right)^{(\alpha - 1)}.
\end{aligned}$$

Appendix C. Quarterly US data

From 1971:q1 to 2006:q1.

Real producer price: Monthly PPI deflated by the monthly CPI. The monthly data are transformed into quarterly ones. Source: BLS. Logged and HP-filtered with a 1600 smoothing weight.

Advertising expenditures: Sum of quarterly advertising expenditures in newspapers (source: http://www.naa.org/, seasonally adjusted using X12) and quarterly advertising expenditures in internet (source: http://www.iab.net/resources/ad_revenue.asp). The sum is GDP-deflated,

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logged and HP-filtered with a 1600 smoothing weight.

GDP: Quarterly gross domestic product. Source: BEA. Logged and HP-filtered with a 1600 smoothing weight.

Consumption: Quarterly total private consumption. Source: BEA. Logged and HP-filtered with a 1600 smoothing weight.

Investment: Quarterly total private investment. Source: BEA. Logged and HP-filtered with a 1600 smoothing weight.

Employment: Quarterly employment in the non farm business sector. Source: BLS. Logged and HP-filtered with a 1600 smoothing weight.

Wages: Quarterly hourly compensation in the non farm business sector. Source: BLS. Logged and HP-filtered with a 1600 smoothing weight.

Interest rate: Monthly 3-month Treasury bill nominal rate. Nominal rates are deflated by the realized 3-month inflation rate. The monthly data are transformed into quarterly ones. Source: Federale Reserve Bank of St Louis. HP-filtered with a 1600 smoothing weight.

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