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A METHODOLOGY FOR INCLUDING THE EFFECT OF A DAMPING TREATMENT IN THE MID-FREQUENCY DOMAIN USING SmEdA METHOD

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An additive damping treatment is an effective tool to control the dynamic response of built-up structures, and it is widely utilized through industrial applications. By applying a viscoelastic layer on a given structure, the vibratory energy is dissipated through shear and in-plane motions at the layer interface. Modeling the effect of such a treatment in a complex mechanical system for the mid frequency domain is of interest. Statistical modal Energy distribution Analysis (SmEdA) has been developed as an alternative approach to Statistical Energy Analysis (SEA) for describing subsystems with low modal overlap. This technique is developed from the knowledge of the uncoupled subsystem modes. In this paper, one proposes to extend SmEdA by including the effect of a damping treatment. A damped subsystem consisting of a composite layer is modeled with the equivalent modulus of a single layer, which gives the same transverse displacement as a multi-layered system. The modal loss factor of a partially damped structure is estimated by the Modal Strain Energy method (MSE), and the results are well agreed with the Complex Eigenvalue Method (CEM). Finally, energy transmission between the damped structure and a coupled cavity can be deduced from SmEdA modeling, knowing the modeshapes and modal loss factors of the equivalent single layer and of the cavity. This method is applied for modeling a rectangular plate partially damped with an unconstrained viscoleastic layer coupled to a small acoustic cavity.

1. Introduction

Deterministic FEM analysis and SEA method are typical means to predict dynamic response of a complicated mechanical system, yet neither method is appropriate for a mid-frequency domain. Over the years, a number of alternative/hybrid techniques have been developed to cover this gap between low and high frequency domains. SmEdA (Statistical Modal Energy Distribution Analysis)[1-5] is among them, where the basic SEA theory is modified on the energy flow exchanged by two oscillators coupled through a gyroscopic element and use of the dual modal formulation for describing coupled subsystems. With modal information of uncoupled subsystems deduced from FEM, individual modal energies between coupled resonant modes can be apprehended rather than ensemble energy of modes that dictates classical SEA theory. Since eigenvectors and natural frequencies of uncoupled subsystems are deduced from FEM, the SmEdA technique can evaluate a system that has a complicated geometry and consists of composite materials.

This paper aims at analyzing energy transmission of a structure-fluid coupled system for a mid-frequency domain. SmEdA can reveal energy distribution of coupled modes in a given frequency band for a rectangular steel plate coupled to an acoustic cavity. This is extended to a case where a plate subsystem is treated with an unconstrained viscoelastic damping layer. The influence of additive damping on the energy transmission is of interest. A rectangular steel plate partially damped with a viscoelastic patch is characterized by equivalent modulus of a single layer plate[6-7] that gives the same transverse displacement as a multi-layered plate. In this approach, multi-layers are replaced by a single homogeneous layer with equivalent mechanical properties calculated by the mixture of classical plate theories. The frequency-dependent properties of a viscoelastic material are considered in the equivalent plate modeling which can compute Young's modulus and damping loss factors at resonant modes for an infinite plate. In order to calculate damping levels of a finite plate, MSE (Modal Strain Energy) method[8,10,11] is employed. The fundamental assumption of MSE is that the modeshapes of a damped structure is the same as those of undamped. Then, a ratio of a damped plate loss factor to the viscoelastic material loss factor can be estimated from a ratio of the elastic strain energies stored in the entire structure to the viscoelastic layer. Finally, with the modal loss factors of a damped plate calculated by series of mentioned techniques, the energy transmission between subsystems can be deduced.

This paper presents numerical analysis of SmEdA on undamped/damped rectangular steel plate coupled to an acoustic cavity. When a plate subsystem is partially damped with a viscoelastic patch, the composite panel is modeled as an equivalent single layer, and modal damping levels of total subsystem are evaluated and compared to those obtained from CEM (Complex Eigenvalue Method)[8-9]. The modal coupling energy loss factors between subsystems and their energy relation is evaluated.

