Resolution of Nonlinear Magnetostatic Problems With a Volume Integral Method Using the Magnetic Scalar Potential

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An integral method using the magnetic scalar potential to solve nonlinear magnetostatic problems is developed. This method uses the range interactions between magnetizable elements and it is particularly well suited to compute field in the air domain which do not need to be meshed. The collocation and Galerkin approaches are presented and compared to solve the nonlinear magnetostatic equation.

Both methods need the construction of full interaction matrices which may be computed with analytical formulae. A Newton-Raphson method, in which the interaction matrix must be built at each solver iteration, is used to solve the nonlinear formulation. A modified fixed point scheme, in which the interaction matrix is built only once, is also proposed. 3-D numerical examples are given and results of the different methods are compared.

Index Terms—Integral method, magnetostatics, nonlinear.

I. INTRODUCTION

For devices with a huge volume free space compared to the active structure or high ratio between the object geometries, the finite element method leads to problems of accuracy and convergence to the solution [1]. In such case, integral methods can be attractive alternatives [2], [3]. Integral formulations of the magnetostatic field problems are particularly advantageous for the numerical solution of open-boundary problems which include ferromagnetic materials since only the active regions containing these materials need to be discretized. This paper gives in the first part a magnetostatic formulation in the framework of the integral volume methods in which the collocation and Galerkin methods are used to solve the magnetostatic equations. The computation of the source potential and of the integrals to build the interaction matrix is presented in this part. The second part proposes the Newton-Raphson and modified fixed point method to solve the nonlinear formulation. The last one presents the results obtained for different applications.

II. VOLUME INTEGRAL METHOD

A. Formulation

Let us consider the following magnetostatic problem. A three-dimensional simply connected region $\Omega_f$ is filled with isotropic ferromagnetic material with the known linear magnetic susceptibility $\chi$. Primary sources of magnetic field in which currents flows are associated to the region $\Omega_f$. Both regions, $\Omega_f$ and $\Omega_j$, are disposed in free space $\Omega_0$ so that these regions do not overlap, $\Omega = \Omega_f \cup \Omega_j \cup \Omega_0$. We note $\Gamma$ the boundary between the ferromagnetic material $\Omega_f$ and the free space $\Omega_0$.

The ferromagnetic behavior law is defined by

$$\mathbf{M}(\mathbf{r}) = \chi(\mathbf{r}, \mathbf{H})\mathbf{H}(\mathbf{r})$$

(1)

where $\mathbf{M}(\mathbf{r})$ is the magnetization, $\mathbf{H}(\mathbf{r})$ the magnetic field at point of coordinates $\mathbf{r}$. At any point of $\Omega_f$, the magnetic field is a sum of the reduced magnetic field created by the ferromagnetic material $\mathbf{H}_{\text{rec}}(\mathbf{r})$ and the magnetic source field created by currents flows $\mathbf{H}_0(\mathbf{r})$:

$$\mathbf{H}(\mathbf{r}) = \mathbf{H}_{\text{rec}}(\mathbf{r}) + \mathbf{H}_0(\mathbf{r}).$$

(2)

The simply connected region $\Omega_f$, containing no current sources, the following volume integral equation [4] using the total scalar potential $\Phi$ is considered:

$$\Phi(\mathbf{r}) + \frac{1}{4\pi} \int_{\Omega_f} \chi(\mathbf{r}', \mathbf{H}) \nabla \Phi(\mathbf{r}') \cdot (\mathbf{r} - \mathbf{r}') / |\mathbf{r} - \mathbf{r}'|^3 \, d\Omega' = \Phi_0(\mathbf{r})$$

(3)

where $\Phi_0$ is the scalar potential deriving whose the magnetic field $\mathbf{H}_0$ produced by sources derives. Only the domain $\Omega_f$ is meshed and the potential $\Phi$ is approximated with first order nodal shape functions:

$$\Phi \approx \sum \Phi_i \alpha_i$$

(4)

where $\Phi_i$ and $\alpha_i$ are respectively the degree of freedom of $\Phi$ and the first order nodal shape function associated to the $i$th mesh node.

