Vibration model of piano soundboards

Xavier Boutillon, Kerem Ege

WAVES 2013 — Invited communication at the *Piano* minisymposium (June 2013, Tunis)

Vibration model of piano soundboards

X. Boutillon[1,*], **K. Ege**[2]
[1] Laboratoire de mécanique des solides, École polytechnique, F-91128 Palaiseau Cedex, France.
[2] Laboratoire Vibrations Acoustique, INSA-Lyon, 25 bis avenue Jean Capelle, F-69621 Villeurbanne Cedex, France.
*Email: boutillon@lms.polytechnique.fr

Abstract

Modal observations of a piano soundboard are compared with results predicted by a model consisting of weakly coupled homogeneous sub-structures. The model is entirely determined by the coarse geometry of the soundboard (main plate, ribs, bridges, cut-off corners) and by the elastic parameters of the wood species. It can also be used to predict the point-mobility at the bridge (where strings are attached) or far from it. The agreement between observations and model predictions is excellent, both in the low- and high-frequency regimes (respectively below and above $\approx$ 1 kHz). Applications include a comparison between the characteristics of different pianos as well as the influence of the wood properties on the point-mobility. Some consequences in terms of acoustical radiation will also be presented.

Introduction

In a piano, the soundboard is the plate-like structure on which the strings are attached. It radiates sound (the strings are too thin to radiate efficiently) and rules the sound-decay which is an essential part of the piano sound. Coupling between the string and the soundboard is described by the point-mobility $Y_Q(\omega) = V(\omega)/F(\omega)$ where, ω is the angular frequency, F the force applied by the string(s) at point Q and V the resulting velocity of the soundboard at that point. $Y_Q(\omega)$ can be written as the sum of the mobilities of the modes of the soundboard at a given point. We consider that modal shapes are sinusoids along the bridge and products of sinusoids across the soundboard (see § 1 for experimental observations and FEM results). Modal frequencies are obtained in average by a model presented in § 2. Modal dampings are given by observation. Ignoring fine geometrical details and local pecularities, these ingredients are sufficient to predict $Y_Q(\omega)$ at any point, according to Skudrzyk's theory of the mean-value of the point mobility [1]. Results pertaining to modal density and to the reciprocal of the frequency-averaged point-mobility are given in § 3, for different pianos.

1 Experimental and numerical observations

The following observations (see [2] for a complete report) have been made on an upright piano soundboard (Atlas, .91 m $\times$ 1.39 m) and result from a high-resolution modal analysis technique [3]. For results below 350 Hz, the soundboard was excited locally by a impact hammer and above that limit, the soundboard was excited globally by a strong acoustical field. The vibration was observed locally with accelerometers. The modal analysis also yielded the modal dampings with an excellent precision in a frequency range not accessible with Fourier-based techniques (modal overlap approaching 100%). It appears that above $\approx$1 kHz, not all the modes are observed at any given observation point, hence the use of the concept of *apparent* modal density, defined as the reciprocal of the average modal spacing and represented in Fig. 1. Below 1 kHz, the apparent modal density does not depend on the point of observation and looks similar to that of a plate (or a combination of plates). Above that limit, the apparent modal density decreases and depends on the point of observation.

A typical modal shape for the so-called low-frequency regime (below 1 kHz) is represented in the top of Fig. 2. The vibration extends over the whole soundboard except, eventually, in one or another cut-off corner. In the high-frequency regime

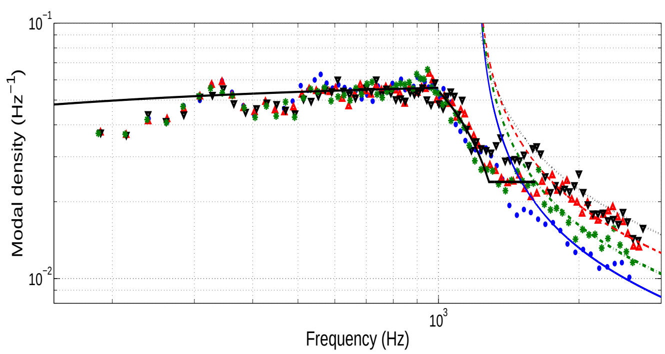

Figure 1: Modal density of the Atlas soundboard. Dots: observed values at various points of the soundboard. Lines: prediction of the model (§ 2).

(above 1 kHz), modal shapes have been obtained by finite-element modeling of the soundboard [2]. It appears (Fig. 2) that the vibration is both *confined* between ribs and, most often, *localised* in one or a very few areas of the ribbed parts of the soundboard, due presumably to the slightly irregular spacing of ribs across the soundboard.

