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An equation is derived for the linearized electrostatic potential of a system of N electrons with a neutralizing ionic background. Two successive smoothings first reveal this potential to be the sum of the shielded Coulomb potentials of the individual particles, and then yield the classical Vlasovian expression including initial conditions for contour calculations of Landau damping. Thereby, shielding and collisional transport now appear as two related aspects of the repulsive deflections of electrons. This unifies and simplifies the introduction of two important concepts of plasma physics.

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INTRODUCTION

In plasma physics, learning Debye shielding and Landau damping from a fundamental point of view traditionally implies following a hard path through the use of fluid and kinetic models which require themselves specific chapters to be derived from first principles (see e.g. [1] and chapter 5 of [2]), and both phenomena are derived independently. In contrast, this paper provides a novel, compact, unified introduction to these basic phenomena without appealing to fluid or kinetic models, or to a body of extraneous mathematics, but with using Newton's second law for a system of N electrons coupled through Coulomb interaction. This shorter and elementary approach is easily accessible to researchers in other disciplines and opens new avenues for teaching and thinking, as it unifies and simplifies basic microscopic plasma physics.

In particular, this paper proposes a novel intuitive interpretation of Debye shielding for plasma particles. It is shown to occur for a single realization of the plasma, as a mere dynamic consequence of the independent Coulomb deflections of particles. Furthermore, our new path to Landau damping goes first through Debye shielding, a totally unexpected fact as classical texts present these concepts in different, unrelated chapters. Calculations are elementary and start with the derivation of the general fundamental equation (11) for the electrostatic potential.

FUNDAMENTAL EQUATION FOR THE POTENTIAL

This paper deals with the One Component Plasma (OCP) model [3–5], which considers the plasma as infinite with spatial periodicity L in three orthogonal directions with coordinates (x, y, z), and made up of N electrons in each elementary cube with volume L^3. Ions are present only as a uniform neutralizing background, enabling periodic boundary conditions. This choice is made to simplify the analysis which focuses on $\varphi(\mathbf{r})$, the potential created by the N particles at any point where there is no particle. The discrete Fourier transform of φ, readily obtained from the Poisson equation, is given by $\tilde{\varphi}(\mathbf{0}) = 0$, and for $\mathbf{m} \neq \mathbf{0}$ by

$$\tilde{\varphi}(\mathbf{m}) = -\frac{e}{\varepsilon_0 k_{\mathbf{m}}^2} \sum_{j \in S} \exp(-\mathrm{i}\mathbf{k}_{\mathbf{m}} \cdot \mathbf{r}_j), \tag{1}$$

where $-e$ is the electron charge, ε_0 is the vacuum permittivity, $\mathbf{r}_j$ is the position of particle j, $S = \{1, \dots N\}$, $\tilde{\varphi}(\mathbf{m}) = \int \varphi(\mathbf{r}) \exp(-\mathrm{i}\mathbf{k}_{\mathbf{m}} \cdot \mathbf{r}) \mathrm{d}^3\mathbf{r}$, with $\mathbf{m} = (m_x, m_y, m_z)$ a vector with three integer components running from $-\infty$ to $+\infty$, $\mathbf{k}_{\mathbf{m}} = \frac{2\pi}{L}\,\mathbf{m}$, and $k_{\mathbf{m}} = \|\mathbf{k}_{\mathbf{m}}\|$. Reciprocally,

$$\varphi(\mathbf{r}) = \frac{1}{L^3} \sum_{\mathbf{m}} \tilde{\varphi}(\mathbf{m}) \exp(\mathrm{i}\mathbf{k}_{\mathbf{m}} \cdot \mathbf{r}). \tag{2}$$

The dynamics of particle l follows Newton's equation

$$\ddot{\mathbf{r}}_l = \frac{e}{m_{\mathrm{e}}} \nabla \varphi_l(\mathbf{r}_l), \tag{3}$$

with m_{e} the electron mass, and φ_l the electrostatic potential acting on particle l, i.e. the one created by all other particles and by the background charge. Its Fourier transform is given by Eq. (1) with the restriction $j \neq l$. Let

$$\mathbf{r}_l^{(0)} = \mathbf{r}_{l0} + \mathbf{v}_l t \tag{4}$$

be a ballistic approximation of the motion of particle l, and let $\delta\mathbf{r}_l = \mathbf{r}_l - \mathbf{r}_l^{(0)}$. In the following, we first consider cases where the $\delta\mathbf{r}_l$'s are small. So we approximate $\tilde{\varphi}_l(\mathbf{m})$ by its expansion to first order in the $\delta\mathbf{r}_l$'s

