Event-triggered nonlinear control for attitude stabilization of a quadrotor

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Abstract— **Event-triggered control is a ressource-aware sampling strategy that updates the control value only when a certain condition is satisfied, which denotes event instants. Such a technique allows to reduce the control computational cost and communications. In this paper, a quaternion-based feedback is developed for event-triggered attitude stabilization of a quadrotor mini-helicopter. The feedback is derived from the universal formula for event-triggered stabilization of general nonlinear systems affine in the control. The proposed feedback ensures the asymptotic stability to the desired attitude. Real-time experiments are carried out in order to show the convergence of the quadrotor states to the desired attitude as well as the robustness with respect to external disturbances. Results show that the proposed control can reduce by 80 % the communications of the embedded system without sacrificing performance of the whole system. To the best of the authors' knowledge, this is the first time that a nonlinear event-triggered controller is experimentally applied to the attitude stabilization of an unmanned aircraft system.**

I. INTRODUCTION

A cyber-physical system (CPS) is an integration of computation with physical processes. Embedded computers and networks monitor and control the physical processes, usually with feedback loops where physical processes affect computations and vice versa. The intersection between physical and information-driven functions (cyber) represents a challenge and results in innovation, see [1]. For CPS, the use of digital platforms and networks emerges as an obvious trend to save space, weight and energy. However, digital implementations can result in additional challenges, like determining how frequently the control signal needs to be updated and applied such that the stability properties are still guaranteed.
Among many CPSs, Unmanned Aerial Vehicles (UAVs) have received growing interest in industrial and academic research. They may prove useful for many civilian missions such as video supervision of road traffic, surveillance of urban districts, forest fire detection or building inspection. Furthermore, among miniature rotorcraft-based UAVs, the mini quadrotor helicopter gives rise to great interest because of its high manoeuvrability, its payload capacity and its ability to hover, as explained in [2]. Such a Vertical Take-Off and Landing (VTOL) vehicle has some advantages over conventional helicopters: owing to symmetry, it is relatively simple to design and construct. In fact, the quadrotor is an under-actuated dynamic system with four input forces and six output coordinates (attitude and position). However, this system can be broken down into two subsystems, one defining the translation movement and the other one the rotation movement. These subsystems are coupled in cascade since the translational subsystem depends on the rotational one, but the rotational subsystem is independent of the translational one. Self-governing flights require the generation of low-level control signals sent to actuators as well as decision-making related to guidance, navigation. Low-level flight control is known as attitude control and it is responsible for maintaining the desired vehicle orientation. Consequently, the attitude controller design is, in itself, a challenge.
Some linear and nonlinear control techniques have been applied for the attitude stabilization of the quadrotor mini-helicopter, like for example in [3], [4], [5], [6], [7], [8], [9]. This list is of course far from being exhaustive. Actually, all proposed attitude control laws previously listed were developed in continuous time framework and their implementation under digital platforms is carried out by means of "emulation". This procedure consists in implementing a continuous time control algorithm with a constant and sufficiently small periodic sampling period. However, this approach can be constrained by hardware and reducing the sampling period to a level that guarantees acceptable closed-loop performance may be impossible.

On the other hand, in the recent years, some works addressed resource-aware implementations of the control law using event-triggered sampling, where the control value is updated only when some events occur. An event is usually generated by an event-function that indicates if the control signal must be updated or not. Typical event-detection mechanisms are functions on the variation of the state (or at least the output) of the system, like in [10], [11], [12], [13], [14], [15]. In [16] in particular, it is proved that such an approach reduces the number of sampling instants for the same final performance. An event-triggered paradigm hence calls for resources whenever they are indeed necessary. In the same idea, an alternative approach consists in taking events related to the variation of a Lyapunov function – and consequently to the state too – between the current state and its value at the last sampling, like in [17], or in taking events related to the time derivative of the Control Lyapunov function, like in [18], [19]. In this latter case, the updates ensure the strict

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decrease of a Control Lyapunov function, and so is ensured the asymptotically stability of the closed-loop system.

Although the advantages of event-triggered control are well-motivated and theoretical results show its potential, few results in the framework of unmanned aircraft systems have been presented in literature, *e.g.* [20], [21]. In these works linear event-triggered controllers are proposed for attitude stabilization of a 3D helicopter model. Unfortunately, these controllers only work in a limited attraction region of the state-space.

In the present work, we develop an event-triggered non linear control strategy for the attitude stabilization of a mini quadrotor helicopter. The feedback is quaternion-based and it is derived from the universal formula for event-triggered stabilization of general nonlinear systems affine in the control [19]. For sake of simplicity, we only consider in this paper null stabilization with initial time instant $t_0 = 0$. The proposed feedback ensures the asymptotic stability and it is smooth everywhere except at the origin. Moreover, we propose to test such a proposal on a real-time system. The idea is to show that an event-triggered scheme could reduce the number of samples even in such a case where rotor blades have to be actively controlled. To the best of the authors' knowledge this is the first time that such a method is experimentally tested.

