

Diverse Routing with Star SRLGs[†]

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La notion de groupe de liens partageant un risque (*Shared Risk Link Group*, SRLG) a été introduite pour modéliser des problèmes de tolérance aux pannes simultanées d'ensembles de liens d'un réseau. Dans ce contexte, le problème du *routing diversifié* est de trouver un ensemble de chemins SRLG-disjoints entre une paire donnée de nœuds du réseau. Ce problème a été prouvé NP-complet en général [7] et certains cas polynomiaux ont été caractérisés [2].

Nous avons étudié le problème du routage diversifié dans les réseaux satisfaisant la *propriété d'étoile* [9]. Dans un réseau satisfaisant la propriété d'étoile, un lien peut être affecté par plusieurs SRLGs, mais tous les liens affectés par un même SRLG sont incidents à un même sommet. Nous avons trouvé des contre-exemples à l'algorithme polynomial proposé dans [9] pour le calcul de paires de chemins SRLG-disjoints dans les réseaux satisfaisants la propriété d'étoile. Puis, nous avons prouvé que ce problème est en fait NP-difficile. Plus généralement, nous avons montré que le problème du routage diversifié dans les réseaux avec la propriété d'étoile est NP-difficile, APX-difficile, et W [1]-difficile lorsque le paramètre est le nombre de chemins SRLG-disjoints. Enfin, nous avons caractérisé de nouvelles instances polynomiales, en particulier lorsque le degré maximum des sommets est 4, ou lorsque le réseau est acyclique.

Keywords: SRLG, Diverse Routing, Colored Graph

1 Introduction

To ensure reliable communications, many protection schemes have been proposed. One of the most used, called *dedicated path protection*, consists in computing for each demand both a working and a protection path. A general requirement is that these paths have to be diversely routed, so that at least one of them can survive a single failure in the network. This method works well in a single link failure scenario, as it consists in finding two edge-disjoint paths between a pair of nodes. This is a well-known problem in graph theory for which there exist efficient polynomial time algorithms. However, the problem of finding two diversely routed paths between a pair of nodes becomes much more difficult in case of multiple correlated link failures that can be captured by the notion of *SRLG (Shared Risk Link Group)*. In fact, an SRLG is a set of network links that fail simultaneously when a given event (risk) occurs. The scope of this concept is very broad. It can correspond, for instance, to a set of fiber links of an optical backbone network that are physically buried at the same location and therefore could be cut simultaneously (i.e. backhoe or JCB fade). It can also represent links that are located in the same seismic area, or radio links in access and backhaul networks subject to localized environmental conditions affecting signal transmission, or traffic jam propagation in road networks. Note that a link can be affected by more than one risk. In practice, the failures are often localized and common SRLGs are SRLGs verifying the *star property* [9] (coincident SRLGs in [5]). Under this property, all links of a given SRLG share an endpoint. Such failure scenarios can correspond to risks arising in router nodes like card failures or to the cut of a conduit containing links issued from a node (see Figure 1).

Related work

In the context of SRLG, basic network connectivity problems have been proven much more difficult to address than their counterparts for single failures. For instance, the problem of finding a “SRLG-shortest”

[†]Full version of this work is available in [1].

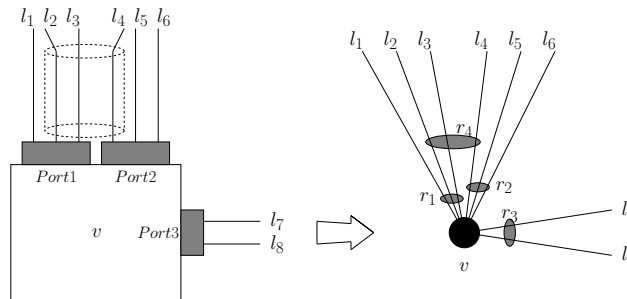


FIGURE 1: Example of localized risks : link l_4 shares risk r_2 , corresponding to Port 2 failure, with l_5 and l_6 , and shares risk r_4 , corresponding to a conduit cut, with links l_2 and l_3 .

st -path which is a path from node s to node t having the minimum number of risks has been proven NP -hard and hard to approximate in general . However, the problem can be solved in polynomial time in two generic practical cases corresponding to localized failures : when all risks verify the star property [3] and when risks are of *span 1* (i.e. when a link is affected by at most one risk and links sharing a given risk form a connected component [2]).

