Multivariate SPC for Total Inertial Tolerancing

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Abstract **-** This paper presents a joint use of the T² chart and Total Inertial Tolerancing for process control. Here, we will show an application of these approaches in the case of the machining of mechanical workpieces using a cutting tool. When a cutting tool in machining impacts different manufactured dimensions of the workpiece, there is a correlation between these parameters when the cutting tool has maladjustment due to bad settings. Thanks to Total Inertial Steering, the correlation structure is known. This paper shows how T² charts allow one to take this correlation into account when detecting the maladjustment of the cutting tool. Then the Total Inertial Steering approach allows one to calculate the value of tool offsets in order to correct this maladjustment. We will present this approach using a simple theoretical example for ease of explanation.

Index Terms **-** Machining, adjustment, T², multivariate, inertial tolerancing

I. INTRODUCTION

The works presented in this paper are based on total inertial tolerancing proposed by Pillet [1] [2]. The objective of this paper is to propose a method to steer a machining process by minimizing the inertia of the surfaces. The proposed method is based on multivariate SPC.

For a given surface, the inertia is calculated using the vector of the deviations between the theoretical position of the surface and the actual position. Its calculation therefore requires several measured points on the surface and their deviations according the normal to the surface**.** The inertia[1] of a surface is calculated using the following relationship (equation 1).

$$I_i = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left(X_{ij} - \tau_j\right)^2} = \sqrt{\sigma_i^2 + \left(\overline{X_{ij} - \tau_j}\right)^2} \quad (1)$$

With

X_{ij}: Point j measured on the surface *i*

σ_i: standard deviation of the points measured on the surface *i*

τ_j: Target point *j* measured relative to the datum system of the part

n: number of points measured on the surface

I_i: surface inertia

Pillet [1] and Adragna [3] show that this measure of variability around the target provides a better representation of statistical behavior during assembly than the conventional zone specification.

The principle behind total inertial steering [4] is to establish a direct link between the parameter settings available on the machine (mainly tools offset) and the position of the points in a coordinate system associated with the machine.

It uses all the available information (the deviation on each measured point) directly. There is no "lost information" induced by the passage through a classic tolerancing method by lengths, diameters, angles, etc... This advantage allows one to obtain a level of accuracy superior to conventional approaches. The Total Inertial Steering approach minimizes the mean square deviations. By the calculating using all the measured points, total inertial steering is – by its very nature – a multidimensional approach.

Statistical process control has long been interested in the control of multidimensional processes. The best known approach is the one proposed by Hotelling which calculates the statistic T² [5]. This approach has been extensively commented on [6] [7], and several improvements have been put forward. Ghute and Shirke [8] have presented a multivariate synthetic control chart consisting of two sub-charts: a T² sub-chart and a CRL (Conforming Run Length) sub-chart. The CRL sub-chart improves the ARL (Average Run length). Champ and Aparisi [9] have proposed two double sampling (Hotelling’s T² charts). Aparisi and Deuna [10] have developed the synthetic T² control chart, which is compared to other control charts. It is shown that it performs consistently better than the T² chart. Boudaoud and Cherfi [11] propose a new statistic for monitoring multivariate trend processes. They focus on the choices of more sensitive statistics than the classical Hotelling T² statistic. The improvement is significant in the case of processes where incipient trends are considered.

This paper focuses on the interest of the Hotelling T² chart in the Total Inertial Tolerancing environment. The originality of this paper is to use the power of inertial steering which allows one to calculate an incidence matrix using the link between the tools offsets and their characteristics, in association with one Hotelling chart per tool offset in order to use the multidimensional information of the incidence matrix.

[1] Inertial tolerances are defined by the French standard XP E04-008 (2009)

II. TOTAL INERTIAL STEERING (TIS)

A. Example

The objective of any production process is to manufacture parts that conform to the requirements of the geometric specifications established by a CAD system (Computer-Aided Design)This requirement is materialized by a digital target that we specify using an acceptable level of variability (tolerances). As with any production processes that induce dispersions, the steering of machines is necessary to satisfy the required level of variability. The TIS approach is a tool that is able to reconcile the real workpiece to its digital model through the measured points on all the surfaces of the workpiece. Inertia is the quality indicator of the surface in inertial steering. Pillet [1] showed that mastering this inertia, allows us to control the process, because the inertia (equation 1) contains both the information concerning the dispersion and the decentering.

