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The Problem of Particle Motion in the Schwarzschild Field: Critical Methodological Analysis, Exact Numerical Solution, and Links to Physics Frontiers

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Abstract

The problem of Schwarzschild dynamics is of fundamental importance in Modern Physics since the Schwarzschild solution is considered the exact one to the Einstein field equation and it is directly testable, in theory. It is widely believed that the solution (a planetary perihelion advance prediction) is flawless and successfully passed the observational test what eventually put General Relativity to the rank of the only true relativistic theory of gravitational field. In this work, we critically analyze the GR prediction methodology and investigate physical meaningfulness of the solution, particularly, in view of relationship of proper versus coordinate time variables and their connections with a relativistic mass and energy concepts. Our results of exact numerical solution to the problem play an essential role in the methodological analysis and assessments. The overall conclusion is made that the GR dynamic equation for point particle motion from Schwarzschild metric is prone to inner contradictions inherited from the GR Physical Foundations. In particular, its exterior solution admits superluminal motion. Strictly speaking, it cannot be used for a treatment of observational data. The work consists of two parts: the first one is devoted to the critical analysis of the Schwarzschild dynamics methodology and the numerical results, the second one to the relevant links with Physics Frontiers. Alternative approach based on a field-dependent proper mass concept is discussed.

Part I

Critique of Schwarzschild Problem Methodology

1 Introductory Comments

1.1 Note about the work

This work of two parts is a continuation of the author's work [1]. The latter contains a history of the GR Mercury problem (including observations), Einstein's methodology of the effect evaluation [2], and a conventional treatment of the GR problem of particle and photon motion in the Schwarzschild field.

The first part is devoted to the extended methodological analysis of the problem with analytical and numerical comparisons (as an addition to that in [1]). A significant attention is given to the GR problem controversies related to the methodological issue of the concept of conserved total energy and the corresponding proper-versus-improper time relationship. The controversy about the time concept is shown to be of principal importance and not resolvable in the GR framework. The major novelty is a conduction of exact numerical computations to solve the problem. A special attention is paid to clarity in the formulation of initial conditions within the GR framework to exclude any physical ambiguity. Though the emphasis is made on the GR problem under weak field conditions, new methodologically important results are presented for the "black-hole" conditions, in particular, the problem of superluminal motion. Unlike in [1], the GR photon problem is not discussed here, though it needs further inquiries.

In the second part, we discuss the results in connections with the GR physical foundations and some relevant issues of Physics Frontiers including relativistic space-time Philosophy.

The main content, novelty, objectives, and results are briefly outlined below in this Section.

1.2 Main historical references

The background of the problem is discussed in details in [1] with an ample literature, and it is only briefed here.

The GR equation of a point mass motion is usually obtained in literature from the Schwarzschild metric, though Einstein derived the equation in 1915 with the use of a number of approximations to the field equations [2], about a year before Schwarzschild suggested his famous “exact field solution” yielding the metric bearing his name [3], [4], (1916). No matter how the equation was obtained, the basic idea of its analytical treatment by Einstein and in numerous GR literature later on remains the same: to find *an approximate solution* of the GR equation in terms of perturbation of the analogous classical equation, as discussed further.

In polar coordinates the GR equation [2] is given by

$$\left(\frac{dx}{d\theta}\right)^2 = \frac{2A}{B^2} + \frac{\alpha}{B^2}x - x^2 + \boxed{\alpha x^3} \quad (1)$$

where (in Einstein’s denotations) $x = 1/r$ is the inverted radius, A and B are the total energy and the angular momentum, correspondingly; $\alpha = 2r_g = 2GM/c^2$ is the Schwarzschild radius (we denote $r_g = GM/c^2$ and call it *the gravitational radius*). Further, the speed of light c is often taken $c = 1$ for brevity.

The term (framed) that causes the GR perihelion advance, is of the second order of smallness, as compared to the main terms in (1). For the circular orbit or the one slightly deviating from a circle of radius r_0 , the known predicted angular advance is $\Delta\theta = 2\pi(3rg/r_0)$ radians per one revolution.

As shown in [1], numerous authors after Einstein brought to the problem (1) nothing new except for some methodological modification of no principal significance. See, for example, Hagihara [5] (1931), who suggested the solution in the form of elliptic functions with the following reduction to Einstein’s solution. Ironically, this work was literally reproduced recently by another authors [6] (2009) without even mentioning the previous publication.

Another example related to our topic is the work by Moller [7] (1972), which is a repetition of Einstein’s approach except for the fact that Moller in his methodology used the coordinate time t in the equation (instead of the proper time τ , as in [2]). The problem discussed below concerning proper-versus-improper time is the one of GR controversial issues.

A derivation of (1) in [8], [9] (Landau) is worth separate commenting. The authors use a semi-classical approach to the problem, namely, the Hamilton-Jacobi equation in the form

$$g^{ik} \frac{\partial S}{\partial x^i} \frac{\partial S}{\partial x^k} - m^2 c^2 = 0 \quad (2)$$

where the contravariant tensor g^{ik} conjugate to the metric tensor g_{ik} , $g_{il}g^{lk} = \delta_l^k$. Then the action S is “classically” given by

$$S = -E_0 t + l_0 \theta + S_r(r) \quad (3)$$

where the radial part of it S_r is obtained after substitution of (3) into (2). Eventually, this leads exactly to (1) though the expression for ϵ_{sch} (or its analogy) is not derived. In fact, the variable t is introduced in a classical manner as a replacement for τ . Thus, the proper-versus improper time controversy remains.

In the monograph written by Bergmann under Einstein’s guidance [10] (1942), published about 27 years after Einstein’s work, a different, seemingly more strict methodological approach (than Einstein’s one) to the derivation and solution of the equation, is applied. However, again, the basic Einstein’s idea remains the same: to treat the problem in terms of perturbation of the analogous classical equation. Thus, the results of our analysis is valid in the latter case as well.

We omit discussions of the so-called PPN approximation approach to the N -body problem (Misner [11], *and elsewhere*) for obvious reason. The matter is that, in the GR literature, the equation (1) is claimed to follow exactly from the Schwarzschild metric, while the latter in its turn is understood as “the exact solution” to the field equations. Thus, it would be unreasonable in our case to consider any Schwarzschild metric “approximation” as “a better approach” to the problem.

1.3 The content, objectives, results, and novelties

It should be recalled that we discuss the Schwarzschild particle dynamics but not a classical field theory of scalar, vector, tensor, or any types in association with quantum carriers possessing a spin. In this work, such a terminology as “field action”, “field strength”, “potential field” is considered, following relevant literature, in relationship with the corresponding “forces” acting on a spinless point particle within the GR treatment in terms analogous to those in Newtonian theory (as was firstly suggested by Einstein and, since then, widely used in GR applications, mostly, in the Newtonian limit). Clearly, entity of a field subjected, in principle, to quantization with radiation is not our issue. “A potential” is meant the velocity-independent one (in this sense, a type of 3D scalar field potential).

General Relativity prediction of perihelion advance in a planetary motion is commonly known from an approximate solution to the GR equation of point mass motion in the Schwarzschild field. In the present work, given the equation, typical assumptions made in derivations of the GR prediction are analyzed, a rigorous method of obtaining of an exact numerical solution is developed, the corresponding numerical integration of the GR equation of motion strictly in the GR framework is conducted, finally, the results are compared with the known in literature approximate solutions and evaluations of the effect. Methodological criticism of “approximations” includes the issues, first, illegitimate replacement of proper time variable with the improper one; secondly (given the first issue), ambiguity in the initial condition formulation.

It is shown that the known approximate solution in the form of uniformly precessing elliptic orbit is incompatible with the original GR equation of particle motion (that is, obtained from the Schwarzschild metric) for the following reasons. First, the classical concept of elliptic orbit loses its physical sense in the GR framework. Second, a rate of GR angular advance is actually not constant within a period: it rises, as a particle comes closer to the center of gravity. It is also shown that the GR solution is governed by a set of two independent physical parameters from the initial conditions. The GR orbital advance follows from the solution without any arguments based on the perturbative comparison with Newtonian problem. In the limit of circular motion in a weak field, the known approximate GR evaluation of angular advance coincides with the exact value. For a non-circular orbits, however, it differs from the GR approximate solution.

It is claimed that an alternative approach based on the SR dynamics methodology can resolve the problem controversies. Such an approach requiring a revision of conventional proper mass concept is presented and briefly discussed in different aspects.

There are three main objectives of the methodological analysis of the problem.

The first objective is to reveal controversies about a derivation of predicted value of the GR perihelion advance and its dependence on the eccentricity, as well as the so-called “analytical approximate solution” (the GR concept of precessing elliptic orbit). These issues are discussed in Section 2 (in some parts, they are raised in [1]).

The second objective is to investigate and explain comparative results

and controversies about the approximate solutions and compare them with results from the exact numerical solution *in the GR framework* leaving the controversies unresolved.

Finally, the third objective (pursued throughout the work, mainly in Part two) is to perceive the problem in its controversial state in connections with GR physical foundations and Modern Physics problems.

This is the first time in GR literature that:

a) Inconsistency of the known approximate solution (the expression of GR uniform rotation of orbital plane) with the GR original equation of motion is revealed, proved theoretically, confirmed numerically, and explained physically;

b) The exact numerical solution is formulated and investigated in the strictly GR framework given with the problem formulation and initial condition setting in terms of (minimal) two independent physical parameters. New analytical, numerical and graphical results are presented for both a weak and strong field and decisive conclusions are made about the GR predictive validity.

2 Controversy about the proper time τ versus the coordinate time t

2.1 GR equation of motion and Schwarzschild metric

The controversy in Schwarzschild orbital dynamics concerns the issue of coordinate-versus-proper time and how it affects the GR concepts of conserved angular (non-zero) momentum B and total energy A as parameters in (1). The problem of radial motion (the zero angular momentum case) is a different one discussed later.

We start with the question: what A and B in (1) are meant? This question is further discussed in the form: should physical quantities in (1) be treated in terms of the proper time τ or the coordinate time t and derivatives with respect to it? The fact is that the radial and angular velocities (and so A and B) in the equation (1) are expressed in terms of derivatives with respect to the proper time τ , in accordance with the equation derivation from Schwarzschild metric, but predictions from (1) are practically treated in terms of the coordinate time t to associate them with astronomical ob-

servations, [11] *and elsewhere*. We argue that a conventional GR problem formulation with the use of (1) has no physical sense in any approximation because the proper time is the time coordinate in the coordinate system of a comoving observer (as discussed further in more details).

From the Schwarzschild metric, the following quantity ϵ_{sch} is identified as the conserved total energy of a test particle per a unit proper mass $m = 1$ ($c = 1$).

$$\epsilon_{sch} = (1 - 2r_g/r)(dt/d\tau) = (1 - 2r_g/r_{ic})(dt/d\tau)_{ic} \quad (4)$$

where a subscript “ic” stand for “initial conditions”. For unbounded motion, it should be $\epsilon_{sch} > 1$ or $(dt/d\tau) < (1 - 2r_g/r) < 1$ but it is not clear how specifically.

The quantity ϵ_{sch} comes out as a result of introduction of the so-called GR “effective potential”, or from the GR metric with the use of Christoffel symbol technique, Bergmann [10], or, equivalently, when one applies the Variational Principle, Fock [12], *and elsewhere*.

The second integral of motion found is the angular momentum l_{sch}

$$l_{sch} = r (d\theta/d\tau) = r_{ic} (d\theta/d\tau)_{ic} \quad (5)$$

which looks similar to the Newtonian expression.

The GR action S for the Schwarzschild field is usually given by

$$\delta S = \delta \int L dt = 0 \quad (6)$$

while the Lagrangian $L = d\tau/dt$ is a square root of the Schwarzschild metric when the latter divided by dt^2 . The metric is given (in the polar coordinates, with the time-like signature $[+, -, -, -]$, ($c = 1$) by

$$d\tau^2 = (1 - 2r_g/r)dt^2 - (1 - 2r_g/r)^{-1}dr^2 - r^2d\theta^2 \quad (7)$$

The Lagrangian depends on derivatives with respect to τ but does not *explicitly* depend on time and angle coordinates, what manifests space-time symmetries. Correspondingly, the following integrals of motion are found, the two independent quantities: $l_0 = r^2(d\theta/d\tau)$ and ϵ_{sch} in (4). The first quantity is intuitively associated with the conserved angular momentum while the second one is thought to be the total energy.

One can also look at the problem in the context of the Killing vectors ξ^μ [13], [14], which preserve the Schwarzschild metric. It is independent of

time variables, so it has time translation symmetry and the Killing vector $\xi_{(t)}^\mu = (1, 0, 0, 0)$ singles out the conserved quantity $\epsilon_{sch} = (1 - 2r_g/r)(dt/d\tau)$. This is actually a way analogous to the Variational Principle scheme (6).

One has eventually come to the equation (1), which describes GR particle trajectories in polar coordinates. “Kinetic energy” and angular momentum terms in the equation contain derivatives of the coordinates with respect to the proper time τ : this is the issue, as discussed further in details.

2.2 Meaning of the proper and improper time

The derivatives $dr/d\tau$ and $d\theta/d\tau$ have a meaning of spatial components of the the proper unit 4-velocity vector tangential to the world line, and as such, they are not and cannot be related to kinetic energies of radial and angular motion in 3D space: the latter are defined with derivatives of special coordinates with respect to t (in polar system, it is dr/dt and $r d\theta/dt$). The proper time τ there determines a variable, – the world line path

$$s = c\tau = \int_{x_1^\mu}^{x_2^\mu} ds(x^\mu) \quad (8)$$

between two points in 4-coordinate space.

It is useful to consider the GR problem formulation in analogy to the SR Dynamics methodology, Synge [15], [16], *and elsewhere*. In SR Kinematics (in the time-like metric), the effects of the time dilation and the corresponding length contraction are derived in the scheme of imaginary experiments, in which two *inertial* observers carrying standard clocks and rods synchronize their clocks at $x = x' = 0$ and further in flight keep exchanging observational information.

Consider two observers in relative motion with respect to each other, both making periodic “flashes” during the *proper* time interval Δt_p . Each observer is capable to measure the corresponding *improper* time interval, which is determined from records of, at least two clocks placed along a trajectory s of a moving partner. One has to detect “flashes” at an instance t_1 from the position s_1 and then t_2 , at s_2 . By definition, the *improper* interval is $\Delta t_i = t_{i+1} - t_i$. The SR time dilation is determined from the connection of intervals via Lorentz factor $\Delta t_i = \gamma \Delta t_p$. Thus, the improper time is “symmetrically constructed” by each observer from records of, at least, two clocks appropriately positioned at different coordinates along a trajectory of observation. It should be compared with observer’s wristwatch records.

The symmetry of two inertial observer frames is lost, however, in SR Dynamics but the scheme remains: the improper time is measured from records of a set of clocks positioned along the trajectory while the concept of proper time τ arises from the world-line (affine) parametrization in the comoving 4-coordinate system.

As a result of a field influence, the proper time interval $d\tau$ depends on a particle position on the world line. A particle passes a length on the world line proportionally to τ with the speed of light. Proper and improper time intervals are connected, as before, via Lorentz factor, now a function of 4-coordinates of the far-away observer's system.

To derive an equation of motion, one has to establish a relationship between $d\tau$ (affected by a field) and dt within consistent relativistic dynamics. Thus, we need more accurately to define an operational concept of the improper (coordinate) time t . It is subject to “book-keeping” by the rest “far-away” observer who uses a set of resting standard clocks, rates of which are not affected by the field (as if a field is “turned off”). A state of rest with respect to the center of gravitational source is meant.

Thus, the (coordinate) time t (contrarily to the proper time τ) is not recorded by a wristwatch but rather “constructed” in such a way that it runs uniformly in the “far-away” observer's coordinate system. The latter is actually an inertial one everywhere; the idea is that the curved (proper) 4-spacetime coordinates can be projected (mapped) onto the inertial (improper) coordinate system in 4-space. In the ratio $d\tau/dt$ only $d\tau$ is a function of 4-coordinates.

2.3 SR-based particle dynamics methodology, and GR

In SR Relativistic Dynamics, the proper 4-velocity vector is a tangent unit vector. For the particle at rest, it is viewed by the comoving observer in the proper coordinate system (with zero spatial components)

$$U_0^\mu = (1, 0, 0, 0) \tag{9}$$

A photon cannot be at rest. Compare its 4-velocity vector with (9)

$$U_{ph}^\mu = (1, 1, 0, 0) \tag{10}$$

The improper 4-velocity can be determined by the far-away observer from spatial coordinate derivatives dx^i/dt , ($i = 1, 2, 3$) with respect to the coordi-

nate time t

$$U^\mu = \gamma(1, dx/dt, dy/dt, dz/dt); \quad U_\mu U^\mu = 1 \quad (11)$$

with the following time connection $dt = \gamma d\tau$, $\gamma = \sqrt{1 - \beta^2}$. $\beta^2 = (dx/dt)^2 + (dy/dt)^2 + (dz/dt)^2$. This is the result of the Lorentz transformation applied at 4-point x^μ for small coordinate intervals dx^μ that gives a relationship $dt = \gamma d\tau$. Here, $(dx/dt) = \beta_x$, $(dy/dt) = \beta_y$, $(dz/dt) = \beta_z$ are components of instantaneous 3-velocity, $\gamma = \sqrt{1 - (dx/dt)^2 - (dy/dt)^2 - (dz/dt)^2}$. The formulas are similar to those in SR Kinematics, but differ in the important feature: they express non-trivial functions of space and time coordinates; components of speed, of course, are dependent, too.

The 4-velocity (11) plays an important role in any relativistic dynamics as it connects 4-coordinate (interval) Δx^μ and 4-momentum P^μ spaces at any 4-point x^μ while $\gamma(x^\mu)$ and $dx^\mu/d\tau$ are functions of the particle 4-coordinates. By definition, the connection is given by

$$dx^\mu = d\tau U^\mu, \quad P^\mu = mU^\mu \quad (12)$$

(here, again, $c_0 = 1$). For $m = const$ the following scalar product is zero

$$U_\mu \frac{dU^\mu}{d\tau} = 0 \quad (13)$$

Finally, the scalar product $dx_\mu \cdot P^\mu$ (what is the 4-phase)

$$dx_\mu P^\mu = m\Delta\tau \quad (14)$$

The goal, the relativistic SR dynamics equation, is derived in terms of Minkowski (4-vector) force K^μ

$$K^\mu = dP^\mu(s)/ds \quad (15)$$

where the interval $ds = d\tau(x^\mu)$ is function of dynamic variables. The equation should be unfolded to find relationships between components of Minkowski and “ordinary” forces in combination with conserved total energy (in terms of coordinate time). The Lorentz factor is, actually, a field functional at a point in 3-space x^i , ($i = 1, 2, 3$) and a given instance t , Synge [15], *and elsewhere*.

$$\Delta\tau/\Delta t = 1/\gamma(\vec{r}, t) \quad (16)$$

This is the first step to establish dynamical connections between coordinate systems of the comoving and far-away observers.

In GR, the analogous 4-velocity follows from the Schwarzschild metric through the metric tensor

$$|U_{gr}|^2 = g_{\mu\nu}(dx^\mu/d\tau)(dx^\nu/d\tau), \quad (c_0 = 1) \quad (17)$$

Unlike in SR Dynamics, the proper time, supposedly, runs uniformly. The 4-velocity is not a unit vector unless a formal normalization is made. However, the normalization “constant” is not a constant; it depends on physical problem conditions and varies from point to point. What we focus attention to is that derivatives (dx^μ/ds) for $\mu = (1, 2, 3)$ are *not* components of physical 3-velocity in the observable space; they are components of spatial part of the proper 4-velocity. At the same time, *they are* components of physical 3-velocity in expressions of kinetic energy constituting the part of total energy $\epsilon_{sch} = A$ entering the equation (1). This is one of controversial issues related to the τ problem. Another controversial issue is the GR interpretation of the gravitational redshift and the gravitational time dilation [17].

