Accelerated Life Testing : Analysis and Optimization

Seyyedeh Zohreh Fatemi, Fabrice Guerin, Laurent Saintis

Accelerated life testing : Analysis and optimization

Seyyedeh zohreh Fatemi, Fabrice Guerin, Laurent Saintis
University of Angers
LASQUO laboratory, ISTIA
Angers-France
seyedehzohreh.daghighi@univ-angers.fr
fabrice.guerin@univ-angers.fr
Laurent.saintis@univ-angers.fr

***Abstract*— This paper describes a methodology to conduct a sequential test defined by an optimal accelerated testing plan. This test plan is based on an economic approach defined in previous work, a prior knowledge on reliability parameters (choice of reliability function, scale and shape parameters …) and acceleration model (choice of model, model parameters …) to evaluate the proportions of failure at each accelerated level. When conducting a test, it is possible to verify the compatibility of results with prior knowledge from a consistency criterion that measures the compatibility between prior distribution and likelihood provided by the data. We can also reduce the censoring time in case of "Good results" while keeping the same level of risk. This possibility is authorized because the robust test is longer than basic optimal test plan. In the process of product development, we will use the qualification. Qualification is an application-specific process involving the evaluation of the product with respect to its quality and reliability. The purpose of qualification is to define the acceptable range of variability for all critical product parameters affected by design and manufacturing. The methodology will be illustrated by a numerical example.**

Keywords: Test planning, accelerated test, reliability, optimization, Bayesian estimation, testing cost, operational cost, robustness, Qualification.

I. INTRODUCTION

An Accelerated Life Test (ALT) is the process of determining the reliability of a product in a short period of time by accelerating the use environment. ALTs are also good for finding dominant failure mechanisms. Thus, an accelerated Life Test (ALT) is a test method which subjects test units to higher than use stress levels in order to compress the time to failure of the units. Conducting a Quantitative Accelerated Life Test (QALT) requires the determination or development of an appropriate life-stress relationship model. In the case of robust optimization of an accelerated testing plan, a plan is provided with a sample size, stress levels and censoring time. This plan provides a robust estimate of the probability of failure during the warranty period (reliability metric is selected to estimate the operation cost of the test plan), the robustness to guaranteeing the best cost with a given risk α integrating the uncertainties on input data (reliability, activation energy …). To define the test plan, in some approaches, we consider an objective function based on economic approach, Bayesian inference for optimizing the test plan. The prior knowledge is based on a feedback from expert's opinion, Field data analysis on old product, Reliability Standard... This information contains the uncertainty on the real reliability of the new product tested. During the test, the observation of data makes possible to verify the consistency of these points with the assumptions of prior information having fixed the test plan. The establishment of batch test to launch the tests needs to fix some parameters of the plan as the number of units tested, the level of each stress, and the number of units tested by stress level. However, other parameters can be modified. In this paper, an optimization test plan is proposed integrating the Bayesian inference and an objective function based on economical formulation. The proposed method consists of 3 subsequent steps:

- Definition of a robust accelerated testing plan

Accelerated testing is an approach for obtaining more information from a given test time than would normally be possible. A detailed test plan is usually designed before conducting an accelerated life test. The plan requires the determination of the type of stress, stress levels, number of units to be tested at each stress level and an applicable accelerated life testing model that relates the failure times at accelerated conditions to those at normal conditions. To define these parameters, an optimization procedure is developed to minimize an economical function (testing and operational costs) with taking into account the uncertainties on input data (reliability, activation energy …) by the Bayesian inference. The optimization process is performed using genetic algorithm.

- Consistency of prior knowledge

Among the articles on Bayesian inference for estimating the parameter many of them insist on opposing the behavior of the posterior estimation when the prior information is badly conditioned. The objective knowledge of reality given by the likelihood may be lost by choosing an improper prior. In most cases, articles are brought to a reinforcement of the robustness later. However, the first difficulty is to correctly regard that prior knowledge and likelihood are not contradictory. We propose a prior validation using a criterion of coherence that measures the compatibility between the prior and the likelihood.

- Stopping criteria of test plan

The previous compatibility criterion take a metric between 0 and 1 and is further used as a weighting factor for the prior distribution. The use of the weighting over the prior distribution should be seen as a diminution of confidence over the prior information for the parameters. Periodically (during the test), the posterior reliability metric is estimated and the risk to validate the deisgn is evaluated and compared to target risk. When the confidence level on estimation of reliability metric is reached, the test is stopped.

The verification whether a product meets or exceeds the reliability and quality requirements of its intended application, is the aim of qualification. Qualification plays an important role in the process of product development. Product qualification can be used to baseline the design and processes. It determines the product performance degradation under normal application conditions. It can also be used to compare different designs to help make design decisions. Product qualification is used to meet the requirements of customers with consideration of the intended application and application conditions.

