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Gelatine as a crustal analogue: Determining elastic properties for modelling magmatic intrusions

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Abstract

Gelatine has often been used as an analogue material to model the propagation of magma-filled fractures in the Earth's brittle and elastic crust. Despite this, there are few studies of the elastic properties of gelatine and how these evolve with time. This important information is required to ensure proper scaling of experiments using gelatine. Gelatine is a viscoelastic material, but at cool temperatures ($T_r \sim 5\text{--}10\text{ }^\circ\text{C}$) it is in the solid 'gel' state where the elastic behaviour dominates and the viscous component is negligible over short to moderate timescales. We present results from a series of experiments on up to 30-litres of maximum 30 wt% pigskin gelatine mixtures that document in detail how the elastic properties evolve with time, as a function of the volume used and gel concentration (C_{gel}). Gelatine's fracture toughness is investigated by measuring the pressure required to propagate a pre-existing crack. In the gel-state, gelatine's Young's modulus can be calculated by measuring the deflection to the free-surface caused by an applied load. The load's geometry can affect the Young's modulus measurement; our results show its diameter needs to be $\lesssim 10\%$ of both the container diameter and the gelatine thickness (H_{gel}) for side-wall and base effects to be ignored. Gelatine's Young's modulus increases exponentially with time, reaching a plateau (E_∞) after several hours curing. E_∞ depends linearly on C_{gel} , while T_r , H_{gel} and the gelatine's thermal diffusivity control the time required to reach this value. Gelatine's fracture toughness follows the same relationship as ideal elastic-brittle solids with a calculated surface energy $\gamma_s = 1.0 \pm 0.2\text{ J m}^{-2}$. Scaling laws for gelatine as a crustal analogue intruded by magma (dykes or sills) show that mixtures of 2–5 wt% gelatine cured at $\sim 5\text{--}10\text{ }^\circ\text{C}$ ensure the experiments are geometrically, kinematically and dynamically scaled.

Research Highlights

- 2-5wt% gelatine mixtures at 5-10°C are good crustal analogues for dyke or sill experiments
- The Young's modulus gelatine solutions evolves with time to a plateau
- This Young's modulus plateau correlates linearly with gelatine concentration
- The time to plateau depends on the room temperature, gelatine thickness and its thermal diffusivity
- Gelatine's fracture toughness has a surface energy of $\gamma_s = 1.0 \pm 0.1 \text{ Jm}^{-2}$

Abstract

Gelatine has often been used as an analogue material to model the propagation of magma-filled fractures in the Earth's brittle and elastic crust. Despite this, there are few studies of the elastic properties of gelatine and how these evolve with time. This important information is required to ensure proper scaling of experiments using gelatine. Gelatine is a viscoelastic material, but at cool temperatures ($T_r \sim 5\text{--}10\text{ }^\circ\text{C}$) it is in the solid 'gel' state where the elastic behaviour dominates and the viscous component is negligible over short to moderate timescales. We present results from a series of experiments on up to 30-litres of maximum 30 wt% pigskin gelatine mixtures that document in detail how the elastic properties evolve with time, as a function of the volume used and gel concentration (C_{gel}). Gelatine's fracture toughness is investigated by measuring the pressure required to propagate a pre-existing crack. In the gel-state, gelatine's Young's modulus can be calculated by measuring the deflection to the free-surface caused by an applied load. The load's geometry can effect the Young's modulus measurement; our results show its diameter needs to be $\lesssim 10\%$ of both the container diameter and the gelatine thickness (H_{gel}) for side-wall and base effects to be ignored. Gelatine's Young's modulus increases exponentially with time, reaching a plateau (E_∞) after several hours curing. E_∞ depends linearly on C_{gel} , while T_r , H_{gel} and the gelatine's thermal diffusivity control the time required to reach this value. Gelatine's fracture toughness follows the same relationship as ideal elastic-brittle solids with a calculated surface energy $\gamma_s = 1.0 \pm 0.2\text{ J m}^{-2}$. Scaling laws for gelatine as a crustal analogue intruded by magma (dykes or sills) show that mixtures of 2–5 wt% gelatine cured at $\sim 5\text{--}10\text{ }^\circ\text{C}$ ensure the experiments are geometrically, kinematically and dynamically scaled.

Key words: Gelatine, Dyke, Sill, Magma, Analogue Scaling

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1 1 Introduction

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2 Analogue experimentation is an important technique in science and engineer-
3 ing. In practice, it is the selection of appropriate analogue materials that is
4 often the biggest challenge in developing a set of experiments that are geomet-
5 rically, kinematically and dynamically scaled (*sensu* Hubbert (1937)). Exper-
6 iments that meet these criteria can be considered a laboratory-scale version
7 of the natural counterpart. In this paper we detail a series of experiments
8 carried out to document the properties of gelatine, a widely used analogue for
9 the Earth's crust.

10 Gelatine is an ideal analogue for those modelling homogeneous, isotropic and
11 elastic materials, for example it has been used by mechanical engineers (e.g. Crisp
12 (1952); Richards and Mark (1966)) and as a biological tissue analogue in the
13 medical sciences (e.g. Righetti et al. (2004)). The use of gelatine in geological
14 sciences has taken advantage of both its elastic and viscous properties, prov-
15 ing especially fruitful in developing our understanding magmatic intrusions
16 (dykes and sills) of volcanic feeder systems and their propagation dynamics
17 in the Earth's brittle and elastic crust (e.g. Fiske and Jackson (1972); Pol-
18 lard (1973); Pollard and Johnson (1973); Maaløe (1987); Hyndman and Alt
19 (1987); McGuire and Pullen (1989); Takada (1990); Heimpel and Olson (1994);
20 Takada (1994); McLeod and Tait (1999); Takada (1999); Dahm (2000); Muller
21 et al. (2001); Menand and Tait (2001); Ito and Martel (2002); Watanabe et al.
22 (2002); Menand and Tait (2002); Walter and Troll (2003); Acocella and Tibaldi
23 (2005); Rivalta et al. (2005); Cañón-Tapia and Merle (2006); Kavanagh et al.
24 (2006); Mathieu et al. (2008); Kervyn et al. (2009); Menand et al. (2010);
25 Maccaferri et al. (2010); Taisne and Tait (2011); Taisne et al. (2011)). The
26 photoelastic properties of gelatine have been of particular use to experimental

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4 27 geologists (e.g. Taisne and Tait (2011)) and civil engineers (e.g Farquharson
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6 28 and Hennes (1940); Crisp (1952); Tan (1947); Richards and Mark (1966)),
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8 29 where the internal stresses of a deformed gelatine can be visualised with the
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10 30 aid of polarised light. The prolific use of gelatine in the food industry has made
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12 31 a wealth of information available on its rheological properties (e.g. Watase and
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14 32 Nishinari (1980)). However, relatively few studies have documented the elastic
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16 33 properties of gelatine or how these evolve with time (e.g. Di Giuseppe et al.
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18 34 (2009))
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31 35 We present results from a series of experiments that investigate the elastic
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33 36 properties of gelatine over a range of concentrations and volumes. Firstly the
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35 37 material properties of gelatine are detailed, followed by a description of the
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37 38 experimental setup and the theoretical basis for our measurements. The ac-
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39 39 curacy to which the experimentalist can determine the Young's modulus of
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41 40 the gelatine is evaluated by considering the uncertainties involved in the mea-
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43 41 surement, the effect of the properties of the applied load used to make the
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45 42 measurements and any apparatus side-wall or floor effects. In particular, our
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47 43 experimental results are focused on how the Young's modulus of the gelatine
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49 44 evolves with time. We also determined the gelatine's fracture toughness, a
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51 45 measure of the material's resistance to the growth of a crack. To aid the ap-
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53 46 plication of the results, we present some scaling laws that are appropriate for
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55 47 the use of gelatine as an analogue for the Earth's crust in geological studies
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57 48 focused on the formation controls and propagation dynamics of magma-filled
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59 49 fractures.
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50 2 Material Properties

51 Gelatine is a polypeptide formed from the hydrolytic degradation of colla-
52 gen (Ross-Murphy, 1992). It is classified as a 'physical gel' (e.g. Peyrelasse
53 et al. (1996)), meaning that during gelification Van der Waals forces lead to
54 the development of a complex and continuously connected three-dimensional
55 network (lattice) of macromolecules (Djabourov et al., 1988a). The hydrogen
56 bonds that are formed in this process are reversible and can be broken by
57 changing temperature or pH (Djabourov et al., 1988b). From the onset, those
58 working with gelatine have commented on its "fickle" nature (Richards and
59 Mark, 1966). In order to use this material for quantitative modelling purposes,
60 control needs to be kept on a range of factors including temperature, pH and
61 gelatine concentration.

62 Gelatine is a viscoelastic material so during deformation it can display both
63 elastic and viscous behaviour. High stresses applied for a short timescale cause
64 the gelatine to behave elastically, whereas small stresses applied over a long
65 time period will produce a viscous response. Viscoelasticity is traditionally
66 modelled with an arrangement of springs and dashpots that can reproduce
67 a measured creep curve (e.g. Richards and Mark (1966)). The proportion of
68 elastic to viscous behaviour can be quantified by a phase shift δ angle, also
69 known as the "loss angle" (Mezger, 2002):

$$70 \quad \delta = \arctan \frac{G''}{G'} \quad (1)$$

71 where G'' is the energy loss (viscous-related) and G' is the energy stored
72 (elastic-related) for a given strain or strain rate. δ is equal to 0° for an ideal-
73 elastic material and 90° for an ideal-viscous material. The transition from

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5 74 viscously dominated to elastically dominated behaviour (or *vice versa*) occurs
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7 75 at the 'gel-point' (Djabourov et al., 1988b), which is the condition where elastic
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9 76 and viscous energies are equal ($G'' = G'$, $\delta = 45^\circ$). Gelatine is in the 'sol-state'
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11 77 (fluid) when $G'' > G'$ and $\delta > 45^\circ$, but is in the 'gel-state' (solid) when $G' > G''$
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13 78 and $\delta < 45^\circ$ (Ross-Murphy, 1992; Nelson and Dealy, 1993; Mezger, 2002). For
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15 79 gelatine, this marked change in mechanical properties can be brought about
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17 80 by changing the extent of deformation (strain) or temperature; the gel-point
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19 81 itself depends on time, temperature and concentration (Askeland et al., 2010;
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21 82 Di Giuseppe et al., 2009).

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27 83 The focus of this paper will be on the ideal-elastic behaviour of gelatine.
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29 84 When a 2.5 wt% gelatine mixture at 10 °C is deformed at low strain it has
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31 85 G' two orders of magnitude higher than G'' and $\delta < 1^\circ$ (Di Giuseppe et al.,
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33 86 2009). At these conditions the material is in the 'gel-state' and it is possible
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35 87 to assume an almost ideal-elastic behaviour. When this is the case, Hooke's
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37 88 Law is obeyed and deformation is recoverable when high stresses are applied
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39 89 over short timescales: the applied stress (σ) is proportional to strain (γ) and
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41 90 independent of the strain rate ($\dot{\gamma}$).

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48 91 The elastic properties of a homogeneous and isotropic solid can be described
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50 92 fully by a combination of the Young's modulus E (ratio of tensile stress to
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52 93 tensile strain) and the Poisson's ratio ν (the relative contractive to expansive
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54 94 response of the deformed material). For gelatine, $\nu \simeq 0.5$ (e.g. Farquharson
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56 95 and Hennes (1940); Crisp (1952); Richards and Mark (1966); Righetti et al.
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58 96 (2004)) and is theoretically incompressible such that deformation results in no
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60 97 net volume change.