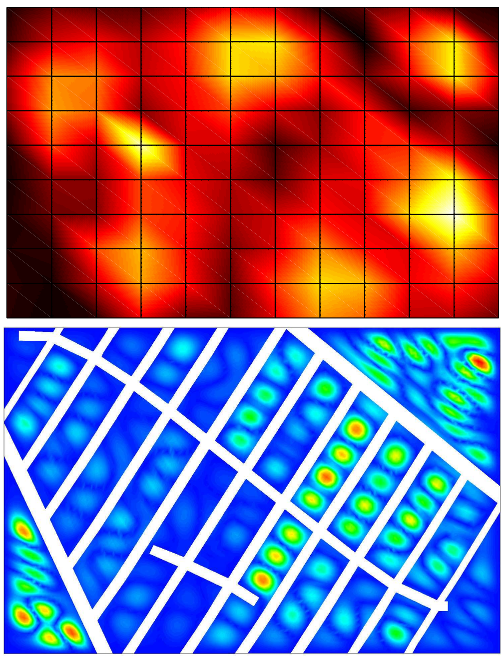

Figure 2: Typical modal shapes. Top: observed in the low-frequency regime (mode 10, 303 Hz). Bottom: numerically obtained in the high-frequency regime (mode 167, 2733 Hz).

2 Model

The different parts (cut-off corners, if any, the two main parts of the soundboard, as limited by the main bridge, the rim and the cut-off bars, the main bridge) are conidered as weakly coupled homogeneous substructures. The bass bridge is described as a simple mass added to the corresponding part of the soundboard. Each plate-like structure is considered with clamped boundary conditions. The main bridge is described as a bar, the cut-off corners as orthotropic plates, as well as the the ribbed parts of the soundboard in the low-frequency regime, following the homogenisation proposed by [4].

In the high frequency domain (where the apparent modal density depends on the point of observation), we consider that the two main parts of the soundboard (ribbed areas, extending on each side of the main bridge) vibrate only in the vicinity of the observation points, namely within three inter-rib spaces. Each inter-rib space of width p is seen as a structural wave-guide where the wave-number in the direction orthogonal to the ribs is $k_x = n\pi/p$, with $n \in \mathbb{N}^*$. A transition has been devised between the two regimes.

Under the weak-coupling hypothesis, the modal density is the sum of the modal densities of the substructures. The agreement between observations and the results given by the model (Fig. 1) is striking.

3 Applications

The model has been used to to analyse the influence of wood parameters (not shown here) and to characterise different pianos (Fig. 3).

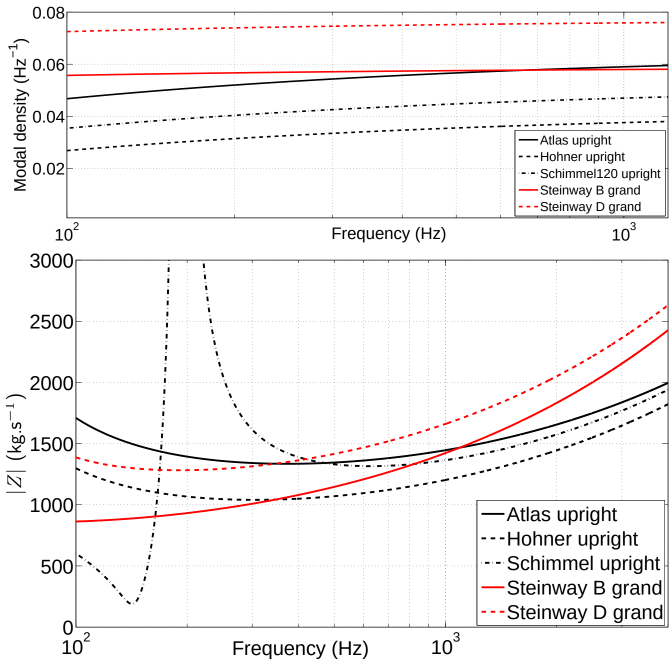

Figure 3: Comparison of three upright pianos and two grand pianos. Top: apparent modal density in low-frequency. Bottom: characteristic impedances (model artefact at 200 Hz for the Schimmel upright).

References

[1] E. J. Skudrzyk. *J.Acoust. Soc. Am.*, **67**(4):1105–1135, 1980.

[2] K. Ege, X. Boutillon, and M. Rébillat. *J. Sound Vibration*, **332**(5):1288-1305, 2013.

[3] K. Ege, X. Boutillon, and B. David. *J. Sound Vibration*, **325**(4-5): 852-869, 2009.

[4] J. Berthaut, M. N. Ichchou, and L. Jezequel. *Applied Acoustics*, **64**(11):1113–1136, 2003.