II. Estimation in Parametric ALT Model

We consider the parametric ALT model has been described in [1]. After selecting the model, in order to provide estimates for the model's parameters, we apply maximum-likelihood estimation as point estimators. The maximum likelihood estimators return a single point estimate for a given data set. In contrast, the Bayesian posterior is an entire distribution over the parameter space. We can turn this in to a point estimate by taking some measure of central tendency, such as the conditional mean of the parameter given the data. In Bayesian Inference by Maximum of A Posteriori, the Bayesian approach is based on the concept of subjective probability depending on the degree of belief in the occurrence of an event [2]. This is not a point value, which is estimated, but the probability distribution of the random variable (probability of non-functioning), the degree of belief that each probability value can be true. In Bayesian statistics, the uncertainty about the unknown parameters is quantified used probability so that the unknown parameters are regarded as random variables.

To simplify the parametric model, we develop the methodology with one accelerating variable and a linear relationship between the location parameter and the stress level. Without adding complexity, the methodology can be generalized to multiple accelerating variables with linear relationship.

It is assumed that the survival function *R (t)* belongs to a class of functions depending only on the parameters of scale η and shape β [2]:

$$R_{S_0}(t) = R_0\left(\left(\frac{t}{\eta}\right)^{\beta}\right), (\eta, \beta > 0) \quad (1)$$

Several models, such as Weibull and lognormal, are just particular cases of the above form $R_0(t) = e^{-t}, R_0(t) = 1 - \Phi(\ln t)$ respectively as detailed in [3].

In this section, we assume, for a particular case of a constant stress with one accelerating variable, which the logarithm of scale parameter η follow a linear function of transformed stress *S as*:

$$\ln(\eta) = \gamma_0 + \gamma_1 . S$$

For our particular case of constant stress, *S*, with one accelerating variable, the reliability function the equation (1) becomes:

$$R_S(t) = R_{S_0}\left(e^{\gamma_0 + \gamma_1 . S}\, t\right) \quad (2)$$

The notations $R(u) = R_0(e^u)$, $u \in \Re$, $u = \ln(t)$, $\gamma = (\gamma_0, \gamma_1)$, allow us to rewrite the equation (2) as:

$$R_S(t) = R\left(\frac{\ln t - \gamma^T S}{1/\beta}\right) \quad (3)$$

The likelihood function can be written as:

$$L(T \mid \gamma, \beta) = \prod_{i=1}^{k}\prod_{j=1}^{n_i}\left\{\left[\beta.\lambda\left(\frac{T_{ij} - \gamma^T S^{(i)}}{1/\beta}\right)\right]^{\delta_{ij}} R\left(\frac{T_{ij} - \gamma^T S^{(i)}}{1/\beta}\right)\right\}$$

Note: T_{ij} is the life time observed or censored of the j^{th} unit from i^{th} stress level group.

We consider a prior information on unknown parameters modeled by the functions $\pi(\gamma_0)$, $\pi(\gamma_1)$, $\pi(\beta)$. We assume that the variables $(\gamma_0, \gamma_1, \beta)$ are independent and the joint prior distribution can be defined as:

$$\pi(\gamma_0, \gamma_1, \beta) = \pi_{\gamma_0}(\gamma_0) \times \pi_{\gamma_1}(\gamma_1) \times \pi_{\sigma}(\beta) \quad (5)$$

The choice of the form of π□ depends on degree of knowledge on parameter γ_0, γ□ or β.

The continuous form of Bayes theorem for the random variable θ over the Ω domain, having t_i, i = 1...n as test results, is:

$$\pi_{apo}(\theta / t_1, .., t_n) = \frac{L(t_1, .., t_n / \theta)\pi(\theta)}{\int_{\Omega} L(t_1, .., t_n / \theta)\pi(\theta) d\theta} \quad (6)$$

with π (θ) the mathematical form, which formalizes the prior information.

With regard to the aspect of reversing in statistics, we consider a probability density as $\pi_{apo}(\gamma_0, \gamma_1, \beta)$. As consequence, the ML theory can be applied. So a search of values that maximizes the π□$_{apo}(\gamma_0, \gamma_1, \beta)$ and the covariance matrix associated to these estimators will be searched. MAP method considers the a posteriori density function $\pi_{apo}(\gamma_0, \gamma_1, \beta | T)$ and the punctual estimators of unknown parameters $(\gamma_0, \gamma_1, \beta)$ are estimated so that they maximize:

$$(\hat{\gamma}_0, \hat{\gamma}_1, \hat{\beta}) = Arg\max\left(\pi_{apo}(\gamma_0, \gamma_1, \beta | T)\right) \quad (7)$$