98 3 Experimental Set-up and Data Processing

99 3.1 Young's Modulus Experiments

100 A series of twenty-six experiments were carried out to investigate the effect
101 of time, gelatine concentration, volume, experimental apparatus dimensions
102 and applied load properties on the calculated Young's modulus of solidified
103 gelatine.

104 A gelatine solution was prepared by adding a measured quantity of approx-
105 imately 80 °C deionised water to the required weight of gelatine granules
106 (260 Bloom, 20 Mesh, Pigskin Gelatine supplied by Gelita UK) to achieve
107 the desired concentration (see Table 1). The use of deionised water is re-
108 quired to produce a clear and transparent mixture which hinders bacterial
109 growth, which would otherwise produce a cloudy appearance to the gelatine
110 solid. This hot mixture was then poured into a specified container and any
111 bubbles were removed from the surface using a spoon. To prohibit the for-
112 mation of a toughened 'skin' on the gelatine surface by water evaporation, a
113 thin layer of vegetable oil was poured on top. The container was then placed
114 into a temperature-controlled cold room at 5–10 °C (T_r), and the mixture
115 temperature (T_0) and time were recorded. The gelatine was left in the cold
116 room for several hours until the mixture temperature had equilibrated with
117 the surroundings.

118 One way of calculating the gelatine's Young's modulus is to measure the de-
119 flection imposed by a load applied to the gelatine's surface (Timoshenko and

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120 Goodier, 1970):

$$121 \quad E = \frac{M_L g (1 - \nu^2)}{D_L w}, \quad (2)$$

122 where D_L is the diameter (m) of the cylindrical load, M_L is its mass (kg), w
123 is the displacement (m) caused, and g is the gravitational acceleration.

124 Measurement of the Young's modulus commenced once the gelatine was able
125 to support a load placed on its free surface. The container was removed from
126 the cold room to make the measurements and then replaced afterwards. All
127 the oil was carefully removed from the surface of the gelatine prior to any mea-
128 surement being taken, using a spoon and then paper towel in order to achieve
129 complete contact between the load and the gelatine. The load was applied
130 by carefully placing a rigid metallic cylinder of known mass and dimensions
131 onto the gelatine surface (see Figure 1 for a schematic sketch of the experi-
132 mental setup and Table 2 for experimental load properties). Using a digital
133 micrometer attached to a fixed reference position, the displacement of the free
134 surface was measured (with an estimated error of ± 0.1 mm) and recorded by
135 hand. The load was applied just prior to the measurement being made, and
136 the total time in which it was in contact with the gelatine was approximately
137 30 seconds per measurement. E was calculated systematically for the duration
138 of each experiment; measurements were made using each of the loads when
139 possible approximately every one to two hours for up to 140 hours after the
140 gelatine was prepared (nearly six days). For each time interval, the gelatine
141 was at ambient room temperature for an interlude of less than ten minutes
142 before being returned to the cold room. No experimental load was applied to
143 the gelatine surface between time steps. The displacement measurement ' w '
144 and the properties of the load were input into equation 2) to calculate the

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8 146 The experimental series considers gelatine concentration C_{gel} (2-30 wt%), tem-
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10 147 perature of the cold room T_r (5–10 °C), volume of gelatine V_{gel} (0.5 to 30
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12 148 litres), diameter of the experimental container D_C (8.6–40.0 cm), thickness of
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14 149 the gelatine H_{gel} (4.1–27.0 cm), and applied load (with mass M_L of 25.5–2808.5
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16 150 g and diameter D_L 20.0–85.6 mm) (see Tables 1 and 2). These experiments
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18 151 allowed the characterisation of the evolution of the Young's modulus of gela-
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22 153 to be assessed.
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27 154 *3.2 Fracture Toughness Experiments*

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32 155 The fracture toughness K_c is a measure of a material's resistance to the growth
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34 156 of a crack. The fracture toughness of gelatine solids was determined by ex-
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38 158 crack (following the analysis of Sneddon and Das (1971)). This experimen-
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40 159 tal method for calculating the fracture toughness of gelatine solid is briefly
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42 160 described by Menand and Tait (2002). However, the mathematical procedure
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44 161 is not detailed explicitly. Therefore, here we present the experimental proce-
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46 162 dure again and detail in the Appendix the mathematical method so that other
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48 163 experimentalists can replicate this.
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52 164 For these fracture-toughness experiments high-clarity pigskin-derived gelatine
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54 165 (acid, 200 bloom) was supplied in granular form by SKW Bio-Systems. The
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56 166 gelatine mixture was prepared by first hydrating 5 to 8 wt% gelatine powder
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58 167 in distilled water, and then heating the solution to 60 °C until the powder was
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60 168 completely dissolved. Sodium hypochlorite was then added to the solution so
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5 169 that it contained 0.1 wt% of active chlorine, in order to prevent fungal and
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7 170 bacterial growth. This amount was kept small to minimise its potential effect
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9 171 on the gelatine mechanical properties.

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12 172 The gelatine solution was poured into a cubic acrylic tank (30 cm wide) and
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14 173 left to solidify for 48 hours at room temperature. A thin layer of silicon oil was
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16 174 poured on the gelatine surface in order to avoid evaporation during solidifi-
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18 175 cation and prevent the development of a gradient in gelatine properties. The
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20 176 tank was only filled to two-thirds its height, immersing a metallic blade that
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22 177 was elliptical in cross-section and had been inserted 5 cm into the gelatine's
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24 178 base. The blade measured 20 cm in length with a 1 cm thickness at its base.
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26 179 Once the gelatine had solidified, the blade was carefully removed thus creating
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28 180 an empty edge-crack in the gelatine solid. Both the crack and the remaining
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30 181 part of the tank were then filled with water, and the tank was overturned so
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32 182 that in its final position the crack was oriented vertically and at the bottom
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34 183 of the gelatine solid (Figure 2a). An outlet enabled water to bleed off any
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36 184 excess pressure in the lower part of the tank, and so ensured the water pres-
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38 185 sure balanced precisely with the weight of the overlying gelatine. Thus there
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40 186 was no excess pressure within the crack. Moreover, the initial state of stress
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42 187 within the gelatine solid was hydrostatic. (The gelatine solid adheres to the
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44 188 tank walls and so there is no horizontal strain, $\epsilon_x = \epsilon_y = 0$. Using Hooke's law,
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46 189 the relationship between the three stress components is $\sigma_x = \sigma_y = \frac{\nu}{(1-\nu)}\sigma_z$;
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48 190 and given that gelatine has a Poisson's ratio $\nu = 0.5$, $\sigma_x = \sigma_y = \sigma_z$.)
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54 191 These fracture toughness experiments were carried out at a room temperature
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56 192 of $19 \pm 2^\circ\text{C}$. At the beginning of an experiment, the Young's modulus of the
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58 193 gelatine was measured as described in Section 3.1 (using a load with diameter
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60 194 approximately one tenth of the tank width). The crack excess pressure was
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195 then increased by injecting air using a thin capillary which protruded into
196 the crack (Figure 2b). During this injection of air, any excess water bled off
197 ensuring that only the crack buoyancy increased; the excess pressure in the
198 other water-filled part of the tank remained nil. As more air entered the crack,
199 its buoyancy increased until it was sufficient to fracture the gelatine at the tip
200 of the crack (Figure 2c). The process was recorded by video camera, and
201 from this video record the exact amount of air that was present within the
202 crack just prior to the gelatine fracture was measured. The Young's modulus
203 of the gelatine was systematically varied between experiments, by changing
204 the concentration of gelatine used during preparation, and the amount of air
205 needed to propagate the initial crack was recorded, as summarised in Table 3.

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206 The fracturing of the gelatine solid was analysed within the Linear Elastic
207 Fracture Mechanics framework, according to which a crack propagates once
208 the stress intensity factor at its tip K_I exceeds the fracture toughness K_c of
209 the solid host (Griffith, 1921). Measuring the height of air present in the crack
210 enabled us to calculate the pressure distribution within the crack and thus
211 the stress intensity factor at its tip following the method of Sneddon and Das
212 (1971) (see Section 4.2 and Appendix). We thus measured the height of air
213 just prior to the crack propagation, and equated the calculated stress intensity
214 factor with the gelatine fracture toughness.

215 3.3 Data Processing

216 Data processing was undertaken in order to identify and quantify potential
217 sources of uncertainty in the Young's modulus measurements before analysing
218 the results. To account for experimental uncertainties, both the effect of the
219 dimensions of the applied load relative to the size of the experimental con-

220 tainer and also the effect of the propagation of errors in the Young's modulus
 221 calculation have been considered. A data weighting procedure has then been
 222 carried out before modelling the experimental results.

223 3.3.1 Effect of Applied Load and Container Size

224 We calculate the Young's modulus of gelatine by measuring by how much its
 225 free surface is deformed by an applied load. In doing so, we effectively assume
 226 that the gelatine solid is semi-infinite. However, the finite lateral dimensions
 227 of the gelatine container and distance to its base may have an important effect
 228 by restricting the movement of the deformed gelatine.

229 Figure 3 shows the relationship between the calculated Young's modulus and
 230 the relative size of the applied load diameter and the experiment container
 231 (D_L/D_C). Data from eight experiments at one time interval are shown (22
 232 hours curing at 10 °C). These experiments have equal gelatine concentration
 233 (2.5 wt%), but a range of volumes (0.5–30 litres), measured by Loads 1–8 in ten
 234 container sizes (see Table 1). Pearson product-moment correlation coefficients
 235 (r) were calculated for each experiment:

$$236 \quad r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}, \quad (3)$$

237 where

$$238 \quad S_{xx} = \sum (x - \bar{x})^2 \quad S_{yy} = \sum (y - \bar{y})^2 \quad S_{xy} = \sum (x - \bar{x})(y - \bar{y}), (4)$$

239 and x and y are experimentally determined variables (in this case E and
 240 D_L/D_C). The correlation coefficient ranges from -1 to +1; $r=+1$ indicates
 241 a positive linear correlation, $r=-1$ suggests a negative linear correlation, and

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242 $r=0$ when no correlation is found. In the region $D_L/D_C >10\%$ the results
243 show a strong positive correlation ($r \geq 0.65$), implying interaction between the
244 applied load and container walls is producing artificially high Young's modulus
245 calculations (an exception is experiment 3, where $r=0.38$). However, when
246 $D_L/D_C <10\%$ the correlation is poor and in this region the experimentalist can
247 be confident of avoiding sidewall effects. Providing this is the case, equation 2
248 holds and can be used to calculate the Young's modulus of the gelatine. Note
249 that from the experiments shown, only experiments with a larger volume (20–
250 30 litres) with Young's modulus measured with loads 3–8 fall into this category.

251 The Young's modulus measurements may also be affected by the distance to
252 the base of the experimental container. If we assume the gelatine is semi-
253 infinite and behaves as a purely elastic solid, we can estimate the stresses
254 variation with depth induced by a load applied to the surface. The largest
255 stress component induced by a load σ_0 is the vertical component σ_z , which
256 can expressed as (Timoshenko and Goodier, 1970):

$$257 \quad \sigma_z = \sigma_0 \left[1 - \frac{8z^3}{(1 + 4z^2)^{3/2}} \right], \quad (5)$$

258 where z has been normalised by the diameter of the load D_L . Following this
259 expression, the stress induced by the load at a depth ten times its diameter is
260 only 0.4% of that imposed by the load at the surface.