Fig. 1 shows a drawing of the finished part which is specified in inertial tolerancing. This example reminds us of the principle of inertial tolerancing.

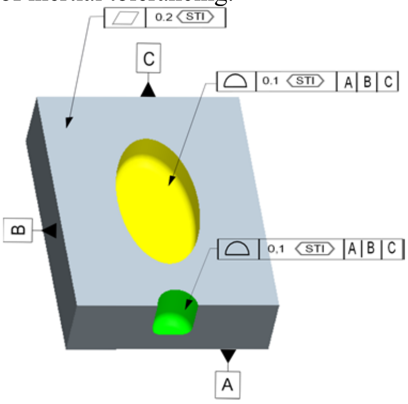

Fig.1. Inertial specification of the part

STI : Symbol of inertial tolerancing

We machine a block of rectangular material (dimensions 25mm x 20mm) on a CNC milling machine, which is fixed to the milling machine table. Three stops are used to position the slug on this table. A clamping system ensures it will not move during the various operations (see Figure 2).

We make an elliptical pocket in the slug by contour milling and create a notch using the same tool, which is a toric milling cutter. The elliptical pocket and the notch have the same inertial tolerance as shown in Figure 1.

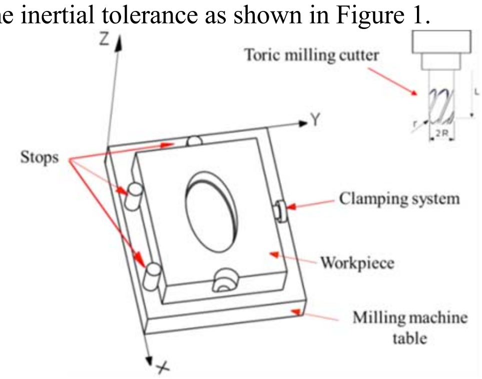

Fig.2. Defining of action axis of the tool – Machining assembly

We decided randomly to measure eleven points on the elliptical pocket (eight on its side (S1) and three on the bottom (S2) and three points on the notch S3 to set the tool (see Figure 3). The surfaces of the fillets generated by the radius at the end of the milling cutter and the bottom of the notch are not probed. If the fillets are not the right shape, the tool will be sharpened or replaced. The points on the notch are measured on its cylindrical portion to allow any repositioning of the notch relative to the ellipse. Fig. 3 shows the measured points on the workpiece.

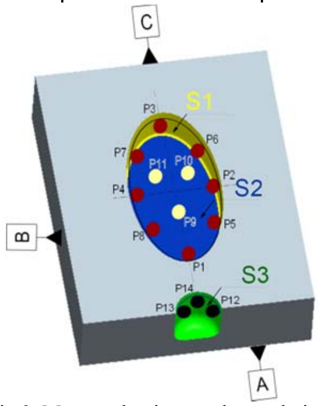

Fig.3. Measured points on the workpiece

Table I gives the coordinates of the points and of the normals vector expressed in the frame of reference of the part, and the deviations of these points along the local normal vectors. The objective of total inertial steering is to minimize the inertia of these deviations.

TABLE I. EXPRESSION OF THE POINTS IN THE REFERENCE PART

Surface	Inertial tolerance	Point	X	Y	Z	Nx	Ny	Nz	e (ni) = $\bar{X}_i$
S1	0.1	P1	20	10	5	-1	0	0	0.43
		P2	12.5	14	5	0	-1	0	-0.61
		P3	5	10	5	1	0	0	-0.44
		P4	12.5	6	5	0	1	0	0.30
		P5	17.8	12.83	5	-0.47	-0.88	0	-0.34
		P6	7.2	12.83	5	0.47	-0.88	0	-0.34
		P7	7.2	7.17	5	-0.47	0.88	0	0.25
		P8	17.8	7.17	5	0.47	0.88	0	0.26
S2	0.1	P9	15	10	3	0	0	1	-0.01
		P10	10	12.5	3	0	0	1	0.01
		P11	10	7.5	3	0	0	1	-0.03
S3	0.1	P12	24	12	4	-0.55	-0.835	0	-0.37
		P13	24	8	4	-0.55	0.835	0	0.56
		P14	22.5	10	4	1	0	0	-0.27

B. Incidence Matrix

Surfaces S1, S2 and S3 are generated using the same cutting tool. This cutting tool can be adjusted by acting on its tool length offset (L) along the Z axis and its tool radius offset (R). Parameters Tx, Ty and Rz are also corrected to enable any necessary repositioning of the shapes on their targets. The program variables of displacement are also used to rebalance the program relative to the workpiece.

