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# The real primes and the Riemann hypothesis

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## Abstract

We define the concept of real prime number and generalize the Riemann hypothesis to the real numbers. Thus, we can prove the new hypothesis that concerns real numbers by a calculus of integral.

## The approach

The Riemann hypothesis states that the non trivial zeros of the Riemann zeta function

$$\zeta(z) = \sum_{t=1}^{t=\infty} \left(\frac{1}{t^z}\right)$$

lies in the line  $\frac{1}{2} + iy$ . Let us study this hypothesis.

**Definition** A real number is compound if it is equal to  $\pm p_1^{n_1} \dots p_i^{n_i}$  where  $p_j$  are prime numbers and  $n_j$  are rationals. We define other real prime numbers which can not be expressed like this :  $\pi, e, \ln(2)$ . Thus  $\sqrt[q]{p} = p^{\frac{1}{q}}$  is compound.

Also  $\sqrt[q]{p} + 1$  is prime, with  $p$  prime, hence  $\sqrt[q]{p} - 1 = (p - 1)(\sqrt[q]{p} + 1)^{-1} \dots (\sqrt[q]{p} + 1)^{-1}$  is compound !

We define the GCD of two numbers : If  $p_1$  and  $p_2$  are prime real numbers

$$p_1 \neq p_2 \Rightarrow GCD(p_1, p_2) = 1$$

$$n_1 n_2 < 0 \Rightarrow GCD(p_1^{n_1}, p_1^{n_2}) = 1$$

$$n_1 n_2 > 0 \Rightarrow GCD(p_1^{n_1}, p_1^{n_2}) = p_1^{\min(n_1, n_2)}$$

$$GCD(p_1^{n_1} p_2^{n_2} \dots p_i^{n_i}, p_1^{m_1} p_2^{m_2} \dots p_j^{m_j}) = \prod_{i,j} (GCD(p_i^{n_i}, p_j^{m_j}))$$

And if  $x = p_1^{n_1} p_2^{n_2} \dots p_i^{n_i}$  and  $y = p_{l_1}^{m_{l_1}} \dots p_{l_j}^{m_{l_j}}$  then  $y$  divides  $x$  if  $1 < l_k < i$  and for  $l_i = j$ ,  $n_j m_{l_i}, |m_{l_i}| < |n_j|$ .

Thus  $\frac{3}{2}$  does not divide the prime 3, for example. Let, now, for  $t$  integer

$$\zeta(z) = \sum_{t=1}^{t=\infty} \left(\frac{1}{t^z}\right)$$

Euler has proved the Euler identity which follows

$$\zeta(z) = \prod_{p=\text{prime}} \left(\frac{1}{1-p^{-z}}\right)$$

It remains true for  $p$  real prime number, but here, for  $p$  real

$$\zeta_1(z) = \prod_{p=\text{prime}} \left(\frac{1}{1-p^{-z}}\right) = \int_{t=1}^{t=\infty} \left(\frac{dt}{t^z}\right)$$

And

$$\zeta_1(z) = \left| \frac{t^{1-z}}{1-z} \right|_1^\infty$$

And there is trivial zeros

$$\zeta_1(-2k) = 0, \forall k \geq 0$$

Thus

$$\zeta_1(0) = |t|_1^\infty = 0 \Rightarrow \lim_{t \rightarrow \infty} (t) = 1$$

And

$$\begin{aligned} \zeta_1\left(\frac{1}{2} + iy\right) &= \left| \frac{t^{\frac{1}{2}-iy}}{\frac{1}{2}-iy} \right|_1^\infty = \zeta_1\left(1 - \frac{1}{2} - iy\right) \\ &= \zeta_1\left(\frac{1}{2} - iy\right) = \left| \frac{t^{\frac{1}{2}+iy}}{\frac{1}{2}+iy} \right|_1^\infty \\ &\Rightarrow \lim_{t \rightarrow \infty} (t^{\frac{1}{2}+iy}) = \lim_{t \rightarrow \infty} (t^{\frac{1}{2}-iy}) \end{aligned}$$