There is no physical similarity of GR Dynamics with the SR-based theory. As a matter of fact, relativistic concepts of potential and kinetic energy are foreign bodies in GR, and Special Relativity theory is considered incompatible with the gravity phenomenon and the GR theory Misner [11], *and elsewhere*. Nevertheless, SR concepts in combination with Newtonian physics are often (illegitimately) introduced in GR applications “by virtue of Newtonian limit”.

2.4 Newtonian limit in GR

Let us consider the general GR geodesic equation based on properties of the world line with the affine parameter τ in the 4-coordinate (curved) space (non-zero Christoffel symbols). A formal procedure is devised for the purpose: the parallel transport (which relates to the covariant derivative and the corresponding operator), [18], *and elsewhere*. As a result, the GR geodesic equation with the Christoffel (Levi-Civita) connection is given by

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma_{\rho\sigma}^\mu \frac{dx^\rho}{d\tau} \frac{dx^\sigma}{d\tau} = 0 \quad (18)$$

A typical procedure of reducing (18) to the Schwarzschild geometry in the Newtonian limit of slow motion suggests $\beta = v/c_0 \ll 1$ and $r_g/r \ll 1$.

The gravitational field is presented in metric as the Minkowski form plus “a small perturbation”

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1 \quad (19)$$

what, in the Schwarzschild field, leads to $h_{00} = -2r_g/r$ in $g_{00} = -(1 - 2r_g/r)$. After plugging Christoffel expressions into (19), one has

$$\frac{d^2 x^\mu}{d\tau^2} = \frac{1}{2} \eta^{\mu\lambda} \partial_\lambda h_{00} \left(\frac{dt}{d\tau} \right)^2 \quad (20)$$

The culminating point comes with “the ultimate approximation” of the partial derivative $\partial_0 h_{00} = 0$ for the temporal component $\mu = 0$ with the result

$$\frac{dt^2}{d\tau^2} = 0 \quad (21)$$

or

$$(dt/d\tau) = \text{const} \quad (22)$$

where the proportionality constant in the context of the problem could have the only meaning of constant Lorentz factor. In other words, a particle is in a state of inertial motion defined in SR Kinematics.

Recall, in the ratio of $d\tau/dt$ only $d\tau$ is a function of 4-coordinates, hence, $d\tau = d\tau(t)$. (Of course, one could plot an inverse function $dt(\tau)$, if needed, and use it appropriately). The τ and t are not replaceable in any circumstances, even in SR Kinematics when $d\tau/dt = \text{const}$. The above properties of SR-based methodology are ignored in the GR methodology in agreement with the “incompatibility statement” that the nature of gravity (subject of the GR theory of gravitational field) does not respect the SR laws [11].

2.5 The root of the problem, and what to do about it

Concerning the incompatibility of SR Dynamics with the GR theory, one can “blame” Nature being different, [11], *and elsewhere*. However, one can equally blame GR. The root of the problem seems to lurk in the GR field equations within the GR curved spacetime methodology, which loses any analogy with Newtonian physics and SR dynamics so that an inevitable introduction of GR physical concepts analogous to the classical mechanics creates controversies about concepts of mass, time, and energy at the fundamental

level. There is nothing what could be done about it except to modify the theory.

In GR practical applications formulas borrowed from SR and other physical branches are often used. They are obtained by a mere physical reasoning to treat the quantity $(dt/d\tau)$ in (4) depending on a specific problem formulation. For example, GPS methodology is based on the combination of the Schwarzschild metric with *ad hoc* introduction of the SR Lorentz factors, [19], and a good portion of classical physics, [20], *and elsewhere*.

The attempted radical enough way to resolve the problem is a reformulation of the theory, as in Moller's work [7]. The concept of "generalized Lorentz factor" is introduced so that the equation (1) becomes presented in terms of coordinate derivatives with respect to the t . GR physics appreciably changes, too, for example, the proper mass becomes dependent on the field strength, namely, it rises as a test particle approaches the center. In our view, this particular way of GR modification remains inherently contradictory and, for this reason, is arguable.

Further, following the related GR literature (with the τ problem historically swept under the carpet), we are to interpret the results of exact numerical solution of (1) as if the equation (1) is given in terms of coordinate derivatives with respect to the coordinate time (pretending no difference between t and τ). This step, though fundamentally wrong, is admitted in the GR framework we deal with.

In particular, we want to investigate whether the exterior solution admits, under certain physical conditions, superluminal motion, $\beta = dl/d\tau > 1$ ($c = 1$), and the interior one – a relativistic motion with the speed less than the speed of light. It would be contradictory to the GR Schwarzschild horizon concept, which states that a particle falling onto a black hole reaches the speed of light at the horizon and keeps moving further with the greater speed.

The long list of widely known academic works of notable authors, who do the same replacement τ with t (in the treatment of (1)) but without comments about it, includes Einstein [2], Schwarzschild [4], Landau, [8], Bergmann [10], Fock [12], Wald [13], Synge [16], Pauli [21], Rindler [22], Chandrasekhar [23], and others. Indeed, just looking at the (1), one has nothing to say about the $d\tau$ or the dt (unless been asked). Making them "equal", Einstein obtained a formula for the GR perihelion advance (discussed later). As Synge noted [16] (p. 296), "Such is its prestige that no new gravitational theory is likely to prove acceptable if it does not yield this

formula, or one practically indistinguishable from it”.

2.6 Comments on the GR framework

To avoid any confusions about our results, it is worth commenting on the GR framework we work in. It includes *mathematics* of the problem formulation, – the GR equation (1) (or its equivalent form) in terms of τ variable and its presentation in terms of minimal independent (two) parameters, for example, $\rho = r_g/r_0$ and β_0 . *Physics* comes to the scene, first, in the formulation of the initial conditions with the use of physical (as opposed to geometrical ones) parameters consistently with the (1); second, in analysis of the equation roots and its approximate and exact numerical solutions in comparison with the corresponding results from the conventional GR methodology.

It is proved that the two independent physical parameters are needed and sufficient. More than that would change the constraints so that the solution could be “spoiled”. In our rigorous formulation of the problem, the elliptic parameters are not valid anymore. For better understanding symmetries, it is important that the equation and the initial conditions are expressed in dimensionless forms.

How the equation (1) was originated is discussed in brief but, actually, this issue goes beyond the scope of this work. Our results are interesting, first of all, in comparison with the corresponding theoretical results from other works rather than with observations.

The concept of field strength is accurately defined by the ρ_0 criterion from the initial conditions for both bounded and unbounded motion: $\rho_0 \ll 1$ for “weak” field; a “strong” field is that is not “weak” as $\rho_0 < 1$.

Physical arguments and assumptions, which are beyond the GR physical foundations, potentially are not consistent with and not germane to the GR framework used in this work. We mean an involvement of quantum physics, thermodynamics, astrophysics, and other disciplines (particularly, used in the Black Hole concept), they are taken from the outside of the GR theory. Strictly speaking, GR approximations in the so-called Newtonian limit are contradictory to the GR curved spacetime philosophy; they are shown to cause contradictions and controversies (the one of the proper-versus-improper time variable in (1)).

The superluminal motion in the BH interior is predicted by GR and is fully consistent with the GR framework. This is one of specific purposes of this work to examine the GR Black Hole concept and superluminality

conditions. The conclusion discussed in this work later can be a surprise for readers: the GR concept of the BH Horizon is actually a misconception since it is a product of some reasoning over the Schwarzschild metric but does not strictly follow from it. This fact alone invalidates the GR theory.

The author takes full responsibility for claiming that the results and conclusions from the GR problem analysis in this work are technically correct and they are the ones of rigorous verified formulation of the Schwarzschild problem in every specific respect needed: the GR framework and its adequacy to Einstein's equation (1), the root analysis of the equation and choice of minimal physical parameters, the corresponding initial conditions and their analytical relationships including the conservation laws, finally, the method of exact numerical solution and its mathematical and physical interpretations.

On the basis of our study, the question arises about to what extent the GR physical picture revealed is a reflexion of physical reality. The question could be unambiguously answered provided the results of this work are viewed together with the results of methodological analysis of GR classical (observational) tests, Mercury's perihelion advance, in particular. Comparisons with observations are actually beyond the scope of the present work. So, our advice would be to critically read and learn more about the most "direct" observations, their models and the corresponding interpretations from the credited scientific works devoted to the GR and BH problems, see our reviewing study [1].

Einstein's problem of the Mercury perihelion advance and the GR "Black Hole" concept is the topic of the following Sections.

3 GR Mercury's perihelion advance problem

3.1 Einstein's perturbation approach

Considering the equation (1) for a relativistic interpretation of observed astronomical data, one might wish to use a "relativistically corrected" Newtonian theory of motion in a central spherically symmetrical gravitational field so that small corrections would be sufficient to account for the GR term impact. For this task, Einstein's idea [2] is to use a perturbation technique to consider the Newtonian equation of motion being "perturbed" by the GR term in (1), as described in the following.

An attentive reader could find the above terminology too loose in view of the fact that Newtonian and GR theories are radically different ones. Later, we shall discuss various aspects related to rigor of the problem formulation, in both the derivation and the solution of the the equation. Now, given the equation, one has to start with a consideration of “the exact general solution” to (1) that is, the integral:

$$\theta_{gr} = \int_{x_1}^{x_2} \frac{dx}{\sqrt{\alpha(x-x_1)(x-x_2)(x-x_3)}} \quad (23)$$

Here θ_{gr} is an angle of planet’s precessional orbit, $x_1 = 1/r_1$, $x_2 = 1/r_2$, and $x_3 = 1/r_3$ are real roots of the cubic equation $(x-x_1)(x-x_2)(x-x_3) = 0$. It is meant here that the third root x_3 appears due to the GR term. The solution requires the initial conditions to be set. The integral cannot be calculated analytically. However, one can compare (23) with the analogous exact solution in the classical (Newtonian) formulation of the problem in the approximation of the “smallness” of the GR term as a perturbation source.

Before making any approximations, one could examine the exact algebraic relationship (from Vieta’s formulas) between the pair of two main roots x_1 , x_2 and the third root x_3 , the one due to the GR-term:

$$(x_1 + x_2 + x_3) = \frac{1}{\alpha} \quad (24)$$

The next step would be to eliminate x_3 in (23) assuming in (24) $x_3 \gg x_1$, $x_3 \gg x_2$; from this, one could “approximate” $x_3 \approx 1/\alpha$, as in [2]. A further step simply comes to algebraically split the integrand in (23) into two additive parts:

$$I_{gr}(x) = I_0(x) + \Delta I(x)$$

where I_0 is the main part, the classical solution, and ΔI is a perturbative correction due to the GR term, correspondingly:

$$\frac{1}{\sqrt{-\alpha(x-x_1)(x-x_2)(x_3-x)}} \approx \frac{1}{\sqrt{-(x-x_1)(x-x_2)}} + \frac{\alpha(x_1+x_2+x)}{2\sqrt{-(x-x_1)(x-x_2)}} \quad (25)$$

As wished, (25) does not explicitly contain the root x_3 and is easily integrated analytically.

The question arises about an impact of the GR term on roots x_1 and x_2 in “the comparison scheme”. It is important that one has to compare the GR roots with the corresponding classical roots \tilde{x}_1, \tilde{x}_2 under “similar conditions”. This is not a trivial procedure at all, as further shown. To make an unambiguous comparison in terms of radial shift, one should consider the case of “nearly circular” orbit (in the limit of circle) in both GR and Newtonian theories: $\tilde{x}_1 + \tilde{x}_2 \approx 2/\tilde{r}_0$, where \tilde{r}_0 is the radius of the classical (Newtonian) circle. Obviously, “the smallness” of radial shift requires the weak-field conditions.

The question arises whether it would be correct to neglect a deviation of the classical circular radius \tilde{r}_0 from the corresponding GR radius r_0 ?

For Mercury conditions, the relative radial shift is very small so that; intuitively, one (naively) expects that the impact of the GR term on the main roots under weak-field conditions must be practically negligible. Under such an assumption, Einstein eventually found [2] the GR perihelion advance (per half a period) as “a perturbative correction” in (25)

$$\frac{1}{2}\Delta\theta = \int_{x_2}^{x_1} \Delta I(x)dx = \pi(3r_g/r_0) \quad (26)$$

to be added to the non-perturbed classical solution as an integral within the main roots

$$\pi = \int_{x_2}^{x_1} I_0(x)dx \quad (27)$$

that explains “the Mercury’s anomaly” (in rad per one revolution) given by

$$\Delta\theta = 2\pi(3r_g/r_0) \quad (28)$$

In general, Einstein considers an elliptic form of a precessing orbit and dependence of the effect on the eccentricity e and the semi-latus rectum p (instead of r_0):

$$\Delta\theta = 2\pi(3r_g/p) \quad (29)$$

or, bearing in mind $p = a(1 - e^2)$ (where a is a semi-major axis)

$$\Delta\theta = 2\pi(3r_g/a(1 - e^2)) \quad (30)$$

Alternative equivalent form can be expressed through a period of revolution. Having all this, he assumes that the impact of the GR term on the main roots can be neglected so that the radial shift could be thought to be negligible.

Definitely, it is not true for strong field conditions. We doubt, however, that this assumption is valid even in the case of weak field.

It is worth noticing that the GR-term causing the effect is of the second order of smallness in (1) (at least, under weak-field conditions) while the magnitude of the effect is of the order of potential energy. This occurrence of apparently high enhancement of the GR effect due a small perturbation of the equation should be paid attention. A physical picture of the GR effect origination and comparative treatment is discussed in the following.

3.2 Solution critique

3.2.1 Impact of the GR term on the solution in “perturbation scheme”

Examining an extremum of the effective potential V_{eff}^2 (with the angular momentum denoted l_0^2)

$$V_{eff}^2 = \epsilon^2 - \beta_r^2 = 1 - 2r_g/r + l_0^2/r^2 - 2l_0^2 r_g/r^3 \quad (31)$$

one can find the approximate relationship between classical and the analogous GR roots \tilde{r}_0 and r_0 , respectively [1]

$$r_0 = \tilde{r}_0 - 3r_g/r_0 \quad (32)$$

when $r_0 \gg r_3$ (or equivalently $x_0 = 1/r_0 \ll x_3$).

The above radial difference makes a direct impact of the GR term on the perturbative solution. Indeed, the GR case differs from the classical one in that a small term αx^3 is added to the main terms in (1). As a result, the orbit is slightly shrinks so that the corresponding difference in circumferences would be $\Delta s = 3\pi\alpha$. Notice that the relative effect of the radial shift $\Delta s/r_0$ is exactly similar to the GR perihelion advance effect. From the above analysis, it follows that the GR perihelion advance is directly related to the radial shift. *It is just believed* that the extreme smallness of the effect in the Mercury case justifies the assumption, but a real significance of assumed “smallness” of the shift (32) in the solution must be carefully investigated.

As noted, the effective potential is used to redefine the total energy, which is not a rigorous concept since potential and kinetic energies themselves are not strictly defined in GR. Besides, its “quadratic form” is suitable only for weak field conditions.

The effective potential is actually used in many GR applications, in particular, for a derivation of the equation (1) from the Schwarzschild metric and in connection with studies of the concept of total energy of a particle in the Schwarzschild field. The redefined total energy matching (1) supposed to be equivalent to (4) is given by

$$\epsilon^2 = 1 - 2r_g/r + \beta_r^2 + l_0^2/r^2 - 2l_0^2 r_g/r^3 \quad (33)$$

where the angular momentum has the classical form $l_0 = r\beta_\theta$. This corresponds to the GR definition of V_{eff}^2 .

Here and further, dimensionless expressions are mostly used (all terms are divided by mc^2). Thus, radial and angular velocity components are, correspondingly, β_r^2 and β_θ^2 , and squared kinetic energy in ϵ_k^2 (a sum or squared kinetic energies of radial and orbital motions) is $\epsilon_k^2 = \beta_r^2 + \beta_\theta^2$. The term “squared” in this case relates to doubled quantities with respect to corresponding classical notions. Later, expressions for total and kinetic energies are specified in terms of two independent physical parameters; they must be given in the initial conditions governing the solution of (1).

3.2.2 The objection to the “perturbation scheme”

We state that the assumption of a radial shift neglect in the problem is wrong. Indeed, when the actual arc shortage caused by the radial shift (32) in the first (presumably, “classical”) integral, is accurately taken into account, one calculates the angle for half a period

$$\frac{1}{2}\theta_0 = \int_{\tilde{r}_1}^{\tilde{r}_2 - \Delta r} I_0(x) dx = \pi - \frac{3}{2}\pi\alpha \quad (34)$$

where $x(r) = 1/r$. The second integral (the perturbation term) is not sensitive to the radius alternation and gives a result

$$\frac{1}{2}\Delta\theta = \frac{3}{2}\pi\alpha \quad (35)$$

This makes totally a half-period angle π , hence, 2π per one revolution with no angular shift.

Contrarily to Einstein’s scheme, one could suggest that the integral (34) had to be considered in the GR framework (rather than “classically”). But how one could know?

In our view, there is a logical breach in the perturbation scheme because a physical criterion of comparison is not strictly formulated consistently with initial conditions. Even more importantly, the perturbation approach based on a comparison of GR and Newtonian physics is meaningless due to radical difference of two theories. In this connection, Synge noted [16] (p. 297): “The mixture of the theory of Newton and Einstein is intellectually repellent since the two theories are based on such a different foundational concepts”. Indeed, to do the prediction, one needs to gain an insight into physical nature of the GR perihelion advance effect.

To evaluate the effect purely within the GR framework, it is important to unambiguously set “the near circle” conditions (in a circle limit) and conduct supporting numerical computations, to find the rigorous numerical solution anyway. Upon fulfilling this task, one can compare the results with classical ones for curiosity rather than as a “perturbation procedure” necessity. Namely such a methodology is realized in this work.

So we conclude that the conventional methodology of the GR perihelion advance evaluation based on a perturbation of the Newtonian equation (due to a small GR term in (1) as a source of perturbation) is wrong. This conclusion is a new methodological result of our study. To evaluate the GR effects, one needs to solve (1) strictly in the GR framework with the consistent initial conditions in physics terms bearing in mind that the classical concept of elliptic orbits, most likely, is not valid in any approximation.

Further, we shall see from the exact root analysis and the corresponding numerical computations that, indeed, the GR orbits are principally not elliptic, and the GR effect appears in the solution to (1) because the GR term leads to a non-classical expression of the virial theorem (or, equivalently, “the area law”) even under the weak-field conditions. The neglect of radial shift immediately makes the expression being a classical one with the GR effect lost.

In this connection, it is interesting to recollect the Landau’s remarks on the rotation of plane of classical elliptic orbit being not closed when the corresponding constraints are imposed on the classical solution by a means of a certain deviation of the potential function from the $1/r$ law, see an example in Fig. 9 in the book [9]. Obviously, such “deviations” must be “non-classical” ones leading to the corresponding deviations from a classical form of the virial theorem. Having not understood this issue, one can be confused about the nature of the GR effect compared to the classical Kepler’s motion. Landau’s finding agrees with the statement of a famous astronomer

Clemence [24] about the GR effect to be observed as “the relativistic (non-classical) rotation of orbital plane in the direction of planetary motion”.

3.3 Critical view of contemporary status of the problem

Our criticism is directed to numerous credited literature devoted to the problem [1] since the majority of contemporary works is to the great extent an ideological reiteration of the original Einstein’s work.

As emphasized, there are two topics (among others) we specially focus attention on:

a) Einstein’s evaluation of the GR perihelion advance effect (discussed above);

b) a conventional derivation of an approximate solution to (1) in terms of precessing ellipse.

Typically, integrals of expressions in (25) are presented in a general form as functions of elliptic parameters e and p related to A , B , and α in (1), [2], [7], and elsewhere. The purpose is to find an approximate solution describing a periodic not closed orbit.

For a nearly circular motion, a solution is approximated by the precessing elliptic orbit with a leading term of precession $2\pi(3r_g/r_0)$ per revolution. To describe the precession analytically, the precession parameter ν is introduced into a classical solution. The latter is taken in the traditional (geometric parameter) form

$$r(\theta) \approx \frac{p}{(1 + e \cos \nu\theta)}, \quad x(\theta) = (1/p)(1 + e \cos \nu\theta) \quad (36)$$

where $x = 1/r$, p is the semi-latus rectum, e is the eccentricity. A graphical illustration of (36) is presented in [1]).