261 We therefore recommend that both the lateral and vertical dimensions of the
262 container be at least ten times the diameter of the load to avoid both container
263 sidewall and base effects.

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264 3.3.2 Propagation of Errors

265 The uncertainty associated with the Young's modulus (equation 2) was calculated according to the principles of the 'Propagation of Errors' (Bevington and Robinson, 2003), where the relative error is expressed as:

$$268 \quad \frac{\Delta E}{E} = \sqrt{\left(\frac{\Delta M}{M}\right)^2 + \left(\frac{\Delta D_L}{D_L}\right)^2 + \left(\frac{\Delta w}{w}\right)^2}, \quad (6)$$

269 where:

$$270 \quad w = \beta + X_1 - X_0. \quad (7)$$

271 β is the thickness of the load, X_0 is the distance to the unloaded surface and X_1 is the distance to the surface of the applied load (both X_1 and X_0 are measured relative to a fixed point of reference). Values of M , D_L , β , X_1 and X_0 used in the calculation are averages of three separate and successive measurements. β , X_1 and X_0 have independent random errors such that calculated values of w have an absolute error (Δw):

$$277 \quad \Delta w = \sqrt{\Delta\beta^2 + \Delta X_1^2 + \Delta X_0^2}. \quad (8)$$

278 Following this, the 'compound uncertainty' associated with each measurement of w is calculated as ± 0.3 mm. As the Young's modulus of the gelatine increases with time, correspondingly the deflection caused by the applied load decreases. Therefore the magnitude of w relative to Δw increases with time, as does the compound uncertainty associated with E ($\Delta E/E$). This is illustrated by the increasing size of the Young's modulus error-bars with time (Figure 4).

284 3.3.3 Weighting the data

285 At each time interval, E was calculated using the deflection caused by each
286 individual load (an average of three successive measurements). So for example,
287 the complete dataset from experiment 25 (4-litres of 2.5 wt% gelatine; Figure
288 5) comprised 14 time intervals at which the Young's modulus was measured
289 by loads 3–8 (where possible). The Young's modulus calculations were thus
290 based on a total of 294 measurements of X_0 and X_1 . In order that all the
291 measurements for each experiment could be considered in the analysis, a data
292 weighting process was carried out.

293 To account for the uncertainties associated with each Young's modulus mea-
294 surement, the data were weighted (W) taking into account both the precision
295 of the measurement and also the applied load used to take the measurement.
296 Table 4 shows the quantitative weightings (depending on the uncertainty in
297 E ; $W_{\Delta E}$) and qualitative weightings (depending on the applied load used;
298 $W_{\Delta Load}$).

299 Weighting the Young's modulus data was straightforward, with high precision
300 data ($\Delta E/E < 5\%$) being weighted most highly ($W_{\Delta E} = 8$). In comparison,
301 weighting the applied loads could only be done qualitatively. Loads 1, 2 and
302 8–11 had low weightings ($W_{\Delta Load} = 1$ or 2) as these had the highest D_L/D_C
303 values and so their data were most likely to suffer from container sidewall ef-
304 fects (see Figure 3). Loads 6 and 7 were also weighted poorly ($W_{\Delta Load} = 2$ and
305 4, respectively), as their relatively high thicknesses causing stability issues).
306 Load 5 exerted the lowest pressure and so inflicted only a small deflection to
307 the gelatine surface; this deflection became increasingly small (and so mea-
308 sured with higher uncertainty) as the gelatine's Young's modulus increased
309 during cooling. Therefore, Load 5 was weighted relatively low ($W_{\Delta Load} = 4$).

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310 Loads 3 and 4 were weighted most highly ($W_{\Delta Load} = 8$), deemed to have the
311 most favourable balance between causing a deflection of the gelatine surface
312 that could be measured to high precision, whilst experiencing minimal inter-
313 action with the container sidewalls.

314 The sum of the weights ($W_{\Delta E} + W_{\Delta Load}$) was used to give an overall weighting
315 for each datum. This procedure enabled the 'best' data to have the strongest
316 influence on the modelling results, whilst enabling all the data to be included
317 in the analysis process.

318 4 Results

319 4.1 *Young's Modulus of Gelatine*

320 By measuring the deflection caused by a load applied to the surface of the
321 solidified gelatine, we have been able to document the evolution of the gela-
322 tine's Young's modulus relative to a number of parameters. These will now be
323 considered separately.

324 4.1.1 *Effect of Time*

325 Figure 5 shows the Young's modulus evolution with time of a 4-litre 2.5 wt%
326 concentrated gelatine mixture kept at 5 °C (Experiment 25). The results show
327 that, over the range of experimental conditions reported here, the gelatine is
328 not able to support an applied load until it has a Young's modulus of approxi-
329 mately 1000 Pa. The Young's modulus then evolves exponentially with time to
330 reach a plateau maximum value after which, as long as the experimental con-
331 ditions are unchanged, the Young's modulus can be considered approximately

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5 332 constant with time. This exponential relationship between Young's modulus
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7 333 of the gelatine and time was documented for all the experiments:
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$$E = E_{\infty}(1 - e^{-\frac{t}{\tau}}), \quad (9)$$

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15 335 where E_{∞} (the Young's modulus plateau; Pa) and τ (hr) are both empirically
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17 336 based constants determined from the exponential fit, and t is time (hr). The
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19 337 values of E_{∞} and τ vary depending on V_{gel} , T_r and C_{gel} (see Table 5). As
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21 338 it is not feasible to wait for E_{∞} to be reached during the timescale of an
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23 339 experiment, we define $0.9E_{\infty}$ as an "effective" Young's modulus plateau and
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25 340 $t_{0.9E_{\infty}}$ as the time taken to reach within 10% of E_{∞} . These values are provided
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27 341 as a guide for the experimentalist in Table 5. The only effects of decreasing
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29 342 the room temperature from 10 °C to 5 °C were to increase the rate of Young's
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31 343 modulus increase with time and so decrease $t_{0.9E_{\infty}}$.
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35 344 The values of Young's modulus plateau reported on Table 5 were all measured
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37 345 with loads 3–8, so that in these experiments the height of gelatine was at
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39 346 least 2.7 times as large as the greatest load diameter. Therefore, according to
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41 347 equation (5), the stress at the base of the gelatine layer induced by the loads
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43 348 was less than 5% of their value, and the potential effect of the base of the tank
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45 349 on these values of Young's modulus plateau was neglected.
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50 350 It should be noted that both the use of deionised water and the storing of the
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52 351 gelatine mixtures in a cold room (set at 5–10 °C) led to the inhibition of bac-
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54 352 terial growth in the media. Thorough cleaning of the experimental container
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56 353 was also vital. Following these methods, our data shows that once the gelatine
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58 354 mixtures have reached their plateau in Young's modulus they can maintain
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60 355 this up to 140 hours after the initiation of the experiment.
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356 4.1.2 *Effect of Concentration*

357 For low concentrations (≥ 2 wt% and < 5 wt%), the Young's modulus plateau
358 (E_∞) of gelatine is linearly correlated with the concentration of the mixture
359 (with a Coefficient of Determination $R^2 = 0.9992$), as shown in Figure 6 for
360 equal V_{gel} and H_{gel} (Experiments 16, 25–28). Values of E_∞ were calculated ac-
361 cording to models fit to weighted Young's modulus data for a range of applied
362 loads (see Section 4.1.1). It is unclear whether or not this linear relationship
363 can be extrapolated to more highly concentrated gelatine mixtures.

364 Highly concentrated mixtures of gelatine (≥ 5 wt%) proved difficult to work
365 with, both in terms of preparing the experiments and then measuring their
366 Young's moduli during the gelification process. During preparation of the mix-
367 tures, difficulties were encountered dissolving such highly concentrated mix-
368 tures and also removing all bubbles from the highly viscous solution was un-
369 achievable so that creating a homogeneous solid was not possible. Once the
370 mixtures were in the 'gel-state' additional problems arose when attempting to
371 measure their Young's moduli. When the loads were applied to these very rigid
372 solids they were insufficient to cause a deflection of the gelatine surface that
373 could be measured precisely; even the heaviest applied loads (Loads 9–11, see
374 Table 2) caused such small deflections that the calculated Young's modulus
375 value would have very large errors.

376 Due to the problems associated with these experiments we present only average
377 Young's moduli for each experiment (Experiments 29–33); these were averaged
378 from measurements taken from the time when the gelatine was deemed to have
379 reached its Young's modulus plateau, an assumption verified by the lack of
380 correlation between Young's modulus and time (indicated by a low r ; see Table
381 6). The results suggest that more strongly concentrated gelatines have a higher

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6 383 of the data were so large we were unable to evaluate whether this relationship
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8 384 continues the linear trend identified in Figure 6.
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11 385 The experimental setup and method described here to measure the Young's
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13 386 modulus of gelatine solids proved unsuitable for highly concentrated mixtures.
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15 387 In order to quantify the Young's modulus of highly concentrated gelatine
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17 388 mixtures (≥ 5 wt% gelatine mixtures, where the Young's modulus $\gtrsim 20,000$
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19 389 Pa), equipment more often associated with measuring the strength of rocks
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22 390 would be required. These tests are however beyond the scope of this study.
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26 27 391 *4.1.3 Effect of Volume* 28

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30 392 Volume appears to have no impact on the Young's modulus plateau (E_∞)
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32 393 of the gelatine mixtures, as experiments that used the same concentration
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34 394 gelatine, stored at the same T_r , evolved to give the same value of E_∞ (± 500 Pa;
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36 395 Figure 7). The small discrepancy between modelled values of E_∞ is assumed to
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38 396 be related to errors associated with the properties of the applied load and the
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40 397 measuring technique, as described above (Section 3.3.1). There is a broadly
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42 398 positive correlation between the volume of gelatine and the time taken to reach
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44 399 the plateau in Young's modulus (modelled from the weighted data), i.e. larger
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46 400 volumes of gelatine take longer to reach their Young's modulus plateau.
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50 51 52 401 *4.1.4 Effect of Layer Thickness* 53

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55 402 The time to reach the Young's modulus plateau value appears to correlate
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57 403 well with the time needed for the gelatine to cool down to T_r , and so we
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59 404 can use this correlation to predict the time an experimentalist would have
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405 to wait until the gelatine Young's modulus has reached its plateau value.
406 The thermal diffusivity of gelatine is assumed to be that of its solvent, that
407 is water: $\kappa = 1.4 \cdot 10^6 \text{ m}^2 \text{ s}^{-1}$. The different containers used in the Young's
408 modulus experiments were made of PMP, PP or Perspex (PMMA), and the
409 thermal diffusivity for these thermoplastic polymers is about $10^{-7} \text{ m}^2 \text{ s}^{-1}$, one
410 order of magnitude lower than that of gelatine. Therefore, to a leading order,
411 a gelatine solid cools down by conducting its heat through its upper surface,
412 and the time t needed for thermal equilibrium is:

$$t = \frac{H_{gel}^2}{\kappa}, \quad (10)$$

414 where H_{gel} is the height of the gelatine solid in the container. Figure 8 com-
415 pares this cooling time with the time $t_{0.9E_\infty}$ taken to reach 90% of the Young's
416 modulus plateau E_∞ for gelatine mixtures of various concentrations (2 wt%
417 to 5 wt%), but all cured at the same temperature of 5 °C (Experiments 13–19
418 and 25–28, Table 1). We find reasonable agreement with a best linear fit:

$$t_{0.9E_\infty} \simeq (29.0 \pm 8.7) + (2.6 \pm 1.2) \frac{H_{gel}^2}{\kappa}. \quad (11)$$

420 Equation (11) gives experimentalists a first-order estimate of the time they
421 would need to wait for before a 2 wt% to 5 wt% gelatine solid cured at 5 °C
422 reaches its Young's modulus plateau.

