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Acoustic wave localization in one-dimensional Fibonacci phononic structures with mirror symmetry

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This paper reports on numerical and experimental results of acoustic transmission spectra of bead chains with symmetric and asymmetric Fibonacci-like structures. As a matter of comparison, perfect periodic acoustic waveguide structures are also examined. This study shows that Fibonacci structures with mirror symmetry can exhibit localized modes with higher amplitude, due to resonant transmission induced by the presence of dimers inside the 1D structure. A good agreement is observed between the theoretical predictions and the experimental power spectra. © 2013 AIP Publishing LLC [<http://dx.doi.org/10.1063/1.4801890>]

I. INTRODUCTION

During the last decades, many theoretical and experimental studies were devoted to the propagation of waves such as electromagnetic or elastic waves in composite materials with periodic structures. In 1993, by analogy to the extensive realizations achieved on photonic crystals,¹ Kushwaha *et al.*,² and Sigalas and Economou³ suggested the concept of “phononic crystals” i.e., artificial structures made of periodic distributions of elastic inclusions embedded in a host matrix. First, full calculation of the band structure of two- and three- dimensional phononic crystals showed the existence of frequencies ranges named as forbidden or stop bands where propagation of elastic waves is not allowed. Further to these pioneering works, by analogy with the electronic properties of natural and artificial semi-conductors, effects of compositional or positional disorder on the dispersion curves and transmission properties of periodic structures have received a great deal of attention. One of the motivations for studying these disordered structures is the localization phenomenon⁴⁻⁶ which can be defined as the spatial confinement of waves associated with disorder. Different degrees of disorder may be considered. Then, the lowest degree of disorder consists of the insertion of a positional or compositional defect inside the periodic structure.⁷⁻⁹ The periodicity is then broken and this may lead to highly localized modes that appear as very narrow pass bands within the band gaps. Potential applications of these defected structures as very selective frequency filters were widely studied theoretically and experimentally, especially in one-dimensional,¹⁰ quasi-one-dimensional,¹¹ two-^{12,13} and three-⁹ dimensional structures. The highest degree of disorder corresponds to the fully disordered system in which randomly distributed perturbations are introduced. As a disordering process is gradually introduced in a periodic structure, the band gaps become partially regions of localized states. The localization of electromagnetic and

elastic waves in 1-D to 3-D samples was widely investigated as well as the frequency dependences of the localization length and the density of states. A numerical study was developed by Williams and Maris¹⁴ to calculate phonon eigenstates for a 2-D square lattice with mass disorder and the phonon localization length was studied versus frequency. In the work reported by Sigalas and Soukoulis,¹⁵ the propagation of elastic waves through disordered multi-layered 1-D systems assumed periodic on the average is investigated. Effect of disorder on the band structure and the transmission profiles is analyzed. In vibration application, Xie¹⁶ calculated the vibration mode localization in randomly disordered single bay and double bay trusses by using the transfer matrix method. More recently, Lu *et al.* demonstrate Anderson localization of ultrasound in a three-dimensional (3D) elastic medium made of randomly distributed brazed aluminium beads.¹⁷

The well known quasi-periodic crystal where the structure is ordered in a non-periodic fashion constitutes an intermediate case between perfectly periodic and random systems.^{18,19} The transmission of classical waves through such systems exhibits very peculiar behaviours, and thus, received a great deal of interest. A lot of works was then reported on the propagation of elementary excitations through quasi-periodic structures according to different geometrical sequences such as Cantor, Fibonacci, Thue-Morse, and double-period sequences²⁰⁻²² that are of particular interest in many domains of physics such as optics, thermodynamics, and acoustics. Among these works, several studies focused on quasi one-dimensional structures made of one-dimensional segments and loops pasted together²³⁻²⁶ and following a Fibonacci series. These structures are constituted of two different building blocks A and B distributed according to the Fibonacci sequence defined recursively as $S_{n+1} = S_n S_{n-1}$ with $S_0 = A$ and $S_1 = B$. For example, in 2006, Hassouani *et al.*²³ reported on theoretical and experimental electromagnetic transmission spectra showing the distribution of the

bandwidths as a function of the Fibonacci generation number (i.e., integer n). Different properties related to the bulk bands such as the fragmentation of the frequency spectrum are discussed and analyzed. The works reveal also the existence of self similarity of order 6 and multi-fractality which are related to the quasi-periodicity of the structure. Effects of these phenomena are much more pronounced when the phase shift along each building block is the same, inducing specific thickness for a given wave velocity in the material. These results are general and remain valid for other kind of excitations and especially for acoustic waves.

