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Surface viscoelasticity and effective properties of thin-walled structures at the nanoscale [☆]

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A B S T R A C T

We discuss the influence of surface viscoelasticity effects on the effective properties of materials such as effective bending stiffness of plates or shells. Viscoelastic properties in the vicinity of the surface can differ from the properties of the bulk material. This difference influences the behavior of nanostructural elements. In particular, the surface viscoelastic stresses are responsible for the size dependent dissipation of nanosized structures. Using the extension of the Gurtin–Murdoch model and the correspondence principle of linear viscoelasticity we derive the expressions of the stress and couple stress resultant tensors for shear deformable plates and shells.

1. Introduction

The interest in the investigation of surface effects is recently grown in relation to the nanomechanics. The surface effects play an important role for such nanosized materials as films, nanoporous materials, etc., see e.g. [Bhushan \(2007\)](#). The model of continuum mechanics which takes into account explicitly the phenomenon of surface stresses is proposed by [Gurtin and Murdoch \(1975\)](#). Within the framework of this theory the elastic body can be considered as a simple elastic body with elastic membrane glued on its surface or its part. Unlike to bulk materials where the surface stresses can be neglected in most cases, at the micro- and nanoscale the surface stresses play an important role. For example they influence the effective or apparent properties of very thin specimens and predict the size effect, i.e. dependence of material properties on a specimen size. Hence, the classical elasticity theory can be extended at the nanoscale taking into account surface stresses acting on the boundary of a nanosized body, see [Duan, Wang, and Karihaloo \(2008\)](#), [Wang, Duan, Huang, and Karihaloo \(2006\)](#). Various applications of the Gurtin–Murdoch model in nanomechanics are presented in the literature, see for example the review by [Wang et al. \(2011\)](#). In particular, the theory of elasticity with surface stresses is applied to modifications of the two-dimensional theories of nanosized plates and shells, see [Altenbach and Eremeyev \(2011\)](#), [Altenbach, Eremeyev, and Morozov \(2009, 2010\)](#), [Dahmen, Lehwald, and Ibach \(2000\)](#), [Eremeyev, Altenbach, and Morozov \(2009\)](#), [Huang \(2008\)](#), [Lu, He, Lee, and Lu \(2006\)](#), [Wang and Zhao \(2009\)](#), [Zhu, Wang, and Karihaloo \(2009\)](#). In most of the cited papers the elastic medium is considered. On the other hand, inelastic behavior analysis is also important for micro- and nanomechanics applications. Dissipative processes in the vicinity of the surface are related to the higher mobility of molecules, surface imperfections,

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absorbates, etc., see e.g. Seoáñez, Guinea, and Castro Neto (2008). In particular, the surface viscoelasticity phenomenon is recognized for both liquids and solids. The experimental methods of investigation of surface viscoelasticity are different than in the case of bulk material, in general, and use various types of microscopies, light scattering, etc., see e.g. Earnshaw, McGivern, McLaughlin, and Winch (1990), Garcia et al. (2006), Kajiyama, Tanaka, Ge, and Takahara (1996), Sahoo, Thakur, Senthilkumar, and Das (2003), Tranchida, Kiflie, Acierno, and Piccarolo (2009), Wang, Xiao, and Tsui (2001). For the description of surface dissipation of nanosized beams, Ru (2009) proposed the one-dimensional constitutive law that is similar to the model of the standard viscoelastic solids but formulated for the two-dimensional surface stresses.

In this paper we consider the influence of surface viscoelasticity on the effective or apparent properties of nanosized thin-walled structures. The paper is organized as follows. In Section 1 we recall the basic equations of the continuum with surface stresses. We use the more general constitutive viscoelastic model for the surface stresses than the proposed by Ru (2009). In Section 2 using the correspondence principle, we present the governing equations of plates and shells with viscoelastic surface stresses. Here we assume that the bulk material is elastic while the surface has the viscoelastic properties. We formulate the two-dimensional (2D) constitutive equations and obtain the 2D relaxation functions for plates and shells. Finally we compare the proposed model of shells with viscoelastic surface stresses with the model of sandwich plate with viscoelastic faces in Section 3.

