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On the localization of straight lines in 3D space from single 2D images

Vincenzo Caglioti, Simone Gasparini
Dipartimento di Elettronica e Informazione
Politecnico di Milano
P.za Leonardo da Vinci, 32
20133 Milano - Italy
{caglioti, gasparini}@elet.polimi.it

Abstract

The reconstruction of 3D scenes constituted by straight lines can find many applications both in Computer vision and in Mobile robotics. Most of the approaches to this problem involve either stereo-vision or the analysis of a sequence of images taken from different viewpoints: in both cases, the solution of the correspondence problem is required. This paper studies the localization of straight lines in 3D space from single 2D images, acquired by a catadioptric camera. In general, using a noncentral camera, the viewing rays starting from the points of a straight line constitute a non-planar surface: if this non-planar surface only contains one straight line, other than the viewing rays, then the straight line can univocally be localized. Some conditions for the univocal localization of straight lines are derived. A simple technique for the straight line localization is presented, and some preliminary experimental results are discussed.

Index terms

Panoramic vision, Omnidirectional sensors, Catadioptric camera, 3D linear scene reconstruction, Space line localization, Ruled quadrics, Noncentral cameras.

1. Introduction

Consider a 3D scene constituted by some straight lines (as, e.g., edges on the ceiling of a room or on furniture objects): the addressed problem is to reconstruct the orientation and position of these straight lines. Laser range scanners can provide sufficient data for the problem solution: however these sensors are invasive, and sometimes their use must be restricted to protect human users. Visual approaches do not suffer from these drawbacks. The large

majority of them are based either on stereo-vision, or on the analysis of a sequence of images taken from different positions: in both cases, the correspondence problem must be solved.

We propose an approach to the localization of straight lines in 3D space based on the use of a single 2D image. In this way, the analysis of the correspondences between different images can be avoided.

Under perspective, all viewing rays are constrained to pass through a common viewpoint. Using a perspective camera, the image of any straight line in the scene has infinite (technically, ∞^2) interpretations as straight lines. In fact, any straight line contained in the plane through the viewpoint and the image line can in principle have produced the image line.

In this work a noncentral camera [11] is adopted: i.e. a camera whose viewing rays are not all concurrent. Non-central cameras can be realized, e.g., by linear push-broom cameras [6], cross-slit cameras [3], catadioptric cameras [8]. A particular type of noncentral camera is the oblique camera [9], where any two viewing rays are skew. Non-central cameras has been used for 3D localization from two views [2]. A method for line localization from single panoramic images has been presented, using prior information about parallelism and coplanarity [10].

In this paper, we study the conditions under which straight lines in 3D space can be reconstructed from single images taken by a noncentral camera.

We prove two sufficient conditions for the univocal straight line localization using an axial-symmetric catadioptric camera: the first property states that univocal localization of a broad set of straight lines is allowed using a single-image axial-symmetric catadioptric camera. The second property states that using a conical mirror, almost any straight line can be univocally localized.

In Section 2 the addressed problem is formulated and some preliminary results are stated. In Section 3, axial-symmetric catadioptric cameras are introduced, and some

sufficient conditions are stated for the univocal localization of straight lines in 3D space. In Section 4, a technique is illustrated for localizing a straight line in space from its image. Preliminary experimental results are presented in Section 5, while Section 6 concludes the paper.

2 Problem formulation and preliminary results

A camera can be represented as an image plane plus a transformation \mathcal{I} (actually, a projection) mapping any point P in the 3D scene onto an “image point” p belonging to the image plane. The *viewing ray* of an image point p is the set of scene points sharing the same image p : since the scene is supposed to be immersed within a homogeneous medium, light propagates along straight lines, and therefore the viewing ray of an image point is actually a straight line. A space curve C is projected by \mathcal{I} onto an image curve c . The *interpretation surface* of an image curve c is the set of scene points, whose image is in c . The interpretation surface of an image curve c is given by the set union of the viewing rays of all the image points contained in c .

If we want to be able to reconstruct scene straight lines from single images, we have to design an optical system such that the interpretation surface of the image of any straight line l (in generic position) does not contain straight lines other than l and other than the viewing rays.

Clearly, central cameras, as e.g. perspective cameras, or even [4] do not allow reconstruction of straight lines, since the interpretation surface of the image of any straight line l is a plane, which contains infinite straight lines other than l .