2. SmEdA Method For The Mid-Frequency Domain

SmEdA is dictated by the basic linear equation of power exchanged between coupled subsystems as the SEA principle. However, the method differentiates by dealing with energy levels of individual modes of subsystems rather than ensemble energy within a considered frequency band Δf. This compensates rather disputable SEA assumption of the energy equipartition among subsystem modes. Two subsystems with natural mode p and q, and powers injected into mode p of subsystem 1 and mode q of subsystem 2 can be written as

$$\begin{aligned}\Pi_{inj}^{p} &= \left(\omega_p\eta_p + \sum_{q=1}^{N_2}\omega_p\eta_{pq}\right)E_p - \sum_{q=1}^{N_2}\omega_c\eta_{pq}\,E_q \\ \Pi_{inj}^{q} &= -\sum_{p=1}^{N_1}\omega_c\eta_{pq}E_q + \left(\omega_q\eta_q + \sum_{p=1}^{N_1}\omega_c\eta_{pq}\right)E_q\end{aligned} \tag{1}$$

where Π_{inj}^{p} and Π_{inj}^{q} are the time-averaged power injected into mode p and q of subsystem 1 and 2, E_p and E_q are the energy of mode p and q of subsystem 1 and 2, η_p and η_q are the damping loss factor of mode p and q of subsystem 1and 2, η_{pq} is the coupling loss factor between mode p and q of subsystem 1 and 2. Consider a rectangular plate coupled to an acoustic cavity illustrated in Fig. 1, the dual modal formulation uses the modal bases of uncoupled subsystems described by displacement and pressure with blocked and free boundary conditions accordingly.

Modeshapes of each subsystem are extracted from FEM, and interactions between coupled modes are evaluated by considering the discretized modes at a coupling surface. The modal coupling loss factor is given by

$$\beta_{ijkmn} = \beta_W \times \beta_\omega$$
$$\beta_W = \frac{(W_{ijkmn})^2}{M_{ijk} M_{mn}} \tag{2}$$
$$\beta_\omega = \frac{\eta_{mn}\omega_{mn}\omega_{ijk}^2 + \eta_{ijk}\omega_{ijk}\omega_{mn}^2}{(\omega_{mn}^2 - \omega_{ijk}^2)^2 + (\eta_{mn}\omega_{mn} + \eta_{ijk}\omega_{ijk})(\eta_{mn}\omega_{mn}\omega_{ijk}^2 + \eta_{ijk}\omega_{ijk}\omega_{mn}^2)}$$

where β_{ijkmn}, β_W and β_ω are the modal coupling loss factor between mode (*m,n*) of the plate and mode (*i, j,k*) of the cavity, the modal spatial coupling factor and the modal spectral coupling factor respectively. W_{ijkmn} M_{mn} and M_{ijk} are the inter-modal work between mode (*m,n*) of the plate and mode (*i, j,k*) of the cavity and the modal mass of mode *(m,n)* of the plate and mode (*i, j,k*) of the cavity respectively. η_{ijk} and η_{mn} are the modal damping loss factors of each subsystem. If a plate is treated with a viscoelastic damping material, the complex modulus of elasticity is considered for the modal damping level. Deducing this term, η_{mn} is rendered by MSE method and is presented in Section 3.3. The inter-modal work (W_{ijkmn}) is given by

$$W_{ijkmn} = \int_{S_p} p_{ijk}(Q) W_{mn}(Q) dQ \tag{3}$$

where $p_{ijk}(Q)$ and $W_{mn}(Q)$ are modeshapes of the cavity and the plate. This term represents the modal interaction between displacement modeshape (*m, n*) of the plate and pressure modeshape (*i, j,k*) of the cavity.

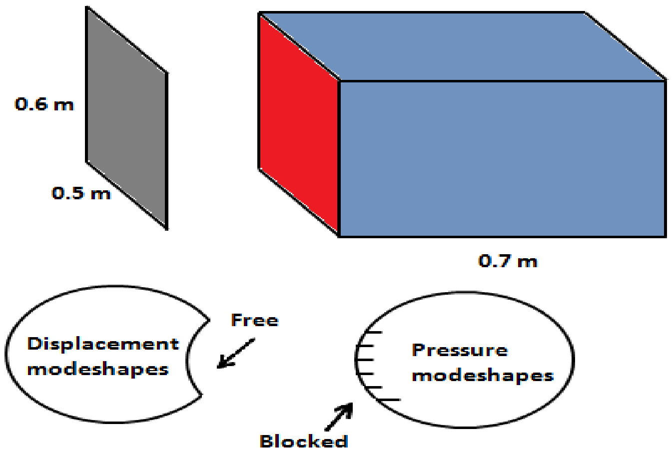

Figure 1. Top: a rectangular plate coupled to a cavity via a coupling surface (red area). Bottom: subsystem descriptions with their boundary conditions of the SmEdA model.

