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*"Risk aversion and multiple breakeven analysis"*

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**RISK AVERSION AND  
MULTIPLE BREAKEVEN ANALYSIS**

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This article focuses on the breakeven point problem for multiproduct firms under conditions of uncertainty. Our aim is to show that this question could be tackled following two distinct methodologies; the first one consists in introducing the manager's attitude toward risk and the second, more simple, is inspired by a portfolio approach. In this latter case, it will be shown that the breakeven point does only represent a lower bound of the efficient set. In the last part of our study, a numerical example is provided to show the practical implications of our reasoning.

## INTRODUCTION

The breakeven point problem holds an important place in managerial accounting literature. From a theoretical point of view, the question has been thoroughly debated in accounting research, from the work of CHARNES, COOPER, IJIRI [1963] to the stochastic formulation developed by CHEN [1980].

Two aspects of the question will be discussed : first, the number of different goods produced by the firm and second, the context in which the firm operates, either stochastic or deterministic. The deterministic, single product model is certainly the oldest and most widely taught in management accounting courses. Literature about single product models in uncertainty is also important but less abundant. Authors like ADAR, BARNEA, LEV [1977], LIAO [1975] and SHIH [1979] are those who most recently addressed that particular topic. The literature focusing on the multiproduct breakeven point in a deterministic context is represented by CHARNES, COOPER, IJIRI [1963]. Finally, the multiproduct breakeven point under uncertainty has, to our knowledge, only been treated by CONSTANTINIDES, IJIRI, LETTCH [1978], JOHNSON, SIMICK [1974] and CHEN [1980].

Our study extends ADAR, BARNEA and LEV's single product model to a multiproduct model and shows the relation between this model and CHEN's. We suggest two alternative formulations. One of these is characterised by known distributions and is assuming normality of the distributions described by their first two moments. In our further models, we will show that the search for the breakeven point can be seen as a portfolio analysis and thus, the case of a decision maker presenting a particular level of risk aversion can be considered. Furthermore, we will focus on the fact that any product mix exceeding the breakeven conditions can be a valid choice as long as the associated risk is also considered. In section 1, we will compare the single product situation with the multiproduct one and clearly show their different natures. Section two develops our two formulations of the multiproducts problem and we conclude our study with a numerical example.

A. FROM A SINGLE PRODUCT TO A MULTIPRODUCT FORMULATION

In order to present the problem we will initially suppose that we have only one product.  
We have :

- $a_i$  : Variable cost by unit,  $i=1,$
- $a_0$  : Total fixed cost,
- $C$  : Total variable and fixed costs,
- $R$  : Total earnings,
- $y_i$  : Sales volume in units,  $i=1,$
- $p_i$  : Selling price unit,  $i=1.$

Total earnings and costs are :

$$R = p_i y_i$$

$$C = a_i y_i + a_0$$

Total production in order to fully cover total fix costs by gross margin is determined as follows :

$$y_i = \frac{a_0}{(p_i - a_i)} \quad (1)$$

Let us now suppose that  $i=1, \dots, n$ ; the relation (1) is transformed into :

$$\sum_{i=1}^n (p_i - a_i) y_i = a_0 \quad (2)$$

This equation can have an infinity of solutions, one of which consists in laying down the  $(n-1)$  variables and solving the problem for the  $n$ th variable. As we know, the breakeven point is reached at the particular production mix for which revenues are covering charges. Since this procedure is totally arbitrary, we have to consider the question differently. In fact, the breakeven point seems to be unique only in the marginal case of single product firms.

One should also be aware that equations 1 and 2 could be seen as constraints rather than goals. Indeed, if we assume that the goal of the decision maker is to maximize the value of the firm, knowing the characteristics of the product mix for which earnings do not exist is

$\begin{aligned} & \underset{y_i}{\text{maximize}} \sum_{i=1}^n p_i y_i \\ \text{SB : } & \sum_{i=1}^n (p_i - a_i) y_i = a_0 \\ & y_i \geq 0, \quad i=1, \dots, n \end{aligned}$	$\begin{aligned} & \underset{y_i}{\text{maximize}} \sum_{i=1}^n (p_i - a_i) y_i \\ \text{SB : } & \sum_{i=1}^n (p_i - a_i) y_i = a_0 \\ & y_i \geq 0, \quad i=1, \dots, n \end{aligned}$	$\begin{aligned} & \underset{y_i}{\text{minimize}} \sum_{i=1}^n a_i y_i \\ \text{SB : } & \sum_{i=1}^n (p_i - a_i) y_i = a_0 \\ & y_i \geq 0, \quad i=1, \dots, n \end{aligned}$
3 (A)	3 (B)	3 (C)