$$\tilde{\phi}_l(\mathbf{m}) = \sum_{j\in S;j\neq l} \delta\tilde{\phi}_j(\mathbf{m}), \tag{5}$$

with (Approximation 1)

$$\delta\tilde{\phi}_j(\mathbf{m}) = -\frac{e}{\varepsilon_0 k_{\mathbf{m}}^2} \exp(-\mathrm{i}\mathbf{k}_{\mathbf{m}}\cdot\mathbf{r}_j^{(0)})(1-\mathrm{i}\mathbf{k}_{\mathbf{m}}\cdot\delta\mathbf{r}_j). \tag{6}$$

We further consider φ to be small, and the $\delta\mathbf{r}_l$'s to be of the order of φ (Approximation 2). At lowest order, the particles dynamics defined by Eq. (3) is given by

$$\delta\ddot{\mathbf{r}}_l = \frac{\mathrm{i}e}{L^3 m_{\mathrm{e}}} \sum_{\mathbf{n}} \mathbf{k}_{\mathbf{n}}\, \tilde{\phi}_l(\mathbf{n}) \exp(\mathrm{i}\mathbf{k}_{\mathbf{n}}\cdot\mathbf{r}_l^{(0)}). \tag{7}$$

We denote with a caret the time Laplace transform which maps a function $f(t)$ to $\widehat{f}(\omega) = \int_0^\infty f(t)\exp(\mathrm{i}\omega t)\mathrm{d}t$ (with ω complex). The Laplace transform of Eq. (7) is

$$\omega^2\delta\widehat{\mathbf{r}}_l(\omega) = -\frac{\mathrm{i}e}{L^3 m_e} \sum_{\mathbf{n}} \mathbf{k}_{\mathbf{n}} \exp(\mathrm{i}\mathbf{k}_{\mathbf{n}}\cdot\mathbf{r}_{l0})\, \widehat{\tilde{\phi}}_l(\mathbf{n},\omega+\omega_{\mathbf{n},l}) + \mathrm{i}\omega\delta\mathbf{r}_l(0) - \delta\dot{\mathbf{r}}_l(0), \tag{8}$$

where $\omega_{\mathbf{n},l} = \mathbf{k}_{\mathbf{n}}\cdot\mathbf{v}_l$ comes from the time dependence of $\mathbf{r}_l^{(0)}$ in the exponent of Eq. (7). The Laplace transform of Eqs (5)-(6) yields

$$k_{\mathbf{m}}^2 \widehat{\tilde{\phi}}_l(\mathbf{m},\omega) = k_{\mathbf{m}}^2 \widehat{\tilde{\phi}}_l^{(00)}(\mathbf{m},\omega) + \frac{\mathrm{i}e}{\varepsilon_0} \sum_{j\in S;j\neq l} \exp(-\mathrm{i}\mathbf{k}_{\mathbf{m}}\cdot\mathbf{r}_{j0})\ \mathbf{k}_{\mathbf{m}}\cdot\delta\widehat{\mathbf{r}}_j(\omega-\omega_{\mathbf{m},j}), \tag{9}$$

where $\omega_{\mathbf{m},j}$ comes from the $\mathbf{r}_j^{(0)}$ in Eq. (6); $\widehat{\tilde{\phi}}_l^{(00)}(\mathbf{m},\omega)$ is the Laplace transform of $\tilde{\phi}_l(\mathbf{m})$ computed from Eqs (5) and (6) by setting $\delta\mathbf{r}_j = 0$ for all j's in the latter. Substituting the $\delta\widehat{\mathbf{r}}_j$'s with their expression, Eq. (9) yields

$$k_{\mathbf{m}}^2 \widehat{\tilde{\phi}}_l(\mathbf{m},\omega) - \frac{e^2}{L^3 m_e \varepsilon_0} \sum_{\mathbf{n}} \mathbf{k}_{\mathbf{m}}\cdot\mathbf{k}_{\mathbf{n}} \sum_{j\in S;j\neq l} \frac{\widehat{\tilde{\phi}}_j(\mathbf{n},\omega+\omega_{\mathbf{n},j}-\omega_{\mathbf{m},j})}{(\omega-\omega_{\mathbf{m},j})^2} \exp[\mathrm{i}(\mathbf{k}_{\mathbf{n}}-\mathbf{k}_{\mathbf{m}})\cdot\mathbf{r}_{j0}] = k_{\mathbf{m}}^2 \widehat{\tilde{\phi}}_l^{(0)}(\mathbf{m},\omega), \tag{10}$$