Both resolution methods are used to solve (3). The first one is a collocation method at mesh nodes and it leads to solve a system of algebraic equations of the form

$$([I] + [A])\Phi = \Phi_0$$

(5)

with

$$I_{ij} = \delta_{ij}$$

(6)

$$A_{ij} = \frac{1}{4\pi} \int_{\Omega_f} \chi(\mathbf{r}', \mathbf{H}) \nabla \alpha_j(\mathbf{r}') \cdot (\mathbf{r} - \mathbf{r}') / |\mathbf{r} - \mathbf{r}'|^3 \, d\Omega'$$

(7)

The weak formulation associated to (3) is

$$\int_{\Omega_f} \alpha \Phi_0(\mathbf{r}) \, d\Omega = \int_{\Omega_f} \alpha \Phi(\mathbf{r}) \, d\Omega + \frac{1}{4\pi} \int_{\Omega_f} \alpha \int_{\Omega_f} \chi(\mathbf{r}', \mathbf{H}) \nabla \Phi(\mathbf{r}') \cdot (\mathbf{r} - \mathbf{r}') / |\mathbf{r} - \mathbf{r}'|^3 \, d\Omega' \, d\Omega, \forall \alpha.$$  

(8)

The second method is a Galerkin’s method. Applied to the weak formulation (8) it leads to the algebraic equation system:

$$([H] + [C])\Phi = \mathbf{d}$$

(9)
Once the problem solved, the magnetic field in the free space is computed from the following relation:

\[ B_{ij} = -\int_{\Omega_i} \alpha_i(r) \alpha_j(r) \, d\Omega \]  

\[ C_{ij} = \frac{1}{4\pi} \int_{\Omega_i} \int_{\Omega_j} \chi(r', \mathbf{H}) \frac{\nabla \alpha_i(r') \cdot (r_i - r')}{|r_i - r'|^3} \, d\Omega' \, d\Omega \]  

\[ D_i = \int_{\Omega_i} \alpha_i(r) \Phi_0(r) \, d\Omega. \]  

Once the problem solved, the magnetic field in the free space is computed from the following relation:

\[ \mathbf{H}(r) = \frac{1}{4\pi} \sum_{r \in \Omega_i} \nabla \left( \int_{\Omega_i} \frac{\mathbf{M}_e \cdot (r - r')}{|r - r'|^3} \, d\Omega_e \right) + \mathbf{H}_0(r) \]  

where \( \mathbf{M}_e \) is the magnetization and \( \Omega_e \) the domain of the \( e \)th mesh element.

**B. Computation of the Source Potential**

The source field \( \mathbf{H}_0 \) can be expressed analytically for some coil geometries ([5] for example). Thus there are no direct analytical expressions for the source potential \( \Phi_0 \). A numerical computation is envisaged and the following minimization problem is considered:

\[ \min_{\Phi_0} (|\mathbf{H}_0 + \nabla \Phi_0|^2). \]  

The weak formulation associated to the minimization problem (14) is

\[ \int_{\Omega_f} \nabla \mathbf{a} \cdot \nabla \Phi_0(r) \, d\Omega = \int_{\Omega_f} \nabla \mathbf{a} \cdot \mathbf{H}_0 \, d\Omega \quad \forall \mathbf{a}. \]  

A finite element method is used on the weak form (15) and it leads to the resolution of an algebraic equation system:

\[ |F| \Phi_0 = F' \]  

with

\[ F_{ij} = \int_{\Omega_f} \nabla \alpha_i \cdot \nabla \alpha_j \, d\Omega \]  

\[ F_i = -\int_{\Omega_f} \nabla \alpha_i \cdot \mathbf{H}_0 \, d\Omega. \]  

This method needs a finite element method applied only to the active materials. However the computation of the source field at Gauss points could be expensive. A fast multipole method could so be used [6].