423 4.2 The fracture toughness of solidified gelatine

424 The stress intensity factor K_I at the tip of a two-dimensional, edge crack of
425 height h can be expressed as:

$$K_I = \alpha \overline{\Delta P} \sqrt{\pi h}, \quad (12)$$

427 where α is a dimensionless factor that accounts for the conditions at the solid
 428 boundary (Sneddon and Das, 1971; Lawn, 1993; Menand and Tait, 2002), and
 429 $\overline{\Delta P}$ denotes the averaged excess pressure within the crack:

$$430 \quad \overline{\Delta P} = \frac{1}{h} \int_0^h \Delta P(z) dz, \quad (13)$$

431 where z is the vertical distance with origin at the reservoir-gelatine interface
 432 (Figure 2a). Determining the value of α is a mixed problem, which simplifies
 433 when the edge of the elastic solid is a free boundary (Sneddon and Das, 1971),
 434 as was the case in our experiments. We measured the value of the coefficient α
 435 using the method of Sneddon and Das (1971), summarized in the Appendix.

436 Griffith (1921) and Irwin (1957) showed that the fracture toughness K_c of
 437 an ideal elastic and brittle solid is related to its Young's modulus E by the
 438 following theoretical relationship:

$$439 \quad K_c = \sqrt{2\gamma_s E}, \quad (14)$$

440 where γ_s is the surface energy of the solid. This is the energy required to create
 441 a unit surface area within that solid, and is thought to depend only on the
 442 composition and temperature of the solid (Griffith, 1921).

443 The calculated values of gelatine fracture toughness are shown in Figure 9.
 444 Despite some scattering, we find that equation 14 fits reasonably well these
 445 values, and that our best fit is:

$$446 \quad K_c = (1.4 \pm 0.1) \sqrt{E}. \quad (15)$$

447 This equation and Figure 9 show that provided the viscous behaviour of gela-
 448 tine solids is negligible and deforms essentially elastically, gelatine solids be-

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449 have as ideal elastic and brittle solids in that their fracture toughness K_c
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450 and their Youngs modulus E follow the theoretical relationship (equation 14)
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451 expected for such solids.

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452 We find a best estimate for the gelatine surface energy:

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$$\gamma_s = 1.0 \pm 0.2 \text{ J m}^{-2}. \quad (16)$$

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454 Remarkably, this value is similar to the surface energy of brittle monocrystals
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455 such as diamond ($\gamma_s = 6 \text{ J m}^{-2}$), silicon ($\gamma_s = 1.2 \text{ J m}^{-2}$), silicon carbide
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456 ($\gamma_s = 4 \text{ J m}^{-2}$), silica ($\gamma_s = 1 \text{ J m}^{-2}$), sapphire ($\gamma_s = 4 \text{ J m}^{-2}$), magnesium
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457 oxide ($\gamma_s = 1.5 \text{ J m}^{-2}$), or lithium fluoride ($\gamma_s = 0.3 \text{ J m}^{-2}$) (Lawn, 1993).
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458 We note, however, that in principle γ_s should depend on the composition and
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459 temperature of the solid (Griffith, 1921), and so the exact value of γ_s may
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460 vary from one type of gelatine to the other. But given the rather small range
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461 of values for brittle monocrystals, which are also similar to that for gelatine,
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462 we believe $\gamma_s = 1.0 \pm 0.2 \text{ J m}^{-2}$ is a fair estimate for acid, pig-skin derived
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463 gelatine with Bloom values between 200 and 260. Experiments carried at lower
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464 temperatures than reported here will either result in higher Youngs moduli
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465 or take less time to reach their Youngs modulus plateau, but their fracture
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466 toughness will scale correspondingly following equation 15.

467 5 Geological Applications

468 Di Giuseppe et al. (2009) summarise the application of gelatine as an analogue
469 material for studying tectonic scale processes. They concluded low concentra-
470 tion gelatine mixtures ($\sim 2.5 \text{ wt}\%$) could be an appropriate analogue for upper
471 crustal deformation experiments. Complementary to this, we now present scal-

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5 472 ing laws appropriate for studying magmatic intrusion dynamics. The scaling
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7 473 for this case is distinct to that presented by Di Giuseppe et al. (2009) as the
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9 474 stress and strain relations are different, and in particular the strain rates for
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11 475 dyke propagation are many order of magnitude faster than those of tectonic
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13 476 processes.

14 15 16 17 477 *5.1 Scaling gelatine for experiments on dyke and sill propagation dynamics*

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22 478 An ideal scaled experiment has an analogue material that obeys geomet-
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26 480 1937); only then can observations and results of the experiment be used to un-
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28 481 derstand the behaviour of the natural system. Others workers have presented
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30 482 simple scalings for the use of gelatine in its elastic-state as a crustal analogue
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32 483 for studying the propagation dynamics of magma-filled fractures (Acocella
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34 484 and Tibaldi, 2005; Cañón-Tapia and Merle, 2006). We now expand on these
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36 485 to present a comprehensive guide for scaling gelatine for this type of geological
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38 486 experiment.

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42 487 Unlike tectonic processes, which occur on a length scale comparable with the
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44 488 thickness of the crust, dyke propagation is characterised by a much smaller
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46 489 length scale. This characteristic length scale is the buoyancy length L_b , as
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48 490 defined by Taisne and Tait (2009), which is the length over which magma
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50 491 buoyancy driving ascent balances resistance from rock fracture:

$$51
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53 492 \quad L = L_b = \left(\frac{K_c}{\Delta\rho g} \right)^{\frac{2}{3}}, \quad (17)$$

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58 493 where L_b is the length of the buoyant head region of the propagating dyke,
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60 494 K_c is the fracture toughness of the intruded medium and $\Delta\rho$ is the density

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5 495 difference between the intruding fluid and its surroundings. Dyke propagation
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7 496 is determined by a local buoyancy balance in the inflated head region of the
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9 497 dike, independent of the total buoyancy of the magma column between source
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11 498 and tip (Lister and Kerr, 1991; Taisne and Tait, 2009). In this case the reduced
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13 499 gravity (g') is the relevant parameter for scaling the dyke driving force:

$$14 \quad 15 \quad 16 \quad 17 \quad 18 \quad 19 \quad 20 \quad 21 \quad 22 \quad 23 \quad 24 \quad 25 \quad 26 \quad 27 \quad 28 \quad 29 \quad 30 \quad 31 \quad 32 \quad 33 \quad 34 \quad 35 \quad 36 \quad 37 \quad 38 \quad 39 \quad 40 \quad 41 \quad 42 \quad 43 \quad 44 \quad 45 \quad 46 \quad 47 \quad 48 \quad 49 \quad 50 \quad 51 \quad 52 \quad 53 \quad 54 \quad 55 \quad 56 \quad 57 \quad 58 \quad 59 \quad 60 \quad 61 \quad 62 \quad 63 \quad 64 \quad 65$$

$$500 \quad g' = \frac{\Delta\rho}{\rho_{solid}}g. \quad (18)$$

501 The timescale for the experiments is obtained by combining L_b (equation 17)
502 and g' (equation 18):

$$503 \quad T = \sqrt{\frac{L_b}{g'}} = \rho_{solid}^{\frac{1}{2}} K_c^{\frac{1}{3}} (\Delta\rho g)^{-\frac{5}{6}}, \quad (19)$$

504 and from this the dyke velocity scale follows easily:

$$505 \quad U = \frac{L_b}{T} = (\Delta\rho g)^{\frac{1}{6}} K_c^{\frac{1}{3}} \rho_{solid}^{-\frac{1}{2}}. \quad (20)$$

506 This approach provides the appropriate scales (length, time and velocity) for
507 each experiment, as one varies one parameter or another, and so provides the
508 appropriate scaling factors $L^* = \frac{L_l}{L_n}$, $T^* = \frac{T_l}{T_n}$ and $U^* = \frac{U_l}{U_n}$:

$$509 \quad L^* = \left(\frac{K_c^*}{\Delta\rho^*} \right)^{\frac{2}{3}}, \quad (21)$$

$$510 \quad T^* = \rho_{solid}^{*\frac{1}{2}} K_c^{*\frac{1}{3}} (\Delta\rho^*)^{-\frac{5}{6}}, \quad (22)$$

$$511 \quad U^* = (\Delta\rho^*)^{\frac{1}{6}} K_c^{*\frac{1}{3}} \rho_{solid}^{*-\frac{1}{2}}, \quad (23)$$

512 where * refers to the ratio of the parameter values measured at the laboratory
513 (subscript l) and natural (subscript n) scale.

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514 Finally, the driving buoyancy pressure (P_b) scale for dykes is:

$$515 \quad P_b = \Delta\rho g L_b, \quad (24)$$

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12 516 which leads to deformation of the host medium around the head of the dyke.

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14 517 The elastic pressure scale (P_e) associated with this deformation is:

$$15 \quad P_e = \frac{E}{2(1-\nu^2)} \frac{\psi}{L_b}, \quad (25)$$

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21 519 where E and ν are the Young's modulus and Poisson's ratio of the elastic host,
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23 520 respectively, and ψ is the thickness (i.e. the opening) of the dyke head. These
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25 521 two stress scales balance each other during dyke propagation (e.g. Lister and
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27 522 Kerr (1991)), which gives:

$$28 \quad E = 2(1-\nu^2)\Delta\rho g \frac{L_b^2}{\psi}. \quad (26)$$

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34 524 The Poisson's ratio for gelatine solids is $\nu \simeq 0.5$, whereas that of rocks lies
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36 525 usually between 0.25 and 0.3. As a result, the factor $2(1-\nu^2)$ varies by 15–20%
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38 526 between nature and laboratory experiments, and the Young's modulus scale
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$$42 \quad E^* = \Delta\rho^* L_b^* \left(\frac{L_b}{\psi} \right)^*. \quad (27)$$

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48 529 Strictly speaking, field measurements made on the geometry of fossilised dykes
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52 531 on the geometry of propagating dykes. The discrepancy between the propa-
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54 532 gating and the final static geometry will certainly be important for those
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56 533 dykes that reached the surface because their thickness will decrease as magma
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58 534 erupts at the surface and elastic deformation of surrounding rocks is released.
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61 535 However, because of mass balance the discrepancy should be marginal for
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the majority of dykes, which stall in the crust and do not reach the surface; notwithstanding potential volume change due to solidification, the volume of a propagating dyke should be the same as the volume of a static dyke. This caveat aside, we can use the geometrical measurements made on solidified dykes as proxies for their geometry during propagation.