3. Analysis of A Structure-Fluid Coupled System

3.1 The damping loss factor of the plate subsystem

The properties of a steel plate are experimentally determined. Young's modulus and the loss factor are measured from modal analysis of a plate with a free-free boundary condition. The plate is driven by an impact hammer at one corner of the plate, and the response is measured with an accelerometer at another corner. The loss factors are obtained from well-separated modes by the high-resolution modal analysis method based on the ESPRIT algorithm[12] on the recorded time signal. Table 1 shows the loss factors averaged over frequency bands of 400 Hz. The total (global) average loss factor is considered for the equivalent single plate modelling as presented in Table 2.

Table 1. Frequency band averaged damping loss factor of a steel plate with a free-free boundary condition obtained from laboratory measurement. The bandwidth (Δf) is 400 Hz.

Center Frequency (Hz)	200	600	1k	1.4k	1.8	Total average
η	0.0027	0.0017	0.0014	0.0019	0.0009	0.0017

3.2 Equivalent single layer modelling of a partially damped plate

Developed by Guyader *et al.*[6-7] and implemented in the software MOVISAND, the equivalent single layer modelling simplifies a multi-layered panel with a single homogenous layer of equivalent viscoelastic moduli. The method is based on transverse waves motion computation (Love-Kirchhoff thin plate theory), the equivalent viscoelastic moduli of a single layer are determined in order to give the same transverse displacements of multi-layered panel. The method considers bending, membrane and shear motions in each layer and uses the continuity condition on displacements and shear stresses at layer interfaces to obtain an equation of motion that is independent of a number of layers. In order to characterize an equivalent single layer material, equivalent moduli e.g. Young's modulus, mass density and Poisson's ratio must be determined. There parameters are given by

$$D = \frac{E_{eqv} h_{total}^3}{12(1 - v_{eqv}^2)} \qquad \rho_{eqv} = \sum \frac{h_i \rho_i}{h_i} \qquad v_{eqv} = \sum \frac{h_i v_i}{h_i} \tag{4}$$

where D and h are bending stiffness of the equivalent layer and thickness of each damping layer respectively. Derivation of the analytical expression of the equivalent bending stiffness D can be found in the literature[7].

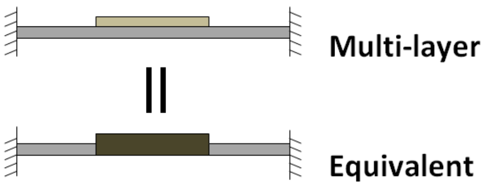

Figure 2. Equivalent single layer modelling. Boundaries are clamped.

A rectangular steel plate subsystem damped with a rectangular PVC (polyvinyl chloride) patch is modelled as an exact multi-layered plate and an equivalent plate as seen in Fig. 2. The PVC patch bears 33% of the base plate, and its location is arbitrarily chosen over the plate surface. Young's modulus of the steel plate presented in Table 2 is obtained from the averaged relative natural frequency differences between numerical FE calculation and experimental results. MOVISAND software calculates equivalent parameters for an infinite plate model according to Eq. 4; Young's modulus, density, Poisson's ratio and damping loss factor. Figure 3 shows the variations of Young's modulus and damping factors with respect to vibration frequency for the equivalent infinite plate model. In fact, neither quantity varies significantly since complex moduli (frequency-dependent) of PVC are neglected in the calculation.