By differentiating after the variables $(\gamma_0, \gamma_1, \beta)$ of the function ln [$\pi_{apo}(\gamma_0, \gamma_1, \beta)$] , the MAP estimators $(\hat{\gamma}_0, \hat{\gamma}_1, \hat{\beta})$ can be obtained by solving the equation system [1]:

$$\frac{\partial \ln\left[\pi_{apo}(\gamma_0,\gamma_1,\beta)\right]}{\partial \gamma_i}=0\ \ (i=1,2),\quad \frac{\partial \ln\left[\pi_{apo}(\gamma_0,\gamma_1,\beta)\right]}{\partial \beta}=0 \qquad (8)$$

Fisher information applies to the function that describes the information on the parameters, $\pi_{apo}(\gamma_0,\gamma,\beta|T)$ [1, 4] becomes:

$$I^{MAP}(\gamma_0,\gamma_1,\beta)=E\left[\left(\frac{\partial \log \pi_{apo}(\gamma_0,\gamma_1,\beta|T|}{\partial\gamma_0\partial\gamma_1\partial\beta}\right)^2 |_{\hat{\gamma},\hat{\beta}}\right]$$
$$=E\left[\left(\frac{\partial \log L(T|\gamma_0,\gamma_1,\beta|}{\partial\gamma_0\partial\gamma_1\partial\beta}\right)^2 |_{\hat{\gamma},\hat{\beta}}\right] \qquad (9)$$
$$+E\left[\left(\frac{\partial \log \pi(\gamma_0,\gamma_1,\beta)}{\partial\gamma_0\partial\gamma_1\partial\beta}\right)^2 |_{\hat{\gamma},\hat{\beta}}\right]$$

So, the estimator of the reliability function $\hat{R}_{S_0}$ is defined by:

$$\hat{R}_{S_0}(t)=R\left(\frac{\ln(t)-\hat{\gamma}^T S^{(0)}}{1/\hat{\beta}}\right) \qquad (10)$$

The parameters $\hat{\gamma}_1,\hat{\gamma}_0,\hat{\beta}$ and $I^{MAP}(\hat{\gamma}_0,\hat{\gamma}_1,\hat{\beta})$ are obtained by Monte Carlo simulation [5].

III. Prior Compatibility and Weighting

Among the articles on Bayesian inference for estimating the θ parameter many of them insist on opposing the behavior of the posterior estimation when the prior information is badly conditioned.

The objective knowledge of reality given by the likelihood may be lost by choosing an improper prior. In most cases, articles are brought to a reinforcement of the robustness later. However, the first difficulty is to correctly regard that prior knowledge and likelihood are not contradictory.

[6] proposes to use a Fisher test between prior and empiric measures of uncertainty. [7] proposes a prior validation using a criterion of coherence that measures the compatibility between the prior π and the likelihood.

[8] proposes a method to validate the prior distribution by studying its compatibility with the likelihood. Further development is carried out in [9].

The methodology proposed by [8] compares the information contained by the prior for each parameter θ_i with the information contained by the likelihood. This is done by using a factor, which is proportional to the convolution product "Fig. 1" between the prior $\pi_{\theta_i}(\theta_i)$ and the likelihood $f(\theta_i)$. It is thus possible to define the real C (δ) by the convolution product $\pi_{\theta_i}(\theta_i)\otimes f(\theta_i)$ as:

$$C_i(\delta)=\int_{D(\theta_i)}\pi_{\theta_i}(\theta_i)\,f(\theta_i-\delta)d\theta_i \qquad (11)$$

The normalized expression of $C(\delta)$, noted as K , is:

$$K_i=\frac{C_i(0)}{C_i^{\max}(\delta)} \qquad (12)$$

with:

- $C_i(0)$ for $\delta=0$
- $C_i^{\max}(\delta)$ maximum of $C_i(\delta)$

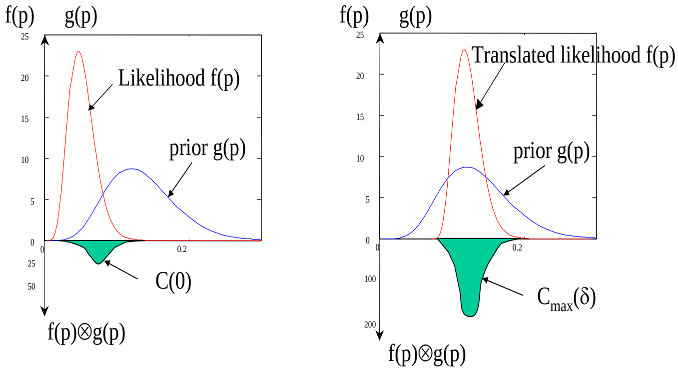

Fig.1. Principle for determination of the weighting coefficient (10)

The resulted K_i takes values between 0 and 1 and is further used as a weighting factor for the prior distribution.

