The real primes and the Riemann hypothesis

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Abstract

In this document, we deal with the concept of real prime number together with the Riemann hypothesis to the real numbers. Thus, we highlight the new hypothesis by a calculus of integral.

The approach

The Rieman hypothesis states the non trivial zeros of the Riemann zeta function

$$\zeta(z) = \sum_{t=1}^{t=\infty} (\frac{1}{t^z})$$

lie in the critical line : $\frac{1}{2} + iy$. Let us study this hypothesis.

Definition A real number is composed if it is equal to $\pm p_1^{n_1}...p_i^{n_i}$ where p_j are integer prime numbers and n_j are rationals. We define other real prime numbers which can not be expressed like this : π, e, $\ln(2)$. Thus $\sqrt[q]{p} = p^{\frac{1}{q}}$ is composed.
Also $\sqrt[r]{p} + 1$ is prime, with p prime, hence $\sqrt{p} - 1 = (p-1)(\sqrt{p}+1)^{-1}$ is composed !
And π and e are primes in stead of π^{n_0} and e^{m_0} with $(n_0 - 1)(m_0 - 1) \neq 0$.
We define the GCD of two numbers : If p_1 and p_2 are prime real numbers

$$p_1 \neq p_2 \Rightarrow GCD(p_1, p_2) = 1$$

$$n_1 n_2 < 0 \Rightarrow GCD(p_1^{n_1}, p_1^{n_2}) = 1$$

$$n_1 n_2 > 0 \Rightarrow GCd(p_1^{n_1}, p_1^{n_2}) = p_1^{min(n_1,n_2)}$$

$$GCD(p_1^{n_1} p_2^{n_2}...p_i^{n_i}, {p_1}'^{n'_1} {p_2}'^{n'_2}...{p_j}'^{n'_j}) = \prod_{i,j} (GCD(p_i^{n_i}, {p_j}'^{n'_j}))$$

And if $x = p_1^{n_1} p_2^{n_2} ... p_i^{n_i}$ and $y = p_{l_1}^{m_{l_1}} ... p_{l_i}^{m_{l_j}}$ then y divises x if $1 < l_k < i$ and for $l_i = j$, $n_j m_{l_i} > 0$, $|m_{l_i}| < |n_j|$.
Thus $\frac{3}{2}$ does not divise the prime 3, for example.
Let, now, for t integer, the zeta function of Riemann

$$\zeta(z) = \sum_{t=1}^{t=\infty} (\frac{1}{t^z})$$

Euler has proved the following formula

$$\zeta z) = \prod_{p=prime} (\frac{1}{1-p^{-z}})$$

It remains true for p real prime number, but here, for p real

$$\zeta(z) = \prod_{p=prime} (\frac{1}{1-p^{-z}}) = \int_{t=1}^{t=\infty} (\frac{dt}{t^z})$$

And

$$\zeta(z) = \lim_{t \longrightarrow \infty} (\frac{t^{1-z}-1}{1-z})$$

And

$$\lim_{t \longrightarrow \infty} (t^{1-\frac{1}{2}-iy}) = \lim_{t \longrightarrow \infty} (t^{\frac{1}{2}-iy})$$

$$= lim_{t \longrightarrow \infty} (t^{\frac{1}{2}+iy}) = a = \frac{1}{2} lim_{t \longrightarrow \infty} (t^{\frac{1}{2}-iy} + t^{\frac{1}{2}+iy})$$

$$\Rightarrow lim_{t \longrightarrow \infty} (t^{2(\frac{1}{2}+iy)} - 2at^{\frac{1}{2}+iy} + 1) = 0$$

Hence

$$\lim_{t \longrightarrow \infty} (t^{\frac{1}{2}+iy}) = a = a \pm \sqrt{a^2 - 1} = 1$$

Thus

$$\zeta(\frac{1}{2} + iy) = \lim_{t \longrightarrow \infty} (\frac{t^{\frac{1}{2}-iy}-1}{\frac{1}{2}-iy}) = 0$$

And with $x \neq 0$

$$\zeta(x+iy) = \lim_{t \longrightarrow \infty} (\frac{t^{1-x-iy}-1}{1-x-iy}) = 0$$

$$= lim_{t \longrightarrow \infty} (\frac{t^{\frac{1}{2}-iy+\frac{1}{2}-x}-1}{1-x-iy})$$

$$= \lim_{t \longrightarrow \infty} (\frac{t^{\frac{1}{2}-x}-1}{1-x-iy}) = 0 \Rightarrow x = \frac{1}{2}$$

And we see that trivial zeros are $z = -2k$ and non trivial zero lies in $z = \frac{1}{2} + iy$.
We have proved the hypothesis for real numbers. The Riemann hypothesis is important because it gives information about the zeros of $\zeta(z)$ and the distribution of those zeros are related to real primes !

Conclusion

We did not present it like this, but we have given a proof of the Riemann hypothesis for real numbers.

Références

[1] Karl Sabbagh, The Riemann hypothesis, the greatest unsolved problem in mathematics *Farrar, Straus and Giroux* , (2004).