**C. Integrals Computation**

The (7), (11) and (13) need the computation of integrals with Green’s kernels. These integrations take the form

\[ I_1(r) = \int_{\Omega_{\epsilon}} \frac{\mathbf{S} \cdot (r - r')}{|r - r'|^3} \, d\Omega' \]  

\[ I_2(r) = \nabla \int_{\Omega_{\epsilon}} \frac{\mathbf{S} \cdot (r - r')}{|r - r'|^3} \, d\Omega' \]  

where \( \mathbf{S} \) is an uniform field on the domain \( \Omega_{\epsilon} \).

Using a vector identity and the Green-Ostrogradsky’s theorem, (19) and (20) can be written as

\[ I_1(r) = \int_{\Omega_{\epsilon}} \frac{\mathbf{S} \cdot \mathbf{n}}{|r - r'|^3} \, d\Gamma' \]  

\[ I_2(r) = \int_{\Omega_{\epsilon}} \frac{(\mathbf{S} \cdot \mathbf{n})(r - r')}{|r - r'|^3} \, d\Gamma'. \]  

A classic approach to compute the previous integrals is a numerical integration with Gauss points. An alternative is to use analytical formulas. Several analytical expressions of the integrals (21) and (22) can be found in the literature for many geometries elements. Thereafter let us consider the analytical expression developed in [7] for any tetrahedral meshes. With (21) and (22) the domain integration can be reduced to the mesh faces.

When the computation point is too close to the integration domain, too many Gauss points are necessary to compute the integration of the Green’s kernels with a good accuracy. Then analytical expressions are required. With the collocation approach, the assemblage of the interaction matrix \( |A| \) of (9) uses an analytical expressions when the computation point is a node of the element integration. Otherwise a numerical integration with Gauss points is used. With the Galerkin approach, the assemblage of the interaction matrix \( |C'| \) of the (9) has two integrals. The first one uses a numerical integration with Gauss points. The second with the Green’s kernel uses the analytical expression when the computation point is a Gauss point which is contained in the integration element. Otherwise a numerical integration with Gauss points is used. Four Gauss points are used on a tetrahedral mesh for numerical integrations with both approaches.

**III. NONLINEAR FORMULATION**

In a general case, the ferromagnetic behavior law of the material is nonlinear. A suitable solver is then required to solve the nonlinear formulation. Newton Raphson and modified fixed point schemes are investigated. In this section, let us consider the collocation approach and the GMRES method is used to solve the matrix systems.

**A. Newton-Raphson Scheme**

The Newton-Raphson scheme is a method to compute the zero of a function. A description of the method could easily be...
found in the literature. The considered function coming from the
behavior law (1) is
\[ F(\chi) = \chi(\mathbf{H}) - \frac{\mathbf{M}[\mathbf{H}]}{\mathbf{H}}. \] (23)
The following equation must be solved at each iteration:
\[ \Phi^k(\mathbf{r}) + \frac{1}{4\pi} \int_{\Omega_f} \chi(\mathbf{H}) \left[ \mathbf{S}[\mathbf{r}', \mathbf{H}^{k-1}] \cdot \frac{(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} \right] \mathbf{d}\Omega' = \Phi_0(\mathbf{r}) \] (24)
where \( k \) is the iteration number. This scheme needs the com-
putation of the interaction matrix at each iteration due to the local
change of magnetic susceptibility in (24). Moreover the com-
putation of the function derivative could be expensive.

