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**COUPLING BETWEEN PEEC METHOD AND AN INTEGRO-DIFFERENTIAL  
APPROACH FOR SOLVING ELECTROMAGNETICS PROBLEMS**

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## **Abstract**

In order to introduce thin electromagnetic shell into quasi-static PEEC method, a coupling with an Integro-Differential Method (IDM) is proposed. This coupling allows us to benefit from both methods. The complex conductor geometry is modelled with the PEEC method while thin shell is taken into account thanks to the IDM.

## **1 Introduction**

In order to analyse eddy-current in electromagnetic devices, an original formulation is developed. It is particularly well-adapted to model devices that include thin electromagnetic shells (electromagnetic shieldings) and complicated conductor systems.

For this kind of application, the direct use of differential methods, such as the finite element method (FEM), is often difficult and may be sometimes inefficient because of the mesh generation difficulty. Indeed, in this method, mesh refinement around conductors is needed in order to take into account the large variation of the magnetic field close to conductors. To generate this mesh can be a hard task. Moreover, even if

this mesh is satisfactorily achieved, such modelling lead to an huge numbers of degrees of freedom and so is limited by memory needs and CPU time requirements if modelling of 3D problems with a large number of conductors is required [1], [2], [3], [4].

The PEEC method (Partial Element Equivalent Circuit) is mainly used for the modelling of complex electrical interconnections and can be applied to a large range of devices where the air region is dominant [5], [6], [7], [8]. In this method, only conductors are described and meshed, so it generally requires less memory and CPU time than the FEM method. However, the classical PEEC method does not enable the 3D modelling of thin electromagnetic regions.

On the other hand, some integral formulations (usually called integro-differential one) have been developed in order to analyze eddy current in conducting shells [9], [10], [11]. Like the PEEC method, it does not require the meshing of the air region.

In this paper, an original IDM/PEEC coupling is presented. The modelling of thin electromagnetic shells will be achieved thanks to the integro-differential method (Part II). The PEEC method allows us to model the contributions of inductors fed (Part III). The coupling of those formulations, in order to compute the interactions between conductors and electromagnetic shells, will be developed in part IV. Finally, a numerical example will be presented in part V. Results obtained with this formulation and the Finite Element Method (FEM) will be compared in this last part.

## 2 Integro-differential approach for modelling thin electromagnetic shells

Low frequency problems are focused (i.e. the skin depth is high in comparison with the thickness of the shell) so the assumption of a uniform current distribution across the thickness of the shell is made. Moreover, eddy currents  $\mathbf{J}_C$  are assumed to flow tangentially to the shell.

Let us now consider a thin electromagnetic shell placed in an inductor field  $\mathbf{H}_0$ . Equations governing the problem in shells are:

$$\text{Curl } \mathbf{E} = -j\omega\mathbf{B} \quad (1)$$

$$\text{Curl } \mathbf{H} = \mathbf{J}_C \quad (2)$$

$$\mathbf{J}_C = \sigma\mathbf{E} \quad (3)$$

$$\mathbf{B} = \mu_0\mu_r\mathbf{H} = \mu_0(\mathbf{H} + \mathbf{M}) \quad (4)$$

where  $\sigma$  is the conductivity,  $\mathbf{J}_C$  is the eddy current distribution in the shell,  $\mu_r$  is the relative permeability of the material and  $\mathbf{M}$  is the magnetization. In our application, we are mainly interested in shielding effects, so the shell will be placed in a quite low magnetic field. So, the permeability will be considered as uniform and the problem as linear.

The Ampere-Maxwell equation (2) implies the existence of an electric vector potential  $\mathbf{T}$  as:

$$\text{Curl } \mathbf{T} = \mathbf{J}_C \quad (5)$$

As the current is tangential and uniform across the thickness,  $\mathbf{T}$  is normal to the shell and can be considered as scalar (i.e. its normal component).

Equation (1) can be rewritten as follows [10]:

$$\text{Curl}\left(\frac{1}{\sigma}\text{Curl}(\mathbf{T})\right) = -j\omega\mu_0\mu_r\mathbf{H}_n \quad (6)$$

where  $\mathbf{H}_n$  is the normal magnetic field. Considering the relation linear between the magnetization  $\mathbf{M}$  and the magnetic field intensity  $\mathbf{H}$ , equation (6) can be rewritten as:

$$\text{Curl}\left(\frac{1}{\sigma}\text{Curl}(\mathbf{T})\right) = -j\omega\mu_0\frac{\mu_r}{\mu_r-1}\mathbf{M}_n \quad (7)$$