Similar investigations were conducted on the light transmission through 1-D quasi-periodic multilayer and quasicrystals arranged by the Fibonacci sequences.^{27–31} Moreover, Macon *et al.*³² presented experimental results on the propagation of surface acoustic waves (SAW) on a quasi periodically corrugated surface with grooves engraved according to a Fibonacci sequence. Reflection and transmission frequency dependences are analyzed and exhibit transport features similarly encountered with disordered systems and related to Anderson localization.⁴ They point out a critical regime of the localization transition and succeed to give an experimental characterization of the critical proper modes. In a recent past, a new type of quasi-periodic structures was considered including the mirror symmetry.^{33–35} Symmetrical Fibonacci sequences with mirror symmetry were studied in photonic aperiodic dielectric multilayers.^{33,34} The symmetric Fibonacci sequence is generated in the following way. The j th generation of the sequence can be expressed as $S_j = G_j H_j$, where G_j and H_j are Fibonacci sequences. G_j and H_j obey the recursion relations $G_j = G_{j-1} G_{j-2}$, $H_j = H_{j-2} H_{j-1}$ with $G_0 = H_0 = B$, $G_1 = H_1 = A$, and consequently $S_j = G_{j-1} G_{j-2} H_{j-2} H_{j-1}$.³³ The structures presented in Refs. 33 and 34 are Symmetric Fibonacci Multilayer (SFM) $\text{TiO}_2/\text{SiO}_2$ with internal symmetry. It was shown that their optical transmission spectra exhibit a series of resonant transmissions induced by the internal symmetry. The resonant transmissions originate from the dimer-like positional correlation in the system, i.e., the occurrence inside the quasi-periodic structure of “CC” pairs, where C is made of building blocks A or B (for example $C = ABA$). This kind of dimers is particularly attractive because the resonant transmission can be adjusted at a specific frequency by tuning aperiodic structure with internal symmetry. Then the mirror symmetry plays an important role in obtaining the perfect transmission features in SFM and may lead to practical applications such as efficient optical filters and multiplexing systems. Nevertheless it has been shown that the mirror symmetry is a sufficient condition but not a necessary one to generate multiple perfect transmission peaks. Indeed asymmetric Fibonacci multilayers were constructed by joining a Fibonacci sequence H_j and a conjugated Fibonacci sequence C_j where the initial conditions are $H_0 = B$, $H_1 = A$, $C_0 = A$, and $C_1 = B$, respectively (for example, the asymmetric structure of order 4 is $H_4|C_4 \equiv ABAAB|BABBA$). Under the assumption of the conservation of the light phase in each medium A and B, the authors³⁶ have shown that while the structure is asymmetric, it exhibits perfect transmission of light at some specific wavelengths. This feature was analyzed with the help

of a rather simple quasi-analytical model based on products of transfer matrices.

In this paper, we focus on one-dimensional chains made of glued spherical solid beads. In recent years, the propagation of longitudinal elastic waves in similar waveguide structures was the subject of several works.^{37,38} In particular, the effect of grafted side branches (or double stub) on the transmission properties of such waveguides was analyzed numerically and experimentally.³⁸ Results showed that with a proper choice of the constituting materials of the different beads, this waveguide structure may exhibit selective filtering functionalities. Inspired by the works on internal symmetry in photonic and phononic quasiperiodic superlattices, we propose another way to obtain selective filtering device based on symmetric Fibonacci waveguides, made of glass and steel glued beads. We report especially on experimental and numerical results of acoustic transmission spectra of Fibonacci bead chains with mirror symmetry. Due to the attenuation of acoustic waves along too long bead chains, the transmission spectra are specifically investigated for the first Fibonacci generations (i.e., $n \leq 7$). Transmission spectra present the resonance modes, and their eigen frequencies of bead chains sandwiched between two transducers. As a matter of comparison perfect periodic acoustic waveguide structures are first examined. Experimental transmission spectra of symmetric Fibonacci structures exhibit resonance peaks of higher amplitude in good agreement with the theoretical predictions.

The paper is organized as follows. In Sec. II, the samples and the numerical methods are presented. The procedure allowing the measurement of longitudinal elastic waves transmission coefficients together with the experimental results are reported in Sec. III. Conclusions are summarized in Sec. IV.