2. Basic equations of solids with surface stresses

First we consider the problems with mixed boundary conditions for a deformable body with surface stresses. Let $V \in \mathbb{R}^3$ is the volume of the body with the boundary $\Omega = \partial V$.

For quasistatic deformations of linear solids with surface stresses the boundary-value problem is given by

$$\nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{f} = \mathbf{0}, \quad \mathbf{x} \in V, \quad (1)$$

$$\mathbf{u}|_{\Omega_1} = \mathbf{0}, \quad \mathbf{n} \cdot \boldsymbol{\sigma}|_{\Omega_2} = \mathbf{t}, \quad \mathbf{x} \in \Omega, \quad (2)$$

where $\boldsymbol{\sigma}$ is the stress tensor, \mathbf{u} the displacement vector, ∇ the 3D gradient operator (3D nabla operator), ρ the density, \mathbf{f} the density of the volume forces, and \mathbf{n} the external unit normal to $\Omega = \Omega_1 \cup \Omega_2$, $\Omega_1 \cup \Omega_2 = \emptyset$. The surface stress vector \mathbf{t} is expressed through a given load $\boldsymbol{\varphi}$ and the stress vector due the surface stresses \mathbf{t}_S by the formula

$$\mathbf{t} = \boldsymbol{\varphi} + \mathbf{t}_S,$$

where \mathbf{t}_S is determined through the surface stress tensor $\boldsymbol{\tau}$ by the formula of Duan, Wang, Huang, and Karihaloo (2005), Duan et al. (2008), Gurtin and Murdoch (1975), Povstenko (1993)

$$\mathbf{t}_S = \nabla_S \cdot \boldsymbol{\tau}, \quad (3)$$

Here $\boldsymbol{\tau}$ is the surface stress tensor on Ω , ∇_S is the nabla operator on the surface Ω that relates with ∇ by the formula

$$\nabla_S = \nabla \cdot \mathbf{n} \frac{\partial}{\partial z}$$

z is the coordinate along the normal to Ω .

For the sake of simplicity, we restrict ourselves to an isotropic material. We assume that the bulk material is elastic but the surface stresses are viscoelastic. So, the constitutive equation for the bulk material is the Hooke law

$$\boldsymbol{\sigma} = 2\mu \boldsymbol{\varepsilon} + \lambda \mathbf{I} \text{tr } \boldsymbol{\varepsilon}, \quad (4)$$

where

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}(\mathbf{u}) \equiv \frac{1}{2}(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)$$

is the strain tensor, λ and μ are Lamé's coefficients, and \mathbf{I} is the three-dimensional unit tensor, respectively.

For the surface stresses we assume the following constitutive equation

$$\boldsymbol{\tau} = 2 \int_{-\infty}^t \mu_S(t - \tau) \dot{\boldsymbol{\varepsilon}}(\tau) d\tau + \int_{-\infty}^t \lambda_S(t - \tau) \text{tr } \dot{\boldsymbol{\varepsilon}}(\tau) d\tau \mathbf{A}, \quad (5)$$

$$\mathbf{e} = \mathbf{e}(\mathbf{v}) \equiv \frac{1}{2}(\nabla_S \mathbf{v} \cdot \mathbf{A} + \mathbf{A} \cdot (\nabla_S \mathbf{v})^T),$$

where \mathbf{e} is the surface strain tensor, \mathbf{v} the displacement of the surface point \mathbf{x} of Ω_2 , $\mathbf{A} \equiv \mathbf{I} - \mathbf{n} \otimes \mathbf{n}$ the two-dimensional unit tensors, the overdot denotes differentiation with respect to time t , and λ_S and μ_S are the relaxation functions of the surface film Ω_2 , respectively.