The aspect ratio $\frac{\psi}{L_b}$ of solidified dykes in nature is typically of the order of $10^{-4} - 10^{-3}$ (e.g. Gudmundsson (2011); Kavanagh and Sparks (2011)), and on the order of $10^{-2} - 10^{-1}$ in gelatine experiments. Taking the following values as representative for natural dykes: $K_c = 10^7 \text{ Pa m}^{\frac{1}{2}}$, $\Delta\rho = 100 \text{ kg m}^{-3}$, $\rho_{solid} = 2800 \text{ kg m}^{-3}$, and for experimental conditions: $K_c = 100 \text{ Pa m}^{\frac{1}{2}}$, $\Delta\rho = 1000 \text{ kg m}^{-3}$ (air) or $\Delta\rho = 10 \text{ kg m}^{-3}$ (water), $\rho_{solid} = 1000 \text{ kg m}^{-3}$, one gets:

$$L^* = 10^{-4}(\text{air}) \quad \text{or} \quad L^* = 2 \times 10^{-3}(\text{water}), \quad (28)$$

$$T^* = 2 \times 10^{-3}(\text{air}) \quad \text{or} \quad T^* = 9 \times 10^{-2}(\text{water}), \quad (29)$$

$$U^* = 5 \times 10^{-2}(\text{air}) \quad \text{or} \quad U^* = 2 \times 10^{-2}(\text{water}), \quad (30)$$

$$E^* = 10^{-6} - 10^{-5}(\text{air}) \quad \text{or} \quad E^* = 2 \times 10^{-6} - 2 \times 10^{-5}(\text{water}). \quad (31)$$

In the experiments, $L_l \simeq 5 \text{ cm}$ with air or $\simeq 1 \text{ m}$ with water; this corresponds in nature to $L_n \simeq 500 \text{ m}$, which seems reasonable. Likewise, a velocity of a couple of mm/s (water) or cm/s (air) in the experiments would give dyke velocities on the order of 0.1–0.5 m/s in nature, in good agreement with estimates of dykes velocities (White et al., 2011). As for elastic deformation, the Young's modulus of rocks typically lies in the range $E_n = 10^9 - 10^{10} \text{ Pa}$, and so properly scaled experiments should involve gelatine solids with Young's modulus in the range $E_l = 10^3 - 10^5 \text{ Pa}$ when air is used as a magma analogue, or

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560 $E_l = 2 \times 10^3 - 2 \times 10^5$ Pa when water is used instead. Both ranges include
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560 $E_l = 2 \times 10^3 - 2 \times 10^5$ Pa when water is used instead. Both ranges include
561 values that have typically been used in dyke and sill experiments, and the
562 data presented in this paper shows that 2–5 wt% of gelatine is sufficient to
563 reach this range of Young’s modulus plateau (Figure 6).

564 These calculations suggest gelatine experiments for magmatic intrusion prop-
565 agation (dykes or sills) carried out at $\sim 5-10$ °C and with gelatine concen-
566 trations of 2–5 wt% are adequately scaled geometrically, kinematically and
567 dynamically.

568 6 Conclusions

569 We present results from a series of experiments that quantify the evolution of
570 the elastic properties of gelatine with time. At 5–10 °C gelatine is in the ‘gel-
571 state’, over the range of stresses and strain rates presented here, and behaves
572 like a solid, with almost ideal-elastic deformation. The Young’s modulus of
573 gelatine evolves with time, modelled best by an exponential relationship, with
574 E evolving to a plateau value that would theoretically be achieved after an
575 infinite amount of time. At low gelatine concentrations (< 5 wt%) the plateau
576 Young’s modulus depends linearly on the concentration of gelatine, and differ-
577 ent volumes of equally concentrated gelatine evolve to the same plateau value.
578 The method we use to measure the Young’s modulus of the gelatine requires
579 that the diameter of the load is $\lesssim 10\%$ the diameter of the experimental con-
580 tainer and thickness of the gelatine solid in order for side-wall and base effects
581 to be avoided; larger dimensions relative to the gelatine solid will affect and
582 lead to artificially high calculated values. Fracture toughness measurements
583 show the K_c of gelatine follows the same relationship as ideal elastic-brittle

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584 solids: it is proportional to the square-root of the Young's modulus multiplied
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585 by twice its surface energy, which was calculated experimentally as 1.0 ± 0.2
586 J m^{-2} .

587 The transparent nature and photoelastic properties of gelatine mean deforma-
588 tions can be easily visualised and monitored, giving the experimental geologist
589 insight into the propagation dynamics of magmatic intrusions. However, cau-
590 tion needs to be taken when using gelatine as an analogue for the Earth's
591 elastic crust. These type of experiments are best carried out at 5–10 °C in
592 order for the viscous component of gelatine's deformation behaviour to be
593 negligible. At these temperatures gelatine is a good analogue for magmatic in-
594 trusion propagation in Earth's elastic crust; using gelatine concentrations from
595 2–5 wt% will ensure gelatine is adequately scaled geometrically, kinematically
596 and dynamically.

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610 **8 Appendix**

611 In the fracture toughness experiments, the pressure distribution (ΔP) within
612 the crack just prior to its propagation was:

$$613 \quad \Delta P(z) = (\rho_g - \rho_w)gz, \quad 0 \leq z \leq z_l, \quad (32)$$

$$614 \quad \Delta P(z) = \rho_ggz - \rho_wgz_l, \quad z_l \leq z \leq h, \quad (33)$$

615 where z_l is the level of the air-water interface within the crack ($z_l = 0$ when
616 the crack is full of air), ρ_g and ρ_w are the density of the solid gelatine and
617 water, respectively. The density of air ρ_a is assumed to be negligible. Following
618 Sneddon and Das (1971), by expressing this crack excess pressure as $\Delta P(z) =$
619 $\overline{\Delta P}f(z)$, the value of α in equation (12) is then determined by calculating the
620 value $\Lambda(1)$, where Λ is the solution of the following integral:

$$621 \quad \Lambda(z) - \int_0^1 \Lambda(u)L(z, u) du = \frac{2}{\pi} \int_0^z \frac{f(s) ds}{\sqrt{z^2 - s^2}}, \quad 0 \leq z \leq 1, \quad (34)$$

622 where z has been normalised with respect to the crack height h , u and s are
623 integration variables, and:

$$624 \quad L(z, u) = \frac{16zu}{\pi^2} \left[\frac{z^2 + u^2}{(z^2 - u^2)^3} \ln \left(\frac{z}{u} \right) - \frac{1}{(z^2 - u^2)^2} \right], \quad \text{if } z \neq u, \quad (35)$$

625 and:

$$626 \quad L(z, u) = \frac{4}{3\pi^2u}, \quad \text{if } z = u. \quad (36)$$

Equation (34) was solved using the gaussian quadrature method. This leads to n linear equations:

$$\Lambda(x_i) - \sum_{j=1}^n w_j L(x_i, x_j) \Lambda(x_j) = \frac{2}{\pi} \int_0^{x_i} \frac{f(s) ds}{\sqrt{x_i^2 - s^2}}, \quad (i = 1, 2, \dots, n), \quad (37)$$

to be solved in order to determine the values $\Lambda(x_1), \Lambda(x_2), \dots, \Lambda(x_n)$, using the values x_1, x_2, \dots, x_n and their respective weights w_1, w_2, \dots, w_n (as listed in Table 52.8 from Abramowitz and Stegun (1964)). The value of α is then:

$$\alpha = \Lambda(1) = \frac{2}{\pi} \int_0^1 \frac{f(s) ds}{\sqrt{1^2 - s^2}} + \sum_{j=1}^n w_j L(1, x_j) \Lambda(x_j). \quad (38)$$

The gelatine fracture toughness K_c was then equated with the stress intensity factor (12), using the average excess pressure $\overline{\Delta P}$ measured just prior to the crack propagation and the corresponding value of α (equation 38).

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| Exp. | C_{gel} | M_{gel} | V_{gel} | H_{gel} | D_C | T_r (°C) | T_0 (°C) |
|------|-----------|-----------|-----------|-----------|-------|------------|------------|
| 1 | 2.5 | 4 | 4 | 17.0 | 17.3 | 10 | 34.5 |
| 2 | 2.5 | 3 | 3 | 12.7 | 17.3 | 10 | 35.0 |
| 3 | 2.5 | 2 | 2 | 8.5 | 17.3 | 10 | 35.5 |
| 6 | 2.5 | 20 | 20 | 12.5 | 40.0* | 10 | 36.0 |
| 7 | 2.5 | 30 | 30 | 18.8 | 40.0* | 10 | 38.0 |
| 8 | 2.5 | 0.5 | 0.5 | 4.1 | 12.5 | 10 | 35.5 |
| 9 | 2.5 | 0.5 | 0.5 | 6.4 | 10.0 | 10 | 34.5 |
| 10 | 2.5 | 0.5 | 0.5 | 8.7 | 8.6 | 10 | 34.5 |
| 11 | 2.5 | 20 | 20 | 12.5 | 40.0* | 5 | 34.0 |
| 12 | 2.5 | 30 | 30 | 18.8 | 40.0* | 5 | 34.5 |
| 13 | 2 | 2 | 2 | 16.4 | 12.5 | 5 | 37.5 |
| 14 | 2 | 1 | 1 | 8.2 | 12.5 | 5 | 37.5 |
| 15 | 2 | 3 | 3 | 12.7 | 17.3 | 5 | 38.0 |
| 16 | 2 | 4 | 4 | 17.0 | 17.3 | 5 | 38.0 |
| 17 | 2 | 10 | 10 | 19.6 | 25.5 | 5 | 37.0 |
| 18 | 2 | 20 | 20 | 27.0 | 30.7 | 5 | 38.5 |
| 19 | 2 | 30 | 30 | 20.0 | 30.2* | 5 | 44.5 |
| 25 | 2.5 | 4 | 4 | 17.0 | 17.3 | 5 | 40.5 |
| 26 | 3 | 4 | 4 | 17.0 | 17.3 | 5 | 39.5 |
| 27 | 3.5 | 4 | 4 | 17.0 | 17.3 | 5 | 39.0 |
| 28 | 4 | 4 | 4 | 17.0 | 17.3 | 5 | 38.0 |
| 29 | 5 | 4 | 4 | 17.0 | 17.3 | 5 | 35.0 |
| 30 | 5 | 4 | 4 | 17.0 | 17.3 | 5 | 64.0 |
| 31 | 10 | 4 | 4 | 17.0 | 17.3 | 5 | 60.0 |
| 32 | 20 | 4 | 4 | 17.0 | 17.3 | 5 | 65.0 |
| 33 | 30 | 4 | 4 | 17.0 | 17.3 | 5 | 56.0 |

Table 1: Table of experimental conditions. C_{gel} = gelatine concentration (wt%), M_{gel} = mass of tested gelatine plus water mixture (kg), V_{gel} = volume of tested gelatine plus water mixture (litres), H_{gel} = thickness of gelatine mixture (± 0.5 cm), D_C = container diameter (± 0.1 cm), T_r = cold room temperature, T_0 = starting temperature of gelatine mixture (± 0.5 °C). Experimental containers were circular in cross-section, except those indicated by * which were square (measuring 40 cm x 40 cm) and † which were oblong (measuring 50 cm x 30 cm). H_{gel} was calculated retrospectively from the container surface area and tested volume.