The paper is organized as follows. First, in section II we present some mathematical definitions and the event-based control strategy for affine in the control nonlinear systems is detailed. The quaternion notion is also introduced and the quadrotor mini-helicopter model is given. The section III states the problem and presents the design of the control law for the attitude stabilization. Some experimental results are presented in section IV and discussions finally conclude the paper.

II. PRELIMINARIES

In this section some facts for event-triggered stabilization of general nonlinear systems affine in the control [19] are reviewed and the system model is introduced [8].

A. A universal formula for event-triggered stabilization

In this paper, the study will focus on affine in the control dynamical systems defined by:

$$\dot{x} = f(x) + g(x)u \tag{1}$$

where $x \in \mathcal{X} \subset \mathbb{R}^n$, $u \in \mathcal{U} \subset \mathbb{R}^p$, and f a Lipschitz function vanishing at the origin. For sake of simplicity, we only consider in this paper null stabilization with initial time instant $t_0 = 0$. If the system (1) admits an asymptotic stabilizing feedback $k : \mathcal{X} \to \mathcal{U}$ then there exists a Control Lyapunov Function $V : \mathcal{X} \to \mathbb{R}$, that is a smooth function, positive definite and such that:

$$\dot{V} = \frac{\partial V}{\partial x} f(x) + \frac{\partial V}{\partial x} g(x) k(x) \tag{2}$$

It is worth noting that if k is assumed to be smooth, then V is known to exist and to be as smooth as k. In the present paper, only the smoothness of V is required which is less restrictive than the one of k.

Event-triggered feedback usually means a set of two functions:

- an *event function* $e : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$ that indicates if one needs ($e \leq 0$) or not ($e > 0$) to update the control value. Event function e takes the current state x as input and a memory m of x last time e became negative.
- a *feedback function* $k : \mathcal{X} \to \mathcal{U}$. Which is used as in the classical frame.

We recall here the definition of semi-uniform Minimum Sampling Interval (MSI) event-triggered control:

Definition 2.1: [19] An event-triggered feedback (k, e) is said to be semi-uniformly MSI if for all $\delta > 0$, and all x_0 in the ball of radius δ centred at the origin $\mathcal{B}(\delta)$ the inter-execution times, that is the duration between two successive events, can be below bounded by some $\underline{\tau} > 0$.

Remark 2.2: This minimal sampling period is useful for implementation purpose but also when the feedback k is discontinuous for robustness purpose [22] as this one proposed in the present paper.

It is known that a nonlinear system of the form (1) with a semi-uniformly MSI event-based feedback (e, k), the solution of (1) starting in $x_0 \in \mathcal{X}$ at $t = 0$ is defined for all positive t as the solution of the differential system:

$$\dot{x} = f(x) + g(x)k(m)) \tag{3}$$

$$\begin{cases} m = x & \text{if } e(x,m) \leq 0,\ x \neq 0 \\ \dot{m} = 0 & \text{elsewhere} \end{cases} \tag{4}$$

$$\text{with} x(0) := x_0 \text{ and } m(0) = x(0) \tag{5}$$

Theorem 2.3 (Event-Triggered universal formula): If there exists a CLF for system (1), then the event-based feedback (e, k) defined below is semi-uniformly MSI, smooth on $\mathcal{X} \setminus \{0\}$, and such that:

$$\frac{\partial V}{\partial x} f(x) + \frac{\partial V}{\partial x} g(x) k(m) < 0, \quad x \in \mathcal{X} \setminus \{0\} \tag{6}$$

where m is defined in (4) and:

$$k_i(x) \;:=\; -b_i(x)\delta_i(x)\gamma(x) \tag{7}$$

$$e(x,m) \;:=\; -a(x) - b(x)k(m) - \sigma\sqrt{a(x)^2 + \theta(x)b(x)\Delta(x)b(x)^T} \tag{8}$$

where

- $a(x) := \frac{\partial V}{\partial x} f(x)$ and $b(x) := \frac{\partial V}{\partial x} g(x)$,
- $x \to \Delta(x) := \text{diag}(\delta_1(x), \delta_2(x), \ldots, \delta_p(x))$ is a smooth function of $\mathcal{X} \setminus \{0\}$ to $\mathbb{R}^{p \times p}$, positive definite on:

$$\mathcal{S} := \{x \in \mathcal{X} |\ \|b(x)\| \neq 0\}$$

- $x \to \theta(x)$ is a smooth positive function of $\mathcal{X}$ to $\mathbb{R}$, such that $\theta(x)\,\|\Delta(x)\|$ vanishes at the origin, and ensuring on $\mathcal{S} \setminus \{0\}$ the inequality $a(x)^2 + \theta(x)b(x)\Delta(x)b(x)^T > 0$
- σ is a control parameter in $[0, 1[$,

- $\gamma : \mathcal{X} \to \mathbb{R}$ is defined by:

$$\gamma(x) := \begin{cases} \frac{a(x)+\sqrt{a(x)^2+\theta(x)b(x)\Delta(x)b(x)^T}}{b(x)\Delta(x)b(x)^T} & \text{if } x \in \mathcal{S} \\ 0 & \text{if } x \notin \mathcal{S} \end{cases} \tag{9}$$