2.1. Ruled quadrics

Non-degenerate ruled quadrics are surfaces containing two distinct families of straight lines, such that each line of one family crosses (only once) all the lines of the other family, while any two lines of a same family are either identical or skew. At each of its points, a line of one family crosses exactly one line of the other family.

Which is the most general class of non-planar interpretation surfaces, that contain at least three straight lines, other than viewing rays, without being planar? The following property holds.

Property 1. *The set of points belonging to all the lines, that cross three skew straight lines, is a ruled quadric [7].*

Therefore, the most general interpretation surface that contains more than two straight lines, other than the viewing rays, is a ruled quadric. A ruled quadric actually contains infinite straight lines other than the viewing rays.

The following is a reformulation of a classical property of ruled quadrics, which will be used in the subsequent Sections.

Given two skew straight lines l_1 and l_2 , let us consider a family \mathcal{F} of straight lines, not including l_1 or l_2 , such that: (i) each line of \mathcal{F} crosses both l_1 and l_2 , and (ii) for any point P_1 on l_1 (for any point P_2 on l_2) there is only one line of \mathcal{F} that crosses the line l_1 at point P_1 (P_2 at l_2). We investigate the conditions under which there is a third line l_3 , not belonging to \mathcal{F} , that crosses all lines of \mathcal{F} .

The generic straight line f of the family \mathcal{F} is characterized by a correspondence between the points $P_1 \in l_1$ and $P_2 \in l_2$ crossed by the line f . Let z be the abscissa of a generic point on the first given line l_1 , and let t be the abscissa on the second given line l_2 : the family of straight lines is characterized by the function $t(z)$.

The following classical lemma states the condition under which the function $t(z)$ identifies a ruled quadric.

Lemma 1. *Let an interpretation surface associated to a family of straight lines, which cross two given skew straight lines l_1 and l_2 , be characterized by the function $t(z)$: it can cross a third straight line l_3 , other than the two given lines, only if $t(z)$ is in the form*

$$t(z) = \frac{az + b}{cz + d}. \quad (1)$$

In addition, in this case the interpretation surface will cross an infinite number of straight lines.

3. Axial-symmetric catadioptric cameras

A catadioptric camera is obtained by placing a mirror between the scene and a standard perspective camera. In a catadioptric camera, the viewing ray coming from a scene point is specularly reflected by the mirror surface, before it goes through the camera viewpoint and crosses the image plane. The surface orientation plays a key role in the reflection of the viewing rays. In this paper, the mirror surface is assumed to be continuously differentiable, therefore the surface orientation is functionally related to the surface equation.

Clearly, if the mirror is planar, the catadioptric camera reduces to a perspective camera, whose center is given by the reflection of the viewpoint of the employed camera by the mirror plane. Even if the mirror satisfies the *single center constraint*¹, the camera reduces to a perspective camera. The non-degenerate solutions to the single center constraint include hyperbolic mirror, ellipsoidal mirror, and (including a vanishing single viewpoint) a paraboloidal mirror

¹We use the word “center” to designate the concurrence point of the viewing rays, instead of “viewpoint” (used by [1]) by which we designate the viewpoint of the employed perspective camera

[1]. Catadioptric cameras, which do not satisfy the (non-degenerate case) single center constraint, are called *noncentral* cameras [9].

According to the specular reflection laws, the viewing ray of an image point p , is the line l through the point B (where the line Op intersects the mirror surface) such that the normal to the mirror surface at B bisects the angle between OB and l .

A catadioptric camera is said to be a *single-image* camera, if for each point P within the visibility region of the camera, there is a unique viewing ray r through P , that is reflected by the mirror towards the camera viewpoint.

Observation. A catadioptric camera based on a convex mirror is a single-image camera.

Now, axial-symmetric catadioptric cameras are considered, and some properties are studied concerning the univocal reconstruction of straight lines in the 3D space.

An axial-symmetric catadioptric camera consists of an axial-symmetric mirror and a perspective camera, whose viewpoint O is on the symmetry axis of the mirror. With no loss of generality, suppose that the viewpoint O is at the origin (i.e., it has null coordinates), while the symmetry axis coincides with the z -axis,

Lines parallel to the z -axis are said to be vertical, while lines orthogonal to the z -axis are said to be horizontal.