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| | Material | β (mm) | M_L (g) | D_L (mm) |
|--------------------|-----------|--------------|-----------|------------|
| Load ₁ | Aluminium | 27.9 | 393.8 | 81.6 |
| Load ₂ | Aluminium | 18.0 | 255.0 | 81.6 |
| Load ₃ | Brass | 12.2 | 50.6 | 25.1 |
| Load ₄ | Brass | 9.2 | 37.9 | 25.1 |
| Load ₅ | Brass | 6.2 | 25.5 | 25.0 |
| Load ₆ | Brass | 11.3 | 35.9 | 22.6 |
| Load ₇ | Brass | 14.3 | 37.8 | 20.0 |
| Load ₈ | Brass | 8.9 | 48.5 | 30.0 |
| Load ₉ | Steel | 23.9 | 130.2 | 30.0 |
| Load ₁₀ | Steel | 92.8 | 2279.3 | 63.5 |
| Load ₁₁ | Steel | 62.8 | 2808.5 | 85.6 |

Table 2: Properties of the experimental loads: β = thickness (± 0.1 mm), M_L = mass of load (± 0.1 g), D_L = diameter of load (± 0.1 mm). In all cases the data are mean averages of three measurements. Loads are cylindrical.

| Exp. | ρ_g (kg m ⁻³) | ρ_w (kg m ⁻³) | E (Pa) | z_l (cm) | $\overline{\Delta P}$ (Pa) | K_c (Pa m ^{1/2}) |
|------------------|-----------------------------------|-----------------------------------|-------------|---------------|-------------------------------|---------------------------------|
| 123 | 1062.0 | 1000.0 | 1449 ± 14 | 2.25 | 89 ± 27 | 59 ± 18 |
| 124 | 1072.3 | 1000.0 | 3969 ± 100 | 1.25 | 156 ± 26 | 93 ± 15 |
| 125 | 1079.3 | 1000.0 | 7603 ± 125 | 1.00 | 176 ± 26 | 103 ± 15 |
| 126 | 1063.3 | 1000.0 | 1877 ± 36 | 3.10 | 54 ± 29 | 40 ± 21 |
| 127 | 1072.7 | 1001.0 | 3906 ± 161 | 3.15 | 65 ± 29 | 48 ± 21 |
| 128 | 1079.3 | 1000.6 | 7328 ± 116 | 0.00 | 270 ± 25 | 148 ± 14 |
| 129 [‡] | 1025.5 | 1000.3 | 10959 ± 354 | 4.00 | 189 ± 27 | 175 ± 25 |
| 131 | 1015.6 | 999.4 | 2254 ± 57 | 2.20 | 85 ± 27 | 57 ± 18 |

Table 3: The values of gelatine fracture toughness K_c determined from eight successful experiments. z_l is the level of the air-water interface within the crack just prior to its propagation; $z_l = 0$ when the crack is full of air. $\overline{\Delta P}$ is the corresponding averaged excess pressure. The gelatine and water densities, ρ_g and ρ_w , were both measured to within 4 kg m⁻³ and 1 kg m⁻³, respectively, and the air level z_l to within 2.5 mm.

‡ The crack was initially 5.0 ± 0.2 cm high in all experiment, except in experiment 129 where it was 10 cm high.

| Quantitative W | $\Delta E/E$ (%) | $W_{\Delta E}$ |
|------------------|---------------------|-------------------|
| | <5 | 8 |
| | 5–10 | 7 |
| | 10–15 | 6 |
| | 15–20 | 5 |
| | 20–30 | 4 |
| | 30–50 | 3 |
| | 50–100 | 2 |
| | >100 | 1 |
| Qualitative W | <i>Applied Load</i> | $W_{\Delta Load}$ |
| | Load ₁ | 1 |
| | Load ₂ | 1 |
| | Load ₃ | 8 |
| | Load ₄ | 8 |
| | Load ₅ | 4 |
| | Load ₆ | 4 |
| | Load ₇ | 2 |
| | Load ₈ | 2 |
| | Load ₉ | 1 |
| | Load ₁₀ | 1 |
| | Load ₁₁ | 1 |

Table 4: Weightings (W) used to quantify the quality of Young's modulus measurement data. Quantitative-based weightings consider the uncertainty in $\Delta E/E$, whereas the effect of the load used to take the measurements could only be weighted qualitatively based on the results from Figure 2. The combined weightings ($W_{\Delta E} + W_{\Delta Load}$) are then used in the subsequent data analysis.

| Experiment | C_{gel} | E_{∞} (Pa) | τ (hr) | $0.9E_{\infty}$ (Pa) | $t_{0.9E_{\infty}}$ (hr) |
|------------|-----------|-------------------|-------------|----------------------|--------------------------|
| 13 | 2 | 4431±44 | 14 | 3988 | 34 |
| 14 | 2 | 4475 ±58 | 12 | 4028 | 28 |
| 15 | 2 | 4317 ±82 | 15 | 3885 | 35 |
| 16 | 2 | 4172±54 | 17 | 3755 | 38 |
| 17 | 2 | 3972±36 | 30 | 3575 | 70 |
| 18 | 2 | 3628±49 | 22 | 3265 | 52 |
| 19 | 2 | 4106±109 | 33 | 3695 | 75 |
| 25 | 2.5 | 7003±233 | 19 | 6303 | 44 |
| 26 | 3 | 10165±284 | 19 | 9149 | 44 |
| 27 | 3.5 | 12775±548 | 16 | 11498 | 37 |
| 28 | 4 | 15973±441 | 16 | 14376 | 39 |

Table 5: Model results showing E_{∞} and τ values (correct to the nearest hour) for an exponential best-fit model $E = E_{\infty}(1 - e^{-\frac{t}{\tau}})$ of calculated gelatine Young's moduli against time for a select group of experiments with the same T_r (5 °C). As E_{∞} can not be reached within the timescale of an experiment, we define $0.9E_{\infty}$ as an "effective" Young's modulus plateau. $t_{0.9E_{\infty}}$ is the time taken (correct to the nearest hour) to reach within 10% of E_{∞} . See Table 1 for experiment settings.

| Experiment | 29 | 30 | 31 | 32 | 33 |
|------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| wt% | 5 | 5 | 10 | 20 | 30 |
| \bar{E} | 2.9×10^4 | 3.6×10^4 | 1.5×10^5 | 7.1×10^5 | 4.5×10^5 |
| St. Dev. | 1.4×10^4 | 2.0×10^4 | 1.6×10^4 | 1.4×10^6 | 5.6×10^5 |
| r | 0.40 | 0.32 | -0.11 | 0.10 | -0.55 |
| n | 36 | 9 | 12 | 13 | 13 |

Table 6: Average Young's modulus of highly concentrated (≥ 5 wt%) gelatine mixtures. An average of ' n ' measurements of the Young's modulus is shown (\bar{E} , correct to 2 s.f.), measurements were taken periodically using a range of applied loads for several hours after 16.5 hours curing at 5 °C. Calculated standard deviations (St. Dev.) indicate a high degree of uncertainty. The low Pierson Product-Moment Correlation Coefficients (r) suggests no correlation between the Young's modulus measurements and time, supporting the assumption that the measurements were all made when the Young's modulus had plateaued.

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6 Figure 1. Schematic illustration of the Young's modulus measurement procedure on
7 a gelatine solid able to support an applied load. The deflection caused by a load
8 placed on the surface of the solidified gelatine is measured, and this information is
9 combined with the properties of the load to calculate the Young's modulus of the
10 material.
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14 Figure 2. Three successive photographs taken during a fracture-toughness experi-
15 ment. (a) An edge-crack is initially created at the base of a gelatine solid, and filled
16 with water. The initial reservoir pressure matches exactly the weight of the overly-
17 ing gelatine solid. (b) Air is injected through a capillary and within the crack. Any
18 potential reservoir excess pressure is released, so that only the crack buoyancy in-
19 creases during air injection (see text). (c) When the crack buoyancy is high enough,
20 the air-filled crack fractures the gelatine and propagates vertically.
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26 Figure 3. Young's modulus (E) of 2.5 wt% gelatine solids, after approximately 22
27 hours curing at 10 °C, plotted against the diameter of the applied load relative to the
28 diameter of the container (D_L/D_C) for loads 1-8 (Table 2) and five container sizes
29 (Table 1). In the region $D_L/D_C > 10\%$ (unshaded) each experiment individually
30 shows a positive correlation between E and D_L/D_C (see legend for Pearson pro-
31 duct-moment correlation coefficients), indicating interaction between the load and
32 container could produce artificially high calculated Young's moduli. Where D_L/D_C
33 $\lesssim 10\%$ (shaded) there appears to be no correlation between E and D_L/D_C , and
34 here sidewall affects can be neglected. When no error bars can be seen, the error is
35 smaller than the symbol size.
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41 Figure 4. Young's modulus evolution with time of 4-litres of 3 wt% gelatine stored
42 at 5 °C (Experiment 26). The Young's modulus was calculated from the deflec-
43 tion caused to the gelatine surface by Load 3 (see Table 2). Error bars show the
44 uncertainty in E increases with time.
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49 Figure 5. Young's modulus evolution with time of 4-litres of 2.5 wt% gelatine stored
50 at 5 °C (Experiment 25). The Young's modulus was calculated from the deflection
51 caused to the gelatine surface by a range of applied loads (loads 3-8; see Table 2).
52 An exponential relationship best fits the data ($E = E_\infty(1 - e^{-\frac{t}{\tau}})$), where $E_\infty = 7003$
53 Pa and $\tau = 19$ hr). E increases with time to an "effective plateau" ($0.9E_\infty$) of 6300
54 Pa after 44 hours curing ($t_{0.9E_\infty}$). The best-fit model (solid line) takes into account
55 all measurements weighted according to $\Delta E/E$ and the load used (see Table 4). The
56 outliers at ~ 55 hours and ~ 98 hours are from Load 7; these data have low weighting
57 on the fitted trend due to this load having high thickness and small diameter that
58 caused stability issues.
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Figure 6. Modelled plateau Young's modulus (E_∞) of a range of gelatine concentrations C_{gel} (Experiments 16, 25–28). Each test volume was 4-litres and was kept at 5 °C (T_r) in an equivalent container. The best-fit model indicates there is a positive-linear correlation ($R^2 = 0.9992$) between E_∞ and C_{gel} . More concentrated gelatine mixtures reach a higher Young's modulus plateau.

Figure 7. Relationship between the modelled Young's modulus plateau ($E_\infty \pm \Delta E_\infty$) and gelatine mixture volume V_{gel} for 2 wt% gelatine mixtures cured at 5 °C (Experiments 13–19). The mean E_∞ (dashed line) is shown and is most closely modelled by the 4-litre experiment (Experiment 16). The Pearson product-moment correlation coefficient ($r = 0.64$) indicates there is little or no correlation between E_∞ and V_{gel} . Gelatine mixtures of the same concentration (C_{gel}) evolve to the same $E_\infty \pm 500$ Pa independent of volume.

Figure 8. Comparison of the time $t_{0.9E_\infty}$ needed to reach 90% of the Young's modulus plateau E_∞ , with the conductive cooling time H_{gel}^2/κ given by equation 10. Data points correspond to experiments 13 to 19 and 25 to 28 (Table 1). These experiments had gelatine concentrations between 2wt% and 5wt%, and were all cured at 5 °C. The plateau time appears to correlate linearly with the cooling time: the curve is the best linear fit, $t_{0.9E_\infty} \simeq (29.0 \pm 8.7) + (2.6 \pm 1.2) \frac{H_{gel}^2}{\kappa}$ (equation 11, $R^2 = 0.3211$); 95% confidence limits are indicated by dashed lines.