The *diverse routing* problem in presence of SRLGs consists in finding two SRLG-disjoint paths between a pair of vertices (i.e. paths having no risk in common). It has been proven NP -complete in general [7, 6, 8, 10] and many heuristics have been proposed. The problem is polynomial in some specific cases of localized failures : when SRLGs have span 1 [2], and in a specific case of SRLGs having the star property [4] in which a link can be affected by at most two risks and two risks affecting the same link form stars at different nodes (this result also follows from results of [2]).

Our results

We studied the diverse routing problem when SRLGs have the star property and there are no restrictions on the number of risks per link. This case has been studied in [9] in which the authors claim that the diverse routing problem with the star property can be solved in polynomial time. Unfortunately their algorithm is not correct ; indeed we exhibited counterexamples for which their algorithm concludes to the non existence of two SRLG-disjoint paths although two such paths exist. We proved that the problem is in fact NP -complete (again, contradicting the supposed polynomiality of the algorithm of [9], unless $P = NP$). On the positive side, we showed that the diverse routing problem can be solved in polynomial time in particular subcases which are relevant in practice. Namely, we solved the problem when the number of SRLGs is bounded by a constant, when the maximum degree is at most 4 or when the input network is a directed acyclic graph. Finally, we considered the problem of finding the maximum number of SRLG-disjoint paths. This problem has been shown to be NP -hard in [8]. We proved that it is also NP -hard under the star property.

2 Diverse Routing with star SRLGs

Counterexample

Luo and Wang have published in [9] an algorithm to find a pair of SRLG-disjoint paths with minimum total cost from a source s to a destination t in graphs with SRLGs satisfying the star property. We have proved the incorrectness of their algorithm with counterexamples.

Figure 2 shows a counterexample where we have 2 specific risks r_1 and $r_2 \neq r_1$ forming a star in v . We obviously have two SRLG-disjoint paths $P_1 = \{s, a, z, w, u, v, b, t\}$ and $P_2 = \{s, a', w, u', v, b', t\}$ but if we ran the algorithm of [9], it will terminate concluding that no two SRLG-disjoint paths exist (see full version in [1] for more details).

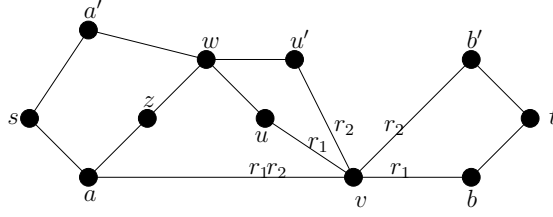


FIGURE 2: Example 2

Model and problem statement

To state precisely our results we use the *multi-colored graph* model [2, 9]. In the multi-colored graph model, the network is modeled by a graph $mG = (V, E)$ and the set of SRLGs by a set of colors \mathcal{R} which is a covering of E . Each SRLG is modeled by a distinct color and an edge modeling a network link subject to different SRLGs will be assigned as many colors as SRLGs. In such a model, the star property corresponds to the fact that all the edges with the same color are incident to a common vertex and so form a star. In what follows we denote by Δ the maximum degree, by Δ_c the maximum colored degree (the number of colors incident to a node), by $E(c)$ the set of edges having color $c \in \mathcal{R}$, by $\mathcal{R}(e)$ the set of colors associated with edge $e \in E$, by $\text{CPE} = \max_{e \in E} |\mathcal{R}(e)|$ the *maximum number of colors per edge*, and by $\text{EPC} = \max_{c \in \mathcal{R}} |E(c)|$ the *maximum number of edges having the same color*.