Proof: Proof was given in [19]. ∎

B. Unit quaternions and attitude kinematics

Consider two orthogonal right-handed coordinate frames: the body coordinate frame, $\mathbf{E}^b = [\vec{e}_1{}^b, \vec{e}_2{}^b, \vec{e}_3{}^b]$, located at the center of mass of the rigid body and the inertial coordinate frame, $\mathbf{E}^f = [\vec{e}_1{}^f, \vec{e}_2{}^f, \vec{e}_3{}^f]$, located at some point in the space. The rotation of the body frame $\mathbf{E}^b$ with respect to the fixed frame $\mathbf{E}^f$ is represented by the attitude matrix $R \in SO(3) = \{R \in \mathbb{R}^{3\times3} : R^T R = I, \det R = 1\}$.
The cross product between two vectors $\xi, \chi \in \mathbb{R}^3$ is represented by a matrix multiplication $[\xi^\times]\chi = \xi \times \chi$, where $[\xi^\times]$ is the well known skew-symmetric matrix.
The n-dimensional unit sphere embedded in $\mathbb{R}^{n+1}$ is denoted as $\mathbb{S}^n = \{x \in \mathbb{R}^{n+1} : x^T x = 1\}$. Members of $SO(3)$ are often parametrized in terms of a rotation $\beta \in \mathbb{R}$ about a fixed axis $e_v \in \mathbb{S}^2$ by the map $\mathcal{U} : \mathbb{R} \times \mathbb{S}^2 \to SO(3)$ defined as

$$\mathcal{U}(\beta, e_v) := I_3 + \sin(\beta)[e_v^\times] + (1 - \cos(\beta))[e_v^\times]^2 \tag{10}$$

Hence, a unit quaternion, $q \in \mathbb{S}^3$, is defined as

$$q := \begin{pmatrix} \cos\frac{\beta}{2} \\ e_v \sin\frac{\beta}{2} \end{pmatrix} = \begin{pmatrix} q_0 \\ q_v \end{pmatrix} \in \mathbb{S}^3 \tag{11}$$

$q_v = (q_1 \;\; q_2 \;\; q_3)^T \in \mathbb{R}^3$ and $q_0 \in \mathbb{R}$ are known as the vector and scalar parts of the quaternion respectively. q represents an element of $SO(3)$ through the map $\mathcal{R} : \mathbb{S}^3 \to SO(3)$ defined as

$$\mathcal{R} := I_3 + 2q_0[q_v^\times] + 2[q_v^\times]^2 \tag{12}$$

Note that $R = \mathcal{R}(q) = \mathcal{R}(-q)$ for each $q \in \mathbb{S}^3$, *i.e.* quaternions q and $-q$ represent the same physical attitude.
Denoting by $\omega = (\omega_1 \;\; \omega_2 \;\; \omega_3)^T$ the angular velocity vector of the body coordinate frame, $\mathbf{E}^b$ relative to the inertial coordinate frame, $\mathbf{E}^f$, expressed in $\mathbf{E}^b$, the kinematics equation is given by

$$\begin{pmatrix} \dot{q}_0 \\ \dot{q}_v \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -q_v^T \\ I_3 q_0 + [q_v^\times] \end{pmatrix} \omega = \frac{1}{2}\Xi(q)\omega \tag{13}$$

The attitude error is used to quantify the mismatch between two attitudes. If q defines the current attitude quaternion and q_d is the desired quaternion, i.e. the desired orientation, then the error quaternion that represents the attitude error between the current orientation and the desired one is given by

$$q_e = q_d^{-1} \otimes q = (q_{e_0} \;\; q_{e_v}{}^T)^T \tag{14}$$

where q^{-1} is the complementary rotation of the quaternion q which is given by $q^{-1} = (q_0 \;\; -q_v^T)^T$ and $\otimes$ denotes the quaternion multiplication [23]. In the case that the current quaternion and the desired one coincide, the quaternion error becomes $q_e = (\pm 1 \;\; 0^T)^T$.

C. System model

The quadrotor is a small aerial vehicle that belongs to the VTOL (Vertical Taking Off and Landing) class of aircrafts. It is lifted and propelled, forward and laterally, by controlling the rotational speed of four blades mounted at the four ends of a simple cross and driven by four DC Brushless motors (BLDC). On such a platform (see Fig. 1), given that the front and rear motors rotate counter-clockwise while the other two rotate clockwise, gyroscopic effects and aerodynamic torques tend to cancel each other out in trimmed flight. The rotation of the four rotors generates a vertical force, called the thrust T, equal to the sum of the thrusts of each rotor ($T = f_1 + f_2 + f_3 + f_4$). The pitch movement θ is obtained by increasing/decreasing the speed of the rear motor while decreasing/increasing the speed of the front motor. The roll movement ϕ is obtained similarly using the lateral motors. The yaw movement ψ is obtained by increasing/decreasing the speed of the front and rear motors while decreasing/increasing the speed of the lateral motors. In order to avoid any linear movement of the quadrotor, these maneuvers should be achieved while maintaining a value of the total thrust T that balances the aircraft weight. In order to model the system's dynamics, two frames are defined: a fixed frame in the space $\mathbf{E}^f = [\vec{e}_1{}^f, \vec{e}_2{}^f, \vec{e}_3{}^f]$ and a body-fixed frame $\mathbf{E}^b = [\vec{e}_1{}^b, \vec{e}_2{}^b, \vec{e}_3{}^b]$, attached to the quadrotor at its center of gravity, as shown in Fig. 1.