With model (A), the goal is to maximize turnover or market share. Using model (B), the firm tries to maximize its gross margin, and model (C) has the objective of minimizing total variable costs. Logically, in order to determine a production vector  $y$  with only positive signs we have to introduce other constraints in each model. The number of constraints to introduce would be at least equivalent to the number of variables and would be specific to each model. It is interesting to note that these three models could be analyzed using the integer approach developed by CHEN [1983], WILSON [1986] and MOLE [1986]. However, these approaches are not considered as useful for solving our problem. In fact, there is no general formulation for the problem of multiple breakeven point in a deterministic context other than the framework established by the above models.

## B. MULTIPLE BREAKEVEN POINT UNDER UNCERTAINTY

To introduce uncertainty in the problem it is necessary to consider the managerial opportunities. Thus, with the certainty models we have only one solution but not information on the quality of these solutions and on their potential realization.

Uncertainty in breakeven point analysis can be done based on several methodologies: first,  $\tilde{y}_i$  and  $\tilde{p}_i$  are stated as random variables with underlying distributions, and their product describes the distribution on net profit; or  $\tilde{p}_i$  is the only random variable while  $\tilde{y}_i$  remain unknown and to be optimized.

The first methodology has been used by many other authors such as JOHNSON, SIMIC [1971], KOTTAS, LAU [1978], SHIH [1979] and LIAO [1975]. In this context, three questions could be asked: first, what is the nature of the  $p_i$  and  $y_i$ 's distributions; second, what would be the distribution of net profit; and finally, what information is given to the manager about these distributions. Without any doubt, if the first two distributions are stated as normal, the product won't qualify as normal. Thus, the information given to the manager won't be useful.

What information can be drawn from a distribution with unknown properties which cannot be defined by its two first moments ? The interesting difference stated by SHIH between demand and production level is useful only in the case of normal distributions and for single product firms; otherwise, we have to refer to equation (2).

ADAR, BARNEA, LEV and CHEN [1980] were more concerned with searching the nature of net profit distribution, considering the vector related to the variables  $y_i$  and  $p_i$  depending on regular distributions qualified by their two first moments. ADAR, BARNEA and LEV treated the single product case without taking into account the manager's attitude toward risk. To extend their model to multiproduct cases, we have to introduce positivity constraints on  $y_i$ , otherwise some solutions could lead to negative values. Furthermore, the above authors' use of breakeven point in investment analysis seems more or less realistic given the fact that the breakeven point is a short term instrument. Thus, it appears impossible to modify the production mix or to create a new mix for products to be developed in a short period of time.

CHEN's model [1980] analyzes a multiproduct case with breakeven and other technical constraints. The model maximizes either  $k_1$ , greater than total fix costs ( $a_0$ ) or  $(\alpha_1)$ , greater than a profit goal or, finally, total gross margin. The author also demonstrates that these models could be applied to a mean-variance framework but only for the gross margin alternative. However, distributions remain normal and stochastic programming problems seem difficult to solve.

The importance of these two models is not to be denied given the fact that they allow us to analyze the following new ideas :

- the introduction of the manager's attitude toward risk in defining the problem.
- the breakeven point is no longer an objective but rather a simple constraint to be satisfied.
- any solution in an "efficient set" greater than the breakeven point could be accepted as long as the manager is ready to assume the risk involved.

In order to study the breakeven point question in a mean-variance framework, we can use two approaches : either stating the problem starting with a clear definition of the utility function of the manager, or stating the problem directly using the mean-variance model. Thus, we will state two alternative formulations of the problem, considering first of all the case of a manager influenced by risk aversion. The second formulation will not make any reference to the manager's attitude toward risk.