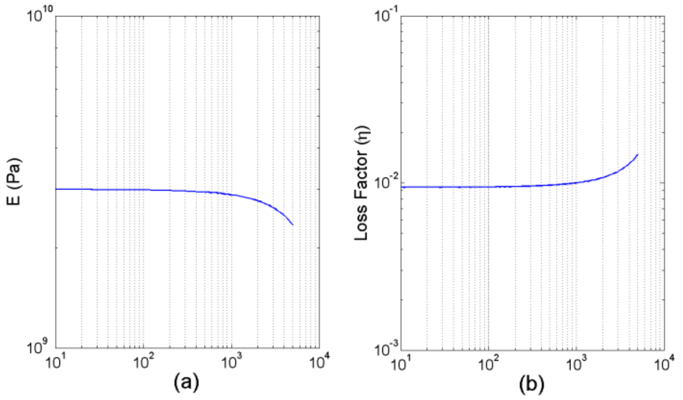

Figure 3. Equivalent moduli of the infinite steel-PVC composite panel: (a) Young's modulus, (b) Loss factor

Table 2. MOVISAND calculation results: equivalent moduli of the infinite steel-PVC composite panel. Frequency-dependent E(f) and η(f) of the equivalent plate model are presented in Fig. 3.

	Steel	PVC	Equivalent
h (m)	0.001	0.003	0.004
E (Pa)	2.03e11	2.4e7	E(f)
ρ (kg/m^3)	7523.1	1200	2780.77
ν	0.33	0.45	0.42
η	0.0017	0.35	η(f)

Once equivalent parameters are determined for an infinite plate model, they can be taken into a computation of the dynamic behaviour of a finite plate partially damped with a PVC patch. To evaluate the equivalent modelling method, the frequency response function of two plate models shown in Fig. 2 is computed. Seen from the equivalent plate model in Fig. 2, the composite core is defined by the equivalent parameters presented Table 2 and Fig. 3, and the rest is the steel plate whose parameters are also given in Table 2. Since complex moduli of the equivalent plate shown in Fig. 3 do not vary significantly in the frequency range of interest (up to 2 kHz), constant averaged values E_{eqv} = 2.95 GPa and η_{eqv} = 0.0096 are considered for the computation. Both plate models with clamped boundaries are driven with a harmonic transverse point force at the same position, and the point response is calculated with NASTRAN FE software. The results are in good agreement between two models, demonstrating a validity of the equivalent single layer modelling.

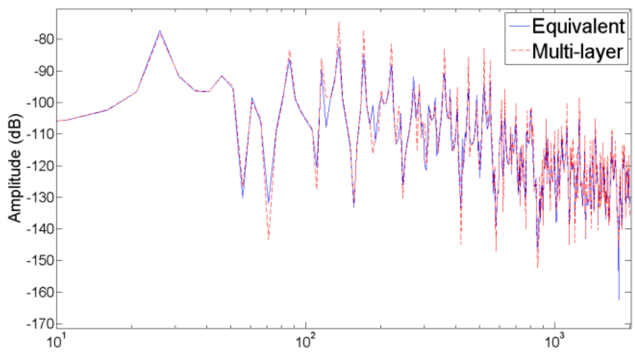

Figure 4. Frequency response function (a single response point) of the equivalent and multi-layered plate models, clamped at their boundaries.

3.3 Numerical estimation of modal loss factor of the finite equivalent plate

Amongst several techniques to predict dynamic behaviours of a damped structure, MSE method has been proved effective and widely used in practice. It relies on the linear dynamics of undamped systems in which a viscously damped linear system is assumed to posses the same real normal modes as undamped systems. In other words, a ratio of the modal composite loss factor to a viscoelastic loss factor can be approximated as a ratio of the elastic strain energy stored in the viscoelastic layer to the total elastic energy stored in the total structure for a given undamped mode ψ. This is given by

$$\frac{\eta_{composite}}{\eta_{viscoelastic}} = \frac{V_{viscoelastic}}{V_{composite}} = \frac{\Psi^T K_{viscoelastic} \Psi}{\Psi^T K_{viscoelastic} \Psi} \tag{5}$$

where K and V are a real part of the stiffness matrix and strain energy. This method is applied to calculate the modal damping levels of a finite damped plate. Note that MSE calculation is performed on the equivalent plate model only. The equivalent parameters calculated for an infinite plate model are taken into FE normal mode calculation on the equivalent plate model with clamped boundaries in order to give the modal damping levels. As the same manner presented in the frequency response function computation given in Section 3.1, constant averaged values of E_{eqv} = 2.95 GPa and η_{eqv} = 0.0096 are considered for the damped part of the finite plate. The results are evaluated with those obtained from the CEM (Complex Eigen Method)[8-9] and are in good agreement.