II. GENERAL DESCRIPTION OF THE DEVICES AND NUMERICAL CALCULATION

A. Description of the samples

The numerical results and experiments are performed on samples made up of two kinds of 8.0 ± 0.1 mm diameter beads: steel and glass beads denominated in the following by capital letters S and G, respectively. The longitudinal C_L and the transverse C_T speeds of sound and the density ρ of the beads used in the calculations are $C_L = 5960$ m/s, $C_T = 3400$ m/s, and $\rho = 7900$ kg/m³ for steel and $C_L = 5600$ m/s, $C_T = 3700$ m/s, and $\rho = 2600$ kg/m³ for glass.

The Fibonacci quasi periodic structures are built with steel (S) and glass (G) beads. The Fibonacci sequences are usually defined with the recursion relation: $S_{n+1} = S_n S_{n-1}$, n being the number of the Fibonacci sequence, with $S_0 = G$ and $S_1 = S$. Unfortunately, self-similarity and multi-fractality phenomena cannot be observed in these bead chains. Indeed as mentioned for electromagnetic Fibonacci structures^{23,33,36} such phenomena are much more pronounced when the phase shift along each building block A or B is the same. This condition is clearly not verified with beads of the same diameter and made of different materials, such as steel and glass. Moreover due to the attenuation of acoustic waves along the

bead chains, our experimental study is limited to chains made of a maximum number of 10 beads.

Different samples have been fabricated and experimentally tested.

- (i) Classical superlattice-like samples, in particular made of 9 beads SGSGSGSGS named Cla-9.
- (ii) The S3 to S7 Fibonacci sequences, in particular the S4 sequence (SGSSG) for further experiments.
- (iii) The Symmetrical Fibonacci sequence based on S4 structure: SGSSG-GSSGS, named Sym-S4 and its reverse obtained from Sym-S4 by placing the mirror on the opposite end of the S4 chain: GSSGS-SGSSG named Rev-S4.
- (iv) A defect has been inserted in the Rev-S4 sequence, i.e., the 4th and the 5th beads (from the left) have been permuted, leading to GSSSG-SGSSG named Def-S4.

The samples are drawn as insets in the figures, the white (respectively, grey) bead being made of glass (respectively, steel).

B. Infinite periodic chain: Calculation of the dispersion curves

In a first time, the case of an infinite periodic chain with alternatively S and G beads is studied. The finite element method, with the help of the ATILA code³⁹ has been used for the calculation of the dispersion curves: only one unit cell of the periodic grating, made of two beads, one made of glass the other made of steel, is meshed and Bloch-Floquet relations are applied on the unit cell.⁴⁰ In further experiments, adjacent beads are pasted together with a very small amount of glue. In numerical calculations, the coupling between the beads is taken into account by introducing a small contact area, which seems to be the simpler way to represent the coupling. The dispersion curves are plotted by varying the wave-number k in the reduced Brillouin zone i.e., for k varying between 0 and $\pi/4R$, where R is the bead radius. Figure 1 presents the corresponding dispersion curves with alternate pass bands and forbidden bands. The pass bands are related to propagating modes whereas forbidden bands are associated to evanescent modes. In the frequency range of interest, a first forbidden band occurs in the 47–83 kHz frequency

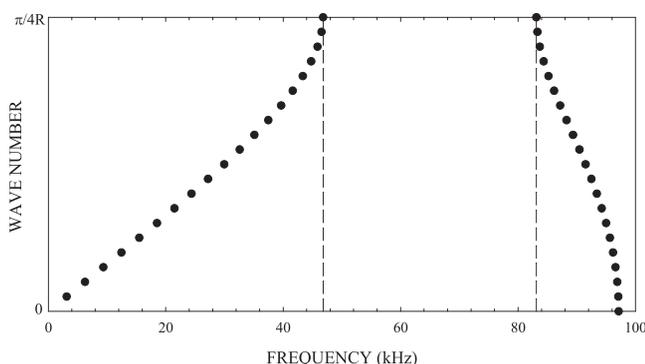


FIG. 1. Band structure (wave number as a function of frequency) for an infinite chain containing alternatively steel and glass beads of radius R . The vertical dotted lines correspond to the limits of the forbidden band.

range. The limits of the forbidden band of the infinite periodic chain are represented as vertical dotted lines in the following figures.