$$\text{Total sale: } \sum_{i=1}^n p_i y_i$$

where  $y_i$  is the volume of each product and  $\tilde{p}_i$  is the unit selling price following a normal distribution (mean :  $\bar{p}_i$ , variance :  $\tau_i^2$ ). If we identify the profit;  $\tilde{W}$ , difference between the two flows of the firm as :

$$\tilde{W} = \sum_{i=1}^n (\tilde{p}_i - a_i) y_i - a_0$$

and if the utility function of the manager is :

$$U[\tilde{W}] = e^{-\delta \tilde{W}}, \delta > 0$$

the manager would normally try to maximize the expected utility of the firm which can be illustrated as :

$$E[U(\tilde{W})] = E(\tilde{W}) + \frac{\delta}{2} \text{Var}(\tilde{W})$$

We will then have :

$$\left[ \begin{array}{l} \text{Maximize} \\ y_i \end{array} \right. \left. \begin{array}{l} \sum_{i=1}^n (\bar{p}_i - a_i) y_i - a_0 - \frac{\delta}{2} \left( \sum_{i=1}^n \sum_{j=1}^n \tau_{ij} y_i y_j \right) \\ \text{SB : } \sum_{i=1}^n (\bar{p}_i - a_i) y_i \geq a_0 \quad (5.1) \\ \sum_{i=1}^n y_i = N \quad (5.2) \\ y_i \geq 0, i = 1, \dots, n. \quad (5.3) \end{array} \right. \quad (5)$$

Constraint (1) relates to a budgeting constraint to attain, where the goal be total fixed cost while constraint (5.2) shows total capacity. As for the other constraint, it simply strengthens the realism of the solutions.

At the same time, we could define the breakeven point problem considering the following relation :

$$r_i = \left( \frac{p_i - a_i}{a_i} \right) \quad (6)$$

which illustrates the return of product  $i$ . The relation (6) would then be treated as a random variable  $\tilde{r}_i$  (mean :  $\bar{r}_i$ , variance  $\hat{\tau}_i^2$ .) Let's now define :

$y_i$  = quantities of the product  $i$

$N$  = Total production (already known)

$x_i$  = product in percentage of total production :

$$x_i = \frac{y_i}{N}$$

$R$  = total return expected by the manager.

We can now define a variation of the MARKOWITZ model [1959] as follows :

$$\left[ \begin{array}{l} \text{Minimize}_{x_i} \sum_{i=1}^n \sum_{j=1}^n \hat{\tau}_{ij} x_i x_j \\ \text{SB : } \sum_{i=1}^n \bar{r}_i x_i = R \quad (7.1) \\ \sum_{i=1}^n \bar{r}_i \bar{a}_i x_i \geq \frac{a_0}{N} \quad (7.2) \\ \sum_{i=1}^n x_i = 1 \quad (7.3) \\ x_i \geq 0 \quad \forall i = 1, \dots, n. \end{array} \right. \quad (7)$$

Equation (7.1) shows the profit objective while equation (7.2) is the breakeven point constraint more traditionally written as :

$$\sum (p_i - a_i) y_i \geq a_0$$

The two preceding models (6) and (7) have their good and bad points. Model (6) explicitly introduces attitude toward risk with the  $\delta$  parameter but  $\tilde{p}_i$  has normal distribution and can then be negative. Inversely, model (7) allows us to use a greater range of distributions but doesn't take into account the risk parameter. In order to introduce this parameter, we would have to use FARRAR's model [1962] written as :

$$\begin{aligned}
 \text{SB : } & \sum_{i=1}^n \bar{r}_i \bar{a}_i x_i \geq \frac{a_0}{N} & (8) \\
 & \sum_{i=1}^n x_i \leq 1 \\
 & x_i \geq 0 \quad \forall \quad i = 1, \dots, n.
 \end{aligned}$$

$\delta$  is introduced to take care of risk aversion. However, if  $\delta$  is chosen in the range  $[0, \alpha]$ , the solution will be found in the efficient set defined by the model 7. We remark that model (7) is closely related to CHEN's [1980] although easier to solve.

### C- A STUDY OF THE MODELS' PROPERTIES THROUGH A SIMULATION

In order to test the preceding models, we now develop a practical application using the case of a firm manufacturing three products (1, 2, 3) within three periods of time. We are preparing the budget for the fourth period.

