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Separating expansion and collapse in general fluid models with heat flux

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In this letter we consider spherically symmetric general fluids with heat flux and give the appropriate set of conditions that define the divide between expansion and collapse in the cases in which heat flux and its evolution vanish at the separating surface. We extend previous works with a fully non-linear analysis in the 3 + 1 splitting, and present gauge-invariant results. The definition of the separating surface is inspired by the conservation of the Misner-Sharp mass, and is obtained by generalizing the Tolman-Oppenheimer-Volkoff equilibrium and turnaround conditions. We emphasize the non-local character of these conditions as found in previous works and discuss connections to the phenomena of spacetime cracking and thermal peeling.

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I. INTRODUCTION

The influence of cosmic expansion on local gravitational structures, or lack thereof, is a long-standing argument of modern gravitation theory, rooted in the works of McVittie and Einstein and Straus [1, 2] (see Ref. [3] for more historical material). It is related to the general problem of assessing the influence of global physics onto local physics [4, 5] and to the Machian local inertia (see e.g. Refs. [6, 7]). Brans-Dicke theory [8–10] was indeed produced from these questions.

Einstein-Straus-type models, in particular, have been used to study the influence of cosmic expansion on the solar system [2, 3]. However, these models have been shown to have a limited scope [11, 12]. Moreover, they involve matching two metrics, which introduces technical challenges [13] and tends to hide difficulties at the junction.

The approach we adopt consists in describing the local region embedded in an expanding cosmology using a single metric. A classic example of this viewpoint is provided by the McVittie solution [1], and the more recent framework of Lasky and Lun [14–16] using generalized Painlevé-Gullstrand (GPG) coordinates provides an adequate formalism for this endeavor.

We follow here recent works where a 3 + 1 spacetime splitting has been used in order to investigate the existence of shells, called *separating shells*, which separate local systems (possibly collapsing) from global expanding regions [17–19].

In particular, we extend the treatment that led to gauge-invariant conditions obtained relating the intrinsic spatial curvature of the separating shells not only to the Misner-Sharp mass [20], but also to a function of the pressure that generalizes the Tolman-Oppenheimer-Volkoff (TOV) relation of hydrostatic equilibrium [21]. We enlarge the validity of previous definitions by generalizing the conditions that relate the existence of a shell separating an expanding outer region from an inner region that allows collapse towards the center of symmetry by requiring: (i) that there be no matter exchange across the shell, and (ii) that the generalized TOV equation be satisfied on that shell, thus ensuring a sort of equilibrium [17, 19]. Previously, models involving perfect fluids [17] and imperfect fluids with anisotropic stress [19] have been considered. Here we give a more complete description of such models by introducing heat flux and studying its role on the existence of separating shells.

In our notation, Latin indices run from 0 to 3, and κ^2 is the usual gravitational coupling.

II. 3 + 1 SPLITTING AND GAUGE-INVARIANT KINEMATICAL QUANTITIES

We present the basic equations in GPG coordinates adapting the works of Lasky and Lun [15, 16] to follow collapse within an underlying overall expansion.

A. Metric and ADM splitting

The spherically-symmetric line element writes

$$ds^2 = -\alpha^2 dt^2 + \frac{(\beta dt + dR)^2}{1 + E} + r^2 d\Omega^2, \quad (1)$$

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with $d\Omega^2 \equiv d\theta^2 + \sin^2\theta d\phi^2$ and α, β, E and r functions of t and R . Notice that the areal radius r differs, in principle, from the R coordinate to account for additional degrees of freedom that are required to cope with a general fluid which includes both anisotropic stress and heat flux. The flow of the fluid is characterized by the timelike normalized vector $u_a \equiv -\alpha t_{,a} = -\alpha \delta_{a,t}$, with $u_a u^a = -1$, which defines a preferred timelike direction, suggesting a 3 + 1 ADM splitting [22] with lapse function $N = \alpha$, and the projection operator $h^{ab} \equiv g^{ab} + u^a u^b$ that determines the 3-metric on surfaces normal to the flow. We define the proper-time derivative along the flow of any tensor X^a_b as $\dot{X}^a_b \equiv u^c X^a_{b;c}$. The covariant derivative of the flow defines kinematics, and in general it can be decomposed as $u_{a;b} \equiv -u_b \dot{u}_a + \frac{1}{3} \Theta h_{ab} + \sigma_{ab} + \omega_{ab}$, whose trace gives the expansion Θ , while the symmetric traceless part gives the shear σ_{ab} . The skew-symmetric vorticity ω_{ab} vanishes due to spherical symmetry.