These equations have to be discretized. The easiest way is to mesh the thin shell into  $n$  triangular prisms elements associated with a constant magnetization (0-order shape functions for  $\mathbf{M}$ ). The unknown current potential variables  $\mathbf{T}$  is approximated with linear functions (1-order shape functions so uniform value of current  $\mathbf{J}$  per element). Let  $p$  denote the number of node. A Galerkin projection method of (7) is then applied in order to get the following matrix system:

$$\mathbf{A} \cdot \mathbf{T} + \mathbf{B} \cdot \mathbf{M} = \mathbf{0} \quad (8)$$

where unknowns are  $\mathbf{T}$  a  $(p \times 1)$  vector and  $\mathbf{M}$  a  $(3n \times 1)$  vector. Matrix  $\mathbf{A}$  is a  $(p \times p)$  matrix and  $\mathbf{B}$  is a  $(p \times 3n)$  matrix.

The total magnetic field  $\mathbf{H}$  is the sum of  $\mathbf{H}_0$ , the source field,  $\mathbf{H}_m$  the reaction of the magnetization and  $\mathbf{H}_e$  the reaction of the eddy current flowing in the thin plate. Let  $V_m$  and  $\Gamma$  denote the volume of magnetic material and its middle surface respectively. An integral equation linking the local field to the magnetization and the electric vector potential can be written [9]:

$$\mathbf{H}(P) = \mathbf{H}_0(P) + \frac{1}{4\pi} \int_{V_m} \left(3\frac{(\mathbf{M} \cdot \mathbf{r})}{r^5} \mathbf{r} - \frac{\mathbf{M}}{r^3}\right) dV_m + \frac{e}{4\pi} \int_{\Gamma} \frac{(\text{Curl } \mathbf{T}) \times \mathbf{r}}{r^3} d\Gamma \quad (9)$$

where  $P$  is a point located in the domain region,  $\mathbf{r}$  is the vector linking the integration point to  $P$  and  $e$  is thickness of shell. Thanks to a point matching approach of this material law at the centre of each element, we get a linear matrix system:

$$\{\mathbf{I}_d / (\mu_r - 1) - \mathbf{F}\} \cdot \mathbf{M} + \mathbf{C} \cdot \mathbf{T} = \mathbf{h}_0 \quad (10)$$

where  $\mathbf{I}_d$  represents the identity matrix;  $\mathbf{h}_0$  is the source fields at each barycentre,  $\mathbf{F}$  is a  $(3n \times 3n)$  matrix and  $\mathbf{C}$  is a  $(3n \times p)$  matrix.

Equations (8) and (10) are solved together to obtain the magnetization  $\mathbf{M}$  of each element and the current potential  $\mathbf{T}$  of each node. This method is a hybridisation between a differential equation and an integral one; this is why it is usually called an integro-differential method.

### 3 Inductive PEEC method

Let us consider  $m$  volume conductors fed with voltage sources placed in a surrounding air region without any magnetic materials. The well-known PEEC method is particularly reliable to solve this kind of problem. It is based on the determination of partial voltage generated on each conductor by electromagnetic sources. To compute these voltages, volume integration on the conductor of the magnetic vector potential created by all the others conductor is provided. Let us assume that the current density in each conductor is uniform. The expression of the magnetic vector potential  $\mathbf{A}_j$  created by conductor  $j$  is [5], [6]:

$$\mathbf{A}_j(P) = \frac{\mu_0 I_j}{4\pi S_j} \int_{V_j} \frac{\mathbf{u}_j}{r} dV_j \quad (11)$$

where  $r$  is the distance between the integration point and the point  $P$ ;  $I_j$  is the current in the conductor  $j$ ;  $S_j$  is the cross-section of the current flow;  $V_j$  is the volume and  $\mathbf{u}_j$  is the unit vector whose direction is that of conventional current  $I_j$ .

The flux created by  $I_j$  through the conductor  $k$  is:

$$\Phi_{kj} = \frac{1}{S_k} \int_{V_k} \mathbf{A}_j \cdot \mathbf{u}_k dV_k = \frac{\mu_0}{4\pi} \frac{I_j}{S_j S_k} \int_{V_k} \int_{V_j} \frac{\mathbf{u}_j \cdot \mathbf{u}_k}{r} dV_j dV_k \quad (12)$$

The mutual inductance between two conductors is defined as:

$$m_{kj} = \frac{\Phi_{kj}}{I_j} = \frac{\mu_0}{4\pi} \frac{1}{S_j S_k} \int_{V_k} \int_{V_j} \frac{\mathbf{u}_j \cdot \mathbf{u}_k}{r} dV_j dV_k \quad (13)$$