The use of the weighting over the prior distribution should be seen as a diminution of confidence over the prior information for the θ_i parameter. Closer to 0 the value of K_i is, less confidence is assigned to the prior $\pi(\theta_i)$. Closer the value of K_i is to 1, the more confidence in the prior $\pi(\theta_i)$. Depending on the value of K_i, a new $\pi_K(\theta_i)$ is obtained and should replace the prior information $\pi(\theta_i)$ (equation 6).

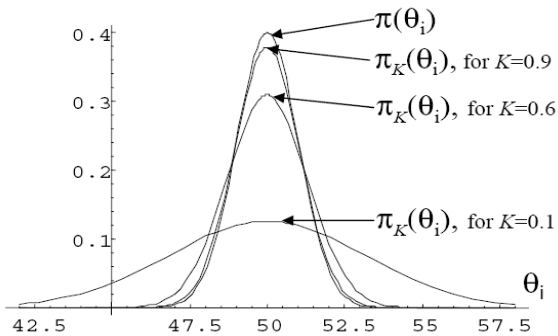

Fig.2. Weighted prior for different values of K

The weighted prior distribution $\pi_K(\theta_i)$ keeps the first order moment of the prior $\pi(\theta_i)$. On the contrary, in order to decrease the confidence we have in $\pi(\theta_i)$, we shall increase the value of the variance associated by $1/K_i$. So, the distribution is to be "stretched" more (K_i close to 0) or less (K_i close to 1), as in "Fig. 2".

IV. SVA MODEL APPLICATION

Research the compatibility between a priori knowledge and the likelihood consists in determining the factors K for each parameters of the SVA model. To do this, we determine the marginal functions (g_θ (θ)) associated with the parameters θ ($f_{\theta i}$ (θ_i) for the parameter θ_i) of model by:

$$f_{\theta_i}(\theta_i) = \frac{\int\limits_{j=1\ldots q,\, j\neq i} \ldots \int L(\theta_1,\ldots,\theta_q).\prod\limits_{j=1..q,\, j\neq i} d\theta_j}{\int\limits_{j=1\ldots q} \ldots \int L(\theta_1,\ldots,\theta_q).\prod\limits_{j=1..q} d\theta_j} \quad (13)$$

Then, we determine the factor $K_{\theta i}$ (method presented in Section III) by considering the marginal distributions $f_{\theta i}$ (θ_i) and a priori $\Pi_{\theta i}$ (θ_i). Once the factors $K_{\theta i}$ are estimated, we determine the prior distributions weighted according to the method detailed in the previous paragraph. This step is not systematic, because the incompatibility can be explained by other causes of a problem a priori. Thus, we obtain a new formulation of Bayesian inference:

$$\pi_{apo}(\theta / t_i) = \frac{L(t_1,\ldots,t_n / \theta).\prod\limits_{i=1}^{q} \pi\left(\theta_i, k_{\theta_i}\right)}{\int\limits_{\Omega} L(t_1,\ldots,t_n / \theta).\prod\limits_{i=1}^{q} \pi\left(\theta_i, k_{\theta_i}\right) d\theta_i} \quad (14)$$

with notations as L $(t_1, .., t_n / \theta)$ the likelihood (defined by equations 4), π_i $(\theta_i, K_{\theta i})$ prior distribution weighted for the parameter θ_i and π_{apo} (θ / t_i) function posteriori.

By the MAP method, we determine the parameters $\hat{\theta} = \left(\hat{\beta}, \hat{\gamma}_0, \hat{\gamma}_1\right)$. Then the matrix Fisher $I^{MAP}\left(\hat{\beta}, \hat{\gamma}_0, \hat{\gamma}_1\right)$ is determined for deduct the matrix variance-covariance $\sum\left(\hat{\beta}, \hat{\gamma}_0, \hat{\gamma}_1\right)$. Finally, we determine the reliability metrics and verify the qualification criteria.

In the case where the qualification criterion is not verified, we continue the initial test plan.

V. ROBUST TEST PLAN BASED ON COST PLANNING CRITERION

The objective of the accelerated testing plan optimization is to determine the test plan parameters (stress levels, sample size allocation at each stress level, …) minimizing the global cost as defined by the costs of testing and operation. This term allows us to introduce a robustness analysis according through an objective function.

$$C_{\text{global}} = C_{\text{testing}} + C_{\text{operation}} \quad (15)$$

The operation cost is defined by considering risk α in terms of target metric m_0. This term allows us to introduce a robustness analysis according to objective function.