Period	Selling price			Variable cost			Fix cost	return		
	1	2	3	1	2	3		1	2	3
1	3	2	2	2.5	1	1.5	50	-	-	-
2	3	3	1	2	2	2	50	-	-	-
3	4	4	2	3	2.5	2.5	50	-	-	-
p	3.33	3	1.66				--	-	-	-
a	--	-	--	2.5	1.83	1.33	--	-	-	-
r	--	-	--	-	--	--	--	.34	.7	.22
N	50						--	-	-	-

TABLE I : Basic data

The variance-covariance matrix for prices and returns is show as :

$$\tau^2 = \begin{bmatrix} .22 & .33 & .11 \\ .33 & .66 & 0 \\ .11 & 0 & .22 \end{bmatrix} \quad \hat{\tau}^2 = \begin{bmatrix} .015 & -.024 & -.017 \\ -.024 & .046 & .022 \\ -.017 & .022 & .024 \end{bmatrix}$$

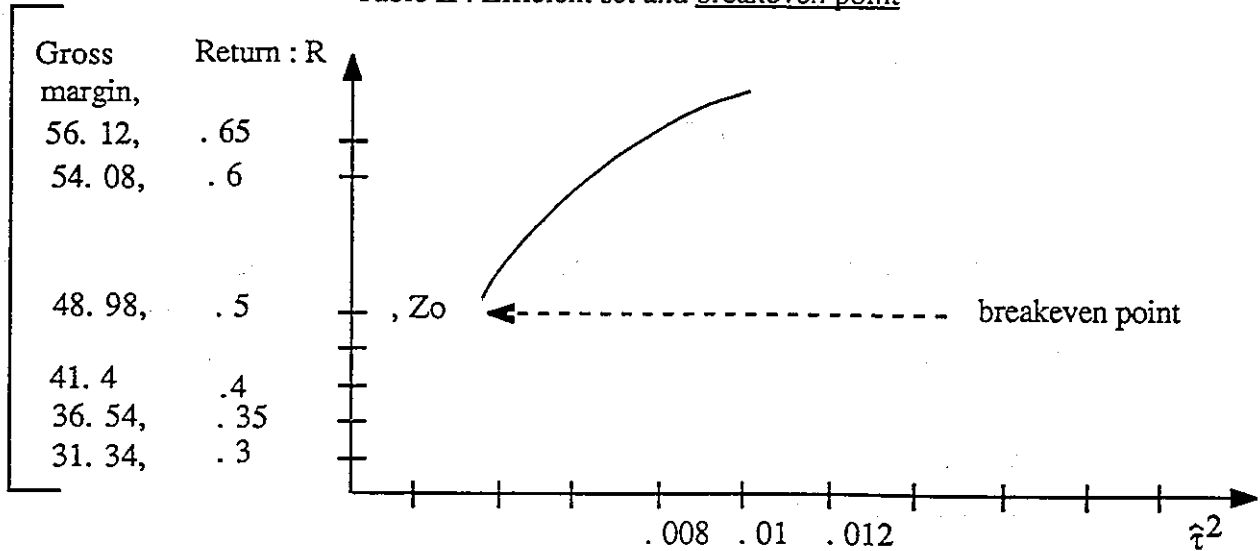
Let us first of all try to solve the problem using model (7) definition :

$$\left[ \begin{array}{l} \text{Minimize} \quad .015 x_1^2 + 0.46x_2^2 + .024x_3^2 - .048x_1x_2 - .034x_1x_3 + 0.44x_2x_3 \\ x_1, x_2, x_3 \\ \text{sb :} \quad .34x_1 + .7x_2 + .22x_3 = R \\ \quad .85x_1 + 1.28x_2 + .2926x_3 \geq 1 \\ \quad x_1 + x_2 + x_3 = 1 \\ \quad x_1, x_2, x_3 \geq 0 \end{array} \right. \quad (9)$$

If we assume that the solutions are found in the range [0.22, 0.7], we will generate all the solutions and analyze them in relation to the breakeven point constraint. Theses solutions can be found using the reduced gradient algorithm credited to WOLF [1963].

rendement	x1	x2	x3	$\hat{\tau}^2$	Fix cost	y1	y2	y3	$r_i a_i x_i$
.30	.55	.03	.42	.0083	50	28	1	21	.63
.35	.60	.12	.28	.0059	50	30	6	14	.74
.4	.66	.21	.13	.0035	50	33	10	7	1.86
.5	.55	.45	0	.0018	50	28	22	0	1.04*
.6	.27	.73	0	.0155	50	13	37	0	1.16
.65	.14	.86	0	.028	50	7	43	0	1.22

Table II : Efficient set and breakeven point



This model can be more detailed by adding more constraints concerning production capacity but would then be more difficult to solve.