Figure 9. The fracture toughness K_c of gelatine solids as a function of their Young's modulus E . The curve is the best fit through the data: $K_c = (1.4 \pm 0.1) \sqrt{E}$ (equation 18, $R^2 = 0.8196$), with the 95% confidence limits (dashed lines).

Figure 1

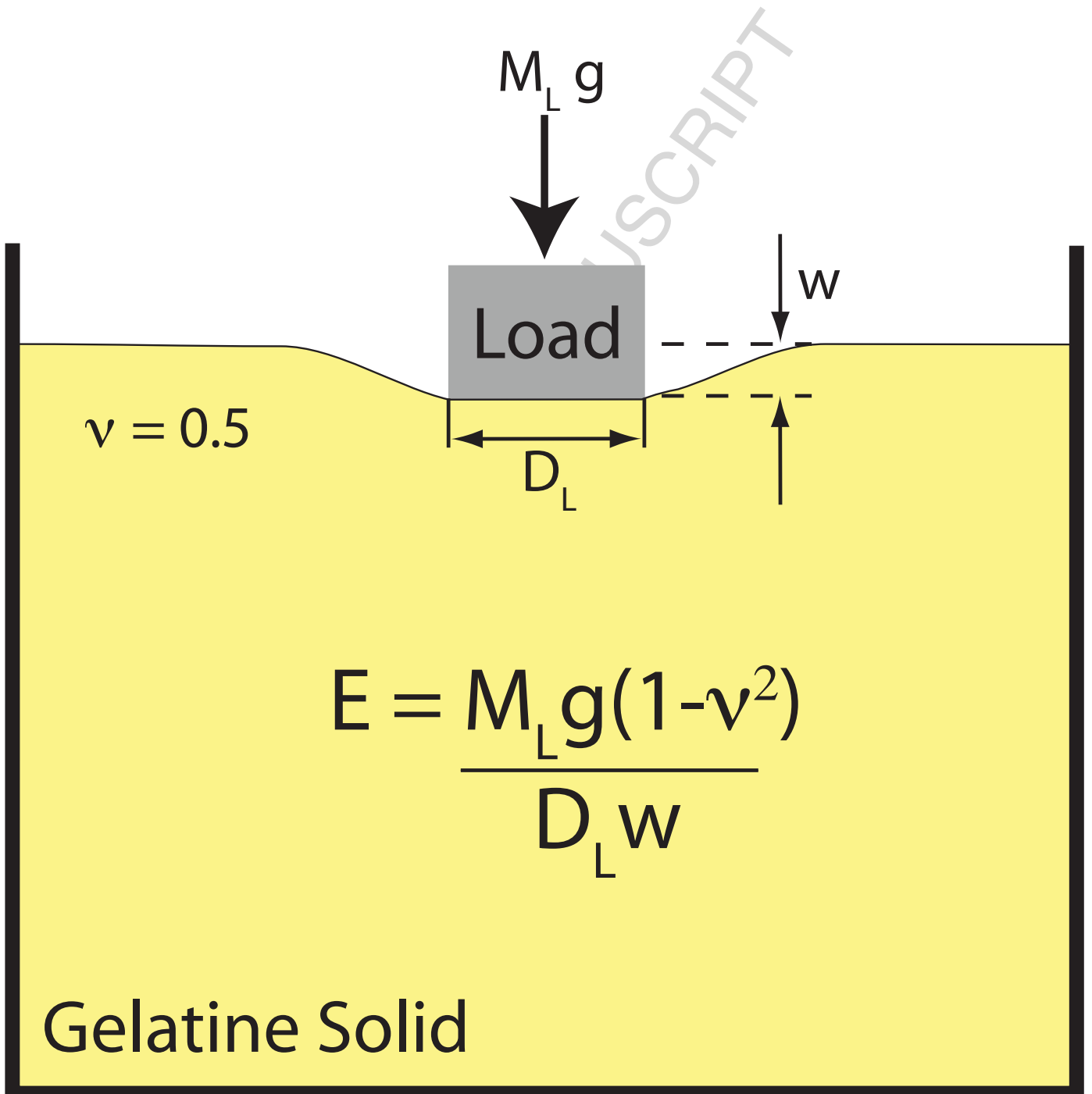