C. Finite size samples: Calculation of the transmission coefficient

For numerical calculations, a prescribed longitudinal force is supposed to be applied on the left end of the chain for simulating the displacement of the front face of the transmitter. The displacements at the right extremity are calculated and the numerical variations of the product frequency (f) times longitudinal displacement (Ux) are plotted in dB.³⁷ These plots represent the variations of the transmission normalized to its highest value as a function of the frequency. Numerical calculations are performed on the above mentioned samples.

Figure 2 presents the variations of the normalized transmission versus the frequency for the Cla-9 chain. Figure 2 highlights two sets of 4 peaks, below and above the stop band that correspond to maxima of the displacement in the x direction (along the main axis of the sample). This has been previously observed in the case of linear chains made of N identical beads where the pass band in the band structure corresponds to $(N-1)$ peaks in the transmission spectrum.³⁷ One can notice that the peaks are sharp because losses are not taken into account in the numerical calculations.

In the same way, the transmission is calculated for several samples with structures according with Fibonacci sequences of different orders. Classically, they exhibit $(N-1)$ peaks in the same range of frequency as the pass band of the periodic analogous structure, when N is the total number of the beads in the finite length chain. This is clearly illustrated in Fig. 3(a) where the frequency locations of the transmission peaks for S3 to S7 Fibonacci sequences are reported. One notes that some of these peaks appear in the frequency range of the stop band of the perfectly periodic Cla-9 chain. These peaks are similar to those observed when structural defects are inserted in a perfectly periodic chain of beads and they correspond to localized modes associated with the Fibonacci sequences. As expected, because the phase shift is not the same across the glass bead and the steel

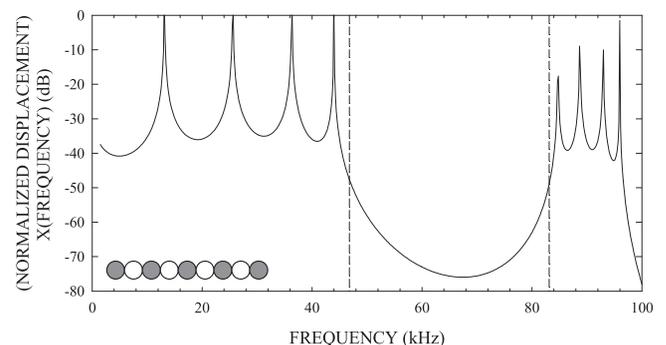


FIG. 2. Chain of 9 beads with alternatively steel and glass beads (Cla-9). Theoretical transmission in terms of Ux as a function of the frequency. The amplitude is normalized to the maximum. The vertical dotted lines correspond to the limits of the forbidden band (see Fig. 1). In the inserted sample, the white (respectively, grey) bead is made of glass (respectively, steel).

bead, the self-similarity between the sequences is not clearly exhibited. Figure 3(b) presents the transmission in the particular case of the S4 chain, for further comparison with experiments.

Figures 4(a) and 4(b) present the normalized transmission for the Sym-S4 and Rev-S4 chains, respectively. As previously, some transmission peaks occur in the frequency range of the stop band of the periodic chain and these transmission maxima can be associated with the mirror symmetry. Moreover, the amplitude of these peaks is higher for the Rev-S4 than the Sym-S4. Resonant transmission induced by dimers may explain the enhancement of the transmission for the Rev-S4 chain. Indeed, one can notice that the sequence (GSSGS-SGSSG) can be arranged under the forms: GS-SGS-SGS-SG, where the dimer (as defined in Ref. 34) SGS-SGS occurs or also as G-SS-G-SS-G-SS-G with the dimer SS. These phenomena can also be observed for higher order of Reverse Symmetrical Fibonacci sequences.³⁴ Therefore, by a proper choice of the Symmetrical Fibonacci sequence, the device can be used as a selective filter at a given frequency. One can notice that symmetry of the Fibonacci chains is sufficient but not necessary for transmission enhancement.³⁶ Because the phase shift is not conserved in each building block, it is not straightforward to propose an analytical model allowing a reliable prediction of the frequency location of the transmission maxima as it was done in electromagnetic Fibonacci superlattices.³⁶

Finally introducing a default in the Rev-S4 chain by inverting the fourth and the fifth beads to lead to the