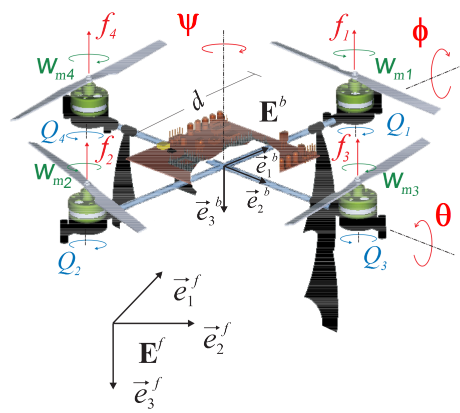

Fig. 1. Quadrotor: fixed frame $\mathbf{E}^f = [\vec{e}_1^f, \vec{e}_2^f, \vec{e}_3^f]$ and body-fixed frame $\mathbf{E}^b = [\vec{e}_1^b, \vec{e}_2^b, \vec{e}_3^b]$

According to [24], [8] and II-B, the six degrees of freedom model (position and attitude) of the system can be separated into translational and rotational motions, represented respectively by Σ_T and Σ_R in equation (15) and (16).

$$\Sigma_T : \begin{cases} \dot{p} = v \\ \dot{v} = ge_3 - \dfrac{1}{m}R^T(q)Te_3 \end{cases} \tag{15}$$

$$\Sigma_R : \begin{cases} \dot{q} = \dfrac{1}{2}\Xi(q)\omega \\ J\dot{\omega} = -[\omega^\times]J\omega + \Gamma \end{cases} \tag{16}$$

where m denotes the mass of the quadrotor and J its inertial matrix expressed in $\mathbf{E}^b$. g is the gravity acceleration and

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$e_3 = (0\ 0\ 1)^T$. $p = (x\ y\ z)^T$ represents the position of the quadrotor's center of gravity, which coincides with the origin of frame $\mathbf{E}^b$, with respect to frame $\mathbf{E}^f$, $v = (v_x\ v_y\ v_z)^T$ its linear velocity in $\mathbf{E}^f$, and ω denotes the angular velocity of the quadrotor expressed in $\mathbf{E}^b$. $\Gamma \in \mathbb{R}^3$ depend on the couples generated by the actuators, aerodynamic couples and external couples (environmental forces). In this paper, it is assumed that these torques are only generated by the actuators. $-Te_3$ is the total thrust expressed in $\mathbf{E}^b$.
The reactive torque Q_j due to the j^{th} rotor drag, $j \in \{1, 2, 3, 4\}$, and the total thrust T generated by the four rotors can be approximated by an algebraic relationship on function of a PWM control signal applied to the BLDC-drivers:

$$Q_j = k_m u_{mj} \qquad T = b_m \sum_{j=1}^{4} u_{mj} = \sum_{j=1}^{4} f_j \tag{17}$$

where the input signals u_{mi} are expressed in ms, *i.e.* the units of the PWM control signal. $k_m > 0$ and $b_m > 0$ are two parameters that depend on the air density, the dynamic pressure, the lift coefficient, the radius and the angle of attack of the blades and they are obtained experimentally.
The components of the control torque vector Γ generated by the rotors are given by:

$$\begin{aligned} \Gamma_1 &= db_m(u_{m3} - u_{m4}) \\ \Gamma_2 &= db_m(u_{m1} - u_{m2}) \\ \Gamma_3 &= k_m(-u_{m1} + u_{m2} - u_{m3} + u_{m4}) \end{aligned} \tag{18}$$

with d being the distance from one rotor to the center of mass of the quadrotor. Combining equations (17) and (18), the forces and torques applied to the quadrotor are written as:

$$\begin{aligned} \begin{pmatrix} \Gamma \\ T \end{pmatrix} &= \begin{pmatrix} 0 & 0 & db_m & -db_m \\ db_m & -db_m & 0 & 0 \\ -k_m & -k_m & k_m & k_m \\ b_m & b_m & b_m & b_m \end{pmatrix} \begin{pmatrix} u_{m1} \\ u_{m2} \\ u_{m3} \\ u_{m4} \end{pmatrix} \\ &= NU_m \end{aligned}$$

where $U_m = (u_{m1} \quad u_{m2} \quad u_{m3} \quad u_{m4})^T$. Since N is an invertible matrix, the vector of signals control U_m is easily obtained.