The energy-momentum tensor describing the fluid is now

$$T_{ab} = \left(\rho + \frac{\Lambda}{8\pi} \right) u_a u_b + \left(P - \frac{\Lambda}{8\pi} \right) h_{ab} + 2q_{(a} u_{b)} + \Pi_{ab}, \quad (2)$$

where ρ is the energy density, P the pressure, q_a the heat flux, Π_{ab} the anisotropic stress tensor and Λ the cosmological constant. By definition of heat flux, we have $q^a u_a = 0$, so we can write $q^a = q n^a = q(t, R) \sqrt{1+E} \delta_R^a$, where n^a is the spherically-symmetric unit vector in the direction orthogonal to the flow. Moreover, the anisotropic stress tensor satisfies $\Pi^{ab} u_b = 0$ and $\Pi^a_a = 0$, and spherical symmetry implies all projected tensors are proportional to the traceless eigenprojector $P_{ab} \equiv h_{ab} - h^c_{u_a} h^a_{u_b}$, and so they can be defined by their tangential eigenvalues.

For the Lie derivative of scalars X along u , the relations $\mathcal{L}_u X = u^a \partial_a X = \dot{X} = \frac{1}{\alpha} \partial_t X - \frac{\beta}{\alpha} \partial_R X$ hold, and from now on we keep the Lie notation and use $X' = \partial_R X$. In particular, we note that the generalized expansion function $\mathcal{H} \equiv \frac{\Theta}{3} + \sigma = \frac{\mathcal{L}_u r}{r}$, where σ is the tangential eigenvalue of the shear.

B. Field equations

By using the same projections of Einstein's field equations (EFE) as in Ref. [19], we obtain the same forms for the expansion and shear propagations, as well as for the constraint on the Weyl tensor; the latter is induced by the difference in the shear propagation obtained directly from the EFE and Ricci identities. However, the Weyl evolution equation, which comes from the flow-projected Bianchi identity, is modified by the introduction of heat flux, and generalizes the form presented in Ref. [19] as

$$\begin{aligned} \mathcal{L}_u \left(\Xi + \frac{\kappa^2}{2} \Pi \right) &= -\frac{\kappa^2}{2} \frac{q}{\mathcal{A}} \mathcal{L}_u \sigma \\ &- \frac{\kappa^2}{2} \left\{ (\rho + P - 2\Pi) \sigma - q \left[\frac{\mathcal{A}}{3} - \frac{\sigma}{\mathcal{A}} (\Theta - \mathcal{H}) \right] \right\} \\ &- \left[2 \left(\Xi + \frac{\kappa^2}{2} \Pi \right) + \left(\Xi - \frac{\kappa^2}{2} \Pi \right) \right] \mathcal{H}. \quad (3) \end{aligned}$$

Here, Ξ and Π are the tangential eigenvalues of the electric part of the Weyl tensor E_{ab} and the anisotropic stress Π_{ab} , respectively, and $\mathcal{A} \equiv \dot{u}^a n_a = \frac{\alpha'}{\alpha} \sqrt{1+E}$ is the projection of the acceleration \dot{u}^a along the unit vector orthogonal to the flow.

The cross-projection of the EFE provides what we call the radial balance constraint

$$\mathcal{H}' + (3\mathcal{H} - \Theta) \frac{r'}{r} = \frac{\kappa^2}{2} \frac{q}{\sqrt{1+E}}, \quad (4)$$

and the orthogonal projections of the Bianchi identities provide the tidal force constraints [16]

$$\frac{\kappa^2}{6} \rho' + \frac{\left[\left(\Xi + \frac{\kappa^2}{2} \Pi \right) r^3 \right]'}{r^3} = \frac{\kappa^2}{2} \frac{q}{\sqrt{1+E}} \mathcal{H}, \quad (5)$$

whereas the Hamiltonian constraint remains as in Ref. [19]. The Bianchi identities give the energy density conservation

$$\mathcal{L}_u \rho = -\Theta(\rho + P) - 6\Pi \sigma - q \sqrt{1+E} \left[\ln(qr^2 \alpha^2) \right]', \quad (6)$$

and heat flux conservation, projected along its own direction

$$\begin{aligned} \mathcal{L}_u q + 2[\Theta - \mathcal{H}]q &= -(\rho + P - 2\Pi) \mathcal{A} \\ &+ \sqrt{1+E} \left[6\Pi \frac{r'}{r} - (P - 2\Pi)' \right], \quad (7) \end{aligned}$$

which plays the role of the heat transport equation in Ref. [23].