3.4 The damping loss factor of the cavity subsystem

The loss factor of a cavity is experimentally determined. A dimension of the cavity is 0.5 * 0.6 * 0.7 (m) as illustrated in Fig. 1. The cavity is driven with a white noise at a random position, and the pressure decay inside the cavity is recorded after the noise source is turned off. Two methods are used to derive the loss factors. The method presented in the literature[12] based on modal analysis and the standard ISO method[13] are used. For the latter, the decay curve of measured pressure is fitted to a straight line by the linear regression approach in order to calculate the reverberation time. Then the modal loss factor of an acoustic space can be obtained as

$$T_R = \frac{0.16V}{A}, \qquad A = \sum_i S_i \bar{\alpha}_i, \qquad \eta = \frac{2.2}{f T_R} \tag{6}$$

where A, S, $\bar{\alpha}$, V and T_R are total absorption area, surface area of the cavity, angle averaged absorption coefficient, cavity volume and the reverberation time (T_{30}) respectively.

Table 3. Frequency band averaged damping loss factor of an acoustic cavity obtained from laboratory measurement with the high-resolution method[13]. The bandwidth (Δf) is 400 Hz.

Center Frequency (Hz)	200	600	1k	1.4k	1.8	Total average
η_{ijk}	0.0047	0.0037	0.0025	0.0026	0.0011	0.0029

3.5 Energy relation between subsystems

SmEdA computations are rendered on two different coupled systems: undamped steel plate coupled to a cavity (model 1) and partially damped steel plate with a rectangular PVC patch coupled to a cavity (model 2). Dimensions of the plate, cavity and PVC patch are 0.5*0.6*0.001

(m), 0.5*0.6*0.7 (m) and 0.25*0.4*0.003 (m) respectively. The total (global) average loss factor of the cavity shown in Table 3 is considered in the SmEdA computation for both models. For model 2, an average value (η_{mn} = 0.0054) of the modal damping levels of the damped plate subsystem is considered. The plate subsystem is driven with a normal harmonic point force at the same position randomly chosen for both models. The modeshape at the excitation point is considered for the injected power, which is used to compute subsystem energy levels from Eq. 1.

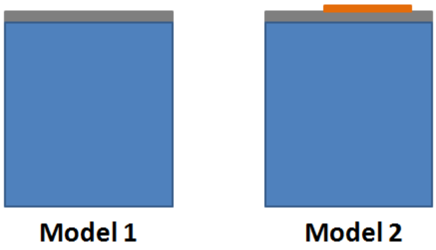

Figure 5. A steel plate coupled to a cavity (model 1) and a steel plate damped with a PVC patch coupled to a cavity (model 2).

SmEdA can reveal the energy transmission between coupled resonant modes in a given frequency band. As seen in Fig. 6 (c1), the coupled modes (11th plate mode at 1097.13 Hz, 10th cavity mode at 1096.33 Hz) of model 1 dominate the energy exchange rate within a frequency band centered at 1 kHz, and this strong coupling occurs when their resonant frequencies coincide. This introduces a considerable modal energy disparity as overlapping between subsystem modes is low. In such case, SEA is not able to describe uneven modal energy distribution and overestimates the energy transfer.

The influence of damping in model 2 can be clearly detected from the modal coupling loss factor (β_{ijkmn}) of model 2 as presented in Fig. 6 (c2). For the same frequency band considered (1 kHz), the resonant coincidence that conducts the highest level of energy transfer is shifted to a different mode couple (19^{h} plate mode at 993.2 Hz, 15th cavity mode at 993.11 Hz) compared to model 1.