III. EVENT-BASED CONTROL STRATEGY

A. Problem statement

The objective is to design a control law that drives the quadrotor attitude to a specified constant orientation and maintains this orientation starting from any initial condition. It follows that the angular velocity vector must be brought to zero and remains null. In this paper, null stabilization is considered. Hence, the inertial coordinate frame is selected to be the desired orientation and the control objective is described by the following asymptotic condition:

$$q \to (\pm 1 \quad 0^T)^T, \quad \omega \to 0 \quad \text{as} \quad t \to \infty \tag{19}$$

Equation (19) represents two equilibrium points ($q_0 = 1$, $q_v = (0\ 0\ 0)^T$) and ($q_0 = -1$, $q_v = (0\ 0\ 0)^T$). These equilibrium points represent the same equilibrium point in the physical space and they yield the same attitude matrix in equation (12). However, they represent two-point set in $\mathbb{S}^3$. This topological obstruction not allows to state any global property for the closed-loop system, using a continuous quaternion-based feedback [25], [26].
In this study, the case $q_d = (1 \quad 0^T)^T$ is considered.
On the other hand, the quadrotor is equipped of an Attitude Heading Reference Systems (AHRS) and an embedded computer system (see Fig. 2). The AHRS continuously monitors the state x (attitude and angular velocity). Based on current state information and the last computed control signal, which is piecewise constant, the event-function decides when to broadcast the current state measurement over the network which is denoted by x_i. Whenever the control block receives a new state value, it updates the control law and the control signal for the actuators (PWM signals). Then, it broadcasts the control signal over the network in order to evaluate the event-triggered function and to detect a new event.
Thus, the problem consists in showing that the attitude of the quadrotor helicopter can be asymptotically stabilized by means of an event-triggered feedback as defined in section II-A, i.e. with the control law (7) together with the event function (8). Another motivation is that other traffic exists between two successive events and after the update and broadcasting of the control signal over the network. Reducing the traffic used for control (thanks to an event-based approach) hence allows i) to reduce traffic congestion in the network and ii) to broadcast other sensors data, for instance GPS or infrared sensors.

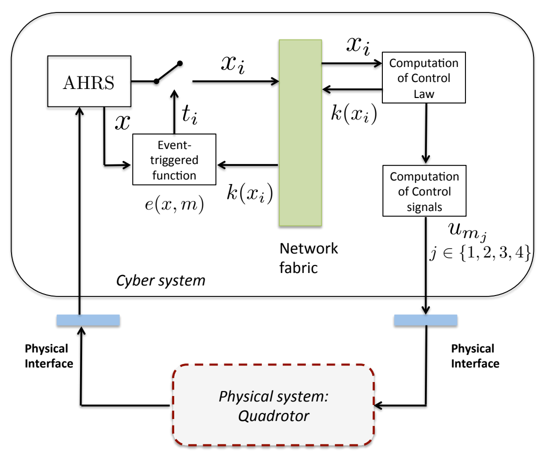

Fig. 2. Quadrotor control system

B. Control design

In order to stabilize the attitude of the quadrotor mini-helicopter, the subsystem Σ_R in (16) is used. Defining the variables $x_1 = q_0 \in \mathbb{R}$, $x_2 = q_v \in \mathbb{R}^3$, $x_3 = \omega \in \mathbb{R}^3$, Σ_R can be rewritten as

$$\dot{x} = f(x) + g(x)u \tag{20}$$

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which is a nonlinear system affine in the control with state $x = (x_1\ x_2^T\ x_3^T)^T$ control $u = \Gamma \in \mathbb{R}^3$ and vectors fields

$$f(x) = \begin{pmatrix} -\frac{1}{2}x_2^T x_3 \\ \frac{1}{2}(x_1 I_3 + [x_2^\times])x_3 \\ -J^{-1}[x_3^\times]Jx_3 \end{pmatrix} \tag{21}$$
$$g(x) = \left(g_1(x)\ g_2^T(x)\ g_3^T(x)\right)^T$$

where $g_1(x) = 0 \in \mathbb{R}^{1\times 3},\ g_2(x) = 0 \in \mathbb{R}^{3\times 3}$ and $g_3(x) = J^{-1} \in \mathbb{R}^{3\times 3}$.
According to (19) the control objective becomes

$$x_0 \to 1, \quad x_2, x_3 \to 0 \quad \text{as} \quad t \to \infty \tag{22}$$

Lemma 3.1: The function $V : \mathbb{S}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}$ defined by

$$V = x_2^T x_2 + (x_1 - 1)^2 + \frac{1}{2}\tilde{x}^T K_3^{-1} J \tilde{x} \tag{23}$$

with $\tilde{x} = x_3 + K_1 x_2$ is a CLF for the system (20) relative to the equilibrium state $x^e = \left(1\ 0^T\ 0^T\right)^T$ with the control

$$u = [x_3^\times]Jx_3 - JK_1\dot{x}_2 - K_2\tilde{x} - K_3 x_2 \tag{24}$$

where $K_1,\ K_2,\ K_3 \in \mathbb{R}^{3\times 3}$ are diagonal positive definite matrices.