In the presence of heat flux, the radial behavior of the Misner-Sharp mass M changes, following Ref. [16], to include the corresponding energy exchanges

$$M' = 4\pi \left(\rho r' + \frac{q}{\sqrt{1+E}} \mathcal{L}_u r \right) r^2, \quad (8)$$

while its definition remains the same. The areal radius velocity and acceleration are derived in the same fashion as in Ref. [19]; however, the perfect-fluid-analogous field equations are modified into

$$r' \mathcal{L}_u E = 2(1+E) \left[\frac{\kappa^2}{2} \frac{q}{\sqrt{1+E}} r - (\mathcal{L}_u r)' - \frac{\beta'}{\alpha} r' \right], \quad (9)$$

$$\mathcal{L}_u M = -\frac{\kappa^2}{2} r^2 \left[(P - 2\Pi) \mathcal{L}_u r + q \sqrt{1+E} r' \right]. \quad (10)$$

Extracting α'/α from Eq. (7), using the definition of \mathcal{A} and assuming a non-vanishing relativistic inertial mass $\rho + P - 2\Pi \neq 0$ (note that a negative value, corresponding to a phantom-like fluid, could in principle be considered), we can rewrite the function gTOV from Ref. [19] into

$$\begin{aligned} \text{gTOV} &\equiv -\mathcal{L}_u^2 r = \frac{M}{r^2} + 4\pi(P - 2\Pi)r + \frac{(1+E)r'}{\rho + P - 2\Pi} \\ &\times \left[(P - 2\Pi)' - 6\Pi \frac{r'}{r} + \frac{\mathcal{L}_u q + 2(\Theta - \mathcal{H})q}{\sqrt{1+E}} \right] - \frac{\Lambda}{3} r, \quad (11) \end{aligned}$$

which provides a general-relativistic generalization of the Navier-Stokes equation. Setting gTOV and $\mathcal{L}_u r$ to zero reproduces the Tolman-Oppenheimer-Volkoff (TOV) equilibrium equation. Eq. (11) shows explicitly the influence of q in the hydrodynamic balance.

III. EXISTENCE OF A SEPARATING SHELL

We give the conditions for separating shells to exist in the presence of heat flux, discuss their non-locality connected to the Misner-Sharp mass and analyze the implication on the behavior of temperature profiles in the neighborhood of the shells.

A. Definition with a general fluid

Now armed with the field equations and the concept of trapped-matter surfaces developed in Refs. [17–19], we recognize the local conditions for the existence of a separating surface r_* to be twofold: firstly, the heuristic guideline is the conservation of the Misner-Sharp mass, which in the cases previously studied with pressure taken to be nonzero everywhere, corresponded to the conservation of the areal radius along the flow. Following the same heuristic argument, many possibilities open, but the simplest requirement which immediately recovers previous results [17–19] is to impose the additional condition $q_* = 0$, where the $*$ denotes a quantity calculated on the separating surface. Here we make this choice, leaving the most general case for future studies. We can see that the requirements of areal radius conservation and vanishing of the heat flux give $(\mathcal{L}_u M)_* = 0$ as from Eq. (10).

Secondly, a relation should come from a generalization of the hydrostatic equilibrium condition which, in parallel to what has been done in Ref. [19], generalizes the TOV equation and requires the appropriate $\text{gTOV}_* = 0$. As can be seen in Eq. (11), the minimal requirement that immediately satisfies the conditions for fluids without heat flux is the vanishing of the flow evolution of the heat flux scalar on the r_* surface, that is $(\mathcal{L}_u q)_* = 0$.

Recalling that the evolution of the areal radius is linked to the generalized expansion, the above definition of the separating surface, expressed in terms of fully gauge-invariant quantities, is

$$\mathcal{H}_* = 0, \quad (12a) \quad (\mathcal{L}_u \mathcal{H})_* = 0, \quad (12b)$$

$$q_* = 0, \quad (13a) \quad (\mathcal{L}_u q)_* = 0. \quad (13b)$$

Eqs. (12a) and (13a) express the conservation of the Misner-Sharp mass or, equivalently, the turning-point condition for the areal radius by analogy with Newtonian dynamics. Eqs. (12b) and (13b) encode the hydrostatic balance on the surface. Although local, this definition involves, in the exact manner discussed in Ref. [19], non-local conditions which are discussed in the next section. Because of the vanishing of heat flux on the r_* surface by Eqs. (13), the dynamics on the r_* shell is governed by the same equations as in the anisotropic stress case [19]. However, as we will see ahead, the difference in dynamics of the neighbouring surfaces will have an impact on the existence of the separating shell.