The quantity ν is a new (third) parameter $\nu \approx (1 - 3r_g/r_0)$, which should be interpreted as a factor of slowing down of the rotational motion comparing with the analogous classical motion. The angular advance is counted, of course, in the positive direction of rotation. By assuming that the angular speed slows down by the factor of $\nu < 1$, one has to think that, to complete a revolution, a planet needs to rotate through the additional angle $2\pi(3r_g/r_0)$ to allow the perihelion to advance. Otherwise, for $\nu > 1$, it will be a retardation. In this way, the parameter ν is introduced in the GR equation (1)

for variable change $\theta \Rightarrow \nu\theta$:

$$\left(\frac{1}{\nu^2}\right)\left(\frac{dx}{d\theta}\right)^2 = -\frac{(1-e^2)}{p^2} + \frac{2x}{p} - x^2 + \boxed{2r_g x^3} \quad (37)$$

and so in (36). All commonly accepted derivations of the GR prediction are intended to describe the GR ν concept of orbital plane precession. This is a mathematical form of the quoted above Clemence's notice [24] that the effect should be observable in the inertial coordinate system as a non-classical rotation of orbital plane in the direction of planet motion.

The scientific logic tells us that the expression (37) appears to the scene as a trial of the GR ν concept. The straightforward way of its verification is a substitution of (37) into (36). The logic was broken when the statement was made without checking that the expressions (37), (36) follow as approximations to (1) and its solution.

The inherent inconsistency of the GR "orbit precession solution" is seen when (36) is substituted into (37): one immediately finds that the solution satisfies the equation if and only if the GR term is removed from the equation. Once it is removed, any value of ν does perfectly fit the equation.

In a number of works, instead of the equation (1), its equivalent second-order form was considered

$$du^2/d\theta^2 = 1/p - u + \boxed{3r_g u^2} \quad (38)$$

with the GR term $3r_g u^2$. One can think that, in this case, the issues of "the main root shift" and "the third root exclusion" is avoided, so that one can formulate the perturbation problem in a more straightforward way. But this would be not correct.

In Bergmann's work [10] (1942), it is suggested that the Fourier's expansion of the solution to (38) be analyzed in terms of a small parameter $\lambda = 3r_g/r_0$:

$$u = a_0 + a_1 \cos(\zeta\theta) + a_2 \cos(2\zeta\theta) + \dots \quad (39)$$

where $\zeta = \sqrt{1 - 2\lambda a^2}$, ζ , a^2 , a_0 , $a_1 \dots$ are to be found. Again, as in Einstein's approach, it is suggested to start with the classical solution as the first approximation

$$u = a(1 - e \cos \theta) \quad (40)$$

After making several (arguable and hardly checkable) approximations, the author finally comes to (36) in full agreement with Einstein's result [2]. One can recheck that the solution is inconsistent with (38) as much as with (1).

Many works in literature exhibit a great similarity with [10]. Strangely enough, references to Einstein [2], and Bergmann [10] are scarcely made in later literature.

A particular (seemingly independent) derivation of the GR effect is given in the known textbook [25], Rashevsky, (1954). The author tries the solution $x = x_0 + x_1$, where $x_0(\theta) = 1/p + (e/p) \cos \theta$ is the classical one with $x_1(\theta)$, a small (perturbative) correction

$$x_1(\theta) = \lambda(e/p^2)\theta \sin \theta \quad (41)$$

Then

$$x = 1/p + (e/p)[\cos \theta + (\lambda/p)\theta \sin \theta] \approx 1/p + (e/p) \cos \nu\theta \quad (42)$$

with $\nu = (1 - \lambda/p)$. There the approximate substitution is made: $\cos \theta - \Delta\theta \sin \theta \approx \cos(\theta + \Delta\theta)$, $\Delta\theta = (\lambda/p)\theta$. Notice, this approximation is not valid: $\Delta\theta$ is not a small quantity since θ indefinitely grows. As noted, a substitution of (42) into (38) shows, as before, inherent inconsistency of the trial. Among other works presenting a version of classical equation perturbation as in [25].

“The small oscillation” approach for nearly a circular orbit was suggested in [13] (see also in [19]) that fully agrees with the ν -concept but appears methodologically original. It is assumed that there must be a difference between orbital and radial frequencies $\Delta\omega = (\omega_\theta - \omega_r)$ due to the GR term in comparison with the classical orbit. From arguable approximations it is shown that the orbital frequency exceeds the radial one just right to explain the effect. However, it is in contradiction with the ν -concept of angular speed slowing down; besides, “the radial harmonic oscillation” does not follow directly from the equation of motion.

Chandrasekhar [23] uses the equation (1) as it is (with the τ) without comments. The work is strictly devoted to the Black Hole model so that Einstein’s problem of the GR planetary shift with the ν concept is not discussed at all, that is why we only mention it. In brief, the author suggests an approximate solution reduced to the Jacobian elliptic integral. Though he makes it differently from [5]), the idea is the same: a parameterization in terms of two independent geometrical parameters and the additional physical one (to be criticized further). Because of special approximations, the results can be obtained only numerically even under weak-field conditions. Clearly, the computed results are not equivalent to the exact numerical solution to (1), and it is hard to compare them with our results; yet, they are not necessarily consistent with the ν concept.

Further, to explain in details why and how the ν concept is wrong (inconsistent with (1)), we conduct a rigorous methodological analysis of the problem and perform the exact numerical solution of (1).

4 Methodological analysis of the GR problem

4.1 The σ -classification of orbits in the classical equation of motion

We start with the *classical* motion to see how it differs from the GR case purely in physical parameters. Let us introduce a new quantity σ characterizing a classification of elliptic orbits, a physical parameter, $\sigma = r_g/r_0\beta_0^2$, which absorbs the initial condition data $r = r_0, \beta_r = 0, \beta_\theta = \beta_0, \theta = \theta_0 = 0$. Further, it will be convenient to apply the concept of σ to the GR problem though GR orbits are not supposed to be elliptic.

The initial angle is fixed $\theta_0 = 0$ that leaves 3 physical parameters (r_g, r_0, β_0) to characterize orbit families with *absolute* orbit sizes (in *meters*, for example). One can find connections between geometrical (elliptic) and physical parameters. We shall see that the classical equation of motion and its unique solution in the dimensionless form need the only one, necessary and sufficient, parameter σ . The orbit type is determined by a value of σ -parameter, as follows, [1].

The σ -classification of classical orbits

(see illustration in Fig. 1)

$0 < \sigma < 0.5,$	hyperbola;
$\sigma = 0.5,$	parabola;
$0.5 < \sigma < 1,$	overcircle ellipse;
$\sigma = 1,$	circle;
$1 < \sigma < \infty.$	subcircle ellipse.

The above (physical) classification of a family of orbits is advantageously different from the conventional (geometrical) one. A family of orbits with a variable eccentricity e and a fixed semi-latus rectum p are not suitable for our analysis. The parameter σ imposes a physically consistent constraint on a classical family of orbits. As a result, a remarkable σ -gauge symmetry of particle dynamics in a spherical symmetric gravitational field takes place:

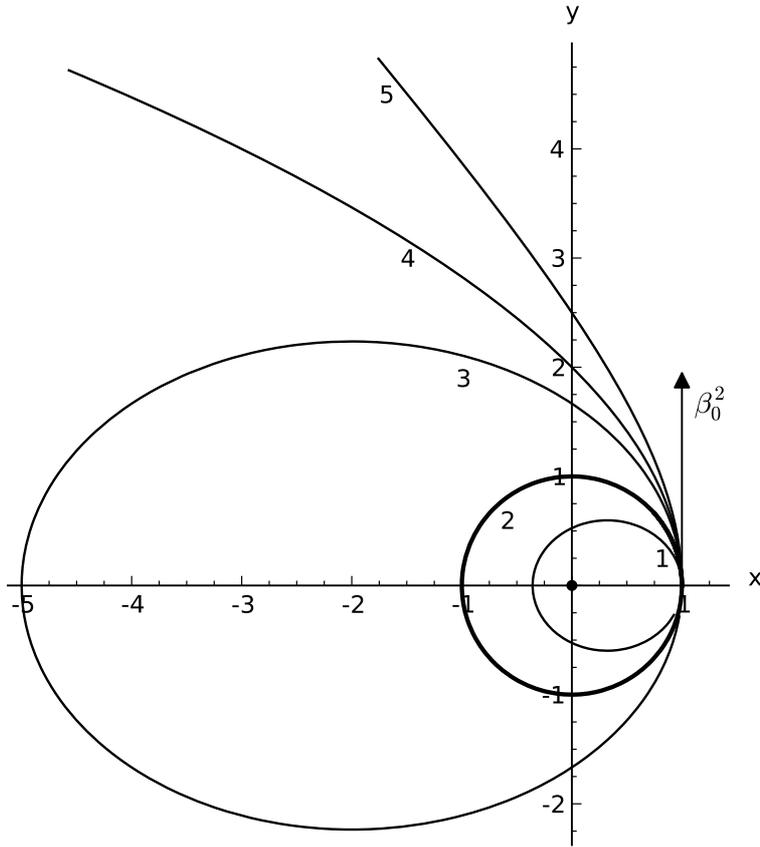


Figure 1: Classical orbits plotted in (x, y) scale, $r = \sqrt{x^2 + y^2}$. Illustration of the σ family of orbits: 1. Sub-circle ellipse, $\sigma=1.9$; 2. Circle (thick line), $\sigma=1$; 3. Over-circle ellipse, $\sigma=0.6$; 4. Parabola, $\sigma=0.5$; Hyperbola, $\sigma=0.4$. The gravity center is placed at the coordinate origin, which is at rest with respect to the far-away stars (an inertial coordinate system). All orbits are produced by launching a test particle at the point $x_0 = 1$ with the initial speed $\beta_0 = \sqrt{r_g/\sigma}$ (the arrow shows the geometry). The meaning of terms “sub-circle” and “over-circle” is seen from the picture, see the text.

any change of initial data (r_g, r_0, β_0 , or equivalently r_g, r_0, l_0^2) preserving σ does not change the character of the particle motion.

Further, we denote $\xi = r_0 x = r_0/r$ where r_0 is not necessary a radius of circular motion. It is defined as a fixed radius in the initial conditions. The conserved momentum is $l_0 = r_0 \beta_0 = r \beta_\theta$. The total (squared) energy ϵ^2 is specified consistently with the effective potential. The classical equation describing a motion in terms of σ -parameter is given by

$$(d\xi/d\theta)^2 = (1 - 2\sigma) + 2\sigma\xi - \xi^2 \quad (43)$$

with the solution

$$\xi(\theta) = r_0/r(\theta) = \sigma + (1 - \sigma) \cos \theta \quad (44)$$

or equivalently

$$r(\theta)/r_0 = [\sigma + (1 - \sigma) \cos \theta]^{-1} \quad (45)$$

and the roots

$$\xi_1 = 1 \quad (\text{always}), \text{ and } x_2 = (2\sigma - 1) \quad (46)$$

The equality $\sigma = 1$ is the exact expression of the classical virial theorem $r_g/r_0 = \beta_0^2$ in the case of a circular motion:

$$r_g/r_0 = \beta_0^2 \quad (47)$$

The theorem can be extended to a non-circular motion.

Thus, given “the launching geometry”, the classical orbit classification, the solution, and the trajectory of motion are uniquely defined by one physical parameter σ .

4.2 GR equation: root analysis and orbit classification

The equation (1) is equivalently presented in terms of physical parameters related to the initial conditions. Comparisons with classical motion can be made just for curiosity. The GR equation and its solution are obtained strictly within the GR framework; no argumentations based on “perturbational comparison scheme” are needed.

Recall our denotation $\xi = r_0 x = r_0/r$ where r_0 is not necessary a radius of circular motion. It is defined as a radius of “particle launching”, in accordance with the following initial conditions (as in the classical case):

$$\beta_\theta(\theta_0) = \beta_0; r(\theta_0) = r_0; \xi(\theta_0) = 1; \theta_0 = 0 \quad (48)$$

It tells that a particle is “launched” at the point $x = r_0, y = 0$ in the counter-clockwise direction perpendicularly to the axis x with the speed β_0 ; the radial speed at this point is $\beta_r = 0$.

Given the geometry, a minimal set of two physical parameters could be the set of two: $\sigma = r_g/r_0\beta_0^2, \rho_0 = r_g/r_0$, or equivalently $\rho_0 = r_g/r_0, \beta_0^2$.

With the above, the GR equation of motion is given in a dimensionless form by

$$(d\xi/d\theta)^2 = [1 - 2\sigma - 2(r_g/r_0)] + 2\sigma\xi - \xi^2 + \boxed{2(r_g/r_0)\xi^3} \quad (49)$$

or equivalently

$$(d\xi/d\theta)^2 = [1 - 2(\rho_0/\beta_0^2) - 2\rho_0] + 2(\rho_0/\beta_0^2)\xi - \xi^2 + \boxed{2\rho_0\xi^3} \quad (50)$$

The parameter ρ_0 characterizes the field strength at the launching point $r = r_0$ while β_0^2 the initial energy of orbital motion; the initial kinetic energy of radial motion is zero. The conserved (squared) quantities, – total energy and angular momentum are

$$\epsilon_0^2 = 1 - 2\rho_0 + \beta_0^2 - 2\rho_0\beta_0^2 = 1 - 2\rho_0\xi + \beta_0^2\xi^2 - 2(\rho_0\beta_0^2)\xi^3 + \beta_r^2(\xi) \quad (51)$$

$$l_0^2 = r_0^2\beta_0^2 = \left(\frac{r_0}{\xi}\right)^2 \beta_\theta^2(\xi) \quad (52)$$

The radial kinetic energy term is

$$\beta_r^2(\xi) = \epsilon_0^2 - [1 - 2\rho_0\xi + \beta_0^2\xi^2 - 2(\rho_0\beta_0^2)\xi^3] \quad (53)$$

The angular kinetic energy term is

$$\beta_\theta^2(\xi) = \beta_0^2\xi^2 \quad (54)$$

The energy balance is completed with the definition of the total kinetic energy $\beta^2(\xi)$ and the effective potential (squared) energy $V^2(\xi)$

$$\beta^2(\xi) = \beta_r^2(\xi) + \beta_\theta^2, \quad V^2(\xi) = \epsilon_0^2 - \beta^2(\xi) \quad (55)$$

Recall, the equation for $(d\xi/d\theta)^2$ is derived from the expression for $\beta_r^2(\xi)$ where $\beta_r = dr/d\tau = (dr/d\theta)(d\theta/d\tau)$ with $\beta_\theta = r(d\theta/d\tau)$ $l_0 = r\beta_\theta = r_0\beta_0$, so that it occurs that $(d\xi/d\theta)^2 = \beta_r^2(\xi)/\beta_0^2$.

To find the exact roots, we use the following Vieta's relationships:

$$\begin{aligned}\xi_1 + \xi_2 + \xi_3 &= 1/2\rho_0, \\ \xi_1\xi_2 + \xi_1\xi_3 + \xi_2\xi_3 &= 1/\beta_0^2, \\ \xi_1\xi_2\xi_3 &= 1/\beta_0^2 - 1/2\rho_0 + 1\end{aligned}$$

or, bearing in mind that $\xi_1 = 1$ is fixed in the initial conditions

$$\begin{aligned}1 + \xi_2 + \xi_3 &= 1/2\rho_0 \\ \xi_2 + \xi_3 + \xi_2\xi_3 &= 1/\beta_0^2 \\ \xi_2\xi_3 &= (1/\beta_0^2 - 1/2\rho_0 + 1)\end{aligned}$$

In the above three relationships, any two of them are independent (third one is redundant).

Thus we complete the links in the chain of relationships: the equation coefficients, the initial conditions, the roots. This allows us to specify root ranges for different types of orbits and classification in terms of

Root ranges in the GR orbits of various types

$0 < \xi_2 < \xi_3$ the GR analog to the classical bounded motion, namely:

$\xi_2 = 1$ makes a GR circle;

$0 < \xi_2 < 1$ the GR analog to the classical overcircle ellipse;

$1 < \xi_2 < \xi_{ed}$ the GR analog to the classical subcircle ellipse;

$\xi_2 = \xi_3 = \xi_{ed}$ "the edge point" (defined below);

$\xi_2 > \xi_{ed}$ the GR spiral fall onto a black hole.

$\xi_2 = 0$ the GR analog to the classical parabolic (unbounded) motion;

$\xi_2 < 0$ the GR analog to the classical hyperbolic (unbounded) motion;

Further, 2-parameter families with parameters σ and ρ_0 (or β_0^2) is considered. One should clearly understand that the GR bounded motion orbits are not elliptic, and the GR circular orbit is physically different from the corresponding classical one. Our purpose is to reveal the corresponding difference in physics.

From the root analysis for the GR circular motion, a formula follows, which is crucially important for the problem study

$$\rho_0/\beta_0^2 = (1 - 3\rho_0) \quad \text{or} \quad \rho_0/(1 - 3\rho_0)\beta_0^2 = 1 \quad (56)$$

Clearly, the classical circularity criterion $\sigma = \rho_0/\beta_0^2 = 1$ must be replaced (at least, under weak-field conditions $\rho_0 \ll 1$) with the corresponding GR criterion

$$\sigma_{gr} = \rho_0/(1 - 3\rho_0)\beta_0^2 = 1 \quad (\text{circle}) \quad (57)$$

This is consistent with the relativistic radial shift $\Delta r = 3r_g$ due to the GR term (32 under conditions “similar” to the case of the classical circular motion (similarity is meant that appears from the consideration of the effective GR potential, as discussed earlier).

A general classification of GR the two-parameter orbits is obtained in analogy to the classical one-parameter equation. The parameter σ_{gr} (instead of σ) is used in the GR orbit classification for convenience though the parameter σ is the one, which can be present in the GR equation of motion. We shall see that the GR bounded orbits cannot be considered elliptic even in a weak field.

The σ_{gr} classification of GR orbits.

$0 < \sigma_{gr} < 0.5 + 2\rho_0$ the analog to a hyperbolic (unbounded) motion;

$\sigma_{gr} = 0.5 + 2\rho_0$ the analog to a parabolic (unstable) motion;

Further are orbits in a bounded motion

$0.5 + 2\rho_0 < \sigma_{gr} < 1$ the analog to the classical overcircle ellipse;

$\sigma_{gr} = 1$ makes a circle;

$1 < \sigma_{gr}$ a classical analog to the subcircle ellipse with a transition to the spiral fall.

Define “the edge point” $\xi_{ed} = \xi_2 = \xi_3$. This is an unstable GR orbit between sub-circle and “over-the-edge” motions, the second one is a spiral fall onto the black hole. By definition, the edge point is determined from the following constraint on ρ_0 and β_0^2 from Vieta’s relationships:

$$1 + 2\xi_{ed} = 1/2\rho_0; \quad 2\xi_{ed} + \xi_{ed}^2 = 1/\beta_{ed}^2 \quad (58)$$

The illustration of edge point formation in the root analysis is given in Fig. 2 where the following algebraic (root) equation for $f(\xi) = (d\xi/d\theta)^2$ is plotted for $\rho_0 = 0.050$

$$f(\xi) = [1 - 2(\rho_0/\beta_0^2) - 2\rho_0] + 2(\rho_0/\beta_0^2)\xi - \xi^2 + 2\rho_0\xi^3 \quad (59)$$

The case 2 (an unstable orbit) shows the edge point $\xi_2 = \xi_3 = \xi_{ed} = 4.500$ occurred at $\beta_0^2 = 0.0341880$. The curve 3 presents the sub-circle orbit with $\beta_0^2 = 0.038$, and the curve 1 with $\beta_0^2 = 0.300$ (the roots ξ_2 and ξ_3 are complex numbers) is the over-edge orbit, a spiral fall.