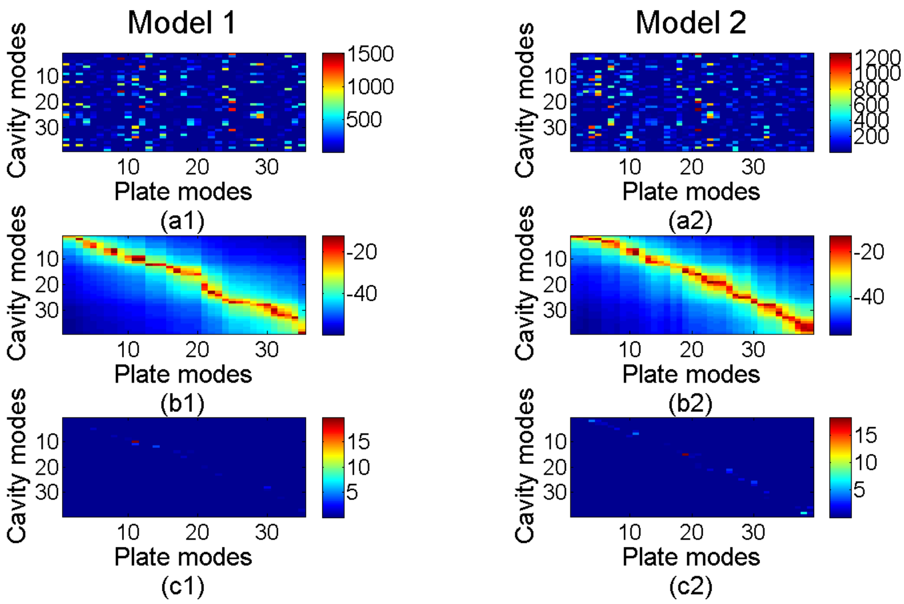

Figure 6. Modal coupling factors of model 1 (left) and model 2 (right) in a frequency band centered at 1 kHz. (a1) (a2) The modal spatial coupling loss factor (β_W). (b1) (b2) The modal spectral coupling factor (β_ω). (c1) (c2) The modal coupling loss factor (β_{ijkmn}).

While an additive damping layer surely reduces the vibratory energy of a plate subsystem, it does not necessarily lead to a reduction of energy transmission to a cavity subsystem. In fact, for the frequency band of 1 kHz an energy ratio of the cavity subsystem to the plate subsystem (E2/E1) of model 2 exceeds that of model 1, which indicates the damped plate permits more energy to be transmitted to a cavity than the case of undamped. As seen in Fig. 6 (a1) and (a2), the interactions between subsystem modeshapes at the coupling surface do not significantly differentiate between two models as total levels (sum) of their modal spatial coupling factors (β_W) within the frequency band are nearly the same. On the other hand, the damping augments the frequency coupling as a sum of the modal spectral coupling factor (β_ω) of model 2 is higher than that of model 1 as presented in Fig. 6 (b1) and (b2). This results in approximately 40 % higher modal damping loss factor (β_{ijkmn}) for model 2, which indicates that the influence of damping over frequency coincidence is more responsible for the strong coupling between subsystems than spatial coincidence. However, such influence of the damping is shown to be disproportionate over different frequency bands, which depends on the modal density.

4. Conclusion

The dynamic behaviour of the structure-fluid coupled system is investigated with SmEdA methodology. Energy transmission between a plate subsystem and an acoustic cavity subsystem is deduced based on energies of individual modes rather than ensemble energy in a given frequency band. When a plate subsystem is partially damped with an additive viscoelastic layer, the composite plate can be modelled as an equivalent single layer which gives the same transverse displacement as a multi-layered plate. Such a method has been demonstrated computationally efficient and proved to yield comparable results. The modal loss factor of a damped plate is obtained from the MSE method. A ratio of the composite system loss factor to the viscoelastic loss factor can be approximated from a ratio of the elastic strain energy store in viscoelastic core and total structure. This provides a rapid estimate of modal damping levels with a reasonable accuracy. With the modal loss factors deduced by MSE, the influence of a damping layer over the vibratory energy flow can be evaluated by the SmEdA technique. The advantage of such technique is that it can reveal the energy transmission between individual coupled modes and evaluate the influence of damping on the modal energies for a frequency band under study. It is shown that it is possible for a damping mechanism to allow high energy transmission to a cavity subsystem largely due to the frequency coincidence of subsystems rather than the spatial coincidence in a given frequency band. However, this is a particular case where a damping layer geometry, thickness and location can yield different outcome.

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