Hence

$$\begin{aligned} \lim_{t \rightarrow \infty} (t^{1-\frac{1}{2}-iy}) &= \lim_{t \rightarrow \infty} (t^{\frac{1}{2}-iy}) \\ &= \lim_{t \rightarrow \infty} (t^{\frac{1}{2}+iy}) = a = \frac{1}{2} \lim_{t \rightarrow \infty} (t^{\frac{1}{2}-iy} + t^{\frac{1}{2}+iy}) \\ &\Rightarrow \lim_{t \rightarrow \infty} (t^{2(\frac{1}{2}+iy)} - 2at^{\frac{1}{2}+iy} + t) = 0 \\ &= \lim_{t \rightarrow \infty} (t^{2(\frac{1}{2}+iy)} - 2at^{\frac{1}{2}+iy} + 1) = 0 \end{aligned}$$

Hence

$$\lim_{t \rightarrow \infty} (t^{\frac{1}{2}+iy}) = a = a \pm \sqrt{a^2 - 1} = 1$$

Thus

$$\zeta_1\left(\frac{1}{2} + iy\right) = \lim_{t \rightarrow \infty} \left( \frac{t^{\frac{1}{2}-iy} - 1}{\frac{1}{2} - iy} \right) = 0$$

And if

$$\begin{aligned} \zeta_1(x + iy) &= \lim_{t \rightarrow \infty} \left( \frac{t^{1-x-iy} - 1}{1 - x - iy} \right) = 0 \\ &= \lim_{t \rightarrow \infty} \left( \frac{t^{\frac{1}{2}-iy+\frac{1}{2}-x} - 1}{1 - x - iy} \right) \\ &= \lim_{t \rightarrow \infty} \left( \frac{t^{\frac{1}{2}-x} - 1}{1 - x - iy} \right) = 0 \Rightarrow x = \frac{1}{2} \end{aligned}$$

And we see that trivial zeros lie in  $z = -2k$  and non trivial zeros lie in the line  $z = \frac{1}{2} + iy$ .

Thus

$$\zeta_1(x + iy) = 0 \Rightarrow x = \frac{1}{2}$$

We have proved the hypothesis for real numbers. For  $t$  integer, Euler identity becomes

$$\zeta_1(x + iy) = \prod_{p-\text{realprimes}} \left( \frac{1}{1 - p^{-x-iy}} \right) = \prod_{p-\text{integerprimes}} \left( \frac{1}{1 - p^{-x-iy}} \right) \cdot B = \zeta(x + iy) \cdot B$$

And

$$\zeta_1(x + iy) = \int_1^\infty \left( \frac{dt}{t^{x+iy}} \right) = \sum_{t=1}^\infty \left( \frac{1}{t^{x+iy}} \right) + A$$

And

$$\zeta(x + iy) + A = \zeta(x + iy) \cdot B \Rightarrow \zeta(x + iy) = \frac{A}{B - 1}$$

$$\zeta_1(x + iy) = \frac{AB}{B - 1}$$

And  $B = A$  Thus

$$\zeta(x + iy) = 0 \Rightarrow A = 0$$

$A = 0$  and  $\zeta(x + iy) = 0$  implies  $\zeta_1(x + iy) = A + \zeta(x + iy) = 0 \Rightarrow (x = \frac{1}{2})$ .  
 And  $B = A = 0$  and  $\zeta(x + iy) = 0$  implies  $\zeta_1(x + iy) = \zeta(x + iy) \cdot B = 0 \Rightarrow x = \frac{1}{2}$ . And in conclusion

$$\zeta(x + iy) = 0 \Rightarrow (x = \frac{1}{2})$$

It is the proof of the Riemann hypothesis for the integers !

### Conclusion

We did not presented like this, but we have proved the Riemann hypothesis for integral numbers after to have proved for the reals.

## Références

- [1] Alan Baker, *Transcendental number theory Cambridge University Press*, (1975).