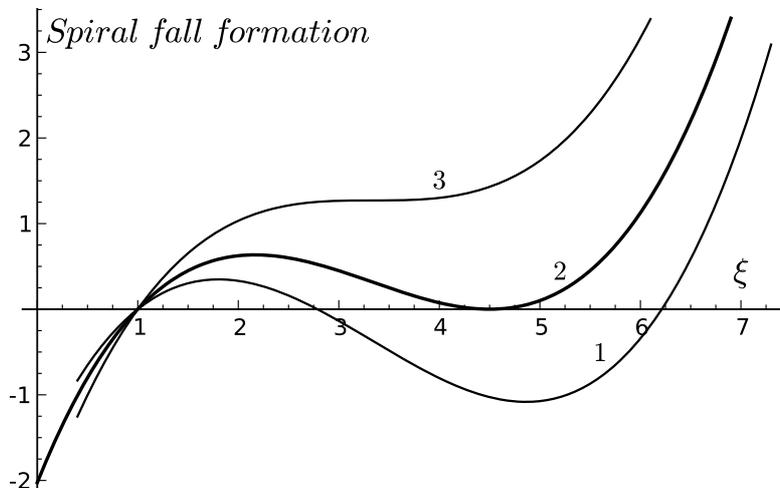


Figure 2: Plotted $f(\xi)$ (see text) for $\rho_0 = 0.050$. Case 2 shows the edge point $\xi_{ed} = 4.5$, $\beta_{ed}^2 = 0.0341880$; the case 1 is a sub-circle orbit, $\beta_0^2 = 0.038$; case 3 an over-edge orbit, $\beta_0^2 = 0.030$. This illustrates the spiral fall that begins with $\beta_0^2 < \beta_{ed}^2$, when the roots x_2 and x_2 become complex (case 3).

Thus, given “the launching geometry”, the GR orbit classification and the trajectory of motion are uniquely defined by two physical parameters (for example, the σ and the ρ_0), while in the classical case, the only parameter σ is necessary and sufficient. From the exact numerical solution, we shall see that the conventional approach of evaluation of the GR effect with the corresponding ν concept of GR orbital rotation based on the method of “perturbation of classical equation” is not needed for solving the problem. Actually, the approach and the results from it cannot be physically substantiated.

5 Results of GR exact numerical calculations

5.1 The law of $\Delta\theta = 3\sigma_{gr}\rho_0$

The major goal of the analysis is to find the GR angular shift per rad $\Delta\theta$ under the weak field conditions $\rho_0 \ll 1$ and compare it with the the exact numerical results. The following formula confirmed by the exact numerical

calculations expresses the $\Delta\theta$ -law having a simple form

$$\Delta\theta = 3\sigma_{gr}\rho_0 \quad (60)$$

where $\Delta\theta$ is a relative angular shift per radian averaged over a period. The shift is doubly proportional: linear with ρ_0 for σ_{gr} being fixed, and linear with σ_{gr} for ρ_0 being a fixed parameter (plotted). Further, this quantity is treated as a relative precessional advance, in %, that is, a relative deviation from π , over half a period interval of integration (unless specified).

All quantitative results are obtained by a high-precision numerical integration of the GR 2-parameter equation of motion (50) in the exact form with the initial conditions discussed. It should be noted that there is no calculational error in our data since we use the method of numerical integration of (50) with the precision in as many digital numbers as needed. In particular, calculations are made for initial conditions in the range of $\rho_0 = [0.100 - 0.001]$ and also 10^{-N} with $N = 4$ up to $N = 10$ when the results reproduce a linear function with the effect strictly proportionally to ρ_0 ; hence, results for any $\rho_0 < 0.001$ are obtained by a mere renormalization of that for $\rho_0 = 0.001$ when one can practically take the limit in the criterion of orbital type classification $\sigma_{gr} = \sigma$. The power of law (60) is its simple analytical form practically exact for any bounded motion under weak-field conditions $\rho_0 \leq 0.001$.

The law (60) is illustrated in Fig. 3. The calculated (plotted) data are positive angular deviations from π (in %). The integration is made in the direction of motion (counter-clock chosen) between two roots $\xi_1 = 1$, ξ_2 .

There are two plots: the line for $\rho_0 = 0.001$ (a weak field), and the curve for $\rho_0 = 0.05$ (a mildly strong field). The line demonstrates the law (60) in the whole range of bounded motion. At $\rho_0 = 0.001$, the angular (relative) shift is 0.300 % at $\sigma_{gr} = 1$.

As for $\rho_0 = 0.050$, the effect must be, at least, 50 times greater but it is even greater than that, and not linear with σ_{gr} because of the effect of a strong field. For convenience of comparison, the normalization coefficient is selected $k = 1$ for $\rho_0 = 0.01$, so that $k = 10$ for $\rho_0 = 0.001$, and $k = 1/5$ for $\rho_0 = 0.050$. It is seen that for $\rho_0 = 0.050$ the law is not valid in the region of the sub-circle motion $\sigma_{gr} > 1$ and partly in the over-circle region $\sigma_{gr} > 0.7$. The deviation progresses while roots ξ_2 and ξ_3 approaching each other to meet at the edge point.

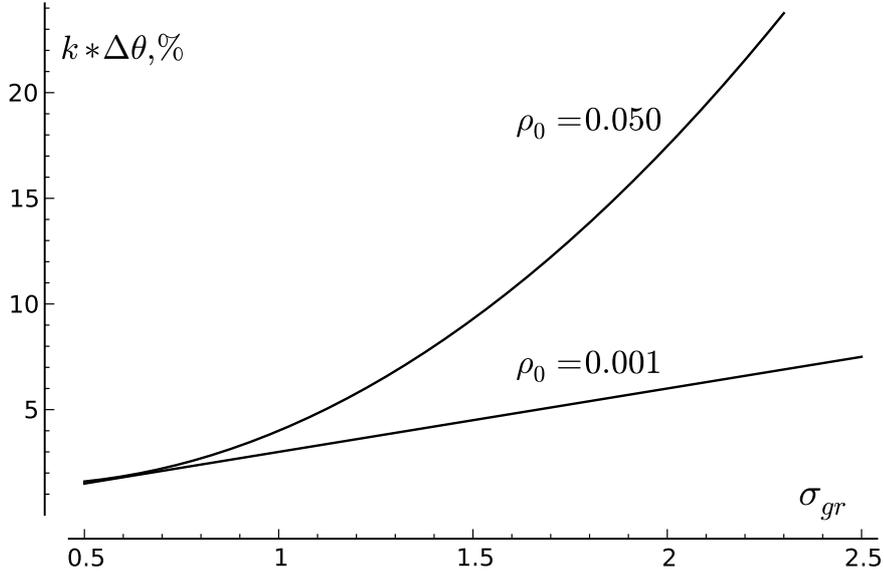


Figure 3: Dependence $\Delta\theta = 3\rho_0\sigma_{gr}$ of relative precessional advance on σ_{rg} as ρ_0 varies (exact numerical integration). Two curves are normalized to the same vertical plotting scale. Normalization coefficient $k = 10$ for $\rho_0 = 0.001$ (weak-field), and $k = 1/5$ for $\rho_0 = 0.050$ (mildly strong field) so that all lines for $\rho < 0.001$ with proportionally however smaller effect coincide with the line for $\rho_0 = 0.001$. Deviated curves appear as the field strength rises with $\rho > 0.001$

5.2 The left-right asymmetry of GR orbits

A variation of the arc stretching with angle causes a non-elliptic form of precessing GR orbits: a classical ellipse loses its left-right elliptic symmetry. Consequently, the classical (geometrical) concepts of eccentricity and semi-latus rectum become invalid.

GR orbits are illustrated in Fig. 4: there are two numerically computed orbits of each kind. The above mentioned left-right asymmetry means non-uniformity of the angular advance. It cannot be seen in values integrated between two roots, that is over half a period. Therefore, we divided half a period into two “quarters” with the intermediate point ξ' , the one of maximal radial energy. In classical orbits, this is a distance from the minor axis center

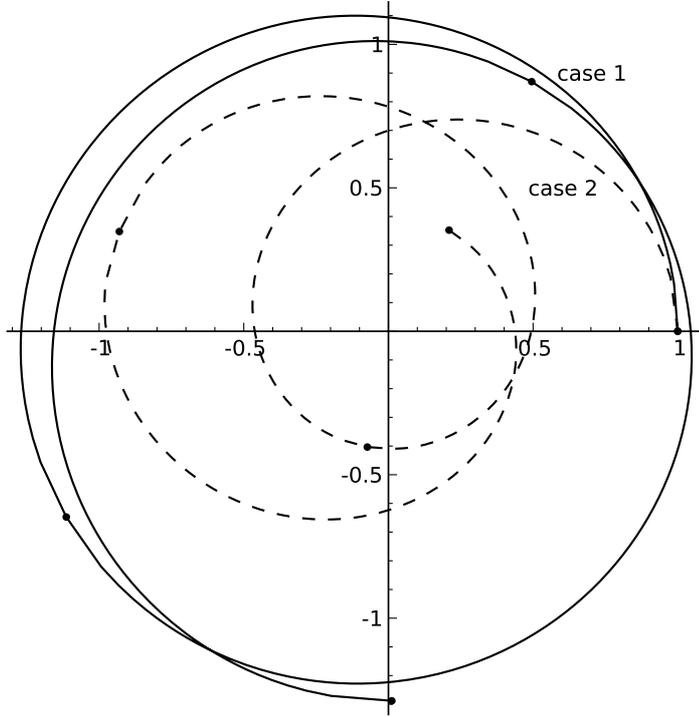


Figure 4: Results of numerical integration. Shown two GR orbits for 3 half periods (here in counter-clock rotation) for $(r_g/r_0) = 0.05$. Case 1: Over-circle, $\sigma = 0.769$, $\sigma_{gr} = 0.809$, $\beta_0^2 = 0.065$, relative advance 16.8 %. Case 2: Sub-circle, $\sigma = 1.250$, $\sigma_{gr} = 1.316$, $\beta_0^2 = 0.040$, relative advance 44.3 %.

to the line of ellipse, the point of classical elliptic left-right symmetry. The point in a GR orbit ξ' is its analog but located asymmetrically.

The results of corresponding calculations for the plotted over-circle and sub-circle orbits are presented in Table 1. It illustrates the law “the closer to the center, the greater the advance” in its local action. Indeed, if a particle is launched into the over-circle (moving farther from the center), the second quarter advance is less than that in the first quarter, and reversely in the sub-circle launch (moving closer to the center). Notice, the parts in radians and in their percentages are proportional to each other and additive. It means that the law (60) is a differential one that is, it is valid for local additive increments, in accordance with the GR angular scaling $d\theta(x) = (\partial x/\partial\theta)d\theta$ from (23).

Table 1: **Non-uniformity of GR angular advance**

Angular advance additives	Over-circle	Sub-circle
Half a period, $[\theta - \pi]$, <i>rad</i>	3.67	4.53
Half a period, $[\theta/\pi - 1]$, %	16.8	44.3
First quarter, $[\theta_1 - \pi/2]$, <i>rad</i>	1.85	2.05
First quarter, $[\theta_1/(\pi/2) - 1]$, %	17.9	30.8
Second quarter, $[\theta_2 - \pi/2]$, <i>rad</i>	1.81	2.48
Second quarter, $[\theta_2/(\pi/2) - 1]$, %	15.6	57.8

5.3 Comments on the law of $\Delta\theta = 3\sigma_{gr}\rho_0$

The fact of the non-uniform GR precession (differentiality of the law of $\Delta\theta(r)/\Delta r = 3\sigma_{gr}(r_g/r)$) and the corresponding left-right asymmetry means that a GR orbit is not an ellipse anymore, even under weak-field conditions due to a physical nature of the GR perihelion advance. It tells us that the concept of ν rotation of elliptic orbital plane is not valid.

From the root analysis, for a circular GR orbit the non-classical relationship was found $(r_g/r_0) = \beta_0^2(1 - 3r_g/r_0)$ (56), and the circularity criterion $\sigma_{gr} = 1$ (57) formulated, which for a weak field $(r_g/r_0) \ll 1$ tends to coincide with the corresponding classical criterion $\sigma = 1$. Therefore, the correction of the σ criterion does not affect the value of the GR advance under weak-field conditions (as shown in Fig. 3).

From the expression (56) considered the form of GR analog of classical virial theorem in the range of σ for a GR bounded motion, also, bearing in mind $\beta_0 = r_0\omega$ ($\omega = 2\pi/T$ – an angular orbital frequency, T – a period), we recognize $r_g = r_0^3\omega^2(1 - 3r_g/r_0)$, the GR form of Kepler’s 2d law (“the area law”). All things considered, we state that the GR perihelion advance comes to the scene as a manifestation of the GR version of the virial theorem, or equivalently, the area law both are affected by the GR radial shift. An instantaneous angular advance depends on a local field strength, so that the advance rate is not uniform except for the GR circular motion. Now it is clear that the radial shift, even however small in the GR virial theorem, is the cause of the effect; neglecting the radial shift would mean destroying the effect. The GR effect is of the order of field strength r_g/r_0 , and it exists as far as the field strength is not zero exactly.

The exact solution predicts the GR perihelion advance rate different from the known “approximate solution” within the concept of uniform GR rotation

of orbital plane (30), (36). Geometrical concepts of eccentricity and semi-latus rectum are shown not valid. There are no analogies of over-circle and sub-circle orbits in the GR “approximate solutions”. That is why, (36) is not consistent with the equation of motion (1), and (30) is not comparable with the law (60). However, at the limit of circular motion under weak-field conditions, the predictions coincide.

5.4 GR particle motion in a strong field

5.4.1 Motion with a zero angular momentum

It should be emphasized that the equation (1) is designed for a GR orbital motion with a non-zero angular momentum, and it cannot be reduced to the limit of a pure radial motion $\beta_0 = 0$.

In literature, a radial motion equation in the Schwarzschild field is derived independently of the equation (1). It is claimed that a solution could be interpreted in both the coordinate (“far-away”) system with the time t and a local (“shell”) rest system with the local time referred to the time “on shell”, t_{sh} . The factor $dt/d\tau$ in the expression (4) is treated as the Lorentz factor $\gamma_0 = 1/(1 - \beta_{r0}^2)$ where β_{r0}^2 is a squared magnitude of radial velocity at infinity; it is directed to the center while the angular component β_0^2 is zero. We know that the introduction of Lorentz factors is not “legitimate” in the GR methodology, especially in view of the mentioned statement that the GR theory is incompatible with SR, [11], *and elsewhere*.

It follows that in the far-away observer’s coordinate system, a particle approaching the center eventually stops at the horizon surface $r = 2r_g$ without crossing it but it takes a however long time. Specifically, the speed of a particle in free radial fall is given by [19], [26]

$$dr/c_0dt = \beta(r) = (1 - 2r_g/r)[1 - (1 - 2r_g/r)/\gamma_0^2]^{1/2} \quad (61)$$

where $\gamma_0 \geq 1$ is initial kinetic energy at infinity.

The formula shows that the particle sent from infinity to the gravitational center begins to accelerate, then at some point $R(\gamma_0)$ starts decelerating and eventually must come to rest at $r = 2r_g$ (the Schwarzschild radius). The higher initial kinetic energy, the farther the deceleration point R from the center. For $\gamma_0 \geq \sqrt{3/2}$, the particle would never accelerate in a gravitational field, $R \rightarrow \infty$. The gravitational force exerted on the particle becomes

repulsive in the entire space. It should be noted that the time of approaching the Schwarzschild surface indefinitely increases.

In the local coordinate system, the following connections with “shell observer” quantities are suggested $dt_{sh} = dt((1 - 2r_g/r)^{1/2})$. Consequently,

$$dr_{sh}/c_0 dt_{sh} = \beta_{sh}(r) = \sqrt{2r_g/r} \quad (62)$$

It is seen now that the particle accelerates and, as $r \rightarrow 2r_g$ (the Schwarzschild radius), the speed approaches the value of the speed of light in vacuum (at infinity), $\beta(r) = 1$ while the energy indefinitely increases [19]:

$$E_{sh} = 1/(\sqrt{2r_g/r}) \quad (63)$$

It is thought that the particle crosses the Schwarzschild surface (“the horizon”) $r = 2r_g$ at the speed of light (in vacuum) $\beta = 1$, and in the interior region moves faster than light.

It should be emphasized that the conclusion is made that the particle in a radial motion, if observed by a local rest observer in a vicinity of the Schwarzschild surface, accelerates and crosses the surface with the speed of light $\beta = 1$. Then, it keeps flying with the increasing speed always being greater than the speed of light. At the same time, it is predicted that the particle, if observed by the far-away observer, stops at the horizon point (therefore, cannot cross the horizon).

As for motion with a non-zero momentum, the situation is not that certain. From case-dependent argumentations, they believe that, outside the Schwarzschild sphere, an observable particle always moves with the speed $\beta < 1$. The non-observable prediction is that the particle crosses the surface at the speed of light $\beta = 1$, and keeps moving inside at the increasing speed greater than the speed of light $\beta > 1$. The prediction cannot be verified/falsified observationally but it can theoretically, as shown in this work.

We argue that the fact of crossing the given surface must be an event having an absolute logical meaning “yes” or “no” regardless of the observer’s viewpoint. There are opposite answers: “yes” from the local observer, and “no” from the the far-away observer. The matter is that the predictions follow from suggested formulas rather than solutions of the equations of motion in different coordinate systems with the rules of GR system transformations. Whether such transformations are derivable in GR would be another question.

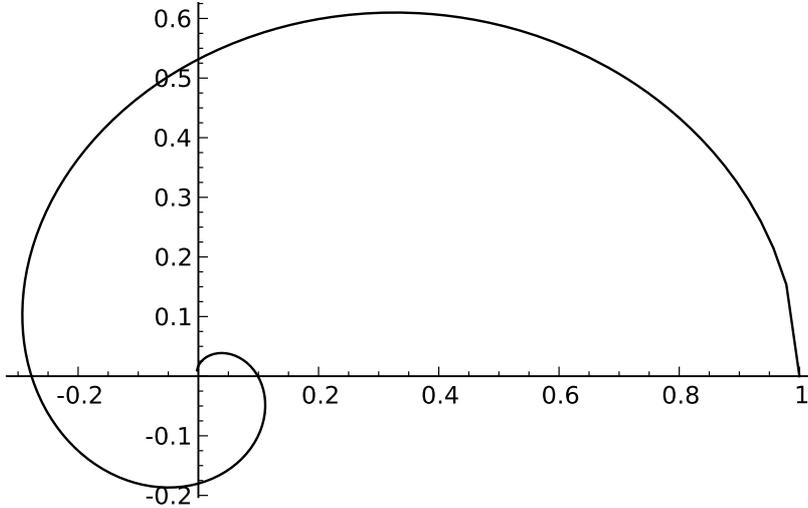


Figure 5: Spiral fall onto the center: crossing the Schwarzschild surface ($\xi = 10$, $r = 0.10$, $r_0 = 1$) in an over-edge orbit, $\rho_0 = 0.0500$, $\beta_0^2 = 0.300$, the case 3 in Fig. 2. The trajectory shown in the scale (x, y) , $r = \sqrt{x^2 + y^2}$ ended up at $r = 1/5 r_g$ ($r_g = 0.05$).

5.4.2 Spiral fall and particle speed at the horizon

Let us apply the GR orbital equation (1) expressed in terms of two independent parameters ρ_0 and β_0 (50) to the “over-edge” motion, which is associated with the spiraling fall onto a Black Hole, see Fig. 5 and Fig 6 with comments there. The results are obtained in exact numerical calculations. A part of the picture shown in Fig. 5 is enlarged in Fig 6 to show proximity of a spiral eternal fall onto the center (the particle can never hit the center). Calculation of the trajectory is ended at $r = r_g/15$ ($r_g = 0.05$).

One would like to know from our results how fast a particle moves in a spiral fall in the exterior and interior regions that is, before and after crossing the Schwarzschild surface (the so-called horizon). This question never arose in literature because of common belief that a particle in spiral fall always crosses the horizon at the speed of light. However, such a belief does not have a rigorous calculational proof, it rather comes from arguments based on suggested formulas with no association with the equation (1); that is why we want to check it.