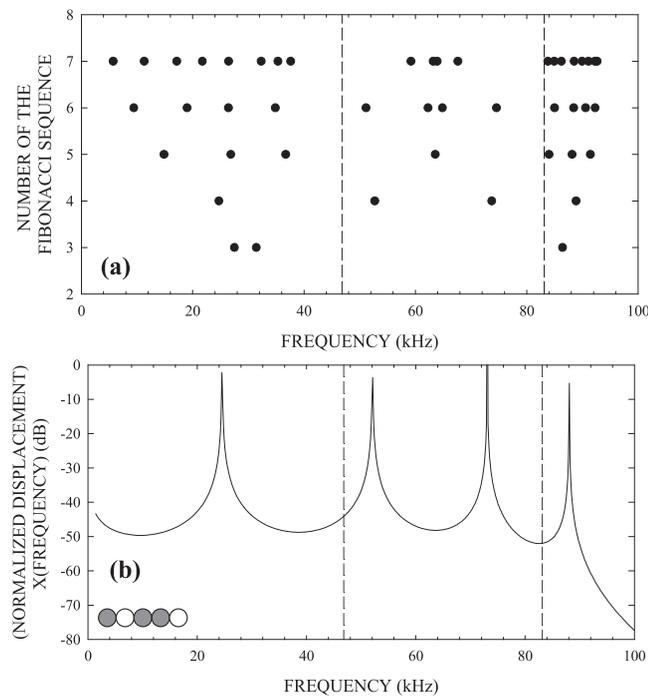


FIG. 3. Chains with steel and glass beads, following Fibonacci sequences, from S3 to S7. (a) Numerical spectroscopic diagram showing the position of the resonance frequencies for different values of n , the number of the Fibonacci sequence. (b) Theoretical transmission in terms of $U_s f$ as a function of the frequency for the S4 sequence. The amplitude is normalized to the maximum. The vertical dotted lines correspond to the limits of the forbidden band. In the inserted sample, the white (respectively, grey) bead is made of glass (respectively, steel).

asymmetric GSSSG-SGSSG structure without dimer, one observes (see Fig. 4(c)) that the higher transmission level observed with Rev-S4 is strongly reduced. Then localized modes with high transmission are favoured in symmetric Fibonacci structures with dimers as already observed for light.³⁴

III. EXPERIMENTAL PROCEDURE AND RESULTS

A. Experimental procedure

Several chains of spherical glued beads were manufactured. Steel and glass beads are quite mono-disperse with a diameter of 8.0 ± 0.1 mm. Two adjacent beads are pasted together with a drop of a bi-component epoxy glue. The amount of glue is chosen as small as possible in order to not affect strongly the transmission properties of the sample. The samples are sandwiched between two longitudinal transducers, used in the [10–80] kHz pass band. The experiments are conducted with short ultrasonic pulses for a frequency analysis via Fast Fourier Transform. The response is electronically selected by eliminating the excitation pulse in order to explore the transmitted signal only. As the amplitude of the wave bursts is usually small, several signals are