Figure 2

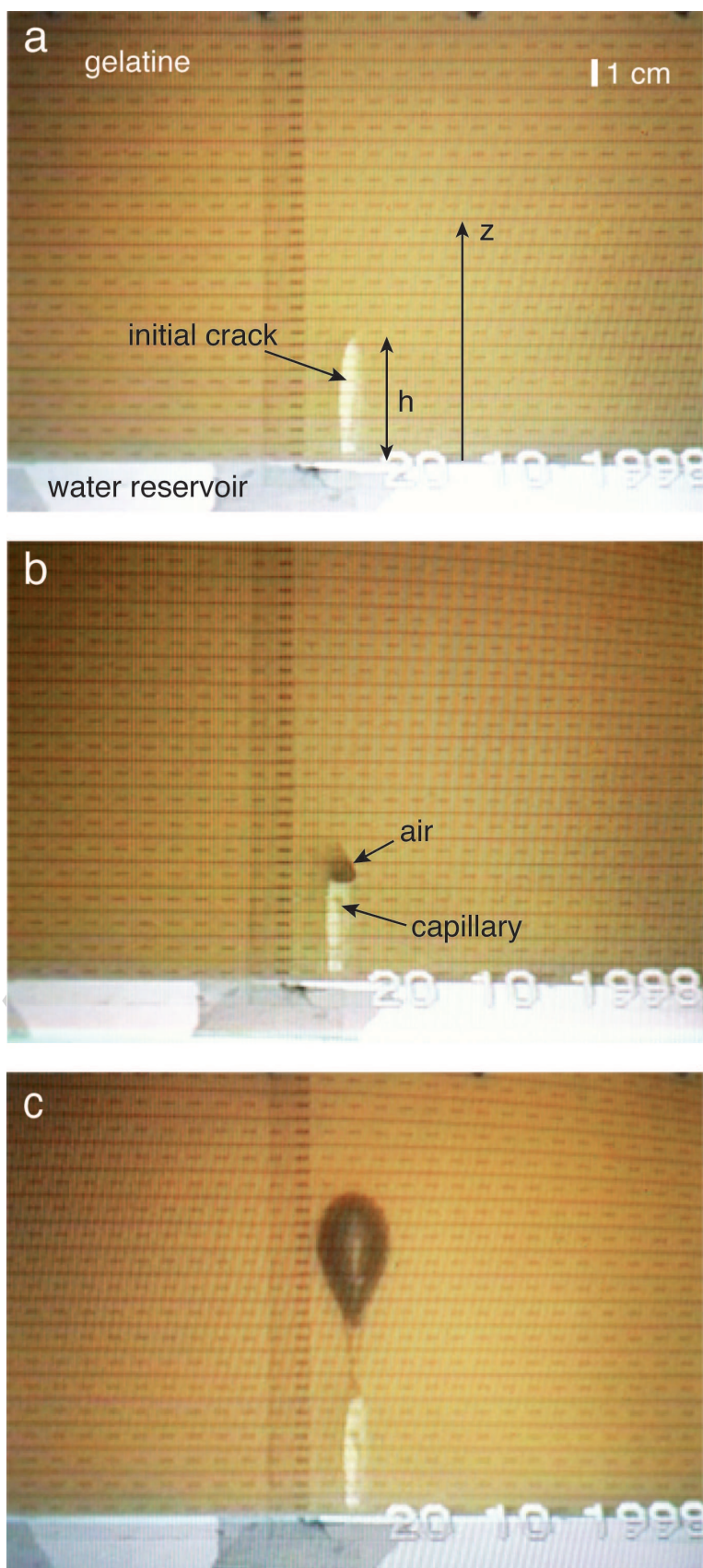


Figure 3

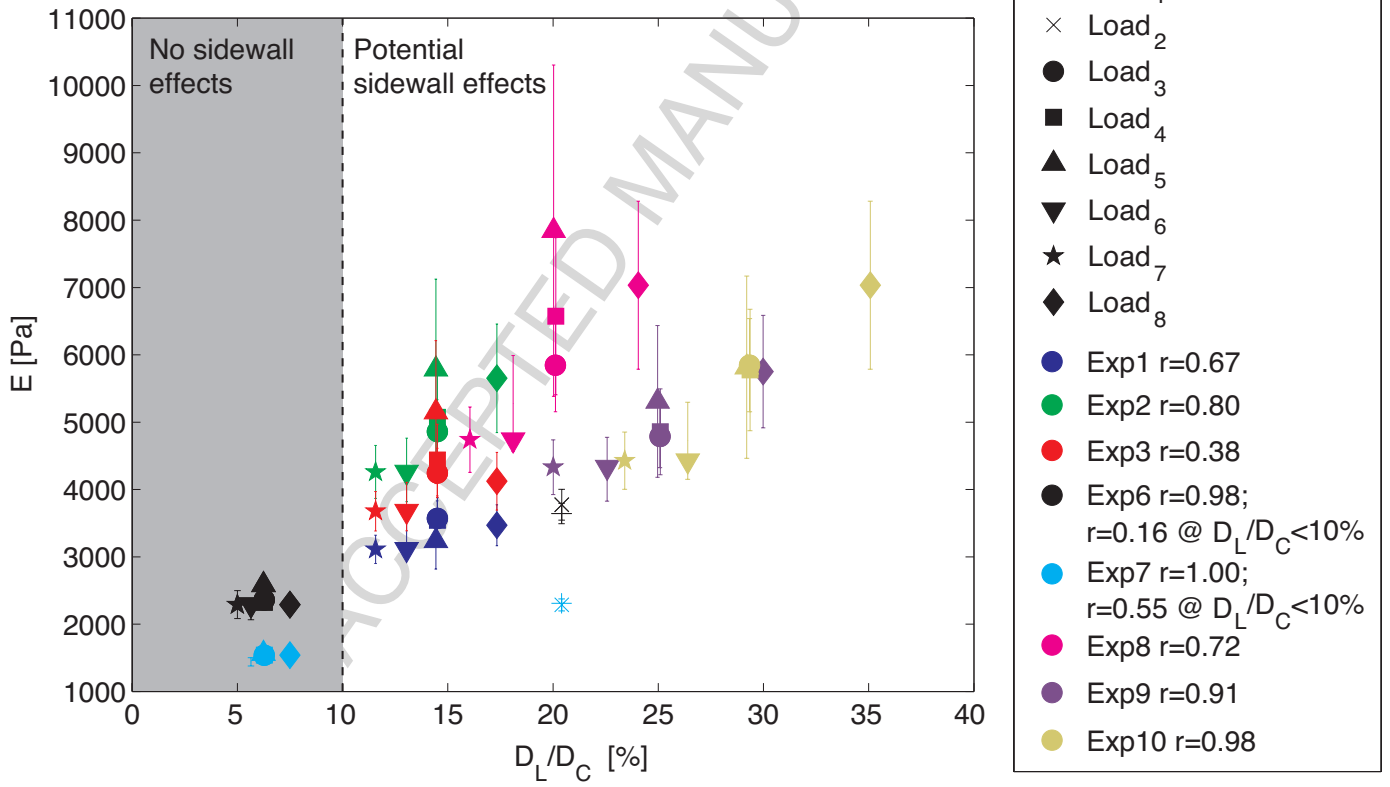


Figure 4

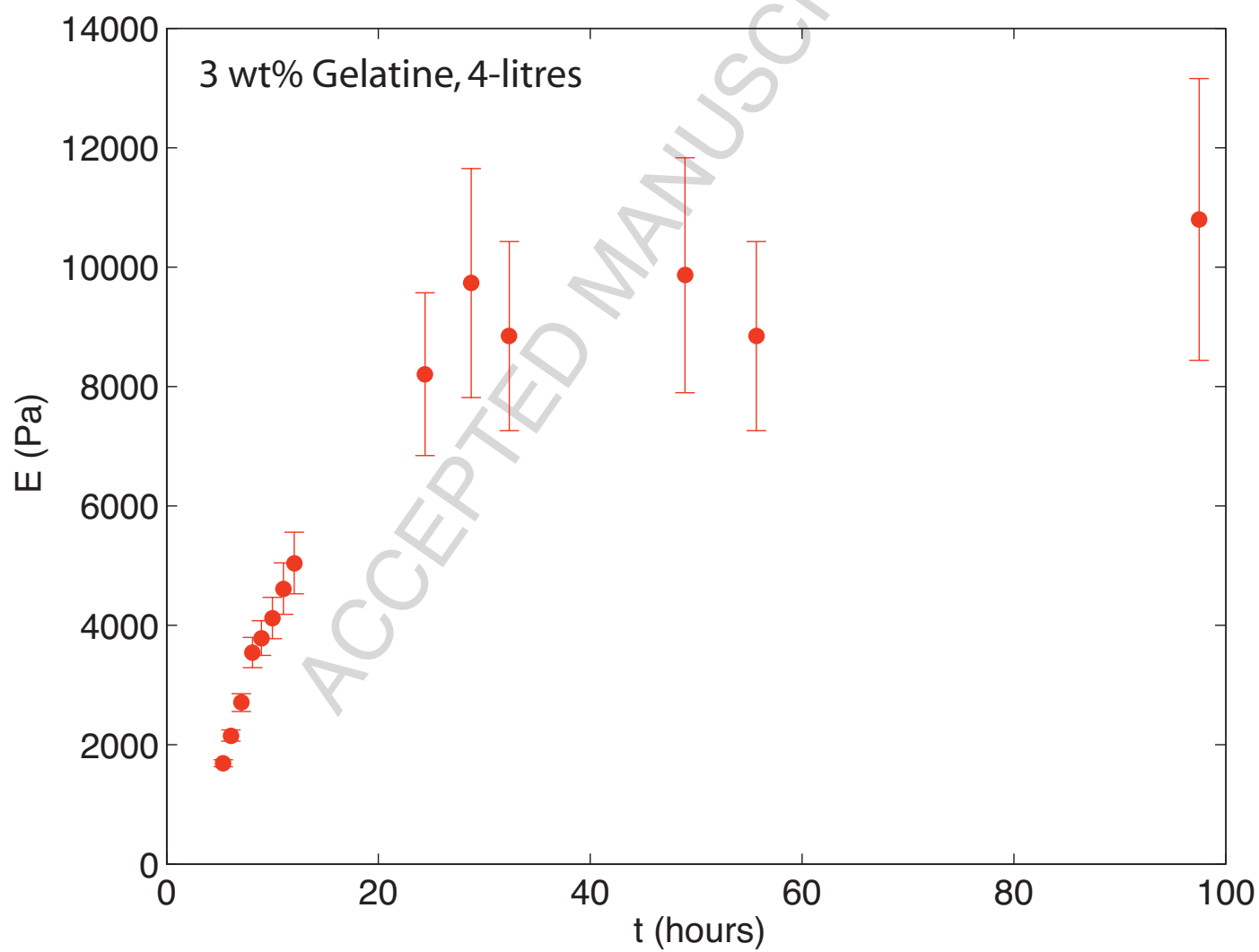


Figure 5

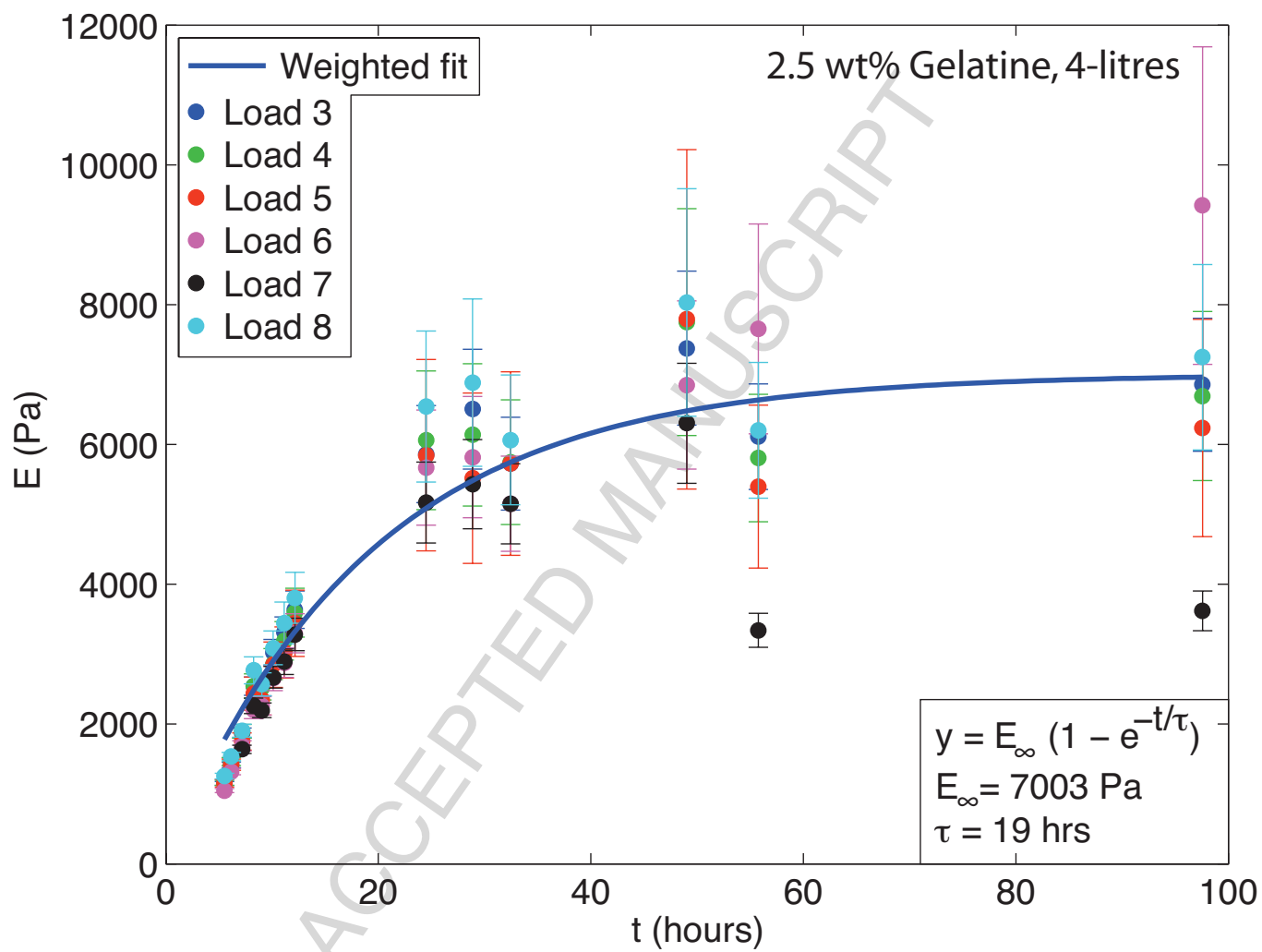


Figure 6

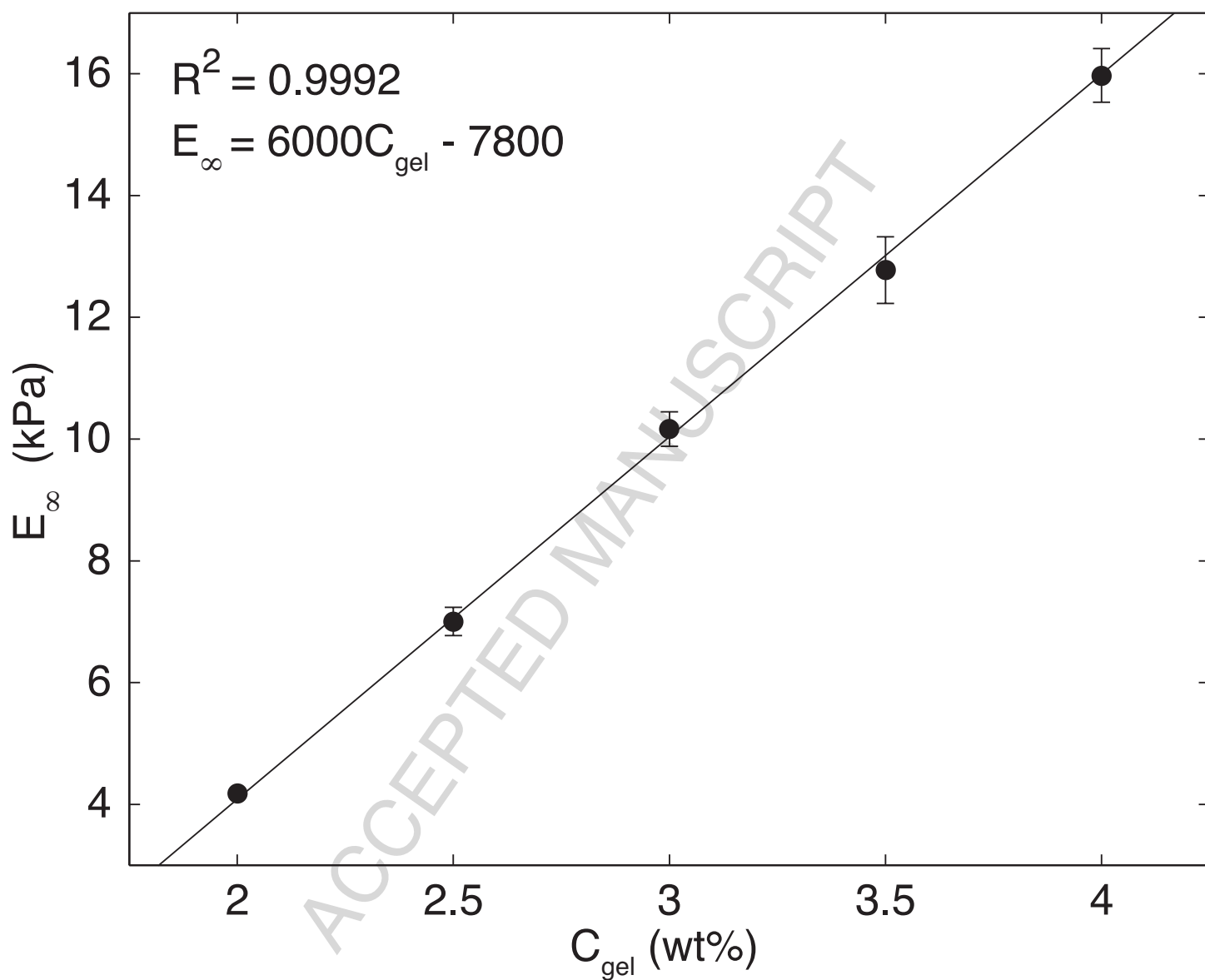


Figure 7