In our analysis, the following relationships and definitions determine prop-

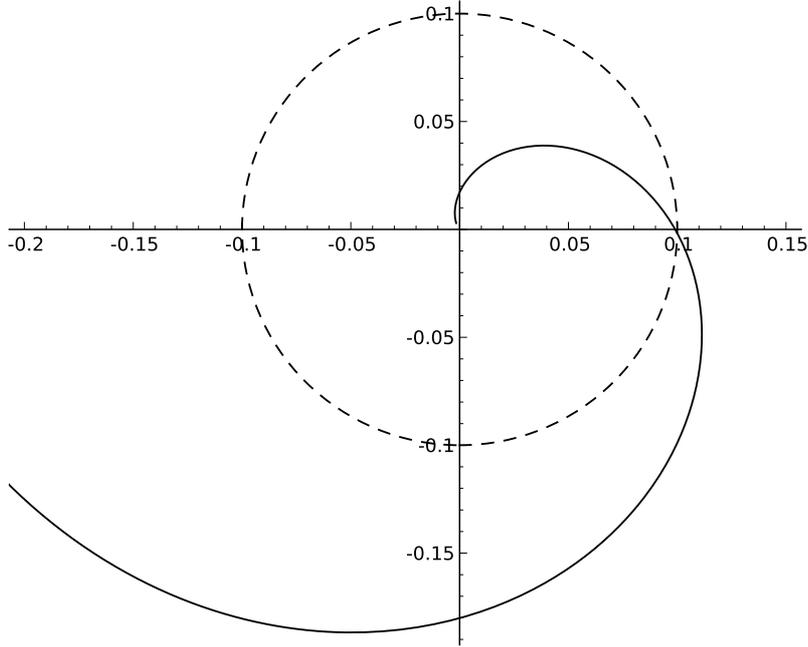


Figure 6: Spiral fall onto the center: crossing the Schwarzschild surface. The picture in Fig. 5 is enlarged to show proximity of a spiral sharp-dive onto the point center $r = 0$. The trajectory shown from some point before crossing of the Schwarzschild surface (dashed line). A particle crosses the Schwarzschild surface at the speed $\beta_{sch} = 1.982$, and, as shown in the picture, ended up deep inside the Schwarzschild sphere, at $r = r_g/15$ ($\xi = 300$), having a speed $\beta(1/300) = 285$. The spiraling particle keeps accelerating and asymptotically approaches the center but never “touches” the point.

erties of (1), and they are further used: (51) (conserved total energy), (52) (conserved angular momentum), (31), (effective potential), (53) (radial component of velocity), (54) (angular component), (55) (total value). For convenience, formulas are compiled below, as follows.

$$\epsilon_0^2 = 1 - 2\rho_0 + \beta_0^2 - 2\rho_0\beta_0^2 = 1 - 2\rho_0\xi + \beta_0^2\xi^2 - 2(\rho_0\beta_0^2)\xi^3 + \beta_r^2(\xi) \quad (64)$$

$$l_0^2 = r_0^2\beta_0^2 = \left(\frac{r_0}{\xi}\right)^2 \beta_\theta^2(\xi) \quad (65)$$

$$\beta_r^2(\xi) = \epsilon_0^2 - [1 - 2\rho_0\xi + \beta_0^2\xi^2 - 2(\rho_0\beta_0^2)\xi^3] \quad (66)$$

$$\beta_\theta^2(\xi) = \beta_0^2\xi^2 \quad (67)$$

The energy balance is completed with the definition of the total kinetic energy

$$\beta^2(\xi) = \beta_r^2(\xi) + \beta_\theta^2 \quad (68)$$

and the effective potential (squared) energy $V^2(\xi)$

$$V^2(\xi) = \epsilon_0^2 - \beta^2(\xi) \quad (69)$$

The equation (1) is derived from

$$\beta_r^2 = \epsilon_0^2 - V_{eff}^2 \quad (70)$$

We are interested in numerical values of velocity components of particle crossing the horizon $\xi_{sch} = 1/2r_g = 1/2\rho_0$, also in exterior and interior regions. On the horizon sphere, the GR effective potential always takes a zero value, $V_{eff}^2 = 0$ (straightforward check); consequently, the squared radial speed is equal to the total (squared) energy, $\beta_r^2 = \epsilon_0^2$ always taking values less than unit, $\beta_{sch}^2 = \epsilon_0^3 < 1$ in the whole range of interior region (including the Schwarzschild surface).

The orbital component could be any, depending on β_0^2 . Indeed, $l_0 = r_0\beta_0 = r\beta_\theta$, ($r = 1/\xi$) or

$$\beta_\theta(\xi) = \beta_0\xi, \quad \beta_\theta^2(\xi) = \beta_0^2\xi^2 \quad (71)$$

At a however small β_0^2 , a however small (squared) addition to the radial speed makes the resultant speed less than the speed of light, $\beta \leq 1$, or, at some greater value, makes $\beta \geq 1$ (as in the example of Fig 6).

To sum up, Fig. 8 and Fig. 7 demonstrate GR predictions of particle motion in the interior (and while crossing the horizon) with speed less and greater than the speed of light. In the example of field strength $rho_0 = 0.050$, we have the following speed (squared) values at the horizon $\xi_{sch} = 10$:

Subluminal case, $\beta_0^2 = 0.0001$, Fig. 8.

$$\beta^2 = 0.910, \beta_r^2 = 0.900, \beta_\theta^2 = 0.010$$

Superluminal case, $\beta_0^2 = 0.0080$, Fig. 7.

$$\beta^2 = 1.707, \beta_r^2 = 0.907, \beta_\theta^2 = 0.080$$

For a greater value of $\beta_0^2 = 0.030$ (still less than the edge value $\beta_0^2 = 0.03419$), it would be $\beta^2 = 3.927, \beta_r^2 = 0.927, \beta_\theta^2 = 0.300$

It is easy to find that the particle crosses the horizon at the speed of light for $\rho_0 = 0.005$, $\beta_0^2 = 0.0010$:

$$\beta^2 = 1,000, \beta_r^2 = 0.900, \beta_\theta^2 = 0.100$$

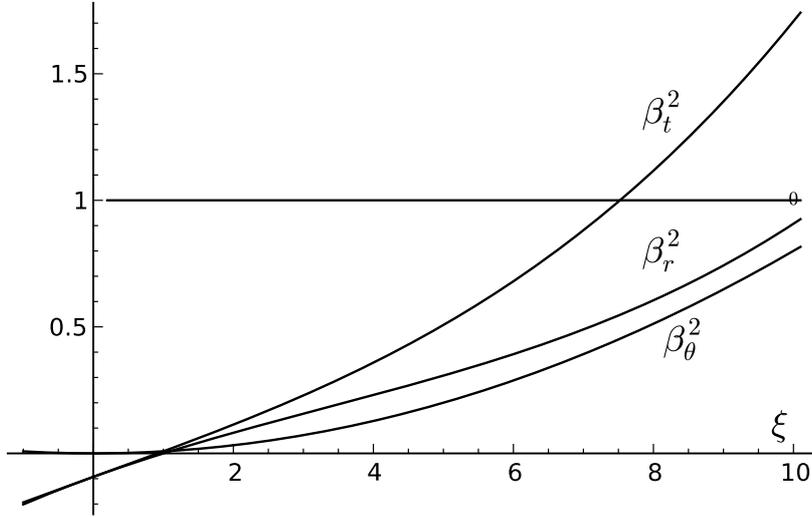


Figure 7: Example of over-edge motion: a spiral fall onto the center; $\rho_0 = 0.050$, $\beta_0^2 = 0.008$. Shown squared relative velocities of the test particle $\beta_r^2(\xi)$ (radial) (53), $\beta_\theta^2(\xi)$ (angular) (54), $\beta^2(\xi)$ (total, their sum); a conserved (squared) total energy $\epsilon_0^2 = 0.907$ (that is, a bounded motion). The particle crosses the Schwarzschild surface $\xi_{sch} = 10$ ($r_{sch} = 0.10$) at the resultant speed $\beta = 1.304$ (faster than light) with the kinetic energies $\beta_r^2 = 0.907$. The angular component of speed is $\beta_\theta^2 = 0.800$ (in this example, it is less than the speed of light), and the resultant one $\beta^2 = 1.707$ (faster than light). The particle reaches the resultant speed equal to the speed of light $\beta = 1$ at the radial point $\xi = 7.54$ ($r = 0.133$), that is outside the interior region.

5.5 Concluding comments

Part One is devoted to the formulation of problem strictly within the GR framework. The latter is defined in accordance with the GR methodology based on the Schwarzschild metric and agrees with the equivalence of τ and t (as commonly accepted but criticized here). Critical analysis of the GR methodology, conventional interpretation of the equation (1) as well as its typical approximations is presented. The exact numerical solution of (1) and the corresponding GR predictions are obtained with no special assumptions beyond the GR framework. Our results are compared with the current conventional ones from approximate models and/or under assumptions known in the GR main-stream literature. Our results are claimed technically cor-

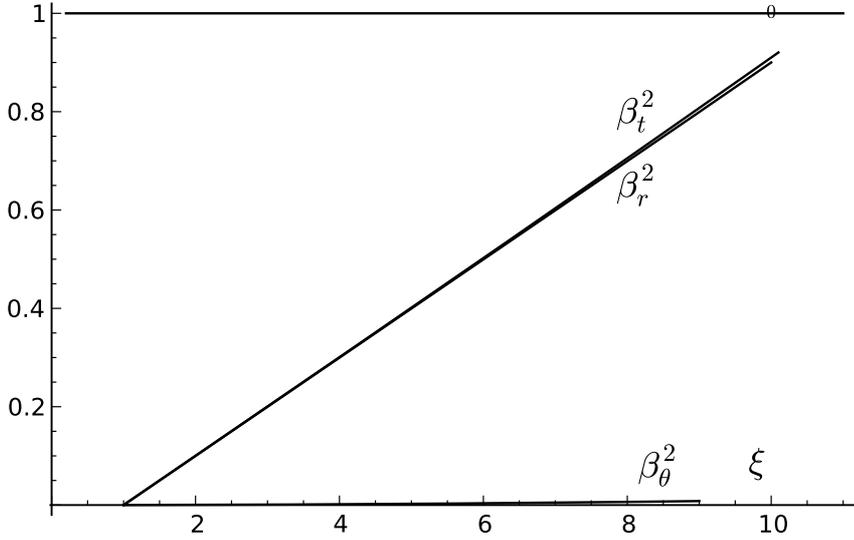


Figure 8: The case of subluminal motion: spiral fall onto the center; $\rho_0 = 0.050$, $\beta_0^2 = 0.0001$. Shown squared relative velocity components $\beta_r^2(\xi)$ (radial), $\beta_\theta^2(\xi)$ (angular) (54), $\beta_t^2(\xi)$ (total, their sum); a conserved (squared) total energy $\epsilon_0^2 = 0.900$. The particle crosses the horizon at the resultant speed $\beta_t = 0.910$ (less than light) with velocity (squared) components $\beta_r^2 = 0.900$, $\beta_\theta^2 = 0.010$.

rect and reproducible by anyone willing to check; they are in disagreement in major points with the conventional ones commonly believed to be true.

Among the GR international society and communities, the belief has gradually built that, as a result of decades of Schwarzschild field studies, a physical problem formulation of (1) is well-posed and cannot be doubted, therefore, the main outcome, – GR predictions of perihelion advance under weak-field conditions (in Astronomy) and BH Physics for a strong field (in Astrophysics and Cosmology), is scientifically grounded and fully understood. However, disputes over GR issues often of secondary significance continue (not to speak about suggestions to modify or refute GR in favor of arguable alternatives).

We challenge this belief and state that the conventional interpretation of GR particle motion in the gravitational field is based on not proven approximations and assumptions rather than the exact solution based on an unambiguous formulation of the problem in relationship with physical initial

conditions.

The criticism concerns, firstly, the controversy about proper versus improper time variables in the equation (1): its solution is supposed to describe astronomical and astrophysical observations in terms of coordinate time t but the equation is expressed in terms of proper time τ , as it is in the Schwarzschild metric. We state that to replace τ with t , even by virtue of Newtonian limit, would be fundamentally wrong. There are no options but stay with (1) as it is.

We follow the GR literature and pretend that there is no difference between the proper and improper time in (1) to be solved to make observational predictions. With this, conventional GR physics is expected to emerge, say, under weak-field conditions, – the GR perihelion advance in the form of orbit plane precessional rotation, or contrariwise, under strong conditions – the Black Hole phenomenon. As the result of exact analytical and numerical solutions, those pictures are proved contradictory and, in major parts, physically and technically flawed. This is the second aspect of our criticism of the GR validity.

Many Physics Frontiers problems are originated from or related, in some or another way, to the GR theory. For this reason, Part Two of the work is presented: it is devoted to study of those types of interfacing problems. The drawn conclusions largely reflect the author's personal experience and point of view, which one is free to competently disagree with or scientifically argue over.

Part II

Links to Physics Frontiers

6 GR Black Hole concept

The Black Hole concept in Astrophysics allegedly is based on the GR theory, but it is much larger concept than the one following from GR. The GR Black Hole concept arises from the fact of the so-called removable divergence in the GR metric form (7) at the Schwarzschild radius $r_{sch} = r_g$ (the horizon sphere), and the central divergence $r \rightarrow 0$ (the matter collapsing point). It should be noted that the original Schwarzschild metric [4] does not have the central divergence, therefore, the GR BH concept as the prediction of the GR theory is questionable. As emphasized further, Black Holes in Astrophysics is a result of arbitrarily made additions to GR of many physical and technical features important for comparison with observations, however, having nothing to do with GR and its framework. The Astrophysical BH concept is more ambiguous due to usage of other disciplines, hypotheses, and mere assumptions.

The academic GR Black Hole picture suggests “new physics”. In the local observer frame, the speed of particle falling onto a Black Hole reaches the speed of light at the Schwarzschild radius, and, in the interior region, the motion becomes superluminal, regardless of initial conditions. It cannot escape back to the exterior region in no circumstances (“the information loss”).

There are numerous claims that “the BH is observed”, though, always indirectly. There must be observable super-dense compact objects (besides neutron stars), which look like “Black Hole” but, again, have nothing to do with the above GR Black Hole concept. In the history of Physics, there are numerous examples of “observations” and “confirmations” of either new phenomena, – a discovery, eventually happened to be fictitious, or it could be known theoretical prediction occurred to be a misconception.

Therefore, we would like to comment on the following (actually, interconnected) GR academic issues concerning the Black Hole concept:

- a) particle motion faster than light ;
- b) no signal return, information loss .

a) Particle motion faster than light.

GR literature teaches that a particle moves at the speed greater than the speed of light only after crossing the Schwarzschild sphere (in the interior region): $\beta = 1$ at $r = r_{sch}$, and $\beta > 1$ for $r < r_{sch}$, while in the exterior region $r > r_{sch}$, $\beta < 1$ always. The particle crosses “the horizon barrier” and becomes superluminal without any traveler’s notice of it until “a final crunch” at the center occurs. An exterior rest observer can see the particle moving in a vicinity of the horizon but never crossing it. The particle motion at the speed of light and faster is, of course, in conflict with the Special Relativity theory, and it shakes General Physics Foundations. To “justify” the GR BH concept, one has to appeal to “new unknown physics”, actually, with no physical idea of it.

From the results of exact solution of the Schwarzschild problem, the following conclusions are drawn. In the non-zero momentum motion in the Schwarzschild field, the resultant speed on the GR horizon sphere could be any (greater or smaller than, or equal to the speed of light). This result undermines the “standard” GR Black Hole picture, which actually follows from a consideration of a “pure” radial motion. No solid proof is given for conditions of motion in a spiral fall. Therefore, to discuss the problem on the basis of constructive scientific logic is hardly possible.

b) Matter trap with no return.

They say, once the horizon crossed, the metric signature changed (from time-like to space-like), and, in connection with this, it is speculated that there could be no motion other than towards the center where the particle (the photon too) would be eventually absorbed by the central (singularity) point, as a manifestation of “gravitational collapse”. Imaginary material “shells” could not be constructed (allegedly, due to “tremendous” forces). As a result, any matter that has crossed the horizon could not return to the outer world, and no information would be retreated about actual events happened to the particle. This phenomenon (including suspiciously concrete technical details) is called “the information loss”, the one of BH phenomena [19], *and elsewhere*; it must be of a new physics category and definitely beyond the described above GR framework for the following reason.

“The no-return” BH concept is inherently contradictory; it does not follow from the equation (1), which must be and actually is used in the BH concept as the exact equation. The statement of impossibility of return back to the outer world contradicts to the time reversibility of the GR equation of

motion (1). This property of the equation cannot be excluded from the GR framework. The point is that the equation provides a natural way of setting an initial velocity at any point of the trajectory equally in “outgoing” and “ingoing” directions. In theory and practical calculations, the particle can be reflected back at each point to the passed part of the trajectory to reach the initial point and go beyond in opposite direction. The “non-return” argumentation might come from some technical considerations, which are not related to the GR theory and must not override the equation time-symmetrical solution. Otherwise, one must change the GR framework, first of all, modify the equation (1) with additional conditions and/or change its solution by imposing special (technical) constraints.

7 Revision of conventional concept of proper mass constancy: Why

7.1 Natural elimination of central divergence

In GR and conventional SR Dynamics, including relativistic and quantum relativistic electrodynamics, the proper mass is considered constant. Further we discuss consequences of relativistic mass revision for the spherical symmetric gravitational field. Arguments for the revision of proper mass constancy is given in [27], see also [28].

The revision leads to deviation of the potential from the law $1/r$ at a small distance, namely, the new law

$$V(r) = \pm m_0 c_0^2 \left[1 - \exp\left(\pm \frac{r_g}{r}\right) \right] \quad (72)$$

where, generally, “plus” stands for a repulsion, “minus” for an attraction. It is seen that the divergence at $r \rightarrow 0$ with $r_g/r \rightarrow 0$ (high momentum harmonics) turns out to be eliminated; this would be true for both the Schwarzschild field and the Coulomb (attractive) fields.

The Schwarzschild field divergence is not renormalizable, while in the relativistic electromagnetic field theory and QED, fields are divergent but normalizable. Over years, “the normalization” as an artificial mathematical procedure became a normal part of the theory, however, great physicists never agreed with it and suggested to keep searching for a natural non-divergent theory. Richard Feynman constantly drew physicists’ attention

to this problem, even in his popular book (“for house-wives”) [29] about “beauty of the theory” felt obligated to talk about its ugly feature. Paul Dirac categorically did not accept the renormalization and hoped for “future theory”. He made the point clear, [30], also [31] saying:

“Sensible mathematics involves neglecting a quantity when it is small - not neglecting it just because it is infinitely great and you do not want it!” For more, see [32], *and elsewhere*).

The potential (72) tells about a binding effect due to “the mass defect” $\Delta m = m_0 - m(r)$ in the r_g/r potential field

$$m(r) = m_0 \exp(-r_g/r) \quad (73)$$

This is a natural way to develop a free-divergence theory starting with relativistic dynamics and extending the methodology to quantum field theory, see more about the theory and tests, [33].

The expression (73) gives a clue for relativistic generalization of the gravitational force

$$F(r) = -\frac{r_g}{r^2} \exp(-r_g/r) \quad (74)$$

Assuming the volume of the source being however small (“BH condition”), we have a principally new behavior of the force at small distances. It rises when the test particle is moved away from the source till the point $r_m = r_g$ of maximal value and after that it approaches the classical form $1/r^2$.

We do not see any “magic” in the fact that the currently used “normalization” procedure practically works since the results can be not much sensitive to “the cut-off” distance as far as it is chosen at some point $0 < r < 2r_e$. The problem with that is that the procedure is inconsistent with the theory, not unique, and physically has no sense.

A similar picture takes place for the Coulomb potential, where the gravitational radius r_g is replaced with the electron radius $r_e = kq^2/m_0c_0^2$, q is a charge of the electron and m_0 a proper mass at infinity. So, the central divergence in Quantum Electrodynamics must be eliminated with the relativistically consistent introduction of field-dependent proper mass. This would be a reformulation of the whole theory.