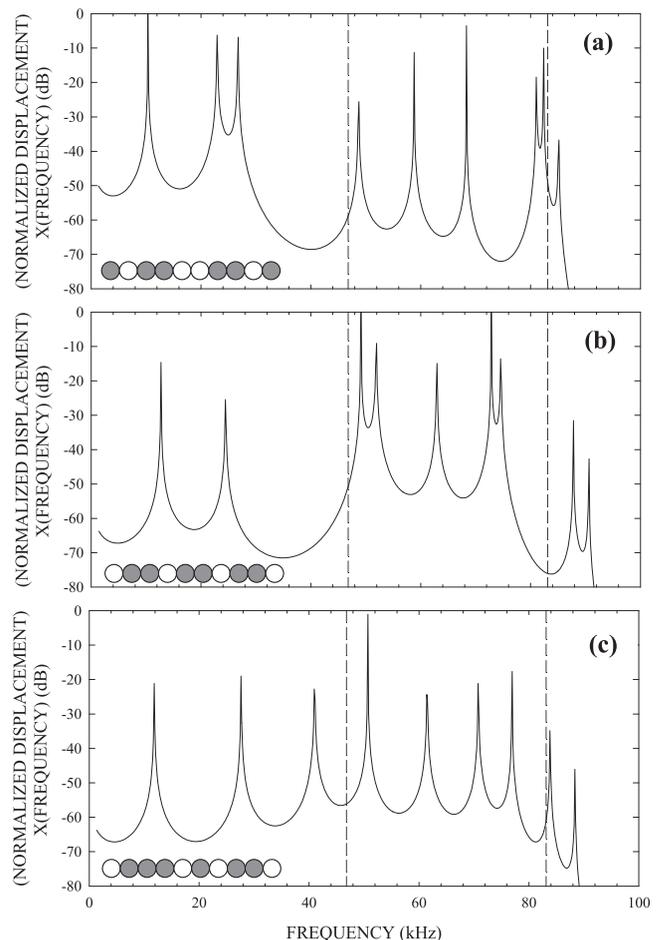


FIG. 4. Theoretical transmissions in terms of $U_s f$ as a function of the frequency for (a) Sym-S4 sequence, (b) Rev-S4 sequence, and (c) Def-S4 sequence. The amplitude is normalized to the maximum. The vertical dotted lines correspond to the limits of the forbidden band. In the inserted sample, the white (respectively, grey) bead is made of glass (respectively, steel).

recorded and averaged to eliminate background noise. The averaged waveforms are stored in a computer for post-treatment, and then power spectra are calculated. This procedure leads to the observation of transmission peaks that emerge in the power spectra.

A longitudinal force (F_0) is applied on both ends of the chain. The frequency position of the modes is sensitive to the value of F_0 and its intensity must be carefully chosen. This dependence may be associated with non-linear effects as already reported.^{41–43} In order to avoid taking into account non-linear effects, F_0 is chosen as low as possible and then of the order of a few Newton. The propagation of the initial perturbation through the chain is created by the intimate contact between the beads. Changes in the arrangement of the beads lead to modifications of the frequency response of the chains.

B. Experimental results

In this section, experimental measurements are reported with finite length specimen namely the S4 sequence (Fig. 5), the Sym-S4 chain (Fig. 6(a)), the Rev-S4 chain (Fig. 6(b)), and the Def-S4 chain (Fig. 6(c)) for comparison with the finite elements results. Compared to the theoretical predictions, experimental results exhibit wider transmission peaks: losses could be adjusted in the numerical calculations in order to fit theoretical transmission and experimental results. The quite low resolution of the experimental setup (2 kHz), does not allow in some cases to separate easily two adjacent peaks in the power spectrum. Frequency shift between theoretical and experimental transmission maxima should be attributed to a defective coupling between adjacent beads.

Figure 5 presents the experimental transmission response of the S4 sequence. Three main peaks occur at 25, 50, and 77 kHz. Due to the attenuation of longitudinal elastic waves at higher frequencies and to the frequency response of the transducers, amplitude of these peaks decreases with increasing frequencies. These three experimental frequencies are in good agreement with the theoretical predicted ones reported in Fig. 3(b), namely at 24.5, 52.0, and 73.0 kHz, respectively. Nevertheless, the fourth peak observed at 88 kHz in Fig. 3(b) does not appear in the experimental spectrum of Fig. 5. This discrepancy between theoretical and

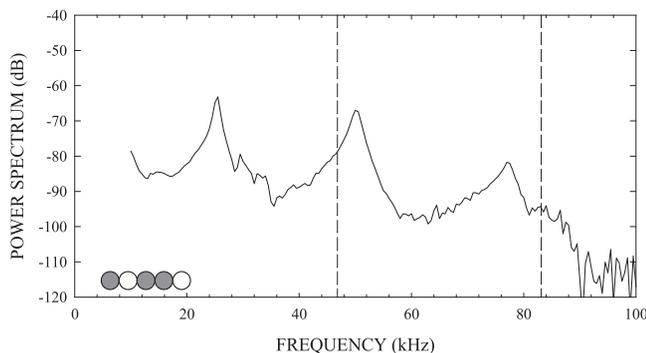


FIG. 5. Experimental power spectrum as a function of the frequency for the S4 sequence. The vertical dotted lines correspond to the limits of the forbidden band. In the inserted sample, the white (respectively, grey) bead is made of glass (respectively, steel).