**B. Modified Fixed Point Scheme**

The behavior law (1) is written
\[ \mathbf{M}(\mathbf{r}, \mathbf{H}) = \chi_{FP} \mathbf{H}(\mathbf{r}) + \mathbf{S}(\mathbf{r}, \mathbf{H}) \] (25)
where \( \chi_{FP} \) is the constant slope of the modified point scheme
and \( \mathbf{S} \) the nonlinear residual. The choice of the slope depends on
the extreme values of the susceptibility \( \chi_{\text{min}} \) and \( \chi_{\text{max}} \) [8]. The
optimal slope \( \chi_{\text{opt}} \) can be evaluated by [9]
\[ \chi_{\text{opt}} = \frac{\chi_{\text{min}} + \chi_{\text{max}}}{2}. \] (26)
The fixed point can be found by iteratively updating the non-
linear residual. Using (25) in (3), the following equation must be
solved at each iteration:
\[ \Phi^k(\mathbf{r}) + \frac{1}{4\pi} \int_{\Omega_f} \chi_{FP} \left[ \frac{\nabla \chi(\mathbf{H}) \cdot (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} \right] \mathbf{d}\Omega' = \Phi_0(\mathbf{r}) + \frac{1}{4\pi} \int_{\Omega_f} \mathbf{S}(\mathbf{r}', \mathbf{H}^{k-1}) \cdot \frac{(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} \mathbf{d}\Omega' \] (27)
where \( k \) is the iteration number. The previous methods (5) and
(9) are used to solve (27) at each iteration. The new value of the
residual \( \mathbf{S} \) at iteration \( k \) is given by
\[ \mathbf{S}^{k+1} = (\mathbf{M}[\mathbf{H}^k] - \chi_{FP}[\mathbf{H}^k]). \] (28)
The algorithm is stopped when the norm of the difference of
magnetic susceptibility between two iterations is lower than a
criterion. This scheme enables the computation of a single in-
teraction matrix during all the resolution process.

**IV. APPLICATIONS**

**A. Steel Plate Problem**

A steel plate with a linear behavior law is placed in a mag-
netic field generated by a coil (Fig. 2). The results obtained by
the volume integral method with collocation and Galerkin ap-
proaches are compared with those obtained by the finite element
method using the software Flux3D® [10]. The evolutions of av-
erage flux density of the magnetic induction along the
axis on the plate are computed with the three methods [Fig. 3(a)].
The relative difference with the FEM results are also presented
in Fig. 3(b).

Both methods give comparable results to the finite element
method. The Galerkin’s approach seems undervalued the
field. An assumption is that the use of Gauss points for the first inte-
gral (11) are not able to compute with accuracy the decreasing of
the integral with the Green’s kernel (11) when both integration
domains are too close. Nevertheless, the collocation is less accu-
rate on the geometry singularities due to the collocation method
at nodes. The assemblage of the interaction matrix with colloca-
tion method is much faster than with Galerkin’s one. That is why
the collocation approach is preferred in order to model more
complex applications like in the next parts.

**B. Contactor-Like Problem**

Let us consider the contactor-like problem described by
Fig. 4(a). The ferromagnetic material has a nonlinear behavior
law with an arctangent form presented in Fig. 4(b). The volume
integral method with a collocation approach (3) is used to solve
the magnetostatic field distribution. The previous modified
fixed point and the Newton-Raphson methods are used to solve
the nonlinear formulation. The value of 550 evaluated by (26)
is assigned to the slope \( \chi_{FP} \) to ensure a good convergence with
the modified fixed point method. The computed magnetostatic
field distribution are presented Fig. 5. The performance of both
methods to solve the nonlinear formulation are compared in
Table I.

In this problem the material is few saturated so the resolu-
tion with the modified fixed point method converges relatively
are compared. The results show a significant deviation between the simulation and experimental results. However the team problem 13 is a benchmark in which existing codes can give significantly different results [11]. That is why the results are hopeful considering the poor quality of the mesh. Compression techniques used on the interaction matrix would increase the number of processable elements.

V. CONCLUSION

A volume integral method using the magnetic scalar potential to solve nonlinear magnetostatic problems has been developed. The collocation and Galerkin’s methods are presented to solve the equations. Both need to take care of the assembly of interaction matrices. The collocation method is preferred in regard to necessary time for matrix assembly. The resolution of the nonlinear formulation using the modified fixed point scheme required a unique assembly of the interaction matrix. So, for any problems with a weakly saturated material, the modified fixed point scheme gives faster resolution than the classic Newton-Raphson one in which the interaction matrix is built at each iteration. For complex problems, compression and acceleration are required to make the volume integral method an efficient method to solve magnetostatic problems without the constraint of the air mesh.

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