The mirror surface of an axial-symmetric catadioptric camera can be obtained by rotating a planar curve (coplanar with the z -axis) about the z -axis. This curve, called *profile*, can be characterized by a function $R(Z)$, where $R = \sqrt{X^2 + Y^2}$ is the distance between the z -axis and the generic point $P = (X, Y, Z)$ on the profile.

By symmetry, any viewing ray is coplanar with the z -axis: therefore each viewing ray crosses the z axis at a point (possibly, at the infinite). The *slope* s of a viewing ray is the ratio between the increment in the z coordinate and the increment in the distance R from the z -axis, of a point moving along the viewing ray.

A point P on the z -axis is said to be *singular*, if it is crossed by infinite viewing rays having distinct values of the slope s .

For instance, in a central catadioptric camera, the common intersection of all the viewing rays is a singular point, since it is incident to many viewing rays with different values of the slope s .

An axial-symmetric catadioptric camera is said to be *nonsingular* if it has no singular points.

A catadioptric camera is nonsingular, if no finite segment of its profile $R(Z)$ coincides with a conic, having one of its foci on the viewpoint.

In an axial-symmetric, non-singular, single-image catadioptric camera, the viewing rays sharing the same slope constitute a cone, whose axis is the Z -axis. The set union of the rays sharing a same slope is said to be a *viewing cone*.

In a single-image, non-singular catadioptric camera, different viewing cones do not intersect.

If there are horizontal viewing rays, the viewing cone constituted by the horizontal viewing rays is a degenerate cone: actually, it is a horizontal plane).

3.1. Conditions for univocal localization of straight lines in space

Firstly, observe that, using an axial-symmetric catadioptric camera, any line that is coplanar with the symmetry axis can not univocally be reconstructed, since its interpretation surface is the plane containing the line and the symmetry axis.

Secondly, if there are horizontal viewing rays, no line lying on the degenerate viewing cone can be reconstructed univocally, since all the viewing rays crossing such a line would be coplanar.

The following theorem states that a broad set of straight lines can be univocally reconstructed using any catadioptric camera within a quite general class.

Theorem 1. *Let us consider a single-image, nonsingular, axial-symmetric catadioptric camera. If a straight line intersects a viewing cone at two distinct points, it can univocally be reconstructed.*

Proof. Let us consider a line l which intersects a viewing cone C at two points, whose abscissae are, respectively, t_1 and t_2 . Let z_o be the abscissa of the cone vertex along the Z -axis. Consider the relationship $z(t)$ between the abscissa t of a point P along l , and the abscissa z of the point where the viewing ray through P intersects the Z -axis. Since $z(t_1) = z(t_2) = z_o$, the function $z(t)$ has an extremum in $[t_1, t_2]$. Therefore it can not be in the form (1). Hence the surface given by the set union of the viewing rays crossing l is not a ruled quadric. Since this surface is not a plane, there are at most two straight lines (other than the viewing rays) contained in the surface. But, the z axis is one of these lines: thus the only other line contained in the surface is l itself. Therefore, the line l is localized univocally. \square

Observe that a line intersects a viewing cone twice only if its slope is less than the slope of the viewing cone.

3.2. Conical Mirror

Considering a specific mirror shape, we can prove that almost any line can be univocally reconstructed.

Lemma 2. *Suppose that a ruled quadric Q contains the Z axis: then it can not include any horizontal semi-circumference centered on the Z axis.*

Proof. Let l_1 be the Z axis, and let l_2 be an other line contained in the ruled quadric Q , such that l_1 and l_2 are skew. Let $\mathcal{F} \subset Q$ be the family of lines crossing both l_1 and l_2 , and let l be a generic line of \mathcal{F} : the relationship between the abscissa t_1 along l_1 of the point where l crosses l_1 , and the abscissa t_2 along l_2 of the point, where l crosses l_2 is in the form

$$t_1(t_2) = \frac{at_2 + b}{t_2 + c},$$

Without loss of generality we can suppose that the line l_2 goes through the point Q of coordinates $[d, 0, h]$ and its direction is given by the vector $[0, 1, \tan \phi]$, where ϕ is the angle on the YZ -plane. Hence the parametric equation for the straight line l_2

$$l_2 : \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d \\ 0 \\ h \end{bmatrix} + \lambda \begin{bmatrix} 0 \\ 1 \\ \tan \phi \end{bmatrix}$$

Given the axial symmetry, let consider the semicircle of radius $r = 1$ with the center on the origin lying on the plane $z = 0$ and on the negative X -half plane (see Figure 1); the general point P of the circle has coordinates $[-\cos \theta, \sin \theta, 0]$, where θ is the angle on the XY -plane. Now, given the family \mathcal{F} of straight lines going through each point of the circle and crossing both l_1 and l_2 , we prove that the function relating the abscissas of intersection points between the family of straight lines and the two skew line is not in the form (1).