When the test plan is defined, it is implemented and the results are collected to estimate the parameters of SVA model. This is then used to deduct the reliability metric and verifies the validity of the qualification criteria. As we precised previously, the qualification criteria is defined by:

$$\text{Prob}(m_\alpha \geq m_0) \leq \alpha \quad (16)$$

with m_0 the reliability metric target to hold, α risk fixed and m_α unilateral terminal.

The optimization problem is formulated as follows:

Search the plan parameters ω from plan chosen P (ω) such that:

Minimize C (ω, θ, m_α (γ, β), m_0)
under constraints:

$$g_i(\omega_i) \geq 0,\ i = 1, \ldots, r$$
$$\beta \in \pi(\beta)\ ;\ \gamma_l \in \pi_l(\gamma_l)\ l = 1, \ldots, m$$

Function g_i (ω_i) is the constraint function on the constraint ω_i. This formulation allows to fix the some parameters of the test plan and to take into account the possible field of SVA model parameters to obtain an optimal plan robust against of the actual reliability of the tested product.

The methodology employed to define the optimal test plan and robust consisted in defining the parameters of the plan guaranteeing the decision making as to qualification whatever the actual reliability of product to given risk α.

VI. DESIGN AND CONDUCTING TEST PLAN METHODOLOGY

The proposed methodology consists of defining a robust accelerated testing plan while considering an objective function based on economic approach, using Bayesian inference for optimizing the test plan, and taking into account the uncertainty on input data (reliability, activation energy …). This will produce a robust, optimal testing plan. To obtain the best test plan, we propose an optimization procedure using a specific genetic algorithm. This GA procedure allows us to improve best compromise test plans by searching the optimum with more freedom variables on ALT plan. The optimization process can be applying to conduct a test. It will allows to verify the compatibility of results with prior knowledge and reduce the censoring time in case of "Good results" while keeping the same level of risk [11].

During the analysis, it is possible to realize that a priori knowledge of the model parameters is not fully compatible with the test results [10]. Thus, it was defined a coefficient of compatibility between a priori and likelihood to measure the degree of coherence between the two sources of information (subjective and objective). This coefficient can be used to weight the a priori in order to give more weight to the data (likelihood) as presented in “section III”.

In cases where the actual reliability corresponds to expected, it is possible to adjudicate more quickly. Also, during the test it is possible to realize periodic estimation of the model parameters and reliability metrics to verify the qualification criteria.

The proposed method is decomposed in different steps as depicted in “Fig. 3”:

Definition of a robust accelerated testing plan
- Three accelerated constant stress levels
- Accelerated life testing model
- Optimization procedure and robustness analysis

↓

Consistency of prior knowledge
- Definition of reliability target
- Prior knowledge on product
- Likelihood provided by the data
- Prior validation

↓

Stopping criteria of test plan
- A weighting factor
- Estimation the posterior reliability metric
- Target risk
- The confidence level & stopping of test

Fig.3. Design test plan methodology

VII. DEFINITION OF DESIGN PROCESS

According to Figure 3, we define a design strategy to obtain the optimal test plan and to conduct the test based on following steps:

A. Information

- *Planning information (Identify and Design)*: a detailed test plan with determining of the type of stress, stress levels, number of units to be tested at each stress level,
- *Test Plan Optimization (Reliability models):* the determination of the mathematical model which best describes the system from which data are measured,
- *Verify system and robustness*: verify the compatibility of results with prior knowledge and reduce the censoring time in case of "Good results" while keeping the same level of risk (Possible when conducting a test),
- *New optimization of test plan*: optimal design to obtain the best parameter estimates.

B. Planning hypothesis and strategy

By planning information, we can plan and perform the test at the highest stress to quickly obtain failures so we have preliminary information on parameters, then we can plan the tests at lower stress levels.

This design strategy will conduct us towards sequential ALT planning. In sequential ALT planning, at the first we plan and run the test at the highest stress, then we verify the compatibility of results (Deduction of prior distributions), and finally we can obtain optimal test plan(Pre-Posterior Analysis and Optimization).

C. How will the testing phase take place?

To illustrate our purpose, we formalize the possible strategy on an example of constant ALT with three stress level. To conduct the test plan, several hypotheses can be taken to define the strategy of setting up testing as:

- A single test bench, Tests on different stress levels are performed one after the other.
- Conduct testing begins with the highest stress levels, and then reduced to the lowest stress levels. This allows observing the maximum of failure initially for a minimum time in order to verify compatibility. Here, there may be several strategies, but it seems to be the best.
- We exploit the intermediate results of the test, once the censoring time completed for a given stress level on a test bench.