Parameters L and R are the dimensional parameters. They are used to modify the dimension of the workpiece.

The displacement of each point can be calculated using the method of small displacements [12] so, by assuming the use of the small displacements methods in relation to the curvatures of the surface, it is possible to linearize the deviation to the point P_i with respect to its target surface towards its displacement, according to the equation 2:

$$ei = \xi i + aiTx + biTy + NiRz + ciL + R \qquad (2)$$

With:

ξi: initial deviations compared to the target points

ei: final deviations after correction

L: Tool length offset

R: Tool radius offset

Tx: X offset

Ty: Y offset

Rz: Z Rotation

ai, bi, ci: direction cosines of the normal $\overrightarrow{ni}$ to the target surface.

Ni: components on the X axis of the vector $\overrightarrow{OPi} \wedge \overrightarrow{ni}$.

If there are n points on the surface carried by the tool, we obtain a system of n equations where the variables are the parameters of the movement of the tool and which can be written in the following matrix form (equation 3):

$$[a](C) + (-e) = -(\xi) \qquad (3)$$

where C is the vector of correctors.

The matrix [a] is called the incidence matrix because it contains the influence coefficients of the corrections L, R, Tx, Ty and Rz on the deviations of the points. The incidence matrix is given in Table II.

TABLE II. INCIDENCE MATRIX CALCULATED IN THE REFERENCE MACHINE

	Tool offsets				
Point	L	R	Tx	Ty	Rz
P1	0	1	-1	0	10
P2	0	1	0	-1	-12.5
P3	0	1	1	0	-10
P4	0	1	0	1	12.5
P5	0	1	-0.47	-0.88	-9.63
P6	0	1	0.47	-0.88	-12.37
P7	0	1	-0.47	0.88	9.7
P8	0	1	0.47	0.88	12.3
P9	1	0	0	0	0
P10	1	0	0	0	0
P11	1	0	0	0	0
P12	0	1	-0.55	-0.835	-13.44
P13	0	1	-0.55	0.835	24.44
P14	0	1	1	0	-10

C. Steering Matrix

The originality of the method proposed is to calculate a T^2 chart for each tool offset. The T^2 chart is built using the non-null term of the incidence matrix for the column associated with the tool offset. If the tool is not on the target, all the characteristics concerned by the tool are probably decentered. The incidence matrix includes this information and the weight of the tool's impact. Thanks to this way of proceeding, the T^2 chart reduced to the implied characteristics is an excellent way to detect a deviation.

The objective of setting the machine is to bring the points on their target positions. It is necessary to calculate the displacement of the tool, i.e. the parameters *Tx, Ty, Rz, L* and *R* that minimize the sum of the squares of the new deviations *ei*. This type of calculation, called multiple linear regression, consists in multiplying the matrix of the initial deviations *(ξi)* by the well-known Gauss pseudo-inverse matrix *[a*]* of the incidence matrix (see equation 4):

$$(C) = [a^*].(\xi) \qquad (4)$$

$$With \;\; [a^*] = ([a]^T.[a])^{-1}.[a]^T$$

By minimizing the sum of the squared deviations, we find the corrections which minimize the inertia of the surface calculated by the equation 1 immediately. The matrix [a*] is called the steering matrix. It is given in Table III.

TABLE III. STEERING MATRIX [a*]

	Point													
Tool offsets	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14
L	0	0	0	0	0	0	0	0	0.33	0.33	0.33	0	0	0
R	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0	0	0	0.09	0.09	0.09
Tx	-0.14	0.09	0.14	-0.09	-0.12	0.29	-0.4	0.24	0	0	0	-0.36	0.21	0.14
Ty	-0.15	-0.32	0.15	0.32	-0.1	-0.47	0.68	-0.11	0	0	0	0.3	-0.47	0.15
Rz	0.01	0.01	-0.01	-0.01	0	0.02	-0.04	0.02	0	0	0	-0.03	0.04	-0.01

III. THE IN/OFF CONTROL IN TIS

A. The IN/OFF control in TIS

The principle behind Statistical Process Control is to dissociate two situations: a process that is "under control" from one that is "out of control". How can these two situations be separated? How can we determine if the deviations measured on the surfaces are the expression of random variations or if these deviations require us to intervene as in the settings?