where $\widehat{\tilde{\phi}}_l^{(0)}(\mathbf{m},\omega)$ is the Laplace transform of $\tilde{\phi}_l(\mathbf{m})$ computed from Eqs (5) and (6) by setting now $\delta\mathbf{r}_j = \delta\mathbf{r}_j(0) + \delta\dot{\mathbf{r}}_j(0)t$ for all j's in the latter.

Summing Eq. (10) over $l = 1, ...N$ and dividing by $N-1$ yields

$$k_{\mathbf{m}}^2 \widehat{\tilde{\phi}}(\mathbf{m},\omega) - \frac{e^2}{L^3 m_e \varepsilon_0} \sum_{\mathbf{n}} \mathbf{k}_{\mathbf{m}}\cdot\mathbf{k}_{\mathbf{n}} \sum_{j\in S} \frac{\widehat{\tilde{\phi}}(\mathbf{n},\omega+\omega_{\mathbf{n},j}-\omega_{\mathbf{m},j})}{(\omega-\omega_{\mathbf{m},j})^2} \exp[\mathrm{i}(\mathbf{k}_{\mathbf{n}}-\mathbf{k}_{\mathbf{m}})\cdot\mathbf{r}_{j0}] = k_{\mathbf{m}}^2 \widehat{\tilde{\phi}}^{(0)}(\mathbf{m},\omega), \tag{11}$$

where $\widehat{\tilde{\phi}}(\mathbf{m},\omega)$ and $\widehat{\tilde{\phi}}^{(0)}(\mathbf{m},\omega)$ are respectively $\widehat{\tilde{\phi}}_l(\mathbf{m},\omega)$ and $\widehat{\tilde{\phi}}_l^{(0)}(\mathbf{m},\omega)$ complemented with the missing l-th term. Equation (11) is the fundamental equation of this paper. *This fundamental equation is of the type* $\mathcal{E}\widehat{\tilde{\phi}} =$ *source term*, where $\mathcal{E}$ is a linear operator, acting on the infinite dimensional array whose components are all the $\widehat{\tilde{\phi}}(\mathbf{m},\omega)$'s.

The above Approximation 1 of φ by ϕ corresponds to substituting the true dynamics in Eq. (3) with an approximate one ruled by

$$\delta\ddot{\mathbf{r}}_l = \frac{e}{m_e}\nabla\phi_l(\mathbf{r}_l^{(0)}+\delta\mathbf{r}_l), \tag{12}$$

where $\phi_l(\mathbf{r}) = \sum_{j\in S;j\neq l}\delta\phi_j(\mathbf{r})$ is the inverse Fourier transform of Eq. (5), so that

$$\lim_{L\to\infty}\delta\phi_j(\mathbf{r}) = -\frac{e}{4\pi\varepsilon_0\|\mathbf{r}-\mathbf{r}_j^{(0)}\|} - \frac{e\,\delta\mathbf{r}_j\cdot(\mathbf{r}-\mathbf{r}_j^{(0)})}{4\pi\varepsilon_0\|\mathbf{r}-\mathbf{r}_j^{(0)}\|^3}. \tag{13}$$

The j-th contribution to the approximate electric field acting on particle l turns out to be due to a particle located at $\mathbf{r}_j^{(0)}$ instead of $\mathbf{r}_j$, and is made up of a Coulombian part and of a dipolar part with a dipole moment $-e\,\delta\mathbf{r}_j$. The cross-over between these two parts occurs for $\|\mathbf{r}_l - \mathbf{r}_j^{(0)}\|$ on the order of $\|\delta\mathbf{r}_j\|$, i.e. when the distance between particle l and the ballistic particle j is about the distance between the latter and the true particle j. For larger values of $\|\mathbf{r}_l - \mathbf{r}_j^{(0)}\|$, the dipolar component is subdominant. For smaller ones, it is dominant, but with a direction which is a priori random with respect to the Coulombian one ($(\mathbf{r}_l - \mathbf{r}_j^{(0)})$ is almost independent from $\delta\mathbf{r}_j$). Since the $\|\delta\mathbf{r}_j\|$'s are assumed small, the latter case should be rare as it corresponds to a very close encounter between particle l and the ballistic particle j. As a result, the approximate electric field stays dominantly of Coulombian nature, but with a small mismatch of the charge positions with respect to the actual ones.