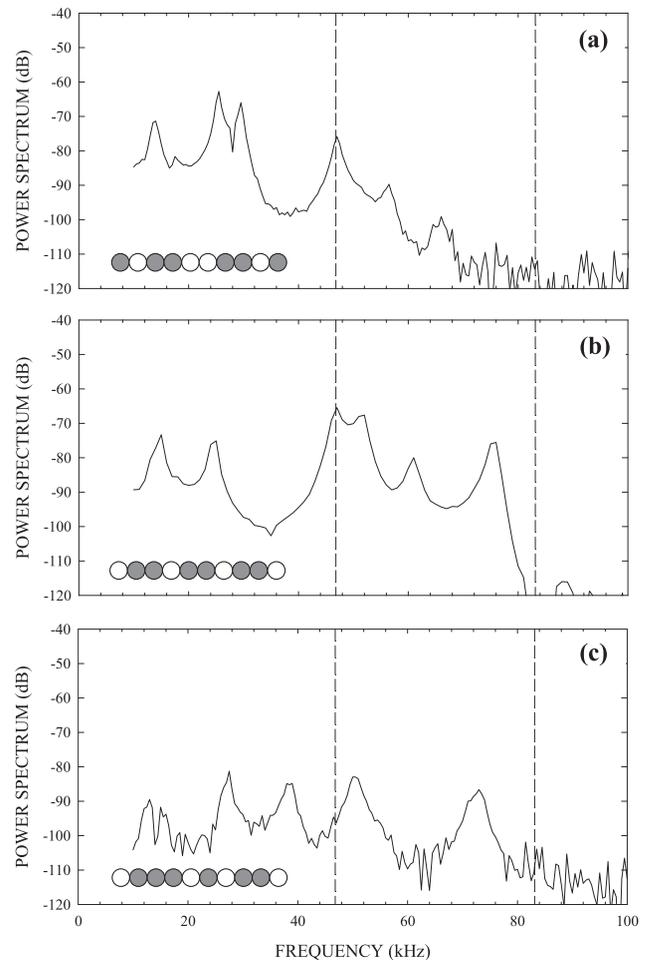


FIG. 6. Experimental power spectra as a function of the frequency. (a) Sym-S4 sequence, (b) Rev-S4 sequence, and (c) Def-S4 sequence. The vertical dotted lines correspond to the limits of the forbidden band. In the inserted sample, the white (respectively, grey) bead is made of glass (respectively, steel).

experimental results could be attributed to the very small pass band of the transducers and to the attenuation of waves with frequency.

Figures 6(a) and 6(b) show the transmission spectra obtained for the Sym-S4 and Rev-S4 chains, respectively. These spectra compare quite well with the finite element calculations reported in Figures 4(a) and 4(b) in terms of frequency locations of the transmission peaks below 80 kHz (i.e., the upper limit of the pass band of the transducers). Larger width and reduced amplitude of the experimental transmission peaks are linked with materials losses. In the frequency range of the stop band of the periodic structure, one observes that the transmission peaks exhibit larger amplitude for the Rev-S4 than for the Sym-S4 chain in agreement with the theoretical predictions (see Figs. 4(b) and 4(a), respectively). The mirror symmetry together with the existence of dimers may explain the enhancement of transmission. In Fig. 6(b), one peak is missing around 73 kHz that could be attributed to the low resolution of the experimental set-up.

In Fig. 6(c), a defect is introduced in the Rev-S4 sequence by permuting the 4th bead made of glass and the 5th bead made of steel. Then, the symmetry of the structure

is disrupted and dimers do not exist. The results present again a good agreement with numerical predictions and, as expected, most of the transmission peaks shrink.

IV. CONCLUSION

In this paper, the propagation of longitudinal acoustic waves is reported through linear chains made of glued steel and glass beads with structures following Fibonacci sequences. Attention is focused on the existence of localized modes associated with the quasi-periodic chain, inside the frequency range of the stop band of the perfectly periodic analogous structure. The existence of such localized modes is of great interest from a fundamental point of view (elastic waves localization) but also for ultrasonic applications such as filtering and demultiplexing of elastic waves.

This study shows that localized modes occur as high amplitude peaks in the transmission spectrum, when the Fibonacci sequence exhibits a symmetrical structure and dimers occur inside the 1D structure. Comparison between numerical and experimental results presents a reasonably good agreement as the frequency location of the resonant peaks is concerned. Numerical results could be improved by taking into account in the finite element calculations the material losses and by modelling the coupling between adjacent beads with a thin slab of glue between beads. From the experimental point of view, it seems necessary to improve the resolution of the experimental setup by using transducers of wider pass band for instance.

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