Following Gurtin and Murdoch (1975), Povstenko (1993), we use the non-separation condition that explicitly states that the displacements of the surface film Ω_2 coincide with the body displacements on the boundary

$$\mathbf{v} = \mathbf{u}|_{\Omega_2}.$$

The integral constitutive law (5) contains the viscoelastic constitutive equation of Ru (2009) as the special case. If μ_s and λ_s are constants then (5) reduces to the elastic constitutive equations used in Duan et al. (2008).

Eqs. (1)–(5) constitute the boundary-value problem (BVP) for the elastic body with viscoelastic surface stresses. In what follows we use this BVP to derive two-dimensional (2D) equations of shear-deformable shells.

3. Reduction to the two-dimensional theory

In the literature there are known various approaches of derivation of 2D equations of plates and shells using the reduction procedure of the equations of elasticity, see e.g. Berdichevsky (2009), Eremeyev and Zubov (2008), Libai and Simmonds (1998), Naghdi (1972). Here we use the through-the-thickness integration procedure presented for example, in Lebedev, Cloud, and Eremeyev (2010) with tensorial notations but applied to the nonclassical BVP (1)–(5).

In the case of viscoelastic material we use the correspondence principle which establishes that if an elastic solution is known, the corresponding viscoelastic solution can be obtained by substituting for the elastic quantities the Laplace transforms of the unknown functions (Christensen, 1971; Tschoegl, 1989). In other words, one can use the solution of BVP for elastic material as the solution of BVP for viscoelastic material but given in terms of Laplace transforms. According to this principle we use the results of 3D to 2D reduction procedure for the elastic shell-like body given by Altenbach et al. (2009, 2010).

In fact, using the Laplace transform of a function $f(t)$

$$f(s) = \int_0^{\infty} f(t)e^{-st} dt,$$

one can write (5) in the form

$$\boldsymbol{\tau} = 2s\mu_s(s)\mathbf{e} + s\lambda_s(s)(\text{tr } \mathbf{e})\mathbf{A}, \quad (6)$$

which coincides formally with the surface Hook's law assumed in Altenbach et al. (2009, 2010).

The through-the-thickness integration procedure applied to shell-like body with surface stresses leads to the following 2D equations, see Altenbach et al. (2009, 2010),

$$\nabla_s \cdot \mathbf{T} + \mathbf{q} = \mathbf{0}, \quad \nabla_s \cdot \mathbf{M} + \mathbf{T}_\times + \mathbf{m} = \mathbf{0}, \quad (7)$$

where \mathbf{T} is the stress resultant tensor, \mathbf{M} the couple stress tensor, \mathbf{T}_\times denotes the vectorial invariant of second-order tensor \mathbf{T} (Lebedev et al., 2010), for example the vectorial invariant of the diad of two vectors \mathbf{a} and \mathbf{b} is equal to their vector product: $(\mathbf{a} \otimes \mathbf{b})_\times = \mathbf{a} \times \mathbf{b}$, \mathbf{q} and \mathbf{m} are the surface force and couple vector fields defined as in Altenbach et al. (2009, 2010).

Tensors \mathbf{T} and \mathbf{M} can be represented each as the sums of two terms, see Altenbach and Eremeyev (2011), Altenbach et al. (2009, 2010),

$$\mathbf{T} = \mathbf{T}_b + \mathbf{T}_s, \quad \mathbf{M} = \mathbf{M}_b + \mathbf{M}_s. \quad (8)$$

Here \mathbf{T}_b and \mathbf{M}_b are the stress resultant and couple stress tensors related to the bulk material while \mathbf{T}_s and \mathbf{M}_s are the stress resultant and couple stress tensors related to the surface stresses. With the accuracy of $O(h/R)$ where h is the shell thickness and R is the maximum of the curvature radius of the shell base surface, one can use the following formulae for \mathbf{T}_b , \mathbf{M}_b , \mathbf{T}_s , and \mathbf{M}_s

$$\mathbf{T} = \langle \mathbf{A} \cdot \boldsymbol{\sigma} \rangle, \quad \mathbf{M} = \langle \mathbf{A} \cdot z\boldsymbol{\sigma} \times \mathbf{n} \rangle, \quad (9)$$