According to the strategy defined, the operating sequence of test is represented in Figure. 4 :

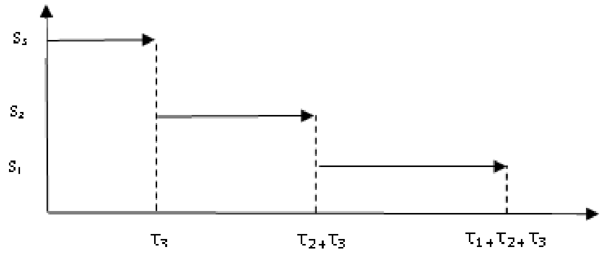

Fig.4. Operating sequence of test bench representing the stress levels as a function of time

VIII. RELIABILITY VERIFICATION TESTING

Reliability verification testing is almost always included as a quality specification for new products. Qualification includes all activities that ensure that the nominal design and manufacturing specifications will meet or exceed the desired reliability targets. Qualification validates the ability of the nominal design and manufacturing specifications of the product to meet the customer’s expectations, and assesses the probability of survival of the product over its complete life cycle. The purpose of qualification is to define the acceptable range of variability for all critical product parameters affected by design and manufacturing. Product attributes that are outside the acceptable ranges are termed defects, since they have the potential to compromise product reliability.

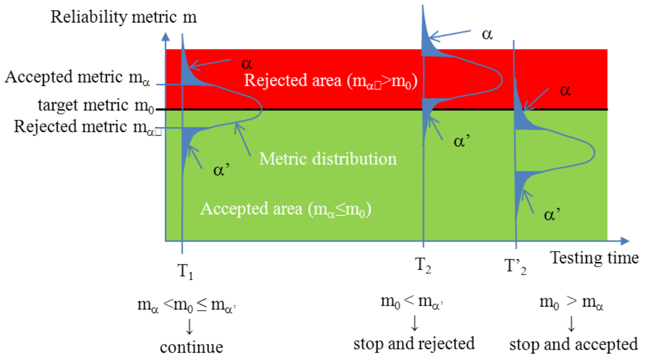

Fig.5. Rejected and acceptance criterion

A. Tests of Hypotheses

Consider the target reliability metric m_0, the test is defined by the hypotheses:

H_0: $m_0 < m_\alpha$

H_1: $m_0 > m_{\alpha'}$

With m_α and $m_{\alpha'}$ the unilateral bounds values estimated with testing results and in using Bayesian approach presented in sections II and III.

Thus these values represent:

m_α: The upper limit of reliability metric estimated for consumer's risk α.

$m_{\alpha'}$: The lower limit of reliability metric estimated for producer's risk α'.

With the estimation obtained on target metric m_0, the bounds m_α and $m_{\alpha'}$ of bilateral confidence interval allow to verify H_0 and H_1 with respectively $m_0 < m_\alpha$ and $m_0 > m_{\alpha'}$.

B. Sequential life testing

Related to operating sequence of test, we define a sequence of time observation (t_n) and the corresponding bounds sequence of bilateral confidence interval $(m_\alpha)_n$ and $(m_{\alpha'})_n$.

At each time t_n, we define acceptation region if $m_0 < m_\alpha$ and rejection region if $m_0 > m_{\alpha'}$ and we continue the test otherwise ($m_\alpha < m_0 < m_{\alpha'}$).

IX. Numerical Example

This example consider an electronic module for pump control that normally operates at 45°C.To estimate its reliability at the use condition, 50 units are to be tested at three elevated temperatures. The high one is 105°C, which is 5°C lower than the maximum allowable temperature. The censoring times are fixed for low, middle and high stress levels respectively at 1080, 600 and 380 hours. The unequal censoring times are considered to be fixed due to industrial constraints (test schedule and total test time fixed).

A Weibull distribution is considered to define the reliability and the Arrhenius model to describe the temperature effect on scale parameter. The SVA model is defined by :

$$R(t)=e^{-\left(\frac{t}{e^{\gamma_0+\gamma_1\frac{1}{273+T}}}\right)^{\beta}}$$

The reliability target is defined by the probability of failure equal to m_0 = 1%.

We use Yang's procedure to determine the best compromise test plans [13]. Thus we have a test plan as can be seen on the following table:

TABLE I. Design test Plan

	Group	1	2	3
Yang best compromise test plan	Number of Test Units	34	5	11
	Temperature(°C)	74	89	105

We define the interval values of prior knowledge for parameters of associated Normal distribution γ_0, γ_1, β as the following table:

TABLE II. Prior knowledge for example

β_{min}	$\hat{\beta}$	β_{max}	N (1.5, 0.03)
1.3	1.5	1.7	
$\gamma_{0\,min}$	$\hat{\gamma}_0$	$\gamma_{0\,max}$	N (-21.7, 1.7)
-31.65	-21.65	-11.65	
$\gamma_{1\,min}$	$\hat{\gamma}_1$	$\gamma_{1\,max}$	N (9980, 413.3)
7500	9980	12460	

We present different possible situations for our example in simulating test results by Monte Carlo.