To find the straight line belonging to \mathcal{F} and going through P , we intersect the plane $\pi : x \cos \theta + y \sin \theta = 0$, which contains the point P and the Z -axis (i.e. l_1), with l_2 and we find the intersection point R : $[d, -d \tan \theta, h - d \tan \phi \tan \theta]$. Considering the triangle $\triangle PRS$ (see Figure 1), this congruence relationship holds for z -coordinate of the intersection point on l_1

$$z = \frac{h - d \tan \phi \tan \theta}{1 + \frac{d}{\cos \theta}} = \frac{h - d \tan \phi \tan \theta}{1 + d \sqrt{\tan^2 \theta + 1}}$$

(remember that $\frac{1}{\cos \theta} = \sqrt{\tan^2 \theta + 1}$). Since it yields that the y -coordinate of intersection points on l_2 is $y = -d \tan \theta$, we can write a relationship $z(y)$ between the z -coordinate on l_1 and the y -coordinate on l_2 as the following:

$$z(y) = \frac{h + y \tan \phi}{1 + \sqrt{y^2 + d^2}}$$

which is not in the form (1). \square

Theorem 2. Consider an axial-symmetric, non-singular catadioptric camera whose mirror surface is conical. Let consider a line l : if (i) l and the Z axis are not coplanar,

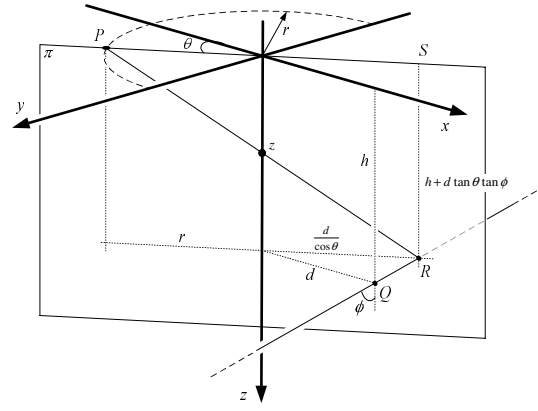


Figure 1. A sketch of the proof of Lemma 2

and (ii) l is not contained in the eventually existing degenerate viewing cone, then l can univocally be localized using the considered camera.

Proof. The viewing rays starting from the line l cross the caustic of the camera. The caustic of an axial symmetric catadioptric camera based on of a conical mirror, is a circle centered on the symmetry axis [1]. However, from Lemma 2 the family of straight line crossing both l and the z -axis, namely the viewing rays of l , can not be in the form (1). Therefore from Lemma 1 the interpretation surface related to the viewing rays of l can not be a ruled quadric. Therefore this surface can only contain at most two lines other than the viewing rays. Since it already contains the z -axis, then it can only contain one other straight line, namely the line l itself. Hence l is localized univocally. \square

4. Straight line localization

To localize a straight line from the image, let us consider four skew viewing rays v_0, v_1, v_2 , and v_3 . Any three of them univocally identify a ruled quadrics. The straight line in space can be determined by intersecting two ruled quadrics, e.g., the ruled quadrics Q_{013} containing all the lines crossing v_0, v_1 , and v_3 , and the ruled quadrics Q_{023} containing all the lines crossing v_0, v_2 , and v_3 . To intersect these ruled quadrics we can proceed as follows:

1. We find a relationship $x'_3 = f_1(x_0)$ between the x -coordinate x_0 of a point P_0 on v_0 and the x -coordinate x'_3 of a point P'_3 on v_3 , such that the line joining P_0 and P'_3 also crosses the line v_1 . The point P'_3 on v_3 is determined by intersecting the ray v_3 with the plane through P_0 and v_1 . The relationship $x'_3 = f_1(x_0)$ is in the form $x'_3 = a'x_0/(b'x_0 + c')$ (from Lemma 1 and since all the four rays cross the Z -axis).