7.2 4-phase dynamic invariance

The field-dependent proper mass leads to invariance of the 4-phase (14) since the scalar product of $\Delta x_\mu P^\mu$ is ended up with $\Delta\tau \cdot m$. Indeed, the

4-momentum space P^μ and the 4-wave vector $(1/h)P^\mu$ can be equally represented by the proportional 4-frequency vector $fU^\mu \propto mU^\mu$ bearing in mind the Einstein-de Broglie relationship in the de Broglie wave concept. For the spherical symmetric field, it is

$$mc_0^2 = hf_0, \quad m(r)c_0^2 = hf(r) \quad f_0 = 1/\Delta t_0 \quad (75)$$

Here, $f_0 = 1/\Delta t_0$ is the proper frequency of particle oscillation inversely proportional to the proper period Δt_0 (“zero” subscripts for infinity, h – Planck’s constant). This leads to the invariant

$$\Delta x_\mu P^\mu = \Delta t_0 \cdot m_0 = h \quad (76)$$

with $c_0 = 1$ and $m_0 = 1$ for simplicity.

The problem of 4-wave phase goes back to the time of de Broglie’s hypothesis on the particle-wave duality. His discovery reveals an inherent connection of QM with the Special Relativity theory through the particle/wave duality. He considered the problem from the relativistic point of view [34] in both particle and wave terms equivalently starting with the relationship between the wavelength and the momentum

$$\lambda = h/p \quad (77)$$

in connection with the corresponding Einstein-Planck fundamental expression (75).

Considering the Lorentz transformation formulas $\Delta t = \gamma\Delta t_0$ and $m = \gamma m_0$, one can notice that the period and the mass grows with the speed, while the mass is proportional to the frequency, and this is a contradiction. At the same time, the wave-particle duality is consistent with Bohr’s condition of angular momentum quantization, if the wave is in a resonance with the phase speed c/β . So, de Broglie tried to find quantum-mechanical relations describing the wave motion with a constant phase (even considered a possibility of Non-Linear Quantum Mechanics for that purpose [35]).

This issue, however, has been given a compromising explanation with a varying wave phase in accord with the Bohr’s model. The Schroedinger’s equation has a solution in its simplest form of dispersion relation

$$\psi = A \exp i(kx - \omega t) \quad (78)$$

which gives the phase and group velocities $v_p = \omega/k$, $v_g = \Delta k/\Delta \omega$. The dispersion relation in its general form of dispersing fields gradually became a special branch of QM, [36], *and elsewhere*.

Concerning de Broglie's question, let us consider again the de Broglie equations in approximations of a small speed and proper mass constancy

$$\lambda_p = h/mv, \quad m_0 c_0^2 = hf_p \quad (79)$$

Here we put the label p for *particle*. The second formula in (79) describes the frequency f_p of a particle oscillation with the following emission of a photon with the same frequency $f_l = f_p$ (the label l for *light*). As for the wavelength being different, from (79), it follows

$$\beta = v/c_0 = \lambda_l/\lambda_p \quad (80)$$

Therefore, the time interval $\Delta t_p = 1/f_p$ subjected to the Lorentz transformation is placed in the first equation while the second one containing the time interval $\Delta t_l = 1/f_l$ relates to the light emitted from the particle.

8 GR Extensions and Applications

8.1 Author's remarks on philosophical distinguishing between true, false, and beliefs in Modern Physics

Physics, as a branch of natural sciences, has a clear *observational* criteria of true versus false, – a compelling branch for modern philosophy. Objects and concepts of Modern Physics could be too elusive and require reconstruction from highly circumstantial, often hypothetical evidences while the cost of break-through observations tends to rise to the scale affordable rather for big international alliances. Besides statistical uncertainties, there is typically a major (systematic) component of error – from a model, its interpretation, and methodological preference. This creates a freedom of imagination beyond reality, hence, arbitrariness in assessments.

Einstein's SR and GR theories are historically burdened with long standing foundational controversies. Some of them are not finally resolved or, at least, commonly agreed on in their formulations. So it is even in the SR theory, – the one, which undoubtedly has become a part of Foundations of Physical Science. Contradictions and controversies make the GR theory arguable in different aspects (though often arguments come wrongly or speculatively in favor of alternatives). Sadly, some obvious controversies are just swept under rug or their existence is plainly forgotten or denied.

In our view, criticism of GR is justified since the theory in many parts contains doubtful (verifiable, in principle) assumptions and approximations. The GR theory is estranged from Quantum mechanics and, strangely enough, the SR theory; though, there are some connections with these and other physical disciplines in GR applications.

The question arises how, on the border of unknown, new genuinely true knowledge comes out from GR philosophy and its speculative extensions into Physics Frontiers and how to scientifically sort out true from false, – the eternal epistemological question, which becomes especially essential in Philosophy of Modern Physics, first of all, relativism and quantization of matter and fields in space-time.

The problem of Quantum Gravity is the one of hot places where Physics meets Philosophy [37], so far, with no progress. Though GR is believed to be non-renormalizable, this is not justification for GR to be non-quantizable.

The issues of indeterminism and acausality in relativistic and quantum worlds (“the problem of measurement”, in particular), continue to be both physical and philosophical problem of fundamental importance, [38] [39], *and elsewhere*). The problem of apparent inconsistency of QM and SR was put forward in the above de Broglie’s works, but it actually turned out to be the inquiry about physical reality and its reflection by the wave function and the corresponding Heisenberg’s uncertainty principle; the latter is firmly considered the inherent property of nature rather than the outcome of model dependent interpretation.

In parallel to the disputes about QM, there were (and continue to be) numerous attempts “to refute” the Special Relativity Theory upon controversial treatments of many “SR paradoxes” (again, “the problem of measurement”). The “critical arguments” in this case, in our view, are mostly based on misunderstanding of the essence of proper/improper quantities, [40], [41]. Contemporary philosophical issues including relativism and quantization, are subject to continued exploration [42], [43], *and elsewhere*).

From the above, it is seen that the methodological and philosophical problem of quest for *true knowledge* should not be underestimated or simplified. The process of interpretation of observations could be vulgarized and beliefs and myths originate, if stakes are high. At the same time, Epistemology, as one of scientific branches, is subjected to variation of concepts and continuing disputes. The difference between “true in god’s nature” and “true on human’s mind” are diffused and criteria incomplete in a “logical chain” of new knowledge origination depending on branch of science. Obviously, the

concept of “truth” in Mathematics has to be formulated differently from that in Physics (whilst most of Physics Frontiers theories are developed on the basis of sophisticated abstract Mathematics).

In our view, there is yet another important fact in epistemological practice. During several past decades, global commercialization took over a societal life and put it under the risk of deterioration of national services including cultural and educational institutions, finally, academic sciences. Access to scientific information is hardened while storage of knowledge corrupted in circumstances of informational noise explosion and “turf protection” freedom against open-minded researchers. Objective criteria of scientific values, particularly, in fundamental physics, are affected by lack of good will and professional competence in circumstances of irreconcilable conflicts of interests.

In spite of all inevitable factors of “academic stagnation”, there are strong criteria of scientific objective proof of truth against false, evidence against belief. “Scientific logic” (“self-consistency” criterion), and “observational confirmation” (“verification and falsification”) based on the Theory of Statistics, remain ones of powerful and convincing tools of scientific judgment.

8.2 GR field quantization problem

The long standing problem of quantization of GR gravitational field is related to the discussed controversial issues of physical interpretation of the Schwarzschild field and its geodesics. Here, we are going to reflect our opinion on the problem status to stimulate further discussions rather than insist on it.

About 80 years of history of the GR quantization of the gravitational field and the corresponding gravitational radiation is marked with numerous approaches, so far, all failed, [44], *and elsewhere*. In particular, the problem was considered in connection with the concept of gravitational waves. The concept of gravitational waves implies the existence of gravitational radiation energy quantum (spin-2 graviton) and suggests that the gravitational field is quantizable.

In reality, this is a theoretical speculation boiled up with unproven assumptions and approximations, see [7], [18], [45], *and elsewhere*. In particular, “the approximation” of substitution of τ with t is necessarily made [12]; a notion of a particle pushed off geodesic due to a mysterious gravitational self-force is put forward, etc.

The gravitational waves have never been directly detected in spite of long (and very costly) efforts. They are “waves” of metric (“ripples in the curvature of spacetime” from, for example, a binary system). We argue that a varying metric and its “disturbances” cannot be energy carriers, as clearly explained by Synge [16]. Strictly speaking, GR does not predict the gravitational waves carrying energy. A belief in GR wave existence was born from a wish rather than a proof and widely spread in scientific popular press.

Meanwhile, an observation of neutron quantum gravitational states in Earth’s gravitational field (in Airy mode approximation) was attempted with the use of ultra-cold neutrons. We criticized the methodology of the experiment and showed that “the neutron gravitational states” could not be observed there in principle [46].

8.3 GR and Big Bang

Contemporary Cosmology (the Big Bang model) is a unique physical discipline, allegedly, born from GR. The gravitational field quantization does not matter for the model, at least, nowadays. As concerns verification, it is based on observations, which are highly circumstantial for treating the whole Universe from looking at a thinnest space-time slice. So, its concepts are hard to distinguish from “expertise beliefs”, and it is admitted that the model could be overtime replaced with a new one [47]. As of today, the Big Bang (BB) untouchable status cracked down under pressure of new observations, in particular, high-precision Cosmic Background Radiation data and Hubble Ultra Deep Field images.

The BB model radically changed over time. Historically, after Einstein’s theory of a static Universe, the BB *expanding* Universe has come from Friedmann’s work (1922) as a solution to Einstein’s field equation under special, actually, arguable assumptions. The solution does not require the cosmological constant Λ (Einstein’s “blunder” of using it). Lemaitre’s quite different later model (1927) suggests to restore the Λ . The Walker-Robinson metric with the scale factor $a(t)$ comes after that in the model of expanding Universe based on, actually, a classical picture of “expandable rubber ball”. This leads to “particle and event horizons” and “energy disappearance”. In our view, those conundra are result of abuse of the SR law requiring to relativistically, rather than classically, to sum up relative velocities. (Recall, GR is not obliged to abide by SR laws).

The Λ (“Einstein’s blunder”) eventually occurs to be quintessence in the

request for “dark energy” to make *the expansion to accelerate*. Altogether with “inflation energy” and “dark matter”, they are important ingredients of the later model, in which the field equation seems to be of secondary importance.

Among leading cosmologists today, it is hard to find a single one who still “believes” in any of BB versions. Quite many other ideas (no review here) are suggested instead to dispute. In our view, the Big Bang concept is not satisfactory, first of all, for the following principal reasons.

- “The singularity at the beginning” is physical absurdity, and it clearly was such starting with the Big Bang first suggestion and in further modifications.

- Explanation of the expansion along with Hubble’s red shift in light from distant galaxies refers to “new physics”, it is hard to follow.

- Big Bang, its modifications, and some new proposals are claimed to originate in their basis from the General Relativity theory. However, GR fails to rigorously formulate concepts of energy and mass, and so it is in the particular BB case. Another example of GR failure is formulated in the simple question: is a moving body more strongly than at rest attracted by the gravitational source? There is no clear answer to that. The fact of GR fundamental deficiencies is simply ignored in cosmological models inevitably using concepts of mass and gravitational interactions between massive objects.

- “Dark matter” is misconception.

Note. The history of “dark matter” discovery goes back to Fritz Zwicky (1933) and later to Vera Rubin (1992). With the use of Doppler technique, they found that the radial profile of speed in spiral galaxies (the galaxy rotation curve) violates the Second Kepler’s law, which says that the farther a star is from the center, the slower its expected orbital speed. The observed curves show that the motion of stars does not slow down with a distance, the picture could be even opposite. The matter is, however, that Kepler’s laws are valid exclusively for the model of point particle of mass $m \ll M$ where M is mass of a source. The observed rotational curve in a spiral or disc-like galaxy must strongly depend on the radial density of stars $\rho(r)$ and it is explainable by Newton’s gravitational law with the function $\rho(r)$ appropriately adjusted when mutual gravitational interactions of stars accounted for; the stronger the gravitational link of galaxy stars, the faster they move at a distance, what, of course, very different from Keplerian motion. The concept of dark matter is taking into account for gravitational lensing fit. Eventually,

the concept has been promoted exclusively as the one of important ingredients of the latest Big Bang modification regardless of earlier (mistreated) astronomical observations of galaxies and lensing data. We made calculations of the rotational curve accounting for stellar interaction in a disc model of Milky Way and, as expected, confirmed our (obvious) understanding that the Dark Matter is a mere misconception.

- “The dark energy” concept, particularly, in association with Λ , comes from circumstantial observations of supernovae; the observations require to correct the Hubble law to fit the novel BB scenario of accelerated expansion. The interpretation of the expansion and other observations cannot be trusted anymore; all BB features (most of them associated with “new physics”) gone with the Big Bang.

- The BB concept has no power of prediction. Its basic features and scales change all the time with new real observations and, equally, with alleged discoveries.

- Besides the traditional observations of Hubble’s redshift and Cosmic Background Radiation, there are some others, known but not incorporated into suggested cosmological models; particularly, they are *baryon asymmetry* and *cosmic rays*.

Note. In our view, without explanation of baryon asymmetry and cosmic rays any idea of cosmological model of Universe would be incomplete and prone to misconceptions. We also insist that the role of relativistic gravitational attraction between massive matter must be principally important in the Universe model, the concept of mass and physical hypotheses should be clear formulated, “new physics” of any kind and GR use avoided. With this line of thoughts, we suggest a new concept of alternative cosmological theory [48], [49], [50], discussed further.

8.4 The Alternative Cosmology

The new cosmological concept, in sketch, suggests a radically new physical idea free of “new physics”. Let us expand our physical imagination far beyond the deadlock of 10 billion light years and hypothesize the existence of the matter-antimatter symmetric *Grand Universe* (GU) in infinite space and time in steady state, on average, on the following premises.

The GU is an open material system (has no boundaries) comprises an infinite number of *Typical Universes* (TU) of finite size (mass) and lifetime. Each of them is a gravitationally linked system of galaxies (and their clusters)

of stars and matter debris. Total mass of TUs varies in a great range. They are made either of matter or antimatter symmetrically and, typically, have a stable structure different in many respects from our Observable Universe (OU).

The GU undergoes a constant recreation of TUs, which evolve in the space of relativistic *GU Background* (GUB). During an evolution, they have a chance to collide with each other. If one sort of matter meets in a collision of a couple of TUs, the result could be a formation of a joint bigger TU; if matter collides with antimatter (as in the case of OU), the result would be a TU smaller by the amount of annihilated matter with a leftover of one sort of matter.

The GU matter-antimatter sustainability (stationarity) is supposed to take place due to the postulated GU openness that is, absence of constraints due to boundary/initial conditions. A process of TU collisions occur with a certain relative probability depending on a free path, a function of TU and GU stationary characteristics. The annihilated matter is balanced in the process of matter recreation due to matter-antimatter creation from energetic gamma radiation. Bearing in mind that the GU is infinite in space and time, its stationary state has a meaning in a somehow big but finite sample volume V_s , in which a large enough number of TUs are characterized by, let it be called, the *local* stationary mass distribution and the corresponding lifetime distribution. The process of matter interaction on a larger scale is much more slower; it can be thought in the form of oscillations of stationary state and waves of average matter density.

The TU lifetime is defined from a moment of “birth” of a smallest mass, say, a matter “seed” having a chance to grow into a local Typical Universe till its “death” in a collision with a bigger antimatter TU. In order the stationary GU state to be reached, the GUB characterized by cosmic gas and debris must come into a consistent equilibrium state in its content and energy spectra bearing in mind the TU interactions and the corresponding causal physics.

On a scale larger than a certain V_s (say, at a distance much greater than the average TU collision distance), a causality relationship between GU remote parts weakens and even vanishes with ultimate distances. However, it is hypothesized that elementary particles, massless radiation, and their products could reach far more distances. As a consequence of causality suppression, any TU is exposed to random particle and photon radiation originated everywhere and having any values of momentum/energy upon arrival. These “cosmological cosmic rays” (CCR) must have a broad energy spectrum up

to so high energy. The process of local annihilation and matter creation is energetically fed up by CCR from far-away sources.

By definition, CCR spectrum is locally Lorenz invariant and could not be generated by any physical mechanism within the “local sample volume”. There is no sense to think about a physical mechanism of CCR “acceleration”, one would rather put the question about a deficit or cut-off of a high-energy tail in a locally observed “cosmic ray” spectrum. The original Cosmic Rays “sources” must be looked for in spaces far beyond and around the Observable Universe.

Our matter-made *Observable Universe* (OU), in accordance with Alternative Cosmology, is a product of matter-antimatter collision of two TUs having different gravitating (that is, proper) masses. The residual mass makes the OU. We studied and confirmed the above GU picture, the process of matter-antimatter statistical separation, and TU evolution, by Monte-Carlo simulation of a simplified model of a Steady State Grand Universe. The model is based on the Cosmological Principal of quasi-equilibrium for massive and massless matter: the equality of energy density of massive matter/antimatter and massless (that is, not gravitating) radiation for a local TU group. As concerns TU evolution, the principle of proper mass minimization in the relativistic Lagrangian approach (within the concept of field-dependent proper mass) is suggested.

There are numerous observable consequences of the TU collision hypothesis.

1. *Expansion as gravitationally unbounded matter recession.* The amount of residual mass occurred to be not sufficient to keep the system gravitationally bounded. In a process of matter-antimatter annihilation, the system began disintegrate. The matter originally in rotation about the center of mass broke up and went off into open space. Parts closer to the center, moved at higher speed, peripheral parts moved respectively slower. At the same time, the conservation of total angular momentum got broken because of the annihilation process; so, to reconstruct a real picture of disintegration, one needs unknown details of how the two TUs collided. Most likely, each of them or, at least, one had enough time to evolve into a super-cluster of galaxies of strongly linked stars.

Thus, the Observable Universe is a finite matter-made system, the result of matter-antimatter annihilation in the process of two TU collision. The receding galaxies are observable since a long time has passed

after the collision beginning; they manifest “a future death” of Our Universe.

2. *Quasars.* A quasar seems to be a matter-antimatter annihilating mix in a process of the collision. The collision has likely happened and lasted, as is seen from quasars activity, in a period about 2-4 billion years after “the beginning” in the BB scale (statistically, there could be earlier and later observable events). This concept also makes understandable huge “voids” in the Observed Universe as well as quasars morphology and “structures” of billions of light-years across.
3. *Hubble’s law.* We think that the observable cosmological redshift is caused, partly, by the Doppler shift from flying away galaxy matter, partly, by the gravitational time dilation. The role of the latter effect is not clear in the BB model; there are not confirmed reports that the absence of the effect in the cosmological redshift is confirmed in observations treated within the BB concept (contrarily to its essential role in Friedmann’s model).

From the above, the Hubble’s law must be regarded as a rude approximation. In the matter disintegration scheme, the relativistic Doppler shift could, indeed, make a contribution assuming that originally more dense, orbiting faster, and during the collision flying away matter tends to pass more slowly moving, less dense peripheral matter. One must bear in mind that, in the Hubble’s law, the redshift is treated in terms of expansion model factor $a(t)/a_0$ having nothing to do with the real Doppler shift; hence, the Hubble’s distance/time scales are not trustful in the extreme.

Besides the disintegration due to loss of gravity, the annihilation radiation creates a pressure gradient forcing material parts fly away; this component must be evaluated.

4. *Baryon asymmetry, isotropy.* The Observable Universe is matter-made and isotropic in the first approximation. More precisely, deviation from those properties must be observable.
5. *Cosmic radiation from CCR, luminous thermal (hot and cold) radiation.* The observable cosmic (particle and gamma) rays are produced from incoming (not observable) CCR, one of the components

of baryon symmetric GUB, as discussed above. The observable cosmic energy spectra and contents are essentially degraded in a process of travel through the Observable Universe. Luminous matter, like stars, are another sources of light, generally, electromagnetic waves from atomic/molecule matter, neutron stars, and hot super-dense galactic cores. Yet, there must be a thermal radiation from cold (non-luminous) materials of great variety of mass, in BG terms, thermal background (alleged “cosmological relic”) radiation.

