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# The real primes and the Riemann hypothesis

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## Abstract

In this document, we deal with the concept of real prime number together with the Riemann hypothesis to the real numbers. Thus, we highlight the new hypothesis by a calculus of integral.

## The approach

The Riemann hypothesis states the non trivial zeros of the Riemann zeta function

$$\zeta(z) = \sum_{t=1}^{t=\infty} \left(\frac{1}{t^z}\right)$$

lie in the critical line :  $\frac{1}{2} + iy$ . Let us study this hypothesis.

**Definition** A real number is composed if it is equal to  $\pm p_1^{n_1} \dots p_i^{n_i}$  where  $p_j$  are integer prime numbers and  $n_j$  are rationals. We define other real prime numbers which can not be expressed like this :  $\pi, e, \ln(2)$ . Thus  $\sqrt[q]{p} = p^{\frac{1}{q}}$  is composed.

Also  $\sqrt[q]{p} + 1$  is prime, with  $p$  prime, hence  $\sqrt[p]{p} - 1 = (p - 1)(\sqrt[p]{p} + 1)^{-1}$  is composed !

And  $\pi$  and  $e$  are primes in stead of  $\pi^{m_0}$  and  $e^{m_0}$  with  $(n_0 - 1)(m_0 - 1) \neq 0$ .

We define the GCD of two numbers : If  $p_1$  and  $p_2$  are prime real numbers

$$p_1 \neq p_2 \Rightarrow GCD(p_1, p_2) = 1$$

$$n_1 n_2 < 0 \Rightarrow GCD(p_1^{n_1}, p_1^{n_2}) = 1$$

$$n_1 n_2 > 0 \Rightarrow GCD(p_1^{n_1}, p_1^{n_2}) = p_1^{\min(n_1, n_2)}$$

$$GCD(p_1^{n_1} p_2^{n_2} \dots p_i^{n_i}, p_1^{m_1} p_2^{m_2} \dots p_j^{m_j}) = \prod_{i,j} (GCD(p_i^{n_i}, p_j^{m_j}))$$

And if  $x = p_1^{n_1} p_2^{n_2} \dots p_i^{n_i}$  and  $y = p_1^{m_1} \dots p_i^{m_i}$  then  $y$  divides  $x$  if  $1 < l_k < i$  and for  $l_i = j$ ,  $n_j m_{l_i} > 0$ ,  $|m_{l_i}| < |n_j|$ .

Thus  $\frac{3}{2}$  does not divide the prime 3, for example.

Let, now, for  $t$  integer, the zeta function of Riemann

$$\zeta(z) = \sum_{t=1}^{t=\infty} \left(\frac{1}{t^z}\right)$$

Euler has proved the following formula

$$\zeta(z) = \prod_{p=\text{prime}} \left(\frac{1}{1-p^{-z}}\right)$$

It remains true for  $p$  real prime number, but here, for  $p$  real

$$\zeta(z) = \prod_{p=\text{prime}} \left(\frac{1}{1-p^{-z}}\right) = \int_{t=1}^{t=\infty} \left(\frac{dt}{t^z}\right)$$

And

$$\zeta(z) = \lim_{t \rightarrow \infty} \left(\frac{t^{1-z} - 1}{1-z}\right)$$

And

$$\begin{aligned} \lim_{t \rightarrow \infty} (t^{1-\frac{1}{2}-iy}) &= \lim_{t \rightarrow \infty} (t^{\frac{1}{2}-iy}) \\ &= \lim_{t \rightarrow \infty} (t^{\frac{1}{2}+iy}) = a = \frac{1}{2} \lim_{t \rightarrow \infty} (t^{\frac{1}{2}-iy} + t^{\frac{1}{2}+iy}) \\ &\Rightarrow \lim_{t \rightarrow \infty} (t^{2(\frac{1}{2}+iy)} - 2at^{\frac{1}{2}+iy} + 1) = 0 \end{aligned}$$

Hence

$$\lim_{t \rightarrow \infty} (t^{\frac{1}{2}+iy}) = a = a \pm \sqrt{a^2 - 1} = 1$$

Thus

$$\zeta\left(\frac{1}{2} + iy\right) = \lim_{t \rightarrow \infty} \left(\frac{t^{\frac{1}{2}-iy} - 1}{\frac{1}{2} - iy}\right) = 0$$

And with  $x \neq 0$

$$\begin{aligned} \zeta(x + iy) &= \lim_{t \rightarrow \infty} \left(\frac{t^{1-x-iy} - 1}{1-x-iy}\right) = 0 \\ &= \lim_{t \rightarrow \infty} \left(\frac{t^{\frac{1}{2}-iy+\frac{1}{2}-x} - 1}{1-x-iy}\right) \\ &= \lim_{t \rightarrow \infty} \left(\frac{t^{\frac{1}{2}-x} - 1}{1-x-iy}\right) = 0 \Rightarrow x = \frac{1}{2} \end{aligned}$$

And we see that trivial zeros are  $z = -2k$  and non trivial zero lies in  $z = \frac{1}{2} + iy$ .

We have proved the hypothesis for real numbers. The Riemann hypothesis is important because it gives information about the zeros of  $\zeta(z)$  and the distribution of those zeros are related to real primes !

## Conclusion

We did not present it like this, but we have given a proof of the Riemann hypothesis for real numbers.

## Références

- [1] Karl Sabbagh, The Riemann hypothesis, the greatest unsolved problem in mathematics *Farrar, Straus and Giroux*, (2004).