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Dynamic clustering of evolving networks: some results on the line†

Cristina G. Fernandes¹, Marcio T.I. Oshiro¹, and Nicolas Schabanel²


²CNRS, Université Paris Diderot, LIAFA, Case 7014, 75205 Paris Cedex 13, France; and IXXI, Université de Lyon, 46 allée d’Italie, 69364 Lyon, France. http://www.liafa.univ-paris-diderot.fr/~nschaban

Understanding the dynamics of evolving social/infrastructure networks is a central challenge in many applied areas such as epidemiology, viral marketing, city planification, etc. During the last decade, a massive amount of data has been collected on such networks that still resist to analysis. In this article, we propose to use the data on the dynamics to find better partitions of the network into groups by requiring the groups to be stable over time. For that purpose, we introduce a dynamic version of the \( k \)-clustering problem which includes a cost for every point that moves from one cluster to another. We show that this yields in many realistic situations better fitting solutions than optimizing independently various snapshots of the network. We present a first non-trivial exact algorithm for this problem when the points move along a line; this algorithm runs in polynomial time when \( k \) and the time horizon are bounded by a constant. We conclude with a series of surprising results on the complexity of the structure of optimal solutions for the line case.

1 Introduction

During the last decade, a massive amount of data has been collected on diverse networks such as web links, nation- or world-wide social networks, online social networks (Facebook or Twitter for example), social encounters in hospitals, schools, companies, or conferences (e.g. [5, 7]), and other real-life networks. Those networks evolve with time, and their dynamics have a considerable impact on their structure and effectiveness (e.g. [6, 4]). Understanding the dynamics of evolving networks is a central question in many applied areas such as epidemiology, vaccination planning, anti-virus design, management of human resources, viral marketing, “facebooking”, etc. Algorithmic approaches have for instance been successful in yielding useful insights on several real networks such as zebras social interaction networks [8].

But the dynamics of real-life evolving networks are not yet well understood, partly because it is difficult to observe and analyze such large networks sparsely connected over time. Some basic facts have been observed (such as the preferential attachment or copy-paste mechanisms) but more specific structures remain to be discovered. In this article, we propose to adapt the problem of \( k \)-clustering to these evolving networks. We show that requiring the solution to be stable over time yields in many realistic situations better fitting solutions than optimizing independently various snapshots of the network.

More precisely, we focus on the \( k \)-clustering problem of points moving in a metric space: we look for the best partition of the points in \( k \) groups (called clusters) over time minimizing a tradeoff between two objectives. The first objective is the span of the clusters (the sum of their diameter), which ensures that each cluster should contain points which are close to each other. The second objective is the instability of the clusters over time, measured as the number of points changing from one cluster to another over time. We argue that incorporating this stability requirement in the objective function helps in many realistic situations to obtain better solutions (see Section 2).

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Our approach differs from the few existing traditional algorithmic approaches to dynamic settings because they ignore the stability of the solution. We show that offline static algorithms that construct an independent optimal solution for each snapshot of the network yield results that, in a large variety of realistic situations, are not only unstable (and thus arbitrarily bad for our objective), but also undesirable with respect to network dynamics analysis. Online greedy-type solutions such as [1] are also excessively pessimistic: we have access to the whole evolution of the network over time (as given by experiments such as [2]) and we can thus anticipate future changes.

**Our results.** After introducing the model, we present a series of examples demonstrating that this problem produces solutions which are fitting better to a dynamic network than previous clustering problems. Then, we focus on the case of points moving along the line and exhibit a first non-trivial exact algorithm, which is polynomial when the number of clusters and the time horizon are bounded by a constant. We then present a series of surprising results on the complexity of the structure of optimal solutions in even very restricted cases of the line case.

2 Dynamic k-clustering problem

2.1 Definition

We present a dynamic extension of the problem studied in [1]. Let \( (X, d) \) be a metric space. For a given \( n \), an \( n \)-configuration in \( X \) is a sequence of \( n \) not necessarily distinct points in \( X \). A \( k \)-clustering is a sorted partition of \( [n] = \{1, \ldots, n\} \) into \( k \) sets, that is, it is a sequence of disjoint subsets of \( [n] \) whose union is \( [n] \).