$$\mathbf{T}_s = \boldsymbol{\tau}_+ + \boldsymbol{\tau}_-, \quad \mathbf{M}_s = \frac{h}{2}(\boldsymbol{\tau}_+ - \boldsymbol{\tau}_-) \times \mathbf{n}, \quad (10)$$

$$\langle (\dots) \rangle = \int_{-h/2}^{h/2} (\dots) dz,$$

where $\boldsymbol{\tau}_\pm$ are the surface stresses acting at the shell faces, i.e. $\boldsymbol{\tau}_\pm = \boldsymbol{\tau}|_{z=\pm h/2}$.

The expressions (9) for the components of the stress resultants and couple stress tensors are widely used in the literature, see e.g. Goldenveizer (1961), Novozhilov, Chernykh, and Mikhailovsky (1991), Timoshenko and Woinowsky-Krieger (1985). Eq. (9) result in the following component representations

$$\mathbf{T}_b = T_{\alpha\beta}\boldsymbol{\rho}^\alpha \otimes \boldsymbol{\rho}^\beta + T_{\alpha 3}\boldsymbol{\rho}^\alpha \otimes \mathbf{n}, \quad \mathbf{M}_b = M_{\alpha\beta}\boldsymbol{\rho}^\alpha \otimes \boldsymbol{\rho}^\beta \times \mathbf{n}, \quad \text{where } \alpha, \beta = 1, 2, \quad (11)$$

$$T_{\alpha\beta} = \langle \sigma_{\alpha\beta} \rangle, \quad T_{\alpha 3} = \langle \sigma_{\alpha 3} \rangle, \quad M_{\alpha\beta} = \langle z\sigma_{\alpha\beta} \rangle,$$

where $\sigma_{\alpha\beta} = \boldsymbol{\rho}_\alpha \cdot \boldsymbol{\sigma} \cdot \boldsymbol{\rho}_\beta$, $\sigma_{\alpha 3} = \boldsymbol{\rho}_\alpha \cdot \boldsymbol{\sigma} \cdot \mathbf{n}$, $\boldsymbol{\rho}_\alpha$ and $\boldsymbol{\rho}^\beta$ are the main and reciprocal bases on the shell base surface ω with the unit normal vector \mathbf{n} .

In what follows we use the linear approximation of the translation vector \mathbf{u}

$$\mathbf{u}(z) = \mathbf{w} - z\boldsymbol{\vartheta}, \quad \mathbf{n} \cdot \boldsymbol{\vartheta} = 0. \quad (12)$$

This approximation is used in the theories of shear-deformable plates and shells, see e.g. Lebedev et al. (2010), \mathbf{w} is the translation vector of shell base surface ω , $\boldsymbol{\vartheta}$ is the rotation vector of the shell normal, that are kinematically independent each other.

For the isotropic shell the constitutive equations, i.e. the dependence of \mathbf{T}_b and \mathbf{M}_b on strain measures, are given by

$$\mathbf{T}_b = \mathbb{C}_b : \boldsymbol{\epsilon} + \Gamma \boldsymbol{\gamma} \otimes \mathbf{n}, \quad \mathbf{M}_b = \mathbb{D}_b : \boldsymbol{\kappa}, \quad (13)$$

where $\boldsymbol{\epsilon}$, $\boldsymbol{\kappa}$, and $\boldsymbol{\gamma}$ are the surface strain measures given by

$$\boldsymbol{\epsilon} = \frac{1}{2}(\nabla_S \mathbf{w} \cdot \mathbf{A} + \mathbf{A} \cdot (\nabla_S \mathbf{w})^T), \quad \boldsymbol{\kappa} = \frac{1}{2}(\nabla_S \boldsymbol{\vartheta} \cdot \mathbf{A} + \mathbf{A} \cdot (\nabla_S \boldsymbol{\vartheta})^T), \quad \boldsymbol{\gamma} = \nabla_S(\mathbf{w} \cdot \mathbf{n}) \quad \boldsymbol{\vartheta},$$