Case 1: Acceptance test at the first step

The SVA parameters values selected for the simulation of the testing results are:

$$\gamma_0 = -21.65,\ \gamma_1 = 10000,\ \beta = 1.5$$

that they are the mean values of each parameter from a priori information.

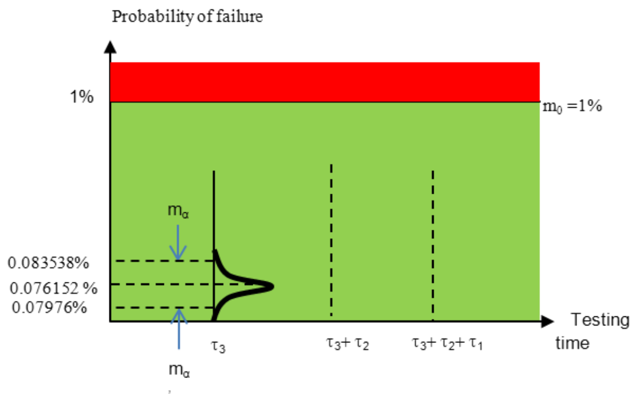

Fig.6. Simulation result in acceptance case at first level

The distribution shown in Figure 6 characterizes the probability of failure estimated from the simulated data and a priori information. The horizontal line at 1% marks an target value and we can find that the confidence interval bilateral symmetrical (at risk level $\alpha = \alpha' = 0.05$) drawn from the distribution of P_f is widely below the threshold. The lower bound allows in certain situations to reject the qualification. It is possible to construct a confidence interval dissymmetrical to take into account the values of different customer and supplier risk.

Tests are realized at the highest stress to cause rapidly of failures. In this case, we obtain early information of our parameters allowing an acceptance of product qualification manner anticipates. The anticipation of the qualification decision will allow preserving a part of the testing cost.

Case 2: Acceptance test in the second step

The initial values selected for the simulation of testing results are:

$$\gamma_0 = -27,\ \gamma_1 = 11500,\ \beta = 1.6$$

The Figure 7 presents the analysis result for two levels of step-stress test. The estimation m_α of failure probability at first level is upper than m_0. At second level, The estimation m_α is in acceptance area (green area).

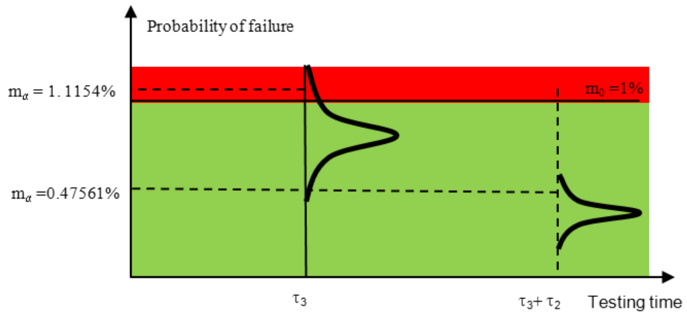

Fig.7. Simulation result in acceptance case at second level

In this case, we obtain early information of our parameters allowing an acceptance of product qualification manner anticipates at the second steps. The anticipation of the decision will again allow preserving a part of the testing cost.

Case 3: Rejection of the test in the third step

The initial values selected for the simulation of testing results are:

$$\gamma_0 = -27,\ \gamma_1 = 11000,\ \beta = 1.6$$

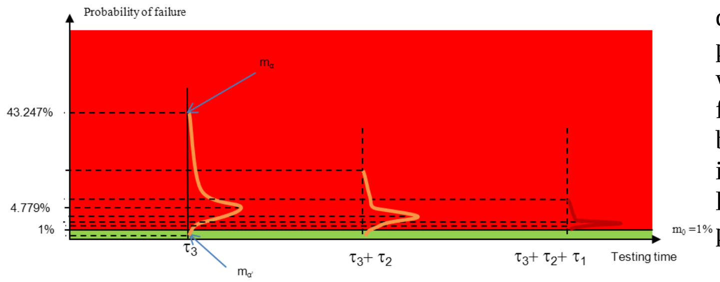

Fig.8. Simulation result in rejected case at third level

In this case, the intermediate analysis does not allow anticipating the result as to the qualification decision. An optimization of test plan in the second step would perhaps increase the number of elements tested at intermediate stress level to provoke failures more quickly and improve the precision of the estimation in the second steps.