SHIELDED COULOMB POTENTIAL

We introduce a smooth function $f(\mathbf{r},\mathbf{v})$, the smoothed velocity distribution function at $t=0$ such that the distribution

$$\sum_{l\in S} \bullet = \iint \bullet f(\mathbf{r},\mathbf{v})\mathrm{d}^3\mathbf{r}\,\mathrm{d}^3\mathbf{v} + W(\bullet), \tag{14}$$

where the distribution W yields a negligible contribution when applied to space dependent functions which evolve slowly on the scale of the inter-particle distance; there the spatial integration is performed over the elementary cube with volume L^3, and the velocity integration is over all velocities.

On replacing the discrete sums over particles by integrals over the smooth distribution function $f(\mathbf{r},\mathbf{v})$ (Approximation 3), Eq. (11) becomes

$$k_{\mathbf{m}}^2 \hat{\tilde{\Phi}}(\mathbf{m},\omega) = k_{\mathbf{m}}^2 \hat{\tilde{\phi}}^{(0)}(\mathbf{m},\omega) + \frac{e^2}{L^3 m_{\mathrm{e}}\varepsilon_0}\sum_{\mathbf{n}} \mathbf{k_m}\cdot\mathbf{k_n}\int \frac{\hat{\tilde{\Phi}}(\mathbf{n},\omega+(\mathbf{k_n}-\mathbf{k_m})\cdot\mathbf{v})}{(\omega-\mathbf{k_m}\cdot\mathbf{v})^2}\tilde{f}(\mathbf{n}-\mathbf{m},\mathbf{v})\ \mathrm{d}^3\mathbf{v}, \tag{15}$$

where $\hat{\tilde{\Phi}}$ is the smoothed version of $\hat{\tilde{\varphi}}$ resulting from Approximations 1 to 3, and $\tilde{f}$ is the spatial Fourier transform of f. We further assume the initial distribution f to be a spatially uniform distribution function $f_0(\mathbf{v})$ plus a small perturbation of the order of Φ (in agreement with Approximation 2). Then operator $\mathcal{E}$ becomes diagonal with respect to both $\mathbf{m}$ and ω (a complex quantity), and linearizing Eq. (15) for $\hat{\tilde{\Phi}}$ yields

$$\epsilon(\mathbf{m},\omega)\hat{\tilde{\Phi}}(\mathbf{m},\omega) = \hat{\tilde{\phi}}^{(0)}(\mathbf{m},\omega), \tag{16}$$

where

$$\epsilon(\mathbf{m},\omega) = 1 - \frac{e^2}{L^3 m_{\mathrm{e}}\varepsilon_0}\int \frac{f_0(\mathbf{v})}{(\omega-\mathbf{k_m}\cdot\mathbf{v})^2}\ \mathrm{d}^3\mathbf{v}. \tag{17}$$

This shows that the smoothed self-consistent potential $\hat{\tilde{\Phi}}$ is determined by the response function $\epsilon(\mathbf{m},\omega)$. The latter is the classical plasma dielectric function. A first check of this can be obtained for a cold plasma: then $\epsilon(\mathbf{m},\omega) = 1-\omega_{\mathrm{p}}^2/\omega^2$, where $\omega_{\mathrm{p}} = [(e^2 n)/(m_{\mathrm{e}}\epsilon_0)]^{1/2}$ is the plasma frequency ($n = N/L^3 = L^{-3}\iint f(\mathbf{r},\mathbf{v})\mathrm{d}^3\mathbf{r}\,\mathrm{d}^3\mathbf{v}$ is the plasma density). The classical expression involving $\partial f_0/\partial\mathbf{v}$ obtains by a mere integration by parts.