The monitoring process using control charts is traditionally described in two phases.

Phase I: Control charts are used to test retrospectively whether or not the process was in control when the first subgroups were measured.

Phase II: Control charts are used for testing whether the process remains in control when future subgroups will be measured.

In TIS, these two phases use different controls charts:

Phase I: Shewhart charts are used to identify the short term standard deviation for each point. Montgomery [13] and Pillet [14] describe this phase.

Phase II: A T^2 Chart is used for testing whether the process remains in control. All the improvements proposed for T^2 charts that were presented in the literature review can be used. Only the basic T^2 chart is presented in this paper.

B. The T^2 Chart

For each measured point, the short-term deviation is calculated. There may be significant differences between the short-term standard deviations. To deal with this kind of data, the multidimensional analysis method chosen must take into account the variability in each different direction. The T^2 Chart method is well-suited for such calculations.

Assuming that the vector x follows a p-dimensional normal distribution, denoted as $Np(\mu_0, \Sigma_0)$, that there are m samples each of size $n \geq 1$ available from the process and that the vector observations X are not time dependent, a control chart can be based on the sequence of the following statistic (equation 5):

$$D_i^2 = n(\bar{X}_i - \mu_0)^t \Sigma_0^{-1}(\bar{X}_i - \mu_0) \quad (5)$$

where

n: The sample size

$\bar{X}_i$: The vector of the sample averages of the i^{th} rational subgroup

μ_0: The known vector of means

Σ_0: The known variance–covariance matrix

The D_i^2 statistic represents the weighted distance (Mahalanobis distance) of any point from the target μ_0. The D_i^2 statistic follows a χ^2 - distribution with p degrees of freedom.

If μ_0 is replaced by $\bar{\bar{X}}_0$, and Σ_0 is replaced by $[\bar{S}]$, and $\bar{X}_i$ is the mean of the i^{th} rational subgroup then, according to Ryan [15], the $D_i^2/co(p,m,n)$ statistic follows an F-distribution with p and $(m*n - m - p + 1)$ degrees of freedom.

where:

$Co(p,m,n) = [p(m-1)(n-1)](m*n - m - p + 1)$

$\bar{\bar{X}}_0$: The overall sample mean vector

$[\bar{S}]$ The pooled sample variance–covariance matrix

$(\bar{S}_{ii} = (\bar{R}_i/d_2)^2$

$\bar{R}$: Moving range

d_2: constant from the range distribution

Thus, a multivariate Shewhart control chart for the process mean, with unknown parameters, is based on the following statistical relation (equation 6):

$$T_i^2 = n(\bar{X}_i - \bar{\bar{X}}_0)^t[\bar{S}^{-1}](\bar{X}_i - \bar{\bar{X}}_0) \quad (6)$$

The control limit in phase II is:

$$Lu = \frac{p(m+1)(n-1)}{nm - m - p + 1} F_\alpha, p, nm - m - p + 1$$

C. Adaptated T^2 Chart for TIS – Phase I

In TIS, phase I is used to calculate the pooled sample variance for each point $\bar{S}_i$ and calculate the $\bar{\bar{X}}_0$ vector. The pooled sample is calculated using the traditional Shewhart chart. Table IV shows the variance for the example.

TABLE IV. RESULT OF PHASE I IN THE EXAMPLE

Surface	Inertial Tolerance	Point	$\bar{\bar{X}}_0$	$\bar{S}_i$
S1	0.1	P1	-0.0120	0.0705
		P2	-0.0312	0.0708
		P3	0.0139	0.0703
		P4	0.0212	0.0703
		P5	-0.0273	0.0686
		P6	0.0429	0.0686
		P7	-0.0068	0.0702
		P8	-0.0193	0.0705
S2	0.1	P9	0.0372	0.071
		P10	0.0231	0.0692
		P11	-0.0603	0.0712
S3	0.1	P12	0.0325	0.0713
		P13	0.0138	0.0703
		P14	-0.0276	0.0695

Assuming that:

1. When the process is in control the variation on each point is purely random.
2. The correlation between different points is the consequence of a decentering of a tool offset.
3. $\bar{\bar{X}}_0$ Represents the best adjustment possible with the Tool offset.

we will suppose that the $[\bar{S}]$ Matrix is diagonal. The appearance of a correlation structure is a symptom of the need to adjust a tool offset.