Case 4: Acceptance test in the third step

The initial values selected for the simulation of the testing results are:

$$\gamma_0 = -26.2,\ \gamma_1 = 11000,\ \beta = 1.6$$

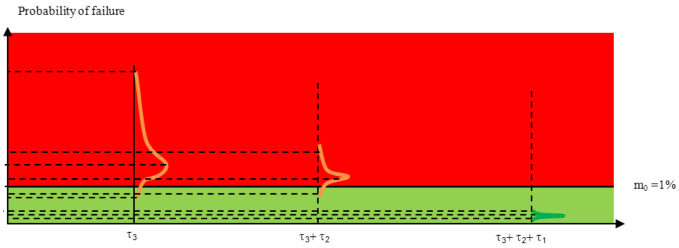

Fig.9. Simulation result in acceptance case at third level

The fourth case, as the previous shows the usefulness of the need for an optimization intermediary of the test plan.

By considering these four cases, it can be said in the first case we selected the initial values equal to mean values and we have $m_0 < m_\alpha$ thus, this will result the acceptation region. In the other three cases, the initial values are different with mean values. In the second and fourth cases, we have $m_\alpha < m_0$ and by continuing test $m_0 < m_\alpha$ that it will result acceptation region. In the third case, $m_\alpha < m_0 \leq m_{\alpha'}$ but by continuing test $m_0 > m_{\alpha'}$ and we have the rejection region. These results are summarized in following table.

TABLE III. Different possible situations for numerical example

Case1	Case2	Case3	Case4
Accept	Continuing test	Reject	Continuing test
$m_0 < m_\alpha$	$m_\alpha < m_0 \leq m_{\alpha'}$	$m_0 > m_{\alpha'}$	$m_\alpha < m_0 \leq m_{\alpha'}$

X. CONCLUSION

In this paper, we are interested in monitoring during of the realization of the tests. In fact, the methodology presented in the section VI can be reemployed to anticipate the qualification decision and improve the cost of testing. To permit this monitoring, the first we use a compatibility factor with a priori data. This factor is used to estimate the reliability for each step of the realization of the plan in consistency between the test data and a priori information. This is illustrated using an example with different cases of test data leading to different conclusions. In some cases (1 2), it is thus possible to make a decision qualifying fastest to backup the spending of test. In other cases, against, requires an intermediary improvement plan from a new optimization test plan.

Reliability is a characteristic inherent for any product or system and in today's world, it’s as one of the measurable quantities of design, construction and operation that in the process should be considered and controlled as an important criterion. Thus according to the previous works [5, 11] we design an optimal accelerated test plan and we were able to develop it. Now, we want to build a qualification plan a new product with a degree of innovation more or less important, it is possible that variability also relates to the choice of model. Also, it would be interesting to study the robustness integrating lack of knowledge of real models.

REFERENCES

[1] S. Voiculescu and F. Guerin, “Bayesian parameter estimation with prior weighting in ALT model”, Reliability and Maintainability Symposium, 2009. RAMS 2009. Annual 53 –58.

[2] P. Congdon, “Bayesian Statistical Modeling”, Wiley Series in Probability and Statistics, John Wiley & Sons, Chichester, 2001.

[3] M. Nikulin, L. Gerville-Réache and V. Couallier, “Statistique des essais accélérés”, Hermes Science, 2007.

[4] M. Manry, C.H. Hsieh, M. Dawson, A. Fung and S.J. Apollo, “Cramer Rao maximum a posteriori bounds on neural network training error for non-Gaussian signals and parameters”, International Journal of Intelligent Control and System,1996. 1, 381-391.

[5] S.Z. Fatemi, F. Guérin and L. Saintis, “Development of Optimal Accelerated Test Plan”, RAMS 2012.

[6] A. M. T. Ide, E. & Lannoy, “Estimation of a lifetime law for equipment on the basis of a highly right multicensored sample and expert assessments”, 2001.

[7] N. Bousqet, “Subjective bayesian statistics: agreement between prior and data”, Institut National De Recherche En Informatique Et En Automatique, Tech. Rep., 2006.

[8] E. Usureau, “Application des méthodes bayessiennes pour l’optimisation des cots de développement des produits nouveau”, PhD thesis, Institut des Sciences et Techniques de l’Ingénieur d’Angers, 2001.

[9] F. Guerin, B. Dumon, and E. Usureau, “New methodology of weighting of prior distribution used bayesian approach applied to reliability estimation”, Probability Method Conference, Los Angeles, USA, Jun. 2003.

[10] S. Voiculescu and F. Guérin, “Fiabilité des systèmes embarqués”, PhD thesis, Institut des Sciences et Techniques de l’Ingénieur d’Angers, 2009.

[11] S.Z. Fatemi, F. Guérin and L. Saintis, “Optimal accelerated test plan: optimization procedure using Genetic Algorithm” ALT 2012.

[12] A. Birolini, “Reliability Engineering, Theory and Pratice”, Fourth edition, Springer, 2004.

[13] G. Yang, “Life Cycle Reliability Engineering”, NJ, USA, John Wiley & Sons, Inc., Hoboken, 2007.