B. Non-locality with q

Introducing heat flux, the expansion reads (here, for convenience, we use $j \equiv \frac{q}{\sqrt{1+E}}$)

$$\Theta = \frac{(\mathcal{L}_u r)'}{r'} + 2 \frac{\mathcal{L}_u r}{r} - \frac{\kappa^2}{2} j \frac{r}{r'}, \quad (14)$$

After choosing a fixed fiducial areal radius r_0 defined as $r_0 \equiv r(t, R_0(t)) = \text{constant}$, and assuming no shell-crossing at least in the range between r_0 and r_* to maintain the bijection between radius and areal radius, Eq. (14) integrates to

$$\mathcal{L}_u r = \frac{1}{r^2} \left[\int_{r_0}^r \Theta r^2 dr + \frac{\kappa^2}{2} \int_{R_0}^R jr^3 dR \right] + \frac{1}{r^2} (r^2 \mathcal{L}_u r)_{r_0}. \quad (15)$$

The turning-point condition (12a) at r_* then yields

$$I_0 \equiv - (r^2 \mathcal{L}_u r)_{r_0} - \frac{\kappa^2}{2} \int_{R_0}^{R_*} jr^3 dR = \int_{r_0}^{r_*} \Theta r^2 dr. \quad (16)$$

If the initial parameter I_0 vanishes at some interior value $r_0 < r_*$, then so does the right-hand-side integral. This requires the vanishing of the expansion Θ at some intermediate value $r_0 < \tilde{r} < r_*$, since it has to change signs within the interval. Differentiating equation (15) with respect to the flow, we obtain

$$\begin{aligned} \mathcal{L}_u^2 r &= \mathcal{L}_u r \left(\Theta - \frac{2}{r} \mathcal{L}_u r \right) + \frac{1}{r^2} \left\{ \int_{r_0}^r \mathcal{L}_u \Theta r^2 dr \right. \\ &\quad \left. + \mathcal{L}_u (r^2 \mathcal{L}_u r)_{r_0} \right\} \\ &\quad + \frac{\kappa^2}{2\alpha r^2} \left[\int_{R_0}^R \partial_t (jr^3) dR - (\beta + \partial_t R_0) jr^3 \right] \\ &= -\text{gTOV}, \end{aligned} \quad (17)$$

which generalizes Eq. (2.48) of Ref. [19] and Eq. (21) of Di Prisco *et al.* [24] and confirms once again the claim that the radial acceleration is non-local. From Eq. (15) we realize that this non-locality is inherent to the radial expansion, and is already present in the energy condition defining r_* [Eqs. (12a) and (13a)] and in our $\text{gTOV}_* = 0$ condition [Eqs. (12b) and (13b)] due to the fact that both expressions implicate M_* , which is an integral between 0 and r_* from Eq. (8).

From the previous equations (15) and (17), we see that at the separating shell we have

$$\begin{aligned} \tilde{I}_0 &\equiv - \frac{\kappa^2}{2\alpha} \int_{R_0}^{R_*} \partial_t (jr^3) dR - \mathcal{L}_u (r^2 \mathcal{L}_u r)_{r_0} \\ &= \int_{r_0}^{r_*} (\mathcal{L}_u \Theta) r^2 dr \end{aligned} \quad (18)$$

so, if \tilde{I}_0 vanishes at an interior value $r_0 < r_*$, then so does the right-hand-side integral. This shows that the vanishing of the proper-time derivative of the expansion $\mathcal{L}_u \Theta$ occurs at some intermediate value between r_0 and r_* . When $\tilde{I}_0 = 0$ at the origin, we recover the vanishing of the radial acceleration, i.e. $\mathcal{L}_u \Theta = 0$ at some $0 < r < r_*$, the result of Di Prisco *et al.* [24].

C. Transport equations

The role of the flows of energy has been analyzed in the literature from the viewpoint of a kinetic theory description of the non-equilibrium processes involved [25]. Originally Eckart and Landau considered a constitutive equation generalizing the Maxwell-Fourier linear relation between heat flow and temperature gradient. However, it is known that the Eckart-Landau law exhibits causality problems arising from the instantaneous propagation of perturbations due to its parabolic nature. To overcome this difficulty, Israel and Stewart [25] put forward a heat transport model involving a finite thermal relaxation time (a more general treatment is given in Refs. [23, 26, 27] based on Ref. [25]), that leads to

$$\tau \mathcal{L}_u q^a + q^a + \tau q^b \left(\sigma_b^a + \frac{\Theta}{3} h_b^a \right) = K h^{ab} (T_{;b} - T \dot{u}_b) - \frac{1}{2} K T^2 \left(\frac{\tau}{K T^2} u^b \right)_{;b} q^a \quad (19)$$

where $K > 0$ is the heat conduction coefficient, T is the fluid temperature and τ is the thermal relaxation time.