Thus, the voltage appearing on the conductor  $k$  is given by:

$$U_{k\_cond} = I_k \cdot R_k + j\omega \sum_{j=1}^m I_j \cdot m_{kj} \quad (14)$$

where  $\omega$  is the angular frequency,  $R_k$  is the resistance of  $k^{\text{th}}$  conductor. By applying equation (14) to all conductors, we get a matrix system known as impedance matrix system:

$$\mathbf{Z} \cdot \mathbf{I} = \mathbf{U} \quad (15)$$

where  $\mathbf{Z}$  is a  $(m \times m)$  matrix,  $\mathbf{I}$  is a  $(m \times 1)$  vector and  $\mathbf{U}$  is a  $(m \times 1)$  vector.

## 4 Coupling principle

Let us now consider a system composed of  $m$  conductors with unknown flowing currents  $I$  and thin magnetic conductive shell. The thin shell is still meshed into  $n$  triangular prism elements associated with a constant magnetization and a constant value of current per element. As in part two, the interactions between each element in the thin shell are calculated thanks the integro-differential method. The PEEC method is used to model interconnection between conductors. It remains to write explicitly the couplings matrix between both methods.

### 4.1 Influence of current conductor in thin electromagnetic shell

The term  $\mathbf{h}_0$  of equation (10) can be computed by using Biot and Savart law. The magnetic field created by  $m$  conductors is given by:

$$\mathbf{H}_0(P) = \sum_{j=1}^m \frac{I_j}{4\pi} \int_{V_j} \frac{\mathbf{u}_j \times \mathbf{r}}{r^3} dV_j \quad (16)$$

with same notations as in (11). We write equation (16) at barycentre of each thin shell element to obtain the following matrix block:

$$\mathbf{h}_0 = \mathbf{BS} \cdot \mathbf{I} \quad (17)$$

where  $\mathbf{BS}$  is a  $(3n \times m)$  matrix,  $\mathbf{I}$  is a  $(m \times 1)$  vector. We substitute (17) into (10) and obtain:

$$(\mathbf{I}_d / (\mu_r - 1) - \mathbf{F}) \cdot \mathbf{M} + \mathbf{C} \cdot \mathbf{T} - \mathbf{BS} \cdot \mathbf{I} = \mathbf{0} \quad (18)$$

## 4.2 Influence of magnetization and eddy currents of thin shell on conductors

Like in the standard PEEC method, we have to integrate the magnetic vector potential  $\mathbf{A}_i$  created by each thin shell elements  $i$  on the conductor  $k$ . The expression of this magnetic vector potential is:

$$\mathbf{A}_i(P) = \frac{\mu_0}{4\pi} \int_{Vm_i} \frac{\mathbf{M}_i \times \mathbf{r}}{r^3} dVm_i + \frac{e\mu_0}{4\pi} \int_{\Gamma_i} \frac{\text{Curl}(\mathbf{T}_i)}{r} d\Gamma_i \quad (19)$$

with same notations as in (9). The magnetic vector potentials of  $n$  shell elements create a new voltage on the conductor  $k$ :

$$U_{k\_shell} = \frac{\mu_0}{4\pi} \frac{j\omega}{S_k} \int_{V_k} \left( \sum_{i=1}^n \int_{Vm_i} \frac{\mathbf{M}_i \times \mathbf{r}}{r^3} dVm_i + e \cdot \sum_{i=1}^n \int_{\Gamma_i} \frac{\text{Curl} \mathbf{T}_i}{r} d\Gamma_i \right) \cdot \mathbf{u}_k dV_k \quad (20)$$

Thus, the total voltage appearing on the conductor  $k$  is given by:

$$U_k = U_{k\_cond} + U_{k\_shell} = I_k \cdot R_k + j\omega \left( \sum_{j=1}^m I_j \cdot m_{kj} + \sum_{i=1}^n M_i \cdot mf_{ki} + \sum_{i=1}^p T_i \cdot mc_{ki} \right) \quad (21)$$

where  $mf_{ki}$  is defined as the mutual inductance between the magnetization  $\mathbf{M}_i$  of shell element and the conductor  $k$ ;  $mc_{ki}$  is the mutual inductance between the current potential  $T_i$  of the shell node and the conductor  $k$ .