To lowest order, the contribution of particle j to $\tilde{\phi}^{(0)}(\mathbf{m})$ is $\delta\tilde{\phi}_j^{(0)}(\mathbf{m}) = -\frac{e}{\varepsilon_0 k_{\mathbf{m}}^2}\exp[-\mathrm{i}\mathbf{k_m}\cdot(\mathbf{r}_{j0}+\mathbf{v}_j t)]$, with Laplace transform

$$\delta\hat{\tilde{\phi}}_j^{(0)}(\mathbf{m},\omega) = -\frac{\mathrm{i}e}{\varepsilon_0 k_{\mathbf{m}}^2}\frac{\exp(-\mathrm{i}\mathbf{k_m}\cdot\mathbf{r}_{j0})}{\omega-\mathbf{k_m}\cdot\mathbf{v}_j}. \tag{18}$$

The corresponding part of $\hat{\tilde{\Phi}}(\mathbf{m},\omega)$ is $\delta\hat{\tilde{\Phi}}_j(\mathbf{m},\omega) = \delta\hat{\tilde{\phi}}_j^{(0)}(\mathbf{m},\omega)/\epsilon(\mathbf{m},\omega)$, which turns out to be the shielded potential of particle j [6–8]. By inverse Fourier-Laplace transform, after some transient whose duration is estimated later at the end of the calculation with Picard iteration technique, the potential due to particle j becomes the shielded Coulomb potential

$$\delta\Phi_j(\mathbf{r}) = \delta\Phi(\mathbf{r}-\mathbf{r}_{j0}-\mathbf{v}_j t,\mathbf{v}_j), \tag{19}$$

where

$$\delta\Phi(\mathbf{r},\mathbf{v}) = -\frac{e}{L^3\varepsilon_0}\sum_{\mathbf{m}\neq\mathbf{0}}\frac{\exp(\mathrm{i}\mathbf{k_m}\cdot\mathbf{r})}{k_{\mathbf{m}}^2\,\epsilon(\mathbf{m},\mathbf{k_m}\cdot\mathbf{v})}. \tag{20}$$

Therefore, after this transient, *the dominant contribution to the full potential in the plasma turns out to be the sum of the shielded Coulomb potentials of individual particles* located at their ballistic positions. Let $\lambda_{\mathrm{D}} = [(\epsilon_0 k_{\mathrm{B}} T)/(ne^2)]^{1/2} = [k_{\mathrm{B}}T/m_{\mathrm{e}}]^{1/2}\omega_{\mathrm{p}}^{-1}$ be the Debye length, where k_{B} is the Boltzmann constant and T the temperature. The wavenumbers resolving scale $\|\mathbf{r}\|$ are such that $k_{\mathbf{m}}\|\mathbf{r}\| \gtrsim 1$. If $\|\mathbf{r}\| \ll \lambda_{\mathrm{D}}$, the corresponding wavenumbers are such that $k_{\mathbf{m}}\lambda_{\mathrm{D}} \gg 1$. Therefore, there is no shielding for $\|\mathbf{r}\| \ll \lambda_{\mathrm{D}}$, since $\epsilon(\mathbf{m},\mathbf{k_m}\cdot\mathbf{v}) - 1 \simeq (k_{\mathbf{m}}\lambda_{\mathrm{D}})^{-2}$.

In the following, Eq. (19) is used by substituting $\delta\Phi(\mathbf{r}-\mathbf{r}_{j0}-\mathbf{v}_j t,\mathbf{v}_j)$ with $\delta\Phi(\mathbf{r}-\mathbf{r}_j,\mathbf{v}_j)$: the shielded potential of particle j is computed by taking into account its actual position, since it is the original Coulomb one close to $\mathbf{r}_j$. The error made for $\mathbf{r}-\mathbf{r}_j$ of the order of λ_{D} is small as long as the mismatch of $\mathbf{r}_j$ from the ballistic orbit is much smaller than λ_{D}. As was done for the bare potential of Eq. (1), the field acting on a given particle l is obtained by removing its own divergent contribution $\delta\Phi_l$ from Φ.

DEBYE SHIELDING AND LANDAU DAMPING

We now apply the smoothing using distribution function f to $\hat{\hat{\phi}}^{(0)}(\mathbf{m},\omega)$ too in Eq. (16). Neglecting $\delta\mathbf{r}_j$ to lowest order in Eq. (6), this yields a $\tilde{\Phi}^{(0)}(\mathbf{m})$ whose Laplace transform is

$$\hat{\tilde{\Phi}}^{(0)}(\mathbf{m},\omega) = -\frac{\mathrm{i}e}{\varepsilon_0 k_{\mathbf{m}}^2}\int\frac{\tilde{f}(\mathbf{m},\mathbf{v})}{\omega-\mathbf{k_m}\cdot\mathbf{v}}\,\mathrm{d}^3\mathbf{v}. \tag{21}$$

This shows that this second smoothing makes Eq. (16) *to become the expression including initial conditions in Landau contour calculations of Langmuir wave growth or damping*, usually obtained by linearizing Vlasov equation and using Fourier-Laplace transform, as described in many textbooks. Therefore, in these calculations, $\hat{\tilde{\Phi}}^{(0)}(\mathbf{m},\omega)$ turns out to be the smoothed version of the actual shielded potential in the plasma.