Each subset \( C \) in a \( k \)-clustering is called a cluster, and the diameter of \( C \) for an \( n \)-configuration \( (p_1, \ldots, p_n) \) is the maximum distance between two points in \( C \), that is, \( \max \{d(p_i, p_j) : i, j \in C\} \). If \( C \) is empty, we say its diameter is zero. The cost of a \( k \)-clustering for an \( n \)-configuration is the sum of the diameters of its clusters.

Given a positive integer \( T \), for each \( t = 1, \ldots, T \), let \( (p'_1, \ldots, p'_n) \) be an \( n \)-configuration in \( X \). Such a sequence of \( n \)-configurations is called a dynamic setting (of \( n \) points) and represents the movement of \( n \) points in the metric space \( (X, d) \). It is denoted shortly by \( P = (p'_1)_{1 \leq i \leq n, 1 \leq t \leq T} \). A dynamic \( k \)-clustering \( C = (C'_i)_{1 \leq i \leq k, 1 \leq t \leq T} \) of the dynamic setting \( P \) consists of a sequence of \( k \)-clusterings of \( [n] \), one for each of the \( n \)-configurations \( (p'_1, \ldots, p'_n) \), \( t = 1, \ldots, T \).

The cost of a dynamic clustering takes into account two objectives: the total span of its clusters and their instability over time. The instability of a dynamic clustering can be measured in several ways. In this paper, we adopt the number of times a point changes from a cluster to another over time, that is, the instability of a dynamic \( k \)-clustering \( C = (C'_i)_{1 \leq i \leq k, 1 \leq t \leq T} \) is the number of pairs \((i, t)\), for \( i \in [n] \) and \( 1 \leq t < T \), such that \( i \in C'_i \) but \( i \notin C'_{i+1} \). For a constant \( c > 0 \), we define the cost of \( C \) as \( c \) times the instability of \( C \) plus the sum of the cost of each of its clusterings (which happens to be the sum of the diameter of each \( C'_i \)). The constant \( c \) allows one to put more or less weight on the instability cost with respect to the clustering cost. A similar formulation of the problem was also introduced in [3].

2.2 Examples

The two examples in Fig. [1] show that dynamic \( k \)-clustering differs significantly from static \( k \)-clustering of every snapshot and furthermore yields more desirable partitions of the network. Example [a] shows the
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3 Exact algorithms for the line

3.1 Dynamic k-clustering on the line

We consider the case where the metric space is the line. That is, the dynamic setting consists of points that move on the line: \( p_l^t \in \mathbb{R} \) for all \( t \) and \( l \). For all \( t \), let \( \pi_t \) be the permutation of \([n]\) that stably sorts \( p_1^t, \ldots, p_n^t \), i.e. such that \( p_{\pi_l^t}^t \leq \cdots \leq p_{\pi_{n}^t}^t \) and, additionally, if \( p_{\pi_i^t}^t = p_{\pi_j^t}^t \) and \( i < j \) then \( \pi_i^t < \pi_j^t \). (In other words, \( \pi_i^t \) is the index of the \( i \)-th point from the left at time \( t \).)

**Lemma 2** The cost of any optimal dynamic \( k \)-clustering for a dynamic setting of \( n \) points on the line of time-length \( T \) is fully characterized by the indices of the two extreme points of each cluster. Moreover, a corresponding optimal dynamic \( k \)-clustering can be recovered in \( O(nk^2T) \) time by dynamic programming.

We are thus left with enumerating all the possible extremities for the \( k \) clusters. There are less than \( (n^2)^2 \leq n^k/k! \) choices for \( t = 1 \) and less than \( k!(n^k)^2 \leq n^{2k}/k! \) choices for \( t \geq 2 \) (since the labels of the cluster matter as soon as \( t \geq 2 \) as the following facts will demonstrate). It follows that:

**Theorem 3** There is an \( O(n^{2kT+1}k^2T/k!^{T+1}) \)-time algorithm that solves exactly the dynamic \( k \)-clustering problem for a dynamic setting of \( n \) points on the line for \( T \) units of time. This running time is polynomial in \( n \) if \( k \) and \( T \) are bounded by a constant.

Note that this algorithm always outperforms the brute-force enumeration of all the \( k^nT/k! \) possible \( k \)-clustering sequences (running in \( O(k^{nT}T) \) time).