If we write this equation for all conductors, we get a new impedance matrix system:



$$\mathbf{U} = \mathbf{Z} \cdot \mathbf{I} + \mathbf{L}_f \cdot \mathbf{M} + \mathbf{L}_C \cdot \mathbf{T} \quad (22)$$

where  $\mathbf{L}_f$  is a  $(m \times 3n)$  matrix and  $\mathbf{L}_C$  is a  $(n \times p)$  mutual inductance matrix between thin shell elements and conductors.

Finally, equations (8), (18) and (22) are brought together in the global  $(3n+p+m) \times (3n+p+m)$  following matrix systems:

$$\begin{bmatrix} \mathbf{MM} & \mathbf{C} & -\mathbf{BS} \\ \mathbf{B} & \mathbf{A} & \mathbf{0} \\ \mathbf{L}_f & \mathbf{L}_C & \mathbf{Z} \end{bmatrix} \times \begin{bmatrix} \mathbf{M} \\ \mathbf{T} \\ \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{U} \end{bmatrix} \quad (23)$$

where  $\mathbf{MM}$  is the matrix  $(\mathbf{I}_d / (\mu_r - 1) - \mathbf{F})$  (see equation (10)).

In (23),  $\mathbf{I}$  and  $\mathbf{U}$  are the current and voltage values of each branch of the conductors system. In order to solve this equation system with a reduced number of degrees of freedom, we use the Kirchhoff's mesh rule. Consequently, current and voltage values become associated to each of independent circuit mesh. We get then a lighter system and after solving it with a direct linear solver (LU decomposition), we obtain magnetizations, current potentials in thin shell and currents in conductors.

## 5 Numerical Example

Let us consider a conducting ferromagnetic plate with the conductivity  $\sigma = 9.6 \times 10^6$  S/m and relative permeability  $\mu_r = 50$  (reversible linear permeability representative of a classical shielding effect). The thickness of the plate  $e = 1.5$  mm and the dimensions are  $200 \times 200$  mm. This plate is placed close to a conductor coil which is fed with a voltage source (1V, 50Hz) (Figure 1).

FIGURE 1 HERE

Our coupling is compared with results given by FEM in Flux software [12]. We focus on the computed eddy current distribution and Joule losses in the plate (Figure 2, 3). Whereas the convergence is quickly reached with our coupling, the problem needs a very fine mesh to be accurately solved with FEM because of the large variation of the field close to the conductor [13].

We can notice small differences between Joule losses computed values. If we consider that FEM method as our reference, the IDM/PEEC coupling leads to an error of 0.8% for the Joule losses in plate (Figure 3). Results provided by our coupling are very encouraging, the convergence being reached with a very few number of elements (around 2000). Of course, the obtained matrix is fully dense, but the computation time is divided per ten in comparison with FEM. Let us notice that this problem of fully dense matrix and associated memory need could be fixed with the use of a compression technique like « Fast Multipole Method » (FMM) [14] for instance.

FIGURE 2 HERE

FIGURE 3 HERE

## **6 Conclusions**

In this paper, we have presented a coupling between PEEC and an integro-differential method in order to introduce the thin electromagnetic shell in PEEC method. Through a simple example, we have validated this formulation. The result shows that the coupling is much faster and more accurate than the FEM method on this simple

geometry. However, our formulation can only be applied for problem where the skin depth is high in comparison with the thickness of the shell ( $\delta \gg e$ ). In further work, formulations enabling smaller skin depth will be investigated.

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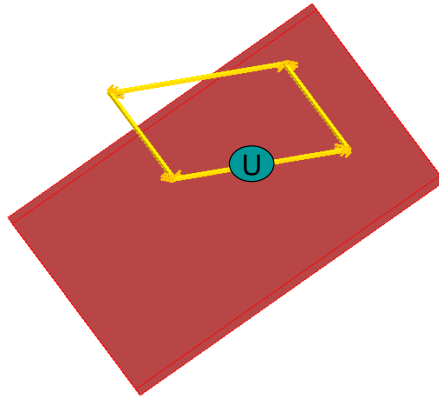