Proof: Clearly V is smooth, positive definite and proper. Now, consider the derivative of (23) along the trajectories of the closed-loop system with any initial condition in $\mathbb{S}^3 \times \mathbb{R}^3 \setminus \left(-1\ 0^T\ 0^T\right)^T$

$$\begin{aligned} \dot{V}(x) &= \frac{\partial V}{\partial x}^T f(x) + \frac{\partial V}{\partial x}^T g(x)u \\ &= -\tilde{x}^T K_3^{-1} K_2 \tilde{x} - K_1 x_2^T x_2 < 0 \text{ for } x \neq x^e \end{aligned} \tag{25}$$

Then this mean that $x_2, \tilde{x} \to 0$. That implies $x_3 \to 0$ and due to the quaternion normality condition $x_0 \to 1$. Consequently V is a Control-Lyapunov Function.

■

Corollary 3.2: Consider the quadrotor mini-helicopter rotational dynamics and the CLF given by (20) and (23), respectively. Then the event-triggered feedback (k, e) defined by (7) and (8) with $\theta = x_2^T x_2 + (x_1 - 1)^2$ and $\Delta(x) = I_3$ asymptotically stabilizes the quadrotor at $\left(1\ 0^T\ 0^T\right)^T$ with a domain of attraction equal to $\mathbb{S}^3 \times \mathbb{R}^3 \setminus \left(-1\ 0^T\ 0^T\right)^T$. Furthermore, the feedback (k, e) is semi-uniformly MSI and smooth on $\mathbb{S}^3 \times \mathbb{R}^3 \setminus \left(1\ 0^T\ 0^T\right)^T$.

Proof: The proof follows the one of Theorem 2.3. ■

Remark 3.3: Note that the stability analysis has been carried out considering the asymptotic condition $q_d = (1\ 0^T)^T$. In the case where the asymptotic condition $q \to q_d$ with $q_d \neq (1\ 0^T)^T$ is considered, the feedback becomes in function of $x_1 = q_{e_0} \in \mathbb{R}$, $x_2 = q_{e_v} \in \mathbb{R}^3$, $x_3 = \omega \in \mathbb{R}^3$, where the q_e is given by (14) which represents the attitude error between the current orientation and the desired one.

IV. EXPERIMENTAL SETUP

This section is devoted to proving the effectiveness of the proposed event-triggered control. Experiments on the quadrotor prototype (Fig. 3) are carried out in real-time.
This prototype is based on the mechanical structure of the 330X-S QUD-Flyer developed by TSH-GAUI Hobby Corporation using four BLDC motors. The control law is executed on a Spartan-6 FPGA LX9 MicroBoard. The Spartan-6 has the ability to implement a "MicroBlaze" soft processor running at 100 MHz. Furthermore, the Spartan-6 has the advantage to develop custom modules such as PWM generators and USARTs ports. An AHRS is used to obtain the attitude quaternion and angular velocity at 73 Hz. A Bluetooth Modem linked to a PC is used to exchange the processed data. The desired attitude q_d is provided by means of a 5-channel Radio-Control Spektrum DX5e with 2.4 GHz radio technology. Four power modules are used to drive the motors by means of a PWM signal. The frequency of the PWM signal is fixed to 500 Hz. The power of the whole system is supplied by a 11.1 Volts Li-Po battery. The specification and parameters of the quadrotor prototype are given in the Table I.

Parameter	Description	Value	Units
m	Mass	0.835	Kg
d	Distance	0.16	m
J_x	Inertia in x-axis	7.80×10^{-3}	Kg· m^2
J_y	Inertia in y-axis	7.80×10^{-3}	Kg· m^2
J_z	Inertia in z-axis	10.22×10^{-3}	Kg· m^2
b_m	Proportionality Constant	39.9	N / ms
k_m	Proportionality Constant	3800.8	N· m / ms

TABLE I

THE SPECIFICATION AND PARAMETERS OF THE QUADROTOR

To evaluate the benefices of the control law defined in the corollary 3.2, two experiments are performed. The objective is to bring the quadrotor from any initial orientation, sufficiently far from the desired attitude defined by $q_d = (1\ 0^T)^T$ *i.e.* $\phi_d = \theta = 0 = \psi = 0$ and hold it there by maintaining the angular velocity to zero. The desired thrust is taken as $T \geq mg$ = 8.19 N such that it guarantees a balance of the quadrotor's weight. Experiments were performed with the following gains: $K_1 = \text{diag}(1, 1, 1)$, $K_2 = \text{diag}(2.5, 2.5, 2.5)$, $K_3 = \text{diag}(0.11, 0.11, 0.12)$. The value for the parameter σ in the event function (8) determinates the frequency of events and it is fixed to 0.94 for these experiments.
In each cases, the first (top) plot shows the Euler angles (since they are more intuitive, however the control law uses quaternions) whereas angular velocities are provided in the second one. The third and fourth plots show the control torques and the Lyapunov function (one can see it decreases while the system is stabilized). Finally, the last (bottom) plots give the event function – an event occurs when this function vanishes to zero, as defined in (8) – and a representation of the sampling instants (1 and 0 in the last plot mean the control is updated or it is kept constant respectively).