It is interesting to compare the above derivation with that used by classical textbooks when they start with the N-body description to derive rigorously both Debye shielding and the combination of Eqs (16) and (21). Debye shielding is exhibited in the equilibrium pair correlation function which is rigorously computed after deriving the first two equations of the BBGKY hierarchy (see e.g. chapter 12 of [9]). The combination of Eqs (16) and (21) is obtained independently by linearizing Vlasov equation about a uniform velocity distribution function, and by using the Fourier-Laplace transform. A prerequisite is the derivation of Vlasov equation by two main rigorous approaches: a mean-field derivation [2], or the BBGKY hierarchy that involves statistical arguments (see e.g. [1]). This path is so hard that most textbooks instead provide more qualitative derivations either of Debye shielding or of Vlasov equation. In contrast, the present derivation performs the Laplace transform in time of the linearized dynamics of a *single realization* of the N-body system. This yields Eq. (11) which keeps the full granularity of the system. A first smoothing involving a velocity distribution function yields Eqs (19-20), and a second one Eq. (21) combined with Eq. (16). This provides a much shorter connection between these equations and the underlying N-body problem. In this derivation, the smoothed velocity distribution is introduced after particle dynamics has been taken into account, and not before, as occurs when kinetic equations are used. This avoids addressing the issues of the exact definition of the smoothed distribution for a given realization of the plasma, and of the uncertainty as to the way the smoothed dynamics departs from the actual N-body one [2].

MEDIATED INTERACTIONS IMPLY DEBYE SHIELDING

In the above derivation of Debye shielding, using the Laplace transform of the particle positions does not provide an intuitive picture of this effect. We now show that such a picture can be obtained directly from the mechanical description of microscopic dynamics with the full OCP Coulomb potential of Eq. (1). To compute the dynamics, we use Picard iteration technique. From Eq. (3), $\mathbf{r}_l^{(n)}$, the n-th iterate for $\mathbf{r}_l$, is computed by

$$\ddot{\mathbf{r}}_l^{(n)} = \frac{e}{m_{\mathrm{e}}}\nabla\varphi_l^{(n-1)}(\mathbf{r}_l^{(n-1)}), \tag{22}$$

where $\varphi_l^{(n-1)}$ is computed by the inverse Fourier transform of Eq. (1) with the $\mathbf{r}_j$'s substituted with the $\mathbf{r}_j^{(n-1)}$'s. The iteration starts with the ballistic approximation of the dynamics defined by Eq. (4), and the actual orbit of Eq. (3) corresponds to $n \to \infty$. Let $\delta\mathbf{r}_l^{(n)} = \mathbf{r}_l^{(n)} - \mathbf{r}_l^{(0)}$ be the mismatch of the position of particle l with respect to the ballistic one at the n-th iterate. It is convenient to write Eq. (22) as $\delta\ddot{\mathbf{r}}_l^{(n)} = \sum_{j\in S;j\neq l} \delta\ddot{\mathbf{r}}_{lj}^{(n)}$, with

$$\delta\ddot{\mathbf{r}}_{lj}^{(n)} = \mathbf{a}_{\mathrm{C}}(\mathbf{r}_l^{(n-1)} - \mathbf{r}_j^{(n-1)}) \tag{23}$$

and

$$\mathbf{a}_{\mathrm{C}}(\mathbf{r}) = \frac{\mathrm{i}e^2}{\varepsilon_0 m_{\mathrm{e}} L^3} \sum_{\mathbf{m}\neq\mathbf{0}} k_{\mathbf{m}}^{-2} \mathbf{k}_{\mathbf{m}} \exp(\mathrm{i}\mathbf{k}_{\mathbf{m}} \cdot \mathbf{r}). \tag{24}$$