Figure 1: Geometry of the numerical example

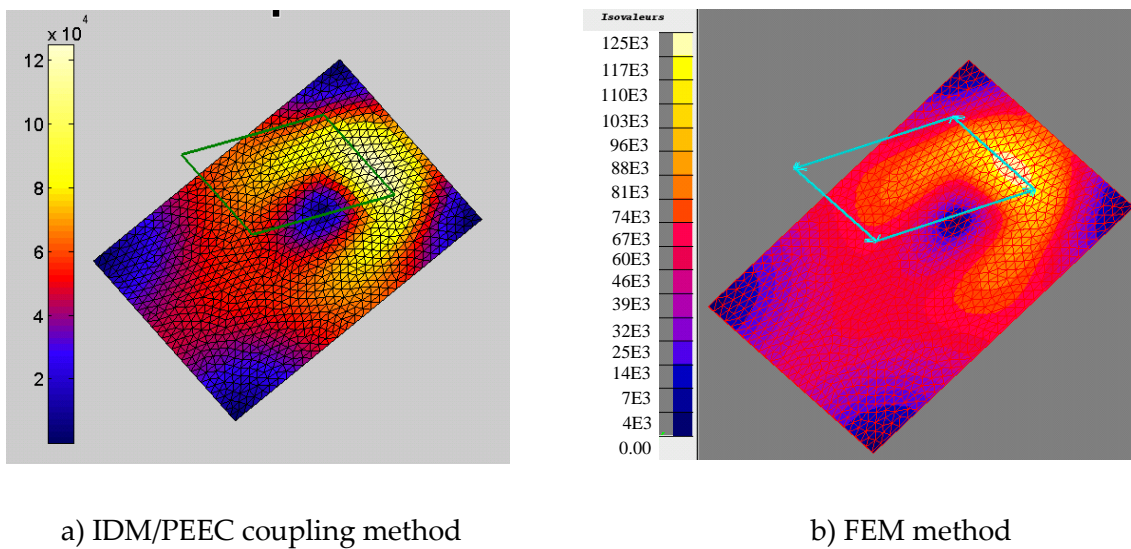


Figure 2: Eddy current in thin plate computed: a) Calculation by our coupling method,  
b) Calculation by FEM method

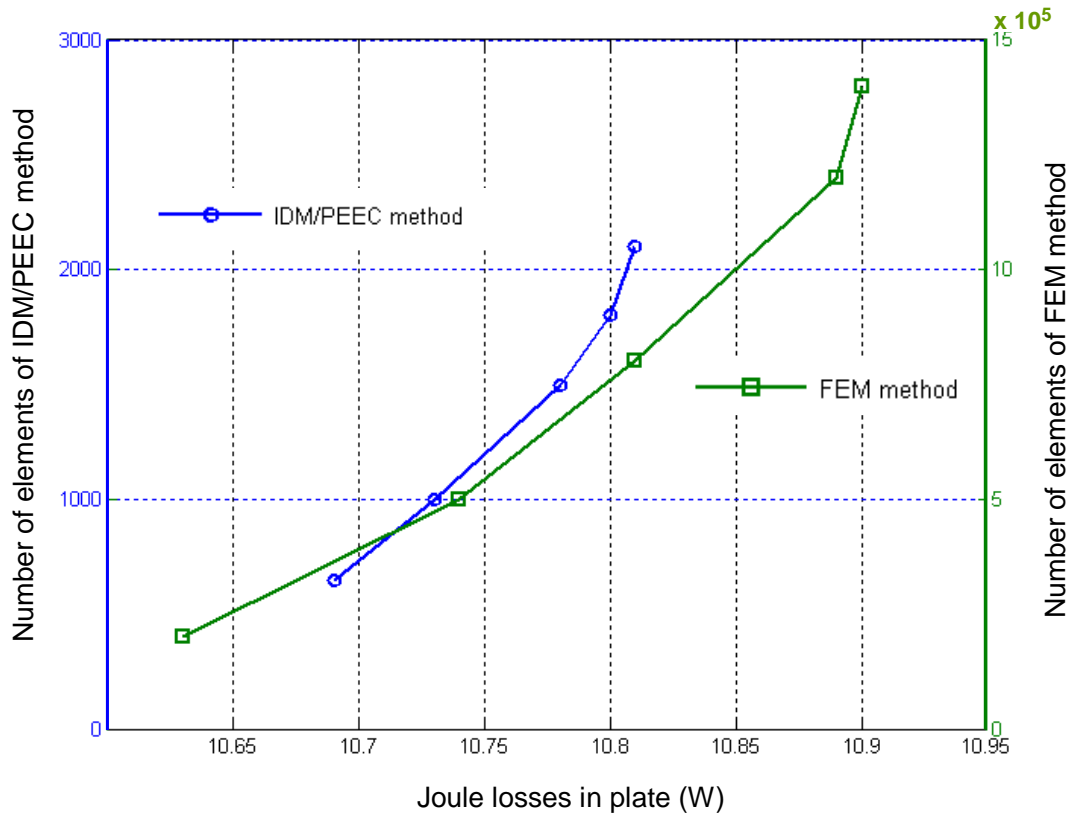


Figure 3: Joule losses in thin plate (W) calculated by IDM/PEEC and FEM method in function of different meshes