

Supplementary material 2

Boltzmann statistics as founding principle of microbial growth

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Fitted and interpolated data supporting Boltzmann sustained microbial growth (figure 2).

Figure 2-A:

We estimate two parameters (μ_{max} and K_S) for our model and Monod model on Monod's data for the growth of *E. coli* on glucose ((Monod 1942) table XVII p 72.):

Our model:

$$\mu = \mu_{max} \cdot \exp\left(-\frac{K_S}{[S]}\right)$$

With $K_S = \frac{E_M + E_{dis}}{V_{harv} \cdot E_{cat}}$

Monod model:

$$\mu = \mu_{max} \cdot \frac{[S]}{K_S + [S]}$$

Estimation is made minimizing the residual sum of squares (RSS) on growth rates with a nonlinear regression.

$$RSS(\mu) = \sum (\mu_{measured} - \mu_{predicted})^2$$

RSS values show that our model fits slightly better on the data than the Monod model.

Parameters values estimated for both model are shown in the table below:

	Our model	Monod model
μ_{max} (1/h)	1.38	1.44
K_S (10^{-4} mol/l)	0.42	0.67

The K_S value estimated for our model enable the calculation of the harvesting volume

$$V_{harv} = \frac{E_M + E_{dis}}{K_S \cdot E_{cat}}$$

For *E. coli* growing on glucose (cf. sup mat 1):

$$E_{cat} = 2.9 \cdot 10^3 \text{ kJ/mol}$$

$$E_{dis} = 0.5 \cdot 10^3 \text{ kJ/C - mol}$$

$$E_M = 0.5 \cdot 10^3 \text{ kJ/C - mol}$$

$$\Rightarrow V_{harv} = 8.2 \cdot 10^3 \text{ l/C - mol}$$

A dry cell of *E. coli* weight $3 \cdot 10^{-13}$ g with 50% carbon and thus contains $1.2 \cdot 10^{-14}$ C-mol/cell.

$$\Rightarrow V_{harv} = 9.9 \cdot 10^{-10} \text{ l/cell}$$

$$\Rightarrow V_{harv} \approx 1 \cdot 10^5 \mu\text{m}^3/\text{cell}$$

Figure 2-B :

Data for isotopic fractionation were taken from (Valentine, Chidthaisong et al. 2004, Penning, Plugge et al. 2005, Kampara, Thullner et al. 2008, Goevert and Conrad 2009).

Values from (Kampara, Thullner et al. 2008) (2-B-4) and (Penning, Plugge et al. 2005) (2-B-2 triangles) were interpolated using ImageJ (Abramoff, Magalhães et al. 2004). Values from (Valentine, Chidthaisong et al. 2004) (2-B-2 circles) and (Goevert and Conrad 2009) (2-B-3) were picked from tables from their articles (Table 6 and Table 1 respectively).

- Data interpolated from (Penning, Plugge et al. 2005) figures 2 and 3:

P(H₂) (kPa)	α	ΔG (kJ/mol)
0,142	1,046	-71,748
1,326	1,031	-91,385
2,036	1,034	-95,105
2,964	1,029	-98,229
2,775	1,04	-98,08
2,424	1,043	-96,89
2,301	1,047	-96,295
1,856	1,053	-93,617
1,108	1,063	-87,815
0,199	1,079	-69,516
0,143	1,053	-70,409
0,143	1,065	-69,814
0,914	1,034	-87,964
2,9	1,031	-98,973
4,957	1,022	-103,882
4,971	1,04	-104,477
4,1	1,043	-102,394
3,757	1,046	-101,353
3,057	1,049	-98,973
1,957	1,053	-93,766
0,686	1,058	-82,459

- Data interpolated from (Kampara, Thullner et al. 2008) figure 2:

Enrichment factors for 50% degradation of substrate (triangles in figure 2-B-4):

log₁₀(C/1μM)	ε
1,961	-958,118

2,011	-941,932
1,977	-938,897
1,994	-924,532
1,78	-938,897
1,77	-944,36
1,02	-917,248
0,991	-901,265
1,044	-894,183
1,011	-874,962
0,035	-754,173
0,058	-781,892
0,08	-789,378

Enrichment factors for 70% degradation of substrate (circles in figure 2-B-4):

$\log_{10}(C/1\mu M)$	ϵ
2,014	-901,062
1,997	-873,343
1,977	-870,916
1,959	-867,274
1,782	-893,171
1,77	-877,997
1,042	-859,939
1,017	-858,877
0,991	-838,392
1,009	-833,991
0,032	-760,04
0,056	-743,045
0,079	-707,84

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