We will use the same data to illustrate model (6) which will be written as :

$$\begin{array}{l}
 \text{Maximize}_{y_1, y_2, y_3} .83y_1 + 1.17y_2 + .33y_3 - 50 - \\
 \frac{\delta}{2} [.22y_1^2 + .66y_1^2 + .22y_3^2 + .66y_1y_2 + .22y_1y_3] \\
 \text{sb : } .83y_1 + 1.17y_2 + .33y_3 \geq 50 \\
 y_1 + y_2 + y_3 = 50 \\
 y_1, y_2, y_3 \geq 0
 \end{array} \quad (10)$$

In order to resolve this problem, we can apply WOLFE's procedure to a maximization case. To stress on the risk aversion parameter, we proceed in two steps; first we develop a solution for a risk parameter equal to 1. Second we produce a simulation for several risk levels.

Table III, give the first step, with a fixed cost of 50, profit goal between 0 to 6.2 and  $\delta = 1$ .

Fixed cost	Projet	y <sub>1</sub>	y <sub>2</sub>	y <sub>3</sub>	$\tau^2$
50	0	25	25	0	962.5
50	1	22	28	0	1029.14
50	2	19	31	0	1099.5
50	3	16	34	0	1173.7
50	4	13	37	0	1252.8
50	5	10	40	0	1333.6
50	6	7	43	0	1419

Table III : Breakeven point and expected utility

Following This table, we may conclude that the two models are equivalent, because they present the same set of solutions . Futhermore, with this results, it is not possible to distinguish the impact of risk parameter on the decision maker.

If we want to point out the risk level, on the production mix, we must define the first constraint in inequality as :

$$.83y_1 + 1.17y_2 + .33y_3 \geq 50 \quad (11)$$

		PROFIT																			
		0			2			4			6										
1		100	1000	2000	1	100	1000	2000	1	100	1000	2000	1	100	1000	2000	1	100	1000	2000	
24.9		24.3	17.2	12.7	14	18.5	13.3	11.8	13.2	12.6	6.7	8.5	7.3	6.8	4.7	4.6					
25		25.4	29.8	32.8	30.8	31.2	34.5	35.7	36.7	37.1	40.9	40.2	42.6	42.9	44.5	44.6					
.1		.3	3	3.5	.2	.2	.2	9.5	.1	.3	1.3	1.3	.1	.2	.8	.8					
962		966	1005	1042	1099	1099	1103	1164	1251	1256	1313	1313	1419	1424	1456	1456					

Table IV, Profit, portfolio and risk level.

level and goal profit. Generally if we increase risk parameter,  $y_3$  with the smallest variance, increase his weight in the portfolio, but his weight is balanced by a high profit goal. In fact, if we search high profit, the risk parameter must be high to influence the solution. At the opposite, if the decision maker is near the breakeven point (corresponding to a fixed charge of 50), the risk of the portfolio is not really sensible at the risk parameter.

We may interpret,  $\delta$  using CHEN's (1980) analysis with (0,2). We define  $\delta/2$  as the probability of reaching the breakeven point. In order, to find CHEN's results, we apply the expected utility model using  $\delta/2 = .84$ . But with this restrictive definition, the risk parameter presents a poor influence on the portfolio. Following this, if it is near 0 the solution is only defined by the profit goal, which minimizes the risk part of the function. For a practical purpose, our approach presents another degree of freedom for the decision maker which is the risk parameter level.

On the level of production, for models (9) and (10) solutions found are perfectly equivalent. This leads us to say that model (7) is more interesting because it allows us to ignore a priori distributions of the variables. We could then use limited distributions, like Pearson's  $\beta$  law, which requires minimal basic information.

#### D - FINAL REMARKS

Our aim was to find breakeven point solutions for multiproduct firms. In deterministic cases, we were able to use one of the three models proposed in paragraph 2 and add other technical constraints. For stochastic cases, it seems that from a conceptual point of view our variation of MARKOWITZ's model is simpler to use than the model using risk aversion. But from a conceptual point of view, it is important to state that the breakeven point, whether it be deterministic or stochastic, leads us to look for minimal levels of production to achieve in order to cover total fixed costs. A situation suitable only for non-profit organizations or public institutions.

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