and the forth-order tensors \mathbb{C}_b and \mathbb{D}_b take the form

$$\begin{aligned} \mathbb{C}_b &= C_{11} \mathbf{a}_1 \otimes \mathbf{a}_1 + C_{22}(\mathbf{a}_2 \otimes \mathbf{a}_2 + \mathbf{a}_4 \otimes \mathbf{a}_4), \\ \mathbb{D}_b &= D_{22}(\mathbf{a}_2 \otimes \mathbf{a}_2 + \mathbf{a}_4 \otimes \mathbf{a}_4) + D_{33} \mathbf{a}_3 \otimes \mathbf{a}_3, \\ \mathbf{a}_1 &= \mathbf{A} \equiv \mathbf{e}_1 \otimes \mathbf{e}_1 + \mathbf{e}_2 \otimes \mathbf{e}_2, \quad \mathbf{a}_2 = \mathbf{e}_1 \otimes \mathbf{e}_1 - \mathbf{e}_2 \otimes \mathbf{e}_2, \\ \mathbf{a}_3 &= \mathbf{A} \times \mathbf{n} = \mathbf{e}_1 \otimes \mathbf{e}_2 - \mathbf{e}_2 \otimes \mathbf{e}_1, \quad \mathbf{a}_4 = \mathbf{e}_1 \otimes \mathbf{e}_2 + \mathbf{e}_2 \otimes \mathbf{e}_1. \end{aligned} \quad (14)$$

Here $\mathbf{e}_1, \mathbf{e}_2$ are arbitrary unit vectors in the tangential plane to ω , $\mathbf{e}_1 \cdot \mathbf{e}_2 = \mathbf{e}_1 \cdot \mathbf{n} = \mathbf{e}_2 \cdot \mathbf{n} = 0$. The components $C_{11}, C_{22}, D_{22}, D_{33}$, and Γ are given by

$$\begin{aligned} C_{11} &= \frac{Eh}{2(1-\nu)}, & C_{22} &= \frac{Eh}{2(1+\nu)}, \\ D_{22} &= \frac{Eh^3}{24(1+\nu)}, & D_{33} &= \frac{Eh^3}{24(1-\nu)}, & \Gamma &= k\mu h, \\ E &= 2\mu(1+\nu), & \nu &= \frac{\lambda}{2(\lambda+\mu)}, \\ C &\equiv C_{11} + C_{22} = \frac{Eh}{1-\nu^2}, & D &\equiv D_{11} + D_{22} = \frac{Eh^3}{12(1-\nu^2)}, \end{aligned}$$

where E and ν are the Young modulus and Poisson ratio of bulk material. C and D are the tangential and bending stiffness of the shell, Γ is the transverse shear stiffness, and k the transverse shear factor.

From (13) it follows the relations

$$\mathbf{T} = C_1 \boldsymbol{\epsilon} + C_2 \text{Atr} \boldsymbol{\epsilon} + \Gamma \boldsymbol{\gamma} \otimes \mathbf{n}, \quad \mathbf{M} = [D_1 \boldsymbol{\kappa} + D_2 \text{Atr} \boldsymbol{\kappa}] \times \mathbf{n}, \quad (15)$$

$$C_1 = 2C_{22}, \quad C_2 = C_{11} - C_{22}, \quad D_1 = 2D_{22}, \quad D_2 = D_{33} - D_{22},$$

or in the component form

$$T_{\alpha\beta} = \frac{Eh}{1+\nu} \left[\epsilon_{\alpha\beta} + \frac{\nu}{1+\nu} a_{\alpha\beta} \epsilon_{\eta}^{\eta} \right], \quad T_{\alpha 3} = \Gamma \gamma_{\alpha}, \quad M_{\alpha\beta} = \frac{Eh^3}{1+\nu} \left[\kappa_{\alpha\beta} + \frac{\nu}{1+\nu} a_{\alpha\beta} \kappa_{\eta}^{\eta} \right],$$

where $a_{\alpha\beta}$ are the metric coefficients, $a_{\alpha\beta} = \boldsymbol{\rho}_{\alpha} \cdot \boldsymbol{\rho}_{\beta}$, $\epsilon_{\eta}^{\eta} = \text{tr} \boldsymbol{\epsilon}$, and $\kappa_{\eta}^{\eta} = \text{tr} \boldsymbol{\kappa}$.