For each tool offset j, it is possible to identify the variance matrix $[S_{TOj}]$ from the Incidence Matrix [a] and the $\bar{S}_i$ Vector. The Null column and row are removed.

$$[S_{TOj}] = \begin{bmatrix} \bar{S}_1^2 & 0 & 0 & 0 & 0 \\ 0 & ... & 0 & 0 & 0 \\ 0 & 0 & \bar{S}_i^2 & 0 & 0 \\ 0 & 0 & 0 & ... & 0 \\ 0 & 0 & 0 & 0 & \bar{S}_{1n}^2 \end{bmatrix}$$

For the T_Y tool offset, the variance matrix is given in Table V.

Ideally, $\bar{\bar{X}}_0$ is a null vector. It means that all the points are in the exact theoretical position. However, some deviation cannot be eliminated with the tools offset available. Some deviation cannot be corrected by tool offset (such as metal deformation, for example). Then a null vector increases the T^2 Value because the deviations vector includes deformation that is impossible to correct.

TABLE V. VARIANCE MATRIX S_{TY}

Variance Matrix S_{TY}								
	P2	P4	P5	P6	P7	P8	P12	P13
P2	0.005	0	0	0	0	0	0	0
P4	0	0.0049	0	0	0	0	0	0
P5	0	0	0.0047	0	0	0	0	0
P6	0	0	0	0.0047	0	0	0	0
P7	0	0	0	0	0.0049	0	0	0
P8	0	0	0	0	0	0.005	0	0
P12	0	0	0	0	0	0	0.0051	0
P13	0	0	0	0	0	0	0	0.0049

$\bar{\bar{X}}_0$ is calculated using the following relation (equation 7), which stems from the possible best fit:

$$\bar{\bar{X}}_0 = \overline{(\xi) - [a](C)} = \bar{e} \qquad (7)$$

i.e. the average residue after adjustment in Phase I. Table IV shows the vector $\bar{\bar{X}}_0$ for the example.

TABLE VI. EXAMPLE FOR THE T² CALCULATION FOR THE T_Y TOOL OFFSET

		$\bar{X}_i - \bar{\bar{X}}_0$				
Surface	Point	1	2	3	4	5
S1	P2	0.031	0.136	-0.030	-0.045	-0.163
	P4	-0.088	0.038	0.090	0.031	0.099
	P5	0.107	0.055	-0.025	-0.184	-0.033
	P6	-0.024	-0.125	-0.079	-0.166	-0.293
	P7	0.051	0.013	-0.053	-0.038	0.117
	P8	0.139	0.099	0.021	0.100	0.151
S3	P12	-0.147	-0.056	-0.098	-0.117	-0.123
	P13	-0.051	0.062	0.180	0.149	0.155
	T²	26.99	22.72	24.74	46.17	81.81

D. Adapted T² Chart for TIS – Phase II

In phase II, for each samples i and each tool offset j, the T_{ij}^2 statistic is calculated by equation 6. If the T_{ij}^2 is upper Lu, an "Off control" situation is detected and a correction is calculated by equation 4. This calculation is illustrated from the mean vector for the T_Y tool offset in Table VI.

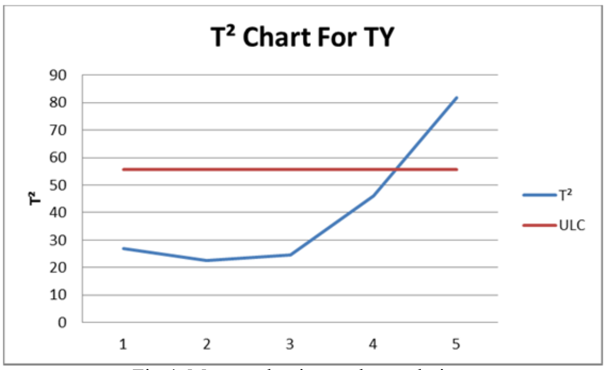

Fig.4. Measured point on the workpiece

With the data in Table VI, where $\alpha=0.0027$, $n = 2$, $p=8$ and $m=25$, the Lu limit is 55.66. The T² Chart is given in Figure 4.