Figure 2. A catadioptric image, with four points used for the line localization

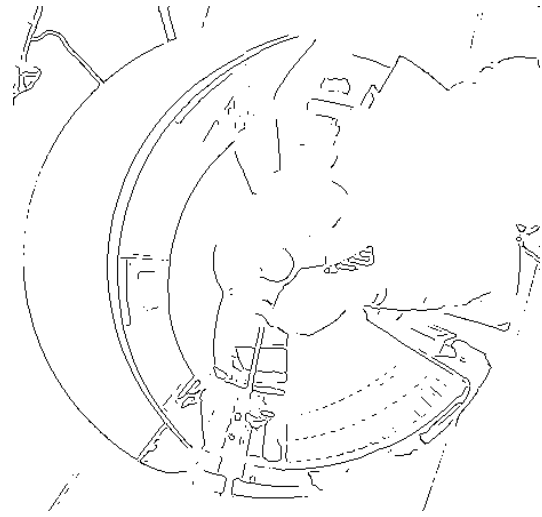


Figure 3. The extracted edges: the contour used for the line localization is emphasized

2. We find a relationship $x_3'' = f_2(x_0)$ between the x -coordinate x_0 of a point P_0 on v_0 and the x -coordinate x_3'' of a point P_3'' on v_3 , such that the line joining P_0 and P_3'' also crosses the line v_2 . To do this, the point P_3'' on v_3 is determined by intersecting the ray v_3 with the plane through P_0 and v_2 . The relationship $x_3'' = f_2(x_0)$ is in the form $x_3'' = a''x_0 / (b''x_0 + c'')$.
3. We find a line joining a point on v_0 to a point on v_3 , that crosses both v_1 and v_2 . To do this, we impose $x_3' = x_3''$: by solving the deriving equation $f'(x_0) = f''(x_0)$ on the unknown x_0 . By eliminating the trivial solution $x_0 = x_3 = 0$ (which corresponds to placing both points onto the z -axis), a linear equation in x_0 is obtained, leading to a single solution.
4. Once the value of x_0 is found, the value of x_3 is given by $x_3 = f'(x_0)$ (or, equivalently, by $x_3 = f''(x_0)$). The searched line is the line joining the point on v_0 , whose x -coordinate is given by the determined value of x_0 , and the point on v_3 whose x -coordinate is given by the determined value of x_3 .

Therefore the localization of the straight lines involves very simple computation.

5. Preliminary experimental results

An axial-symmetric catadioptric camera has been used, constituted by a conical mirror and a perspective camera. Various techniques for the calibration of catadioptric cameras have been suggested [2] [5]. In this preliminary ex-

perimentations, we followed a simple two-step calibration method: first, the intrinsic camera parameters are estimated [12], and then some easily detectable features of the mirror (as its circular base) are localized wrt to the camera. This technique relies on sufficient accuracy of the mirror shape (accuracy specifications are within 10^{-2} mm). The extracted image contours are interpolated to achieve sub-pixel accuracy, and the contours are segmented by splitting them at corners. The contour associated to the mirror base is discarded, and some contour curves are selected based on their length, as longer curves are localized more reliably than short ones. A single contour curve is interpreted by default as the image of a straight line: a localization hypothesis is formulated by applying the technique illustrated in Section 4, based on four viewing rays. In Figure 2 a catadioptric image is shown, together with four points on a selected contour line (the extracted contours are shown in Figure 3). The viewing rays, corresponding to the four points, are used for the line localization (according to the method described in the last Section). The localization hypothesis can then be checked, by verifying that the distance between the line (in its hypothesized position) and the viewing rays of further points of the contour curve is below a certain threshold. Finally, the further rays find to be compatible with the line pose hypothesis, can be employed to refine the estimation.

Localization experiments show that selecting contours that are seen through viewing rays, that form a sufficient angle, quite accurate position estimates can be obtained: about 1 cm translational accuracy, and about 3° rotational accuracy.

6. Conclusions

A study has been presented on the localization of straight lines in 3D space from single 2D images, acquired by an axial-symmetric catadioptric camera. Some properties have been proved on the univocal localization of such lines. Preliminary experimental results, obtained with a catadioptric camera based on a conical mirror, suggest that this can be a promising direction for further investigation. Ongoing research is aimed at 1) studying univocal localization with non-axial-symmetric catadioptric cameras, and 2) correlating the mirror shape to the localization error for design purpose.

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