Let us consider the constitutive equations for \mathbf{T}_s and \mathbf{M}_s . For the sake of simplicity we assume the same viscoelastic properties for both shell faces. From (12) follow the relations

$$\begin{aligned} \boldsymbol{\tau}_{\pm} &= \int_{-\infty}^t \lambda_S(t-\tau) \text{tr} \dot{\boldsymbol{\epsilon}}(\tau) d\tau \mathbf{A} + 2 \int_{-\infty}^t \mu_S(t-\tau) \dot{\boldsymbol{\epsilon}}(\tau) d\tau \\ &\mp \frac{h}{2} \left(\int_{-\infty}^t \lambda_S(t-\tau) \text{tr} \dot{\boldsymbol{\kappa}}(\tau) d\tau \mathbf{A} + \int_{-\infty}^t 2\mu_S(t-\tau) \dot{\boldsymbol{\kappa}}(\tau) d\tau \right). \end{aligned}$$

Thus, we obtain that

$$\begin{aligned} \boldsymbol{\tau}_+ + \boldsymbol{\tau}_- &= \int_{-\infty}^t (2\lambda_S(t-\tau) \text{Atr} \dot{\boldsymbol{\epsilon}}(\tau) + 4\mu_S(t-\tau) \dot{\boldsymbol{\epsilon}}(\tau)) d\tau, \\ \boldsymbol{\tau}_+ - \boldsymbol{\tau}_- &= h \int_{-\infty}^t (\lambda_S(t-\tau) \text{Atr} \dot{\boldsymbol{\kappa}}(\tau) + 2\mu_S(t-\tau) \dot{\boldsymbol{\kappa}}(\tau)) d\tau, \end{aligned}$$

and finally we have

$$\mathbf{T}_S = \int_{-\infty}^t \left[C_1^S(t-\tau) \dot{\boldsymbol{\epsilon}}(\tau) + C_2^S(t-\tau) \text{Atr} \dot{\boldsymbol{\epsilon}}(\tau) \right] d\tau, \quad (16)$$

$$\mathbf{M}_S = \int_{-\infty}^t \left[D_1^S(t-\tau) \dot{\boldsymbol{\kappa}}(\tau) + D_2^S(t-\tau) \text{Atr} \dot{\boldsymbol{\kappa}}(\tau) \right] d\tau \times \mathbf{n}, \quad (17)$$

$$C_1^S = 4\mu_S, \quad C_2^S = 2\lambda_S, \quad D_1^S = h^2 \mu_S, \quad D_2^S = h^2 \lambda_S / 2.$$

As a result from (8), (15)–(17) we establish the constitutive equations for the shell with viscoelastic surface stresses in the form

$$\mathbf{T} = \int_{-\infty}^t [C_1(t-\tau)\dot{\boldsymbol{\epsilon}}(\tau) + C_2(t-\tau)\mathbf{Atr}\dot{\boldsymbol{\epsilon}}(\tau)] d\tau + \Gamma\boldsymbol{\gamma} \otimes \mathbf{n},$$

$$\mathbf{M} = \int_{-\infty}^t [D_1(t-\tau)\dot{\boldsymbol{\kappa}}(\tau) + D_2(t-\tau)\mathbf{Atr}\dot{\boldsymbol{\kappa}}(\tau)] d\tau \times \mathbf{n},$$

$$C_1(t) = 2C_{22} + 4\mu_S(t), \quad C_2(t) = C_{11} - C_{22} + 2\lambda_S(t),$$

$$D_1(t) = 2D_{22} + h^2\mu_S(t), \quad D_2(t) = D_{33} - D_{22} + \frac{h^2}{2}\lambda_S(t).$$

The tangential and bending relaxation functions are given by

$$C = \frac{Eh}{1-\nu^2} + 4\mu_S + 2\lambda_S, \quad (18)$$

$$D = \frac{Eh^3}{12(1-\nu^2)} + \frac{h^2}{2}(2\mu_S + \lambda_S). \quad (19)$$

Let us note that the surface stresses do not influence the transverse shear stiffness.

