



**HAL**  
open science

## About the Riemann hypothesis

Jamel Ghannouchi

► **To cite this version:**

| Jamel Ghannouchi. About the Riemann hypothesis. 2013. hal-00815862v1

**HAL Id: hal-00815862**

**<https://hal.science/hal-00815862v1>**

Preprint submitted on 19 Apr 2013 (v1), last revised 14 Dec 2015 (v13)

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# About the Riemann hypothesis

Jamel Ghanouchi

Ecole Supérieure des Sciences et Techniques de Tunis

jamel.ghanouchi@topnet.tn

## Abstract

We define the concept of real prime number and generalize the Riemann hypothesis to the real numbers. Thus, we can prove the new hypothesis by a calculus of integral.

## The approach

The Riemann hypothesis states the non trivial zeros of the Riemann zeta function

$$\zeta(z) = \sum_{t=1}^{t=\infty} \left(\frac{1}{t^z}\right)$$

are the complex numbers  $\frac{1}{2} + iy$ . Let us study this hypothesis.

**Definition** A real number is composed if it is equal to  $\pm p_1^{n_1} \dots p_i^{n_i}$  where  $p_j$  are integer prime numbers and  $n_j$  are rationals. We define other real prime numbers which can not be expressed like this :  $\pi, e, \ln(2)$ . Thus  $\sqrt[p]{p} = p^{\frac{1}{p}}$  is composed.

Also  $\sqrt[p]{p} + 1$  is prime, with  $p$  prime, hence  $\sqrt[p]{p} - 1 = (p - 1)(\sqrt[p]{p} + 1)^{-1}$  is composed !

And  $\pi$  and  $e$  are primes in stead of  $\pi^{m_0}$  and  $e^{m_0}$  with  $(n_0 - 1)(m_0 - 1) \neq 0$ .

We define the GCD of two numbers : If  $p_1$  and  $p_2$  are prime real numbers

$$p_1 \neq p_2 \Rightarrow GCD(p_1, p_2) = 1$$

$$n_1 n_2 < 0 \Rightarrow GCD(p_1^{n_1}, p_1^{n_2}) = 1$$

$$n_1 n_2 > 0 \Rightarrow GCD(p_1^{n_1}, p_1^{n_2}) = p_1^{\max(n_1, n_2) - \min(n_1, n_2)}$$

$$GCD(p_1^{n_1} p_2^{n_2} \dots p_i^{n_i}, p_1^{m_1} p_2^{m_2} \dots p_j^{m_j}) = \prod_{i \neq j} GCD(p_i^{n_i}, p_j^{m_j})$$

And if  $x = p_1^{n_1} p_2^{n_2} \dots p_i^{n_i}$  and

Thus  $\frac{3}{2}$  does not divide the prime 3 as 2 is prime, for example. Let, now, for  $t$  integer

$$\zeta(z) = \sum_{t=1}^{t=\infty} \left(\frac{1}{t^z}\right)$$

Euler has proved the following formula

$$\zeta(z) = \prod_{p=\text{prime}} \left(\frac{1}{1-p^{-z}}\right)$$

It remains true for  $p$  real prime number, but here, for  $p$  real

$$\zeta(z) = \prod_{p=\text{prime}} \left(\frac{1}{1-p^{-z}}\right) = \int_{t=1}^{t=\infty} \left(\frac{dt}{t^z}\right)$$

And

$$\zeta(z) = \lim_{t \rightarrow \infty} \left(\frac{t^{1-z} - 1}{1-z}\right)$$

And

$$\begin{aligned} \lim_{t \rightarrow \infty} (t^{1-\frac{1}{2}-iy}) &= \lim_{t \rightarrow \infty} (t^{\frac{1}{2}-iy}) \\ &= \lim_{t \rightarrow \infty} (t^{\frac{1}{2}+iy}) = a = \frac{1}{2} \lim_{t \rightarrow \infty} (t^{\frac{1}{2}-iy} + t^{\frac{1}{2}+iy}) \\ &\Rightarrow \lim_{t \rightarrow \infty} (t^{2(\frac{1}{2}+iy)} - 2at^{\frac{1}{2}+iy} + 1) = 0 \end{aligned}$$

Hence

$$\lim_{t \rightarrow \infty} (t^{\frac{1}{2}+iy}) = a = a \pm \sqrt{a^2 - 1} = 1$$

Thus

$$\zeta\left(\frac{1}{2} + iy\right) = \lim_{t \rightarrow \infty} \left(\frac{t^{\frac{1}{2}-iy} - 1}{\frac{1}{2} - iy}\right) = 0$$

And

$$\begin{aligned} \zeta(x + iy) &= \lim_{t \rightarrow \infty} \left(\frac{t^{1-x-iy} - 1}{1-x-iy}\right) = 0 \\ &= \lim_{t \rightarrow \infty} \left(\frac{t^{\frac{1}{2}-iy+\frac{1}{2}-x} - 1}{1-x-iy}\right) \\ &= \lim_{t \rightarrow \infty} \left(\frac{t^{\frac{1}{2}-x} - 1}{1-x-iy}\right) = 0 \Rightarrow x = \frac{1}{2} \end{aligned}$$

And we see that trivial zeros are  $z = -2k$  and non trivial zero is  $z = \frac{1}{2} + iy$ .

## Conclusion

We did not presented like this, but we have proved the Riemann hypothesis for real numbers.

## Références

- [1] Alan Baker, *Transcendental number theory Cambridge University Press*, (1975).