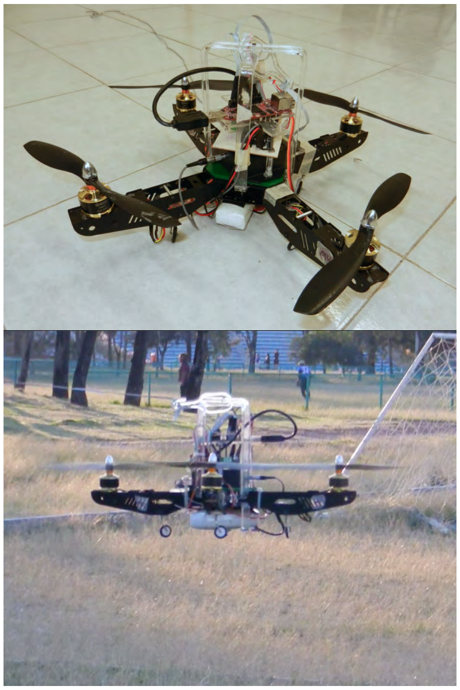

Fig. 3. The quadrotor mini-helicopter in flight

In the first experiment, the control capabilities are tested to stabilize the system, with initial conditions $(-21^o, 26.4^o, -37^o)$. The results are depicted in Fig. 4 where the stabilization takes about 2.5 seconds. In the classical frame (time-triggered control), the control law should be updated 365 times for a span of 5 seconds, since the AHRS continuously provides the state at a frequency of 73 Hz. With the proposed approach, one could note in Fig. 4(f) that some large intervals without any control update exist. Actually, the control law is updated only 72 times during the 5 seconds experimental time, which represents a reduction of 80.2 % w.r.t. the classical frame. It is worth noting that this reduction in the number of updates, reduces the data exchange between AHRS, controller and actuators without sacrificing performance. Also, one could note in Fig. 4(e) that, whereas the event function only vanishes in theory (it could not become negative by construction), the implemented version becomes negative due to the AHRS sampling time. Indeed, an event can only be detected when some data are received and these data are only available every 0.0136 seconds.

In the second experiment, the robustness of the proposed controller towards disturbance rejection is tested. The disturbances along each axis (the three directions) are introduced in the system once achieved the attitude stabilization. The results are depicted in Fig. 5, where a disturbance is performed on roll, pitch and yaw at about 23, 33 and 46 seconds respectively. As one can see, the disturbances produce an error on both the angles and the angular velocities, see Fig. 5(a) and (b). As a consequence, the Lyapunov function in Fig. 5(d) is highly increased when a perturbation occurs. The event-triggered function vanishes or becomes negative in consequence and, as a result, the control law is updated more often to overcome the perturbations, see Fig. 5(c). In this experiment, the amount of samples needed for the event-triggered control for 55 seconds is 801 (instead of 4015 in the classical frame) which represents a reduction of 80 %.

V. CONCLUSIONS

The main contribution of this paper is the development and implementation of a nonlinear event-triggered feedback for the attitude stabilization of a quadrotor mini-helicopter. The attitude is parameterized using the unit quaternion. Firstly, it is proved the existence of a smooth Control Lyapunov Function for the attitude dynamics of the quadrotor. Then, an event-triggered static feedback is derived from the universal formula for event-triggered stabilization of general nonlinear systems affine in the control [19]. The control law ensures the asymptotic stability of the closed-loop system to the desired attitude. The approach is validated in real-time and the experiments show that the event driven controller reduces by 80 % the communication load without deteriorating the closed-loop system performance. The proposed approach still has to be compare with other control schemes. However, to our best knowledge, this is the first time that a nonlinear event-triggered controller is applied for the attitude stabilization of an unmanned aircraft system.