4. Comparison: plate with surface stresses versus three-layered plate

The presented above model of plates and shells with surface stresses is similar to the theories of three-layered plates and shells that are widely presented in the literature, see e.g. Altenbach, Altenbach, and Kissing (2004), Berdichevsky (2009), Reddy (2004). This similarity is mentioned in Altenbach et al. (2010) in the case of elastic materials. To analyze this similarity in the viscoelastic case let us consider the symmetric three-layered plate (sandwich plate) with the thickness $h = h_c + 2h_f$, where h_c is the thickness of core, h_f the thickness of faces, and $h_c \gg h_f$. We assume that the core is made of elastic material with the Young modulus E or the shear modulus μ , and Poisson ratio ν while the faces are viscoelastic with the relaxation function $E_f(t)$ and the constant Poisson ratio ν_f .

Using the approach by Altenbach and Eremeyev (2009), for the viscoelastic sandwich plate we obtain the constitutive equations in the form

$$\mathbf{T} = \int_{-\infty}^t \tilde{\mathbf{C}}(t-\tau) : \dot{\boldsymbol{\epsilon}}(\tau) d\tau + \int_{-\infty}^t \tilde{\mathbf{T}}(t-\tau)\dot{\boldsymbol{\gamma}}(\tau) d\tau \otimes \mathbf{n}, \quad \mathbf{M} = \int_{-\infty}^t \tilde{\mathbf{D}}(t-\tau) : \dot{\boldsymbol{\kappa}}(\tau) d\tau, \quad (20)$$

where

$$\begin{aligned} \tilde{\mathbf{C}} &= \tilde{C}_{11}\mathbf{a}_1 \otimes \mathbf{a}_1 + \tilde{C}_{22}(\mathbf{a}_2 \otimes \mathbf{a}_2 + \mathbf{a}_4 \otimes \mathbf{a}_4), & \tilde{\mathbf{D}} &= \tilde{D}_{22}(\mathbf{a}_2 \otimes \mathbf{a}_2 + \mathbf{a}_4 \otimes \mathbf{a}_4) + \tilde{D}_{33}\mathbf{a}_3 \otimes \mathbf{a}_3, \\ \tilde{C}_{11} &= \frac{1}{2} \left(\frac{2E_f h_f}{1-\nu_f} + \frac{Eh_c}{1-\nu} \right), & \tilde{C}_{22} &= \frac{1}{2} \left(\frac{2E_f h_f}{1+\nu_f} + \frac{Eh_c}{1+\nu} \right), \\ \tilde{D}_{22} &= \frac{1}{24} \left[\frac{E_f(h^3 - h_c^3)}{1+\nu_f} + \frac{Eh_c^3}{1+\nu} \right], & \tilde{D}_{33} &= \frac{1}{24} \left[\frac{E_f(h^3 - h_c^3)}{1-\nu_f} + \frac{Eh_c^3}{1-\nu} \right]. \end{aligned}$$

The transverse shear relaxation function $\tilde{\mathbf{T}}$ can be restored from its Laplace transform given by the relation

$$\tilde{\mathbf{T}} = \ell^2 \tilde{\mathbf{D}}_{22},$$

where ℓ is the minimal positive root of the equation

$$\xi \cos \ell \frac{h_f}{2} \cos \ell \frac{h_c}{2} - \sin \ell \frac{h_f}{2} \sin \ell \frac{h_c}{2} = 0, \quad \xi = \mu/\mu_f, \quad \mu_f = \frac{E_f}{2(1+\nu_f)}.$$