6. *Gamma bursts, nova explosions, and related phenomena.* Similarly to quasars, we suggest to consider gamma bursts, nova explosions, and other “strange” observed phenomena in association with the matter-antimatter annihilation (in bulk, individual stars, smaller pieces and their formations) in the process of collision of two TUs. Overall, the collision hypothesis tells us about the presence of observable (but not recognized) antimatter in our “decaying” (rather than “expanding”) Universe.
7. *Presence of antimatter.* One can hypothesize that antimatter remnants are present in the ultra high-energy tail of cosmic rays and among asteroids and meteorites bombarding the Earth (for example, one can hypothesize that the Tunguska meteorite is composed of antimatter; some scientifically documented UFOs contain antimatter and propulse by it in the atmosphere; etc).

9 Unity of Quantization and Relativism

9.1 Controversies about QM interpretation

Recall, GR is claimed to be *naturally* non-quantizable but it is not clear whether this statement means that the gravitational field could not be *naturally* quantized in the non-relativistic Quantum Mechanics (QM), in principle. Still, numerous efforts continue to find the way “to quantize” the field in the GR framework.

In the QM theory, there are its own, methodological problems. The raised questions reflecting the QM interpretation controversies, roughly, are twofold:

- (a). Based upon “paradoxes/contradictions” mostly arising from the QM indeterministic philosophy: is QM a complete, inherently consistent theory

(for example, in the EPR thought experiment)?

(b). Based upon new propositions: can it be modified or, more radically, replaced with some deterministic theory (for example, “hidden parameters” version)?

As known, the criticism of the theory occurred to be not constructive anyway. Both questions, actually, intend to attack “the Copenhagen School”. The latter, in response, propagates the positive answer to a) and the negative reaction to b), roughly, on the following premises.

Indeterministic uncertainties in quantum measurements arise in the limit of however high precision of measuring tools and reflect, as a consequence of particle-wave duality, the fact that an atomic object being in a quantum state cannot be “observed” without a finite disturbance destroying the state. Apparent contradictions arise from the “language” of macroscopic deterministic world of an idealized isolated systems, while the quantum world requires a new language in terms of wave function. A squared function is a probability density distribution describing non-deterministic (probabilistic) nature of atomic systems.

Basically, we agree with the School’s response in a) and b). However, the indeterminism issue is much more broader when the criticism goes to the origin and validity of the Heisenberg’s Uncertainty Principle (HUP), its applications and the interpretation, as well as new methodological and philosophical definitions of microscopic world concepts.

We argue that the QM theory is “self-consistent” as far as it is considered a non-relativistic theory. The disputes over the probabilistic interpretation of wave function do ignore the fact that Schroedinger’s equation incorporates the de Broglie wave concept reflecting a real deeply relativistic phenomenon of particle-wave duality, as previously discussed. The non-deterministic interpretation of wavefunction in the classical physics approximation not necessarily gives an adequate description of quantum objects in reality of the relativistic 4-space.

Apart from the above disputes, “the miracle” of newly born QM theory (starting with Bohr’s model) was its ability to unite waves and particles in the form of equivalent presentations of the phenomenon in Quantum and Wave Mechanics on the following physical (postulated) principles:

– *Particle is an oscillator.* A particle at rest “oscillates”, in accordance with the SR Kinematics relationship between the proper mass m_0 , the proper

frequency f_0 , and Planck's constant h (the speed of light $c_0 = 1$ for simplicity)

$$E_0 = m_0/h = f_0, \quad \text{or} \quad E_0 \Delta t_0 = h \quad (81)$$

and in motion we use the corresponding “improper quantities” (in spite of the fact that there are no such notions in Schroedinger's framework)

$$E = m/h = f, \quad \text{or} \quad E \Delta t = h \quad (82)$$

The second expression in (81) and (82) is actually a scalar product of 4-coordinate (interval) and 4-momentum proper vectors manifesting the 4-phase invariance in motion by inertia.

– *Particle-oscillator in motion becomes a wave.* The particle, correspondingly, exhibits “a wave” of the length λ depending on particle speed β

$$\lambda = h/m_0\beta \quad (83)$$

that is, the smaller the speed, the greater the wavelength. This is a manifestation “Bohr's quantization” of momentum and angular momentum, in particular, when the quantum effect (the particle as a wave) is more pronounced at smaller speed. High energy particle has a wavelength comparable with or however smaller than “a classical size” (the particle as a point particle), all in accordance with the Lorentz transformations.

The use of the above two principles leads to a conflicting situation, for example, the 4-phase invariance occurs to be broken (or not ensured) in the classical form the Schroedinger's equation.

The separate principle tells about energy quantization:

– *Particle bounded states are quantized.* In particular, the Bohr's atomic model and its full QM version describe angular momentum and total energy quantization. Photon energy is also quantizable.

Clearly, the de Broglie wave concept is consistent with SR only “approximately”, in the first order of β ; this means that the Minkowski real world (we live in) is replaced with the “approximate” Newtonian world (with separated time and space variables) in Schroedinger's framework, [51] *and elsewhere*.

Consequently, the HUP derivation and its interpretation depend on the physical model of Schroedinger's equation formulation and solution (the wave function $\psi(x)$). The latter is characterized by an arbitrary phase:

$$\psi(x) \sim e^{i\theta} \psi(x) \quad (84)$$

where x include all coordinates. The phase $\theta(x)$, as a consequence of separation of time and space variables, can vary with x . This affects the HUP derivation.

A “covariance spoiling” is a usual situation with redundant degrees of freedom in field theories, for example, in a transition from classical Electrodynamics to Relativistic Electrodynamics. There, the problem is “fixed” by replacing the Lorentz gauge with the Coulomb one, see details in [52], [53], [54], *and elsewhere*. As previously discussed, the gauge problem in QED is actually “half-healed”: the symmetry between the 4-coordinate interval and momentum spaces (that is, between temporal components, – the proper time interval and the proper mass, correspondingly) remains broken but it can be fully restored with the field-dependent proper mass in the 4-space covariant formulation.

Further, the HUP problem is discussed in more details.

9.2 On the HUP methodology

The author of HUP [55] made a note expressing similarity of his view to the Copenhagen School QM interpretation but leaving a door to conceptual freedom of micro-world perception when the very concepts of QM measurements and precision loose their classical meaning and require a deeper philosophical analysis and redefinition on the border of Physics and Philosophy. However, this note does not justify the fact that the HUP is not rigorously proved, and its formulation in terms of “simultaneous measurements” of momentum and position is vague.

Presented in literature argumentations leading to the inequality $\sigma_x \sigma_p > \text{constant } h$ vary, they are not clear in many respects and actually do not add rigor to the original derivation [55]. The results are sensitive to a specific form of wave function ψ and the corresponding probability distribution $C\psi^*\psi$; the wavefunction is often chosen without its connection to Schroedinger’s equation. In a particular case of Gaussian probability density, the product of dispersions (or standard deviations) of x and p_x is minimal, that leads to the exact equality in the HUP, Feynman [56]

$$\sigma_x \sigma_p = \frac{1}{2} \hbar \tag{85}$$

For an arbitrary chosen (not specified) distribution, it would be inequality but if specified, one can follow Feynman’s reasoning and obtain an exact

equality with a specific *constant*. Moreover, the Plank's constant could be found in the form of h or \hbar , depending on a physical problem formulation and boundary condition for a solution of Schroedinger equation.

The HUP is usually derived with the use of QM operators, first of all, for a pair of complementary (conjugate) observables, components of 3-vectors: $x \rightarrow \hat{x}$, and $p_x \rightarrow \hat{p} = -i\hbar(d/dx)$. Absolute dispersions and standard "errors" are assessed with the use of the commutation relations

$$[\hat{x}\hat{p}] = i\hbar \quad (86)$$

and

$$\sigma_x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}, \quad \sigma_p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} \quad (87)$$

With the use of the above scheme, the HUP is derived in a general form. The result is independent of absolute values of vector components and their relative precisions.

$$\sigma_x \sigma_p \geq \frac{1}{2} \hbar \quad (88)$$

Often, the inequality in (88) is explained in terms of the mathematical Cauchy-Schwarz inequality for inner products of vectors a, b

$$|(ab)|^2 \leq (aa)(bb) \quad (89)$$

However, the explanation is questionable because of neglect of physical correlation of wave functions in the coordinate and momentum representations of the same particle state, as in the above Feynman's example. The correlation means that uncertainties of complementary variables are not independent, as assumed in the HUP. The similar neglect is also seen in the general methodology of the HUP, especially, in the 3D-space consideration.

The equality/inequality issue is important because the change from "inequality" to "equality" drastically changes the character of philosophical debates. Another important problem is the fact that the HUP for the energy-time pair $\Delta E \Delta t \leq h$ cannot be principally derived since the operator for the time variable as observable does not (and could not) exist. So the derivation is made heuristically.

Let us consider the problem relativistically. From the SR Kinematics equations (81), (82) expressing the 4-phase conservation. A variation $\delta t \approx \Delta t$ of the time variable and the corresponding variation of mass/energy $\delta E \approx E$ of the test particle (quantum-mechanical oscillator) are not independent:

$$\delta E \delta t = h \quad (90)$$

Clearly, variations are correlated with the correlation coefficient in the 4-phase variance $k_{E,t} = -1$. Here, the equality in (90) is a deterministic expression reflecting the de Broglie wave nature of particles in motion. Notice, a coefficient before the Planck's constant is unity.

If the HUP in its conventional form is claimed to be a proven physical law, this is “a strange” one. But there is nothing “strange” about it if to realize that the non-relativistic QM methodology and its relativistic micro-world basis are basically in conflict. The QM physical picture of particle-wave duality rests upon Einstein-de Broglie relativistic relationships between 4-vector components and their products while the Schroedinger equation is formulated in terms of classical mechanics. This circumstances did not preclude the theory from remarkable (though limited) achievements in electromagnetic applications in low speed/energy approximations (especially with implementations of new concepts such as spin, magnetic moment, polarization, quantum statistics, etc) when results are not explicitly subjected to the philosophical treatment of the Schroedinger equation and its solutions.

However, indeterministic (probabilistic) methodology and philosophy lead, in our view, to fundamentally wrong statement admitting violations of conservation laws and causality principle in QM. The cause of it is a classical disconnection of time dimension from 3-space-time; the only remedy would be to restore the Minkowski framework.

9.3 Nowadays status and final remarks

As of today, the HUP is widely accepted as a true law of Nature. We disagree with that in spite of its empirical “confirmations”, in particular, in terms of Bell's theorem. The latter, roughly, asserts that no physical theory of *local hidden variables* can ever reproduce all of the QM predictions. This is basically true because the de Broglie's duality phenomenon (having actually a relativistic basis) cannot be “refuted”. Its amazingly multifaceted consequences in connections with quantum entanglement are studied in numerous works and found new practical applications, in particular, due to works headed by Anton Zeilinger, while the interpretation of the EPR (entanglement) experiment in terms of “hidden parameters” was disproved [57], *and elsewhere*.

There are “paradoxes” in QM, in particular, the one about particles as waves passing through slits. The consequences of involvement of wave diffraction and interference was understood “classically” more than a hundred years

ago in connection with Fresnel bright spot – the one that arises at a center of solid disc’s shadow. This fully deterministic effect of particle-wave duality is confirmed in the similar experiment with de Broglie’s neutron waves, as one deals with in slit and other QM measurements.

Therefore, we state that the indeterministic treatment of HUP propagated to the QM theory does not reflect physical reality. Indeterminism rather comes from an inappropriate treatment of wavefunction and the corresponding probability density in Schroedinger’s framework. Though De Broglie’s hypothesis of particle-wave duality phenomenon is incorporated in the framework, the full understanding of the phenomenon requires a relativistic theory and its quantum extension. The phenomenon originates “natural uncertainties” at the Plank’s level described relativistically without any “hidden parameters”. Those uncertainties, of course, are outlined in the Copenhagen School interpretation. However, in the HUP terms, they should not be treated literally as a manifestation of indeterminism and acausality in the microscopic world. They should be rather clarified logically and philosophically, first of all, in a sense of measurements [58] and encompassed in a generalized deterministic methodology in a spirit of Klein-Gordon and Dirac equations.

Our criticism suggests that the Causality Principle cannot be proved in classical physics; rather, it has to have the eternal value within SR-based (preferably, quantum) dynamics framework.

10 The Alternative Relativistic Dynamics

We propose a relativistic dynamics theory [27] different from GR. In the referred work, the principles of relativistic dynamics are given and illustrated in the example of free radial fall in the spherical symmetric field. The orbital motion problem is described in brief in the present work, as follows.

A new concept of the relativistic proper mass $m(r)$ depending on field strength is introduced. From the Lagrangian problem formulation, it follows $m(r) = m_0/\gamma_r$ where $m(r) \rightarrow m_0$ as $r \rightarrow \infty$, with $\gamma_r = \exp(r_g/r)$. The revision of the proper mass concept is motivated by several reasons, one of them, a necessity to introduce the 4-momentum vector P^μ in the form complementary to the 4-coordinate vector X^μ . The temporal component in X^μ is the proper time depending on the gravitational potential $\tau = \tau(r_g/r)$. Therefore, the temporal component m in P^μ should be $m = m(r_g/r)$. This

explains the gravitational time dilation.

Thus, the gravitational dynamics is formulated in the Minkowski space in the presence of gravitational sources. In polar coordinates, the 4-coordinate interval and the 4-momentum vectors are $dX^\mu(r) = \gamma d\tau(r) (1, \beta_r, \beta_\theta)$ and $P^\mu(r) = \gamma m(r) (1, \beta_r, \beta_\theta)$, where 3-velocity components and the Lorentz factor are functions of r and θ , $c_0 = 1$. The Minkowski 4-force $K^\mu = dP^\mu/d\tau$ acts on the test particle, and it naturally has the tangential component (with respect to the world-line s) and the orthogonal one, while s is a function of 4-position.

There are two conservation laws, – for total energy ϵ_0 , and the angular momentum L_0 formulated below for initial conditions $r(r) = r_0$, $\theta = 0$, $\beta_r = 0$, $\beta_\theta = \beta_0$

The total energy and the angular momentum are

$$\epsilon_0 = \gamma_0 \gamma_{r_0} = \gamma \gamma_r \quad (91)$$

$$L_0 = \gamma_0 \gamma_{r_0} r_0 \beta_0 = \gamma \gamma_r r \beta_\theta \quad (92)$$

Instead of (92), it is convenient to use a conserved quantity $l_0 = \epsilon_0/L_0$:

$$l_0 = r \beta_\theta \quad (93)$$

Here, a squared inverted Lorentz factor is $1/\gamma^2 = (1 - \beta_r^2 - \beta_\theta^2)$, $\beta_r = dr/dt$, $\beta_\theta = r(d\theta/dt)$, and we are going to use, as usual, the formula $\beta_r = (dr/d\theta)(d\theta/dt)$, and $\beta_\theta^2 = l_0^2/r^2$. After introducing a variable $\xi = r_0/r$, we arrive to the exact relativistic equation of orbital motion of confined particle. The equation is valid for a however strong field (by the criterion r_g/r) (compare with (1)):

$$\left(\frac{d\xi}{d\theta}\right)^2 = \left(\frac{1}{\beta_0^2} - \xi^2\right) - \left(\frac{1}{\gamma_0^2 \beta_0^2}\right) \exp\left(\frac{2r_g}{r_0}(1 - \xi)\right) \quad (94)$$

The Newtonian limit (weak field conditions) follows as a linear approximation of the exponential function

$$(d\xi/d\theta)^2 = (1 - 2\sigma_r) + 2\sigma_r \xi - \xi^2 - \sigma(1 - \xi)^2 (r_g/r_0) \quad (95)$$

where $\sigma_r = r_g/r_0 \gamma_0^2 \beta_0^2$ is the σ criterion in the relativistic case. The last term in (95) presents the strong field contribution comparable with the relativistic kinematics effect.

In the strong field, the potential is

$$V(r) = - [1 - \exp(-r_g/r)] \quad (96)$$

The next order corrections to the Newtonian approximation are due to the deviation of the gravitational potential from the classical law $V(r) \sim 1/r$.

Definitely, the theory does not confirm the GR precession proportional to $3r_g/r_0$. It predicts a free radial fall without the deceleration, as seen from a comparison with (61)

$$\beta(r) = [1 - (1/\gamma_0^2) \exp(-2r_g/r)]^{1/2} \quad (97)$$

The theory requires a revision of the conventional relativistic concept of mass. The revision results in an elimination of divergence of gravitational potential at $r \rightarrow 0$. It is shown that the introduction of the concept of variable proper mass also resolves the long standing problem of divergence in relativistic electrodynamics. For years, the problem has been tackled by the artificial procedure of mass/charge renormalization (subtractions of infinite numbers).

The theory adopts a field concept as an optically active refracting medium for propagation of electromagnetic waves and de Broglie waves. In this way, it has quantum connections needed for further development of relativistic gravitational field theory.

The theory explains GR classical tests in a new way and leads to new practically verifiable predictions, but it is in disagreement with the GR predictions of perihelion advance and ‘‘Black Hole’’.

Details are out of the scope of the present work.

11 Speculations

11.1 GR and Modern Physics

A quantum relativistic gravitational field theory remains a central problem in Modern Physics. One could hardly expect Einstein’s GR to have extensions to Quantum Mechanics and Particle Physics, or QCD. On the other side, QCD is aimed to describe particles in the quantum world but with no claim to incorporate the gravitational force neither at quantum nor relativistic level. Moreover, QCD failed to rigorously formulate the particle problem

in 4D spacetime so that the question arises if the famous gap mass problem could be well posed; the example of exactly soluble problem in rigorous terms of 4-space relativistic quantum field is still to come (or rather never would) [59].

GR and QCR have a fundamental methodological commonality, though: they failed to rigorously formulate the mass concept; consequently, both are inconsistent with SR-based dynamics of matter. In both theories, relationship of predictions with physical reality to the great extent is the matter of belief.

As concerns QCD, it satisfactory describes the phenomenological multi-parameter classification of particle zoo. In fact, those successes are real as far as QCD model believed to be valid, at least, in the perturbative range of low-energy quark-gluon interactions. Assumptions and approximations become more and more vague as energy rises. As Wilczek quoted about QCD rigor [60], “it is more blessed to ask forgiveness than permission”.

The (alleged) observation of the predicted Higgs boson is considered a big success among particle physicists. Meanwhile, it is still not clear what kind of bosons and how many of them would serve the task. The belief is spread in scientific media that the boson is important to explain masses of particles making the whole visible Universe. In a shorter version, the boson is needed to explain all *elementary* particles in QCD; there are plenty of them including the electron (but the Higgs itself). In the first place, however, it was all about the spontaneously broken chiral symmetry in the so-called “electroweak unification” where only W^\pm and Z bosons must be given masses by means of the Higgs field. Now, when the Higgs was reconstructed from sophisticated observations, nothing happened in science; yet more years are requested to find out what the particle is good for and why. It looks like QCD is quite resilient in the framework with too many narrowly targeted parameters making the model, in Popper’s terms, more verifiable rather than falsifiable.

11.2 QCD critique

Similarly to GR and QED, in QCD the proper mass of observable particles is constant. As discussed, this postulate deeply affects a relativistic theory in general. One of a typical consequences of it – a central divergence and a need for artificial mathematical procedure, “renormalization”.

Recall our suggestion of the relativistic mass concept revision and the in-

roduction of the alternative, – the general concept of field dependent proper mass valid in all type of interactions, in the gravitational and e/m interactions, in particular. For attractive forces, it leads to a principally new behavior of particles at a small distance (comparable with a particle size) between them. In the QCD case, this is a size of quark confinement due nuclear forces. A similar effect is illustrated below in the example of the test particle of field dependent proper mass.