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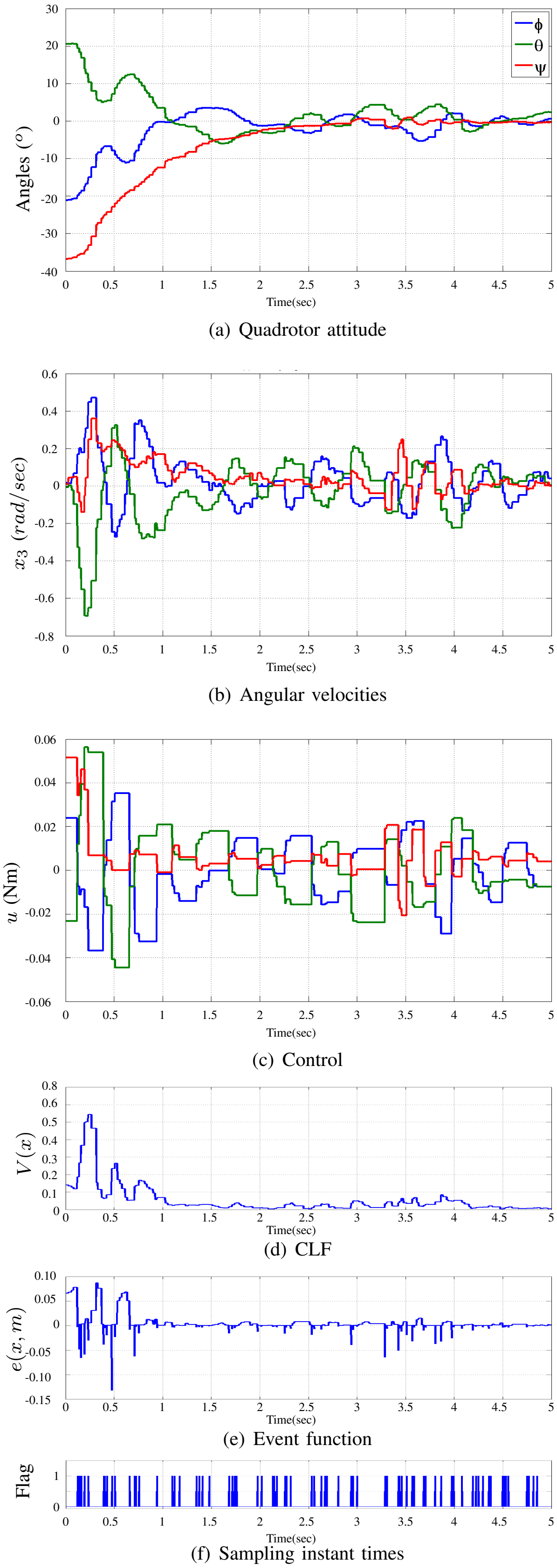


(a) Quadrotor attitude

(b) Angular velocities

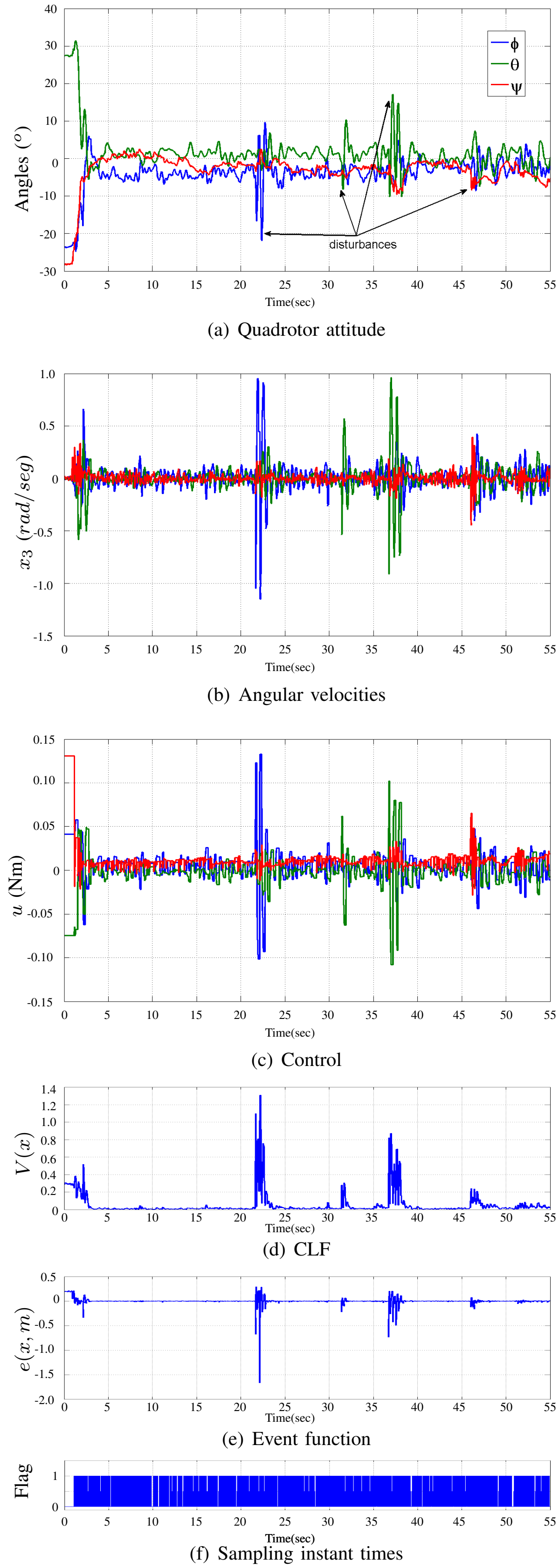
(c) Control

(d) CLF

(e) Event function

(f) Sampling instant times

Fig. 4. Stabilization to the origin of the quadrotor.



(a) Quadrotor attitude

(b) Angular velocities

(c) Control

(d) CLF

(e) Event function

(f) Sampling instant times

Fig. 5. Robustness of the event-based control no-lineal to disturbances.

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