3.2 Surprising facts on optimal dynamic clusterings on the line

For each \( t \), an interval \((k,n)\)-cover of \( n \) points at time \( t \) is a sequence of \( k \) pairs \( (a^t_l, b^t_l)_{l=1}^k \) such that \( a^t_l, b^t_l \in [n] \), the \( a^t_l \) are pairwise distinct for \( l \in [k] \), the \( b^t_l \) are pairwise distinct for \( l \in [k] \), and \( \cup_{l=1}^k [a^t_l, b^t_l] \supseteq [n] \). We say \( (a^t_l, b^t_l)_{l=1}^k, t=1, \ldots, T \) is an interval \((k,n)\)-cover sequence if, for \( t = 1, \ldots, T \), each \( (a^t_l, b^t_l)_{l=1}^k \) is an interval \((k,n)\)-cover of the \( n \) points at time \( t \). By Lemma 2, a dynamic \( k \)-clustering on \([n]\) can be represented by an interval \((k,n)\)-cover.

The following lemma shows that almost every interval \((k,n)\)-cover sequence can correspond to an optimal dynamic \((k+1)\)-clustering of a dynamic setting of \( n \) points on the line (including sequences where all intervals are concentric!).

**Lemma 4** For every \( n \geq 2k \) and every interval \((k,n)\)-covers sequence \( (a^t_l, b^t_l)_{l=1}^k, t=1, \ldots, T \), there exists a dynamic setting of \( n \) points on the line of time-length \( T + 1 \) for which, for \( t = 1, \ldots, T \), \( a^t_l \) and \( b^t_l \) for \( l = 1, \ldots, k \) are the extremities of the \( k \) first clusters of the only optimal dynamic \((k+1)\)-clustering (of the optimal dynamic \( k \)-clustering when \( n = 2k \)).

We restrict attention to the particular case in which the trajectories of the \( n \) points in the line do not cross each other. That is, the permutation \( \pi_t \) is the identity at every time \( t \).

One may hope that, if the trajectories of the \( n \) points never cross, the \( k \) clusters may keep the same relative order. This is unfortunately not the case, as the next example shows.
Fact 5 There exists a dynamic setting of five points on the line with non-crossing trajectories for which the order of the clusters in the only optimal dynamic 2-clustering changes. This can be generalized to a dynamic setting of n points on the line with n ≥ 5, k ≥ 2, and T ≥ 2.

So, even when the trajectories are non-crossing, it is not straightforward to design a dynamic programming algorithm that runs in time polynomial in n, k, and T. Non-crossing trajectories have however a simpler structure for T = 2 or k = 2, but, surprisingly, not as soon as k ≥ 4 and T ≥ 3! An interval (n,k)-cover is non-overlapping if its intervals are pairwise disjoint.

Fact 6 Consider a dynamic setting of n points on the line having non-crossing trajectories. If T = 2 or k = 2, then there is an optimal dynamic k-clustering which consists in a sequence of non-overlapping interval (n,k)-covers.

Fact 7 There is an instance with n = 7 points, k = 4 clusters, and T = 3, with non-crossing trajectories, for which all optimal dynamical 4-clusterings have two clusters that overlap at some time t.

When trajectories are non-crossing, the span of a dynamic k-clustering may have some laminar structure, i.e., at each time step the spans of every pair of clusters are either disjoint or included one into the other, and moreover with every point belonging to the cluster with smallest span that contains it. If this were true, then we would have a much better algorithm, running in polynomial time in n and linear time in T.

Theorem 8 When trajectories are non-crossing, there is an algorithm that computes an optimal laminar dynamic k-clustering in \(O(k! n^{k-4} T)\) time and \(O(k! n^{2k-2} T)\) space.

4 Conclusion

We believe that this problem is a key to the understanding of real-life evolving network, such as social interactions. It appears to have very intriguing behavior even in the simpler case of the line. Although running in polynomial time for fixed k and T, our algorithms need still to be improved to be fitted for real-life data collections which are typically huge. One may thus consider aiming at faster algorithms approximating the optimum. In other ongoing work, [3] develops an approximation algorithm and suggests that there might be some strong inapproximability results related to this problem but the situation needs yet to be clarified.

Références


