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# Multi-scale modeling of imperfect interfaces

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## Abstract

In [1–6] we study the mechanical behavior of thin films between elastic adherents. The analysis is based on the classic idea that a very thin adhesive film can be replaced by a contact law.

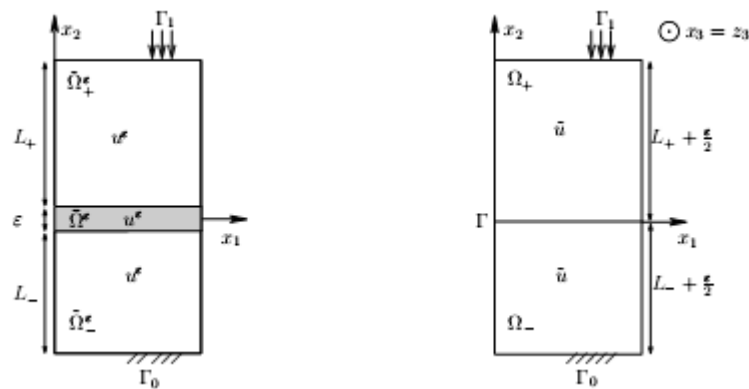
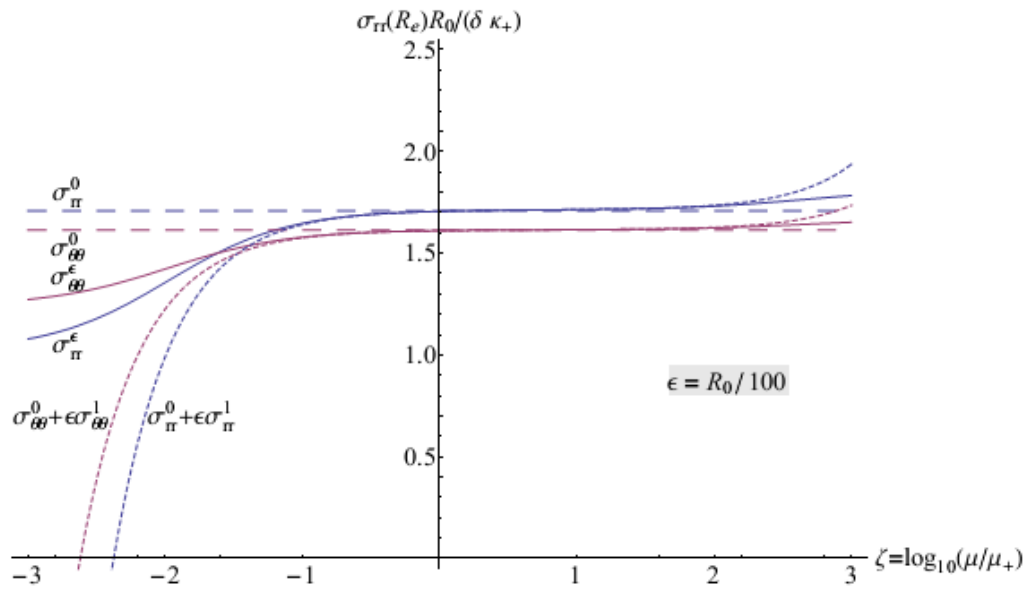


Figure 1. Initial and limit configuration of two solids glued together.

The contact law describes the asymptotic behavior of the film in the limit as its thickness goes to zero and it prescribes the jumps in the displacement (or rate of displacements) and traction vector fields at the limit interface. The formulation of the limit problem involves the mechanical and the geometrical properties of the adhesive and the adherents, and in [1–6] several cases were considered: soft films [1]; adhesive films governed by a non convex energy [2]; flat linear elastic films having stiffness comparable with that of the adherents and giving rise to imperfect adhesion between the films and the adherents [3,4]; joints with mismatch strain between the adhesive and the adherents [6]. Several mathematical techniques can be used to perform the asymptotic analysis:

- $\Gamma$ -convergence,
- Variational analysis,
- Matched asymptotic expansions,
- Energy approaches [5],
- Numerical studies [7].

In this lecture, we present some mathematical contributions in elasticity, with a focus on the energy approach, and numerical results in this field. In particular, it will be shown that the energy approach is very efficient to compute the singularities on the edge of the glue. Plane and curved interfaces will be analyzed, and it will be demonstrated the necessity of a “at least first order asymptotic theory” in the case of similar rigidity between the adherents and the glue.



**Figure 2.** Bonding of two spheres: comparative plots of the radial and circumferential stresses on the external surface calculated from the exact three-phase solution, the perfect (order 0) and the imperfect (order 1) interface approximations.

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