The 5th sample gives a T^2 upper than the Lu limit, an adjustment on the Ty is necessary.

Same calculations are made on the other tool offset. For the 4th sample (Table VII), the T^2 for each tool offset is presented in Table VIII.

TABLE VII. VECTOR $\bar{X}_i - 5^{th}$ SAMPLE

Vector $\bar{X}_i - \bar{\bar{X}}_0 - 5^{th}$ sample						
Point 1	Point 2	Point 3	Point 4	Point 5	Point 6	Point 7
0.035	-0.163	-0.083	0.099	-0.033	-0.293	0.117
Point 8	Point 9	Point 10	Point 11	Point 12	Point 13	Point 14
0.151	-0.022	-0.043	0.084	-0.123	0.155	-0.028

TABLE VIII. EXAMPLE FOR THE T² CALCULATION

Offset	L	R	Tx	Ty	Rz
p	3	11	9	8	11
Lu	21.53	93.6	66.0	55.7	93.6
T²	3.75	85.32	70.82	81.81	85.32
Situation	OK	OK	KO	KO	OK

The tool offset adjustment is calculated by equation 4 from the vector ξi and the matrix $a*$ reduced to the "Off control" Tool Offset (T_X, T_Y).

TABLE IX. STEERING MATRIX [a*] REDUCE FOR T_X T_Y

-0.22	0.00	0.22	0.00	-0.10	0.10	-0.10	0.10	0.00	0.00	0.00	-0.12	-0.12	0.22
0.00	-0.15	0.00	0.15	-0.14	-0.14	0.14	0.14	0.00	0.00	0.00	-0.13	0.13	0.00

The best fit is given by equation 4:

Table X gives the values of the Tx and Ty tool offsets.

TABLE X. TX AND TY VALUE

Tool Offset	Adjustment
T_X	0.06
T_Y	-0.156

The expected situation after adjustment is given by equation 7. Figure 5 shows the adjustment for each point.

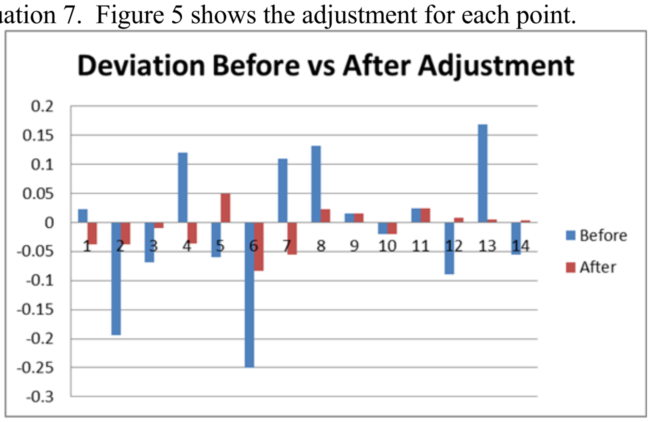

Fig.5. Deviations before and after adjustment

IV. Discussion & Conclusion

Total Inertial Steering brings a solution to carry out statistical process control without a parameterization by lengths, diameters, angles, etc... The information used is the root information: the deviation from the target surface. The paper shows the way to calculate the best adjustment directly from the root information, and to identify the situations needing adjustment thanks to the use of a T^2 chart.

In example used in this paper, 14 points are measured on the workpiece. In real-life cases, the number of measured points runs into hundreds or even, thousands. It is impossible to use an individual control chart for each point. With the T^2 control chart, the number of control charts is limited and equal to the number of tools offset. Each control chart is calculated using a high quantity of data. Thus, the precision of the steering is very high.

The process described in this paper needs to use a very large matrix with dynamic calculations of an a* matrix. However, even if the dimensions of the matrix are expressed in thousands of lines, the calculation is instantaneous with modern-day computers.

T^2 Control charts use incidence matrix information. This matrix gives the correlation structure which can be used to detect maladjustment. By using the raw information for the deviation and the expected correlation structure given by the incidence matrix, Total Inertial Steering and T^2 Charts offer a very efficient method to guarantee very high quality in 3D workpiece machining.

Many improvements in the method presented could be made, beginning with looking at the different possible evolutions of the T^2 Chart in a TIS environment.

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