The tangential and bending relaxation functions of the sandwich plate are given by

$$\tilde{\mathbf{C}} = \tilde{C}_{11} + \tilde{C}_{22} = \frac{2E_f h_f}{1-\nu_f^2} + \frac{Eh_c}{1-\nu^2}, \quad (21)$$

$$\tilde{\mathbf{D}} = \tilde{D}_{22} + \tilde{D}_{33} = \frac{1}{12} \left[\frac{E_f(h^3 - h_c^3)}{1-\nu_f^2} + \frac{Eh_c^3}{1-\nu^2} \right]. \quad (22)$$

Comparing (21) with (18) we can conclude that the surface relaxation functions λ_S and μ_S can be expressed through the relaxation function of faces E_f , Poisson ratio ν_f , and the thickness h_f . With accuracy of $O(h_f^2)$ we obtain that

$$\mu_S \approx \frac{E_f h_f}{2(1+\nu_f)} \equiv \mu_f h_f, \quad \lambda_S \approx \frac{\nu_f E_f h_f}{1-\nu_f^2} \equiv \lambda_f h_f \frac{1-\nu_f}{1-\nu_f}, \quad (23)$$

where λ_f is the second relaxation function of faces. Comparison of (19) with (22) results in the same formulae. Hence, we get

$$\mu_S = \lim_{h_f \rightarrow 0} \mu_f h_f, \quad \lambda_S = \lim_{h_f \rightarrow 0} \lambda_f \frac{1}{1} \frac{2\nu_f}{\nu_f} h_f. \quad (24)$$

The latter equations interpret the surface viscoelastic properties μ_S and λ_S through the relaxation functions of plate faces and their thickness.

Unlike to the tangential and bending relaxation functions the transverse shear relaxation function for the sandwich plate with viscoelastic faces \tilde{T} differs from the function T for any values of h_f .

For linear and nonlinear sandwich plates and shells Berdichevsky (2009, 2010a, 2010b) proposed the model of hard-skin structures using two additional small parameters

$$A_1 = \frac{\mu h_c}{\mu_f h_f}, \quad A_2 = \frac{h_f}{h_c}. \quad (25)$$

In the case of shell with surface stresses we use (24)₁ and $h_f = 0$. Thus, we $h = h_c$ and A_1, A_2 became

$$A_1 = \frac{\mu h}{\mu_S}, \quad A_2 = 0. \quad (26)$$

A_1 depends on the shell thickness and the characteristic length parameter $d = \mu_S/\mu$. It is obvious that $A_1 \ll 1$ if $h \ll d$. As an example let us take the values used in Duan et al. (2005, 2008), Wang et al. (2006) for anodic aluminum. We set $\mu = 34.7$ GPa, $\nu = 0.3$, and $\mu_S = 6.2178$ N/m. As in Altenbach et al. (2009, 2010), one can prove that the influence of surface stresses is negligible when $h > 50$ nm. This means that the elastic and viscoelastic plates and shells with surface stresses can be considered as the hard-skin structures at the nanoscale only.

5. Conclusions

In this paper we extend the constitutive relations of elastic thin-walled structures with surface stresses (Eremeyev et al., 2009; Altenbach et al., 2009, 2010) taking into account the surface viscoelasticity. Similar to the Gurtin–Murdoch model of surface elasticity the linear surface viscoelasticity contains the surface stresses which depends on the surface strains. But here the 2D constitutive equations express dependence of the surface stresses on the prehistory of surface strains. In linear isotropic case these dependencies are given by the relation (5). Using the correspondence principle and the through-the-thickness integration technique of reduction of 3D equations to 2D ones, it was shown that

- the well-known elastic approach can be extended to the linear viscoelastic case,
- the transverse shear stiffness plays a specific role and must be computed carefully, and
- the surface behavior is not affected by the transverse shear behavior with respect to the thinness of the influence zone.

It was also shown that influence of the surface viscoelastic properties depends on the values of material parameters in the relaxation functions and may be significant for such objects as nanoplates and nanoshells.

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