In the spherical symmetric attracting field characterized by interaction radius r_i , the proper mass $m(r)$ of the test particle probing a field is given by

$$m(r) = m_0 \exp(-r_i/r) \quad (98)$$

In SR-based Dynamics, this variation implies the appearance of the Minkowski force component tangential to the world line $K_{tan}^\mu = U^\mu dm/ds$, which is absent in the conventional SR Dynamics

$$K^\mu = dP^\mu/ds = d(mU^\mu)/ds = mU^\mu/ds + U^\mu dm/ds \quad (99)$$

The corresponding scalar potential $V(r)$ takes the form

$$V(r) = -m_0 c_0^2 [1 - \exp(-r_i/r)] \quad (100)$$

Consequently, the force $F(r)$ acting on the particle is

$$F(r) = -m_0 c_0^2 (r_i/r^2) \exp(-r_i/r) \quad (101)$$

In Fig. 9, the relativistic force in the concept of field-dependent proper mass (in the example of Newtonian gravitational force) is shown in comparison with the law $1/r^2$, as follows from the potential (100) versus $1/r$. One can notice an apparent resemblance of the graph with what is expected from the hypotheses of quark confinement and the corresponding asymptotic freedom if the source mass is concentrated at the center of mass. The force acting on the test particle rises from zero (in its absolute value) with a distance from the center of mass. It reaches the maximal value at the interaction radius $r = r_i$ and drops after that; its asymptotic (weak-field) graph coincides with the Newtonian form. The principal difference of this example from QCD is that the energy required to pull the particle out of the “confinement zone” is limited (in QCD, quarks are firmly confined and do not exist free).

If the central divergence is eliminated relativistically, it must be eliminated in the corresponding quantum field theory. Thus, we propose to revise the conventional proper mass concept in GR, QED, and QCD.

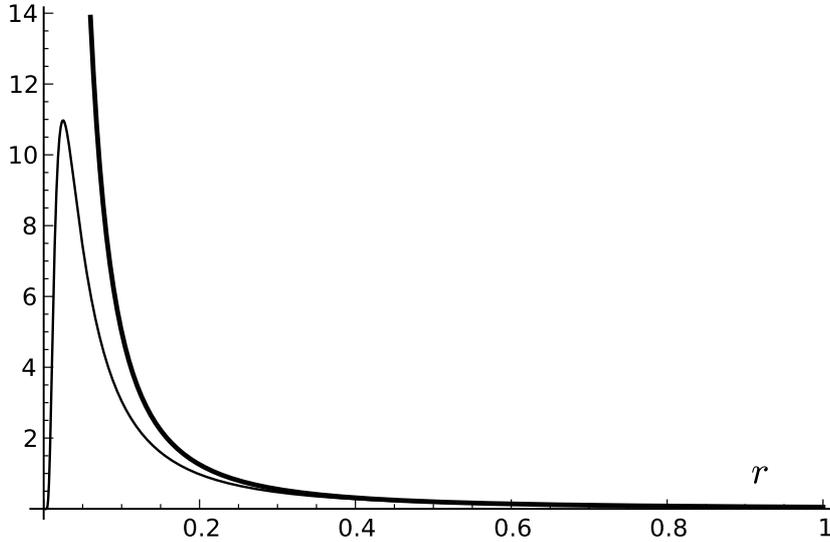


Figure 9: Newtonian gravitational force $|F(r)| \sim r_g/r^2$ (thick line) and its relativistic generalization $|F_R(r)| \sim r_g/r^2 \exp(-r_g/r)$ in the concept of field-dependent proper mass, in the example of a strong interaction $r_g = 0.05$. It has a maximal (absolute) value at $r = r_g$, and $|F_r(r)| \rightarrow 0$ as $r \rightarrow 0$; in the range $r < r_g$, it rises with distance and asymptotically $|F_R(r)| \rightarrow |F(r)|$ as $r \rightarrow \infty$.

As a great contemporary philosopher pointed out, “the growth of knowledge depends entirely on disagreement” [58]. So we disagree with the proper mass constancy, in general, and with the following QCD basics, in particular:

1. Divergences treated with renormalization in quark-gluon model;
2. Introduction of “elementary particle” concept;
3. Electroweak unification calling for Higgs;
4. Three families of neutrinos and neutrino’s proper mass possession.

Our concrete argumentations are left for further discussions outside of this work.

11.3 Speculations about particle masses and the role of neutrinos

Next, we present, in a very sketchy, intuitive rather than mathematically concrete way, our speculations about a relativistic gravitational field theory in 4D spacetime, first of all, about the issues of gravitational force carriers, force “unification”, and mass origination.

It is reasonable to hypothesize that there is a generic connection between forces in the scalar-type gravitational potential and a vector-type electromagnetic field. In practice, the gravity is observed in electrically neutral media but naturally always there: the Nature unified both in the first place. This hypothesis actually requires GR to be abandoned. Yet, one has to agree with our field-dependent proper mass concept in the SR Dynamics framework of unified field theory (in this case, gravitation and electromagnetism).

We would like to look for a unification of tensor-spinor type with extension to combined scalar-vector fields bearing in mind that suitable mathematics is mainly developed (though, essential changes could be needed). It is suggested that massless force carriers formed of neutrinos and their composites should be deliberated. We argue that neutrinos of all kinds are massless and physically identical (that is, not distinguishable) [61].

The idea of neutrino pairs as gravitational force carriers is not original; it goes back, at least, to Feynman’s lectures on gravitation [62]. He rejected the idea since it requires the potential to deviate from $1/r$ law. However, with the proper mass concept revised, the potential must deviate as $r \rightarrow 0$ in (100). There are old disputes about a neutrino theory of light [63], *and elsewhere*, which are worth revisiting. Neutrino composites of “quasiboson” or “quasifermion” types can be hypothesized as intermediate formations in the particle inner structure, so that a photon could originate from those structures of interacting particles, in particular, when an electron-positron pair (both in the pair being “elementary particles”) annihilates. When, reversely, a high-energy photon creates a massive pair in a vicinity of nucleus, how particle inner possessions (including proper mass) emerge, and what is a real difference between the proper and kinetic mass, – those kind of questions remain intriguing.

Consider an atomic electron of mass m_e in a Hydrogen ground state. When “pushed” into the sub-Bohr’s region, the electron is subjected to the so-called degenerate (or quantum) repulsive pressure, the nature of which is

related to the smallness of volume in terms of de Broglie wavelength. The same is true for an incident electron having a sufficient kinetic energy to penetrate there. In accordance with the concept of field-dependent proper mass, the electron proper mass must increase due to an internal force working on the electron. What is a mechanism of mass variation?

Eventually, the electron could convert into a bounded muon. The latter being unstable has greater mass $m_\mu > m_e$. Consequently, its orbit is respectively closer to the proton. The muon can be considered “the excited electron” not only in the quantum atomic states but at every moment of motion on the road to the muon formation. While the excited electron is gravitationally heavier than the electron, in the usual Hydrogen, contrarily, the bounded (stable) electron exhibits the mass defect; because of the structural change, it is gravitationally lighter than in a free state.

The term of “point mass particle” should not be understood literally: the particle having mass, charge, spin, magnetic moment, etc must have an internal (yet unknown) structure requiring some room. At some values of mass, “an elementary particle” can exist in the stable or decaying mode, and can be “excited”. We state that constant proper masses of particles are, strictly speaking, “imaginary” fundamental physical constants when the particles are “far away” from field sources (how “far” and from “what” is not clear).

Thus, we hypothesize that particles are made of boson-fermion condensates, which have “a gravitating core” of the variable proper mass, possibly, of boson type, and “a shell” of variable “mass of motion” related to kinetic energy, so that the total mass is conserved in the conservative field.

Generally, the particle in a field can absorb/emit a single neutrino or neutrino composite of boson or fermion type, therefore, it can convert from ‘a quasiboson’ to a fermion, and back during a force transient. It is not clear if the neutrino composite is massive or massless in free motion but it must acquire a proper mass property after the absorption that is, it must add/remove a small quantum portion of particle proper mass in the absorption/emission process. This would mean that the electron can exist momentarily or some period of time in the bosonic form. In similar circumstances, a dynamic spin change in protons and neutrons could be speculated too.

While our “bare speculations” could or could not have a sense, we assume QCD to be ignored anyway.

12 Sum up and General Conclusion

The multi-aspect problem of gravitational field theory is the main thread throughout the work. The problem could not be rightly approached out of the context of Physics Frontiers. It is commonly believed that GR applications in Cosmology, Astronomy, and Astrophysics demonstrate successes of GR as the only one true theory of gravitational field.

The theory possesses strange properties, which have been under scrutiny and discussed for decades: it is totally disconnected from the SR dynamics theory, Quantum and Particle Physics, it is divergent and not renormalizable, it is not quantizable either.

Part One is devoted to GR particle dynamics. We present proofs of GR inherent contradictions at the fundamental level and discussed controversial arguments allegedly validating the theory and its applications.

In fact, a large number of physicists involved in GR studies express criticism of various aspects of the GR problem but it is not heard for different reasons, first of all, because of lack of critical analysis concerning GR self-consistency and GR classical tests. We investigated the conventional GR problem formulation, its technical execution and predictions analytically and numerically; the exact numerical solution of the GR particle dynamics equation is conducted, the results are compared with that in “conventional GR methodology”. The conclusion is made that the belief in GR being the only true gravitational field theory is not justified.

In Part Two, a sketchy review of potentially vital links of Modern Physics with GR and an alternative gravitational theory are given as the author’s view on the current status of the problem. Doubts about and disagreement with some relevant existing concepts concerning the future theory development in connections with Physics Frontiers are expressed in the context of Physics and Philosophy. We argue that basic cosmological, quantum-mechanical and Particle Physics observations are often used to justify theoretical models, which are believed to be a true reflection of physical reality but actually rest, figurally speaking, upon quicksand.

The proposal to introduce the concept of field dependent proper mass in relativistic and quantum relativistic field theories is marked by the red flag throughout the work and demonstrated in SR-based Relativistic Dynamics of particles in the gravitational field, – the alternative to GR particle dynamics. Some other ideas of revision of conventional concepts in Modern Physics are suggested as an invitation for further healthy constructing discussions among

Physics Community, first of all, interested open-minded physicists and scientists of new generation who enjoy curiosity, critical thinking, and recognition of difference (sometimes fuzzy or arguable but often crude) between pseudoscience and plain beliefs, on the one hand, and science itself and scientific hypothesis, on the other hand.

References

- [1] A. Vankov. “General Relativity Problem of Mercury’s Perihelion Advance Revisited”. arXiv[physics.gen-ph] August 2010, 1008.1811v1.
- [2] A. Einstein. “Erklärung der Perihelwegung Merkur der allgemeinen Relativitätstheorie”. *Königlich Preussische Akademie der Wissenschaften*, Berlin, 1915. 831-839 . See also: “The Collected Papers of Albert Einstein”. vol. 6, A., J. Kox, M. J. Klein, and R. Schutmann, Editors. Princeton University Press, (1996).
- [3] Letter from K. Schwarzschild to A Einstein dated 22 December 1915, in *The Collected Papers of Albert Einstein*, vol. 8a, doc.169, (1996).
- [4] K. Schwarzschild. “Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie”. Reimer, Berlin 1916, (Sitzungsberichte der Königlich-Preussischen Akademie der Wissenschaften; 1916).
- [5] Y. Hagihara. “Theory of relativistic trajectories in a gravitational field of Schwarzschild”. *Jap. Journal Astron. Geophys.* Vol. 8, 67 (1931).
- [6] Feng He, Fan Zhao. “Exact Analytical Solution to Equations of Perihelion Advance in General Relativity”. *International Journal of Theoretical Physics.* Vol. 48, 2198, (2009) .
- [7] C. Moller. “The Theory of Relativity”. The international series of monographs on physics. Delhi, Oxford University Press, 1972.
- [8] L. D. Landau, E. M. Lifshitz. “The Classical Theory of Fields”, 4-th English edition, Pergamon Press, 1975.
- [9] L. D. Landau, E. M. Lifshitz. “Mechanics”. 3d Edition (reprinted), Butterworth-Heinemann, Oxford, Boston, 2000.

- [10] P. Bergmann. “Introduction to the Theory of Relativity”. 1942. Translated to Russian, 1947. In English: Kessenger Publishing House, 2008.
- [11] C. W. Misner, K. S. Thorne, J. A. Wheeler. “Gravitation”. W. H. Freeman and Company, San Francisco, 1973.
- [12] V. Fock. “The Theory of Space, Time, and Gravitation” (translated from the original Russian Edition, 1955, by N. Kemmer). Pergamon Press, 1959.
- [13] R. Wald, “General Relativity”. University of Chicago Press, Chicago and London, 1984.
- [14] S. Weinberg. “Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity”. John Wiley and Sons, Inc., New York, London, 1972.
- [15] J. L. Synge. “Relativity: The Special Theory”. North Holland Publishing Company, Amsterdam, 1965.
- [16] J. L. Synge. “Relativity: The General Theory”. North Holland Publishing Company, Amsterdam, 1964.
- [17] L.B.Okun, K.G.Selivanov, and V.L.Telegdi On the interpretation of the redshift in a static gravitational field. Am.J.Phys. 68 (2), 115-119. (2000).
- [18] Sean M. Carroll, “Lecture Notes on General Relativity”, arXiv: gr-qc/9712019 (1997)
- [19] E. F. Taylor, J. A Wheeler. “Exploring Black Holes; Introduction to General Relativity”. Addison Wesley Longman, 2000.
- [20] N. Ashby. Relativity and Global Positioning System. Physics Today, May. 2002.
- [21] W. Pauli. “Relativitätstheories”, Encyklopadie der mathematischen Wissenschaften, Band V2 Heft IV Art. (1921). Translated to English: “Theory of Relativity”, Pergamon Press, New York, 1958.
- [22] W. Rindler. “Relativity: Special, General, and Cosmological”. 2d Edition, Oxford University Press, 2006.

- [23] S. Chandrasekhar. “The Mathematical Theory of Black Holes”. Clarendon Press, 1983.
- [24] Clemence G. M. “The Relativity Effect in Planetary Motions”. *Review of Modern Physics*, 1947, vol. 19, 361.
- [25] P. K Rashevsky. “Riemannian Geometry and Tensorial Analysis”. (in Russian). GITTL, Moscow, 1953.
- [26] S. Blinnikov, L. Okun, M. Vysotsky. “Critical velocities in General Relativity”. arXiv: gr-qc/0310020 (2003).
- [27] A. Vankov. “On relativistic generalization of gravitational force”. *Found. Phys.* 2008, vol. 38, 523-545.
- [28] A. Vankov. “Different Look at Relativistic Gravity and Related Phenomena”, 2005. In “Relativity, Gravitation, Cosmology: New Development” (Series: Contemporary Foundational Physics), Nova Publisher, p. 169-194, 2010.
- [29] Richard Feynman , “QED: The Strange Theory of Light and Matter”. Princeton University Press, 1986.
- [30] P. Dirac, “The Evolution of the Physics of Nature”, In *Sci. Am.*, May, 1963.
- [31] H. Kragh, “Dirac: “Scientific Biography”, Cambridge University Press, 1990.
- [32] Rodger Penrose. “The Road to Reality: A Complete Guide to the Laws of the Universe”, Jonathan Cape, London, 2004.
- [33] A. Vankov. “Testing relativistic mass-energy concept in Physics of gravity and electricity.” *Ann. Found. Louis de Broglie*, Vol. 29 (2), 1035, 2004. The special issue dedicated to the 50th anniversary of the Yang-Mills 1954 paper).
- [34] Louis de Broglie. “Radiation Waves and Quanta” . Note of Louis de Broglie, presented by Jean Perrin. (Translated from *Comptes rendus*, Vol. 177, 1923, pp. 507-510)

- [35] Louis de Broglie. “Non-Linear Wave Mechanics: Casual Interpretation”. Translated by A.Knodel and J.Miller. Elsevier Pub. Co., 1960.
- [36] L. D. Landau, E. M. Lifshitz. “Quantum Mechanics, Non-Relativistic Theory”. 3d Edition, Elsevier Science Ltd, 2003.
- [37] Craig Callender, Nick Huggett, “Physics Meets Philosophy at the Planck Scale: Contemporary Theories in Quantum Gravity”, Cambridge University Press, 2001.
- [38] T. Mauldun. “Philosophy of Physics. Space and Time”, Princeton University Press, 2012.
- [39] J. Earman, “Determinism: What We Have Learned and What We Still Dont Know”. In “*Freedom and Determinism (Topics in Contemporary Philosophy)* . Vol. II, p. 21. A Bradford Book, The MIT Press, 2004.
- [40] A. Vankov. “Mass, time, and clock (twin) paradox in relativity theory”. arXiv: physics/0603186v1 (2006).
- [41] A. Vankov. “On controversies in Special Relativity”. arXiv: physics/0606130 (2006).
- [42] J. Norton. “Is there an Independent Principle of Causality in Physics?”. Br. j. Philos. Sci, 60(3), 475-486 (2009).
- [43] J. Cambell, H. O'Rourke, D. Shier (Editors). “Freedom and Determinism (Topics in Contemporary Philosophy). Vol. II. A Bradford Book, The MIT Press, 2004.
- [44] Steve Carlip. “Quantum Gravity in 2+1 Dimensions”. Cambridge Monographs on Mathematical Physics, 2003.
- [45] Yu-Quing Lou and Zu-Hui Fan. “Gravitational Radiation from Binary Systems”. Astrophysics and Space Science, **207**, 151, 1993.
- [46] A. Vankov, “On observation of neutron quantum states in the Earths gravitational field”. Phys. Rev. **D 81**, 052008 (2010).
- [47] P. J. Peebles, “Principles of Physical Cosmology”, Princeton University Press, 1993.

- [48] A. Vankov, “On Alternative Cosmology”, arXiv: astro-ph/9811165v1 (1998).
- [49] A. Vankov, “Baryon Asymmetry of Observed ”, arXiv: astro-ph/9812230v1 (1998).
- [50] A. Vankov, “On Matter-antimatter Separation in Open Relativistic Material System ”, arXiv: astro-ph/9906279v1 (1999).
- [51] A. Vankov. “On the de Broglie wave nature”. Ann. Foud. Louis de Broglie, Vol. 30 (1), 15, 2005.
- [52] C. Quigg, “Gage Theories of the Strong, Weak, and Electromagnetic Interactions”. Advanced Book Classics. Westview Press, 1997.
- [53] A.Capri, “Non-Relativistic Quantum Mechanics”, 3d Ed., World Scientific, 2002.
- [54] R. T. Leung, K. Yong, “Gauge invariance and reciprocity in quantum mechanics”, Phys. Rev. A 81, 032107, 2010.
- [55] W. Heisenberg, “The Physical Principles of the Quantum Theory” (translated by C. Eckart and F. Hoyl). Chicago, Dover Publications, 1930.
- [56] R. Feynman, R. Leighton, M. Sands, “Feynman Lectures on Physics”. Vol. 3, Quantum Mechanics. Addison-Wesley Publishing Company, Massachusetts, 1963.
- [57] G. Weihs, T. Jennewein, C. Simon, A. Zeilinger. “Violation of Bell’s inequality under strict Einstein’s locality conditions”. Phys. Rev. Lett. Vol. 81, (23), 5039-5043 (1998).
- [58] K. R. Popper, “The Logic of Scientific Discovery”, Taylor and Felicia Group, London, New York, 2005.
- [59] “Nonperturbative QCD: The promise of a bridge from elementary particle interactions to nuclear force”. Workshop in hadroparticle physics: Matalescanas, Huelva, Spain, October 2012.
- [60] F. Wilczek, “QCD Made Simple”. Physics Today, August 2000, p. 22-28.

- [61] A. Vankov, “On Neutrino Family Problem”, arXiv: hep-ph/0012036v1 (2000).
- [62] Richard Feynman *et al*, “Feynman Lectures on Gravitation”, Edited by Brian Hatfield. Addison-Wesley Publishing Company, 1995.
- [63] W. A. Perkins .“Quasiboson”. International Journal of Theoretical Physics, 41 (5), 2002, p. 823838. Also: W. A. Perkins .“Quasiboson”, arXiv: hep-th/0107003v1 (2001).