Let $\delta\mathbf{r}_{lj}^{(n)} = \int_0^t \int_0^{t'} \delta\ddot{\mathbf{r}}_{lj}^{(n)}(t'')\mathrm{d}t''\mathrm{d}t'$. For $n \geq 2$ one finds

$$\delta\ddot{\mathbf{r}}_l^{(n)} = \sum_{j\in S;j\neq l} [(\delta\ddot{\mathbf{r}}_{lj}^{(1)} + M_{lj}^{(n-1)}) + 2\nabla\mathbf{a}_{\mathrm{C}}(\mathbf{r}_l^{(0)} - \mathbf{r}_j^{(0)}) \cdot \delta\mathbf{r}_{lj}^{(n-1)}] + O(a^3), \tag{25}$$

where a is the order of magnitude of the total Coulombian acceleration, and

$$M_{lj}^{(n-1)} = \nabla\mathbf{a}_{\mathrm{C}}(\mathbf{r}_l^{(0)} - \mathbf{r}_j^{(0)}) \cdot \sum_{i\in S;i\neq l,j} (\delta\mathbf{r}_{li}^{(n-1)} - \delta\mathbf{r}_{ji}^{(n-1)}) \tag{26}$$

is the modification to the bare Coulomb acceleration of particle j on particle l due to the following phenomenon: particle j modifies the position of all other particles, which implies the action of the latter ones on particle l is modified by particle j. Therefore $M_{lj}^{(n-1)}$ *is the acceleration of particle l due to particle j mediated by all other particles.* The last term in the bracket in Eq. (25) accounts for the fact that both particles j and l are shifted with respect to their ballistic positions.

Since the shielded potential of the previous paragraph was found by first order perturbation theory, it is felt in the acceleration of particles computed to second order. This acceleration is provided by Eq. (25) for $n = 2$. Therefore its term in brackets is the shielded acceleration of particle l due to particle j. As a result, though the summation runs over all particles, its effective part is only due to particles j typically inside the Debye sphere (sphere with radius λ_{D}) about particle l. Starting from the third iterate of the Picard scheme, the effective part of the summation in Eq. (25) ranges inside this Debye sphere, since the $\delta\mathbf{r}_{lj}^{(n-1)}$'s are then computed with a shielded acceleration. This approach clarifies the mechanical background of the calculation of shielding using the equilibrium pair correlation function which shows shielding to result from the correlation of two particles occurring through the action of all the other ones (see e.g. section 12.3 of [9]).

The preceding calculation yields the following interpretation of shielding for a particle in the bulk of the distribution function. At $t = 0$, consider a set of randomly distributed particles. Consider a particle l. At a later time t, it has deflected all particles which made a closest approach to it with an impact parameter $b \lesssim v_{\mathrm{th}}t$ where v_{th} is the thermal velocity. This part of their global deflection due to particle l reduces the number of particles inside the sphere $S(t)$ of radius $v_{\mathrm{th}}t$ about it. Therefore the effective charge of particle l as seen out of $S(t)$ is reduced: the charge of particle l is shielded due to these deflections. This shielding effect increases with t, and thus with the distance to particle l. As a result, the typical time-scale for shielding to set in, when starting from random particle positions, is the time for a thermal particle to cross a Debye sphere, i.e. ω_{p}^{-1}, which sets the duration of the transients occurring in the inverse Laplace transform leading to Eq. (19); this order of magnitude is correct for a plasma close to equilibrium. Furthermore, *shielding is a cooperative dynamical process: it results from the accumulation of almost independent repulsive deflections with the same qualitative impact on the effective electric field of particle l* (if ions were added, the attractive deflection of charges with opposite signs would have the same effect). It is a cooperative effect, but not a collective one (it does not involve any synchronized motion of particles). Basic plasma physics textbooks show the accumulation of almost independent repulsive deflections to produce collisional transport of particles in plasmas. *Unexpectedly, it turns out that Debye shielding is another aspect of the same two-body repulsive process.*

CONCLUSION

One might think about trying to apply the above approach to plasmas with more species, or with a magnetic field, or where particles experience trapping and chaotic dynamics. The first generalization sounds rather obvious, and the third one is under way, at least in one dimension (see a pedestrian introduction in [10] and more specific results in [11–13]). As in many textbooks, linearization was applied in this paper without questioning deeply its range of validity. However, as reviewed in Ref. [14], the smallness of the perturbation is not a sufficient criterion. In reality, there is a (more intricate) version of the fundamental equation (11) that does not involve linearization [15]. It might be used to study the effect of the coupling of Fourier components with both coherent and incoherent effects.

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