

Supplementary material 2

Boltzmann statistics are underlying microbial growth

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Fitted and interpolated data supporting Boltzmann sustained microbial growth (Figure 1b and Figure 2).

Figure 1b:

We estimate two parameters (μ_{max} and K_S) for our model and Monod model on Monod's data for the growth of *E. coli* on glucose ([1] table XVII p 72.):

Our model:

$$\mu = \mu_{max} \cdot \exp\left(-\frac{K_S}{[S]}\right)$$

With $K_S = \frac{E_M + E_{dis}}{V_{harv} \cdot E_{cat}}$

Monod model:

$$\mu = \mu_{max} \cdot \frac{[S]}{K_S + [S]}$$

Estimation is made minimizing the residual sum of squares (RSS) on growth rates with a nonlinear regression.

$$RSS(\mu) = \sum (\mu_{measured} - \mu_{predicted})^2$$

RSS values show that our model fits slightly better on the data than the Monod model.

Parameters values estimated for both model are shown in the table below:

	Our model	Monod model
μ_{max} (1/h)	1.38	1.44
K_S (10^{-4} mol/l)	0.42	0.67

The K_S value estimated for our model enable the calculation of the harvesting volume

$$V_{harv} = \frac{E_M + E_{dis}}{K_S \cdot E_{cat}}$$

For *E. coli* growing on glucose (cf. sup mat 1):

$$E_{cat} = 2.9 \cdot 10^3 \text{ kJ/mol}$$

$$E_{dis} = 0.5 \cdot 10^3 \text{ kJ/C - mol}$$

$$E_M = 0.5 \cdot 10^3 \text{ kJ/C - mol}$$

$$\Rightarrow V_{harv} = 8.2 \cdot 10^3 \text{ l/C - mol}$$

A dry cell of E. coli weight $3 \cdot 10^{-13}$ g with 50% carbon and thus contains $1.2 \cdot 10^{-14}$ C-mol/cell.

$$\Rightarrow V_{harv} = 9.9 \cdot 10^{-10} \text{ l/cell}$$

$$\Rightarrow V_{harv} \approx 1 \cdot 10^5 \mu\text{m}^3/\text{cell}$$

Figure 2 :

Data for isotopic fractionation were taken from [2-5].

Values from [3] (Figure 2c) and [4] (Figure 2b triangles) were interpolated using ImageJ [6]. Values from [5] (Figure 2b circles) and [2] (Figure 2d) were picked from tables from their articles (Table 6 and Table 1 respectively).

- Data interpolated from [4] figures 2 and 3:

P(H₂) (kPa)	α	ΔG (kJ/mol)
0,142	1,046	-71,748
1,326	1,031	-91,385
2,036	1,034	-95,105
2,964	1,029	-98,229
2,775	1,04	-98,08
2,424	1,043	-96,89
2,301	1,047	-96,295
1,856	1,053	-93,617
1,108	1,063	-87,815
0,199	1,079	-69,516
0,143	1,053	-70,409
0,143	1,065	-69,814
0,914	1,034	-87,964
2,9	1,031	-98,973
4,957	1,022	-103,882
4,971	1,04	-104,477
4,1	1,043	-102,394
3,757	1,046	-101,353
3,057	1,049	-98,973
1,957	1,053	-93,766
0,686	1,058	-82,459

➤ Data interpolated from [3] figure 2:

Enrichment factors for 50% degradation of substrate (triangles in Figure 2c):

$\log_{10}(C/1\mu\text{M})$	ϵ
1,961	-958,118
2,011	-941,932
1,977	-938,897
1,994	-924,532
1,78	-938,897
1,77	-944,36
1,02	-917,248
0,991	-901,265
1,044	-894,183
1,011	-874,962
0,035	-754,173
0,058	-781,892
0,08	-789,378

Enrichment factors for 70% degradation of substrate (circles in Figure 2c):

$\log_{10}(C/1\mu\text{M})$	ϵ
2,014	-901,062
1,997	-873,343
1,977	-870,916
1,959	-867,274
1,782	-893,171
1,77	-877,997
1,042	-859,939
1,017	-858,877
0,991	-838,392
1,009	-833,991
0,032	-760,04
0,056	-743,045
0,079	-707,84

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