Imposing conditions (13) at the separating shell reduces the Israel-Stewart equation to

$$(h^{ab} T_{;b})_* = (T \dot{u}^a)_*, \quad (20)$$

which allows us to determine the temperature distribution on the shell.

Note that if $\Pi_* = \Pi'_* = P'_* = 0$ then, from Eq. (7), we get $\alpha'_* = 0$ and the geodesic condition $(\dot{u}^a)_* = 0$, as expected. In turn, this implies $(h^{ab} T_{;b})_* = 0$, which reflects more clearly the local thermodynamical equilibrium at r_* .

An interesting result, that also illustrates the important role of the flows of energy in connection with separating shells, is provided by the work of Herrera and collaborators [28] who introduced the concept of *thermal peeling* as an effect that contributes to fluid cracking. Their analysis relied on the Maxwell-Fourier law, which is a particular case of Eq. (19) for $\tau = 0$, and reads

$$q^a = K h^{ab} (T_{;b} - T \dot{u}_b). \quad (21)$$

This equation can be projected along n_a to give

$$q = K \sqrt{1+E} \left(T' - \frac{\alpha'}{\alpha} T \right), \quad (22)$$

which, by substituting in (15), leads to

$$\mathcal{L}_u r = -\frac{1}{r^2} \left[\int_r^{r_*} \Theta r^2 dr + \frac{\kappa^2 K}{2} \left((T_* r_*^3 - T r^3) - \int_R^{R_*} T \frac{(\alpha r^3)'}{\alpha} dR \right) \right]. \quad (23)$$

In the proximity of the separating shell, the leading contribution from the second term goes as $(T_* - T) r_*^3$; this corroborates the results in Ref. [28] that sufficiently large negative temperature gradients $T_* - T < 0$ favor cracking and tend to increase

the radial velocity of collapse $\mathcal{L}_u r > 0$ in the interior neighbourhood of r_* . It is important to note though, that cracking shells [29] do not have to coincide with separating shells. In fact, comparing the conditions for cracking [19, 24, 29] and Eqs. (12) and (13), we conclude that cracking surfaces also need to satisfy Eq. (13b) to be separating shells.

IV. DISCUSSION AND CONCLUSION

In this work we investigated the existence of separating shells dividing expanding and collapsing regions by using a single metric describing a spherically symmetric inhomogeneous universe with imperfect fluids. The introduction of heat flux has allowed tackling the problem for a wider class of spacetimes than in previous works [19]. Although not fully general, the framework here presented allows treating all cases in which heat flux is segregated on both sides of a separating surface. That is, it makes use of the simplifying conditions (13), which can be interpreted as a dynamical constraint on the fluid. The aim of this work has been to explore the main consequences of such an extension, and further discussion will follow in a more general study.

Extending the techniques used previously in Refs. [14, 15, 17–19], we proposed local conditions characterizing the existence of a separating surface. Such a surface, across which heat flux and its evolution along the flow vanish, is defined by locally setting (i) balance between analogues to total and potential energies, and (ii) a generalized TOV hydrostatic equilibrium. All this ensures no matter or energy transfer across the separating shell, justifying the present definition to be a valid generalization of the concept of *trapped-matter surfaces* introduced in Ref. [18]. The formulation of the intrinsic non-locality of the shell was also extended, and we constructed the governing equations and separation conditions in terms of gauge invariant scalars as in Ref. [19].

We made contact with kinetic theory by using the Israel-Stewart [25] relativistic transport equation for dissipative fluids, which has been used before in related problems (see e.g. Refs. [23, 30]). The transport equation applied to our model gives the temperature profile across the separating shell and is consistent with our conditions of local equilibrium. Furthermore, in the limit of vanishing relaxation time, we were able to conclude, as in Ref. [28], that temperature gradients can contribute to spacetime cracking. By comparing these conditions with those from Ref. [19], we also established the relationship between cracking and separating shells.

These interesting aspects, especially the role of the temperature gradient in the existence of separating shells, will be studied in more detail in future works.

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