Given a multi-colored graph mG and two vertices s and t . We say that two st -paths P_1 and P_2 are *color-disjoint* if $(\cup_{e \in P_1} \mathcal{R}(e)) \cap (\cup_{e \in P_2} \mathcal{R}(e)) = \emptyset$.

The diverse routing problem defined in the introduction consists then in finding k color-disjoint paths and can be formulated formally as follows :

Problem 1 (*k-Diverse Colored st -Paths, k -DCP*). *Given a multi-colored graph mG and two vertices s and t , are there k color-disjoint st -paths from s to t ?*

Another related problem we studied aims at finding the maximum number of color-disjoint paths and can be stated formally as follows :

Problem 2 (*Max Diverse Colored st -Paths, MDGP*). *Given a multi-colored graph mG and two vertices s and t , find the maximum number of color-disjoint st -paths.*

NP-completeness

Theorem. *The k -DCP problem is NP-complete for any fixed constant $k \geq 2$, even if all the following properties hold :*

- the star property ;
- the maximum degree Δ is fixed with $\Delta \geq 6 + k$;
- CPE, EPC and Δ_c are fixed with either $[\text{CPE} \geq 4, \text{EPC} \geq 2, \text{ and } \Delta_c \geq 14 + k]$ or $[\text{CPE} \geq 2, \text{EPC} \geq 4 \text{ and } \Delta_c \geq 2 + k]$.

we proved the above theorem by using a reduction from the problem of finding a T -compatible path (or a path avoiding forbidden transitions), which was proved NP-complete in [11].

Let $G = (V, E)$ be an undirected graph. A transition in $v \in V$ is a pair of edges incident to v . To each vertex v we associate a set $T(v)$ of admissible (or allowed) transitions in v . We call transition system the set $T = \{T(v) \mid v \in V\}$. Let $G = (V, E)$ be a graph and T a transition system. A path $P = \{v_0, e_1, v_1, \dots, e_k, v_k\}$ in G , with $v_i \in V$, $e_i \in E$, is said to be T -compatible if, for every $1 \leq i \leq k - 1$, the pair of edges $\{e_i, e_{i+1}\}$ is an admissible transition, i.e. $\{e_i, e_{i+1}\} \in T(v_i)$.

Problem 3 (*T-Compatible path, T-CP*). *Given a graph $G = (V, E)$, two vertices s and t in V , and a transition system T , does G contain a T -compatible path from s to t ?*

The reduction consists in transforming an instance T -CP on a graph G to an instance of 2-DCP on a colored graph mG . The transformation is such that every edge in G is substituted by 2 paths, of length 2 each, in

Graph	Δ	EPC	CPE	k -DCP	MDCP	
Undirected	≥ 8	≥ 2 ≥ 4	≥ 4 ≥ 2	Strongly NP -hard for $\Delta \geq 6 + k$	Not approximable within $O(V)$	
Directed	≤ 3	any	any	Solvable in $O(V + E)$ time	Optimum in $O(V + E)$ time	
	$= 4$	any	any	Solvable in $O(V + E)$ time for $k = 2$	2-approximation in $O(V + E)$ time	
$ \mathcal{R} = O(1)$, even without star	any	any	any	Solvable in $O(V + E)$ time	Optimum in $O((V + E) \log \Delta)$ time	
DAG	unbounded	≥ 3	≥ 3	Solvable in $O(V E ^{2k})$ time	Strongly NP -hard	
		≥ 2	≥ 6		Not approximable within $O(V)$	
		≥ 2	unbounded			
		unbounded	3			APX -hard
		unbounded	unbounded			$W[1]$ -hard

TABLE 1: Summary of complexity results.

mG and colors are assigned in mG such that two color-disjoint paths can cross a vertex in mG if and only if a path in G can cross the same vertex using an allowed transition (see full version in [1] for more details).

3 Conclusion

Our results give an almost complete characterization of the problem of finding SRLG-disjoint paths in networks with SRLGs satisfying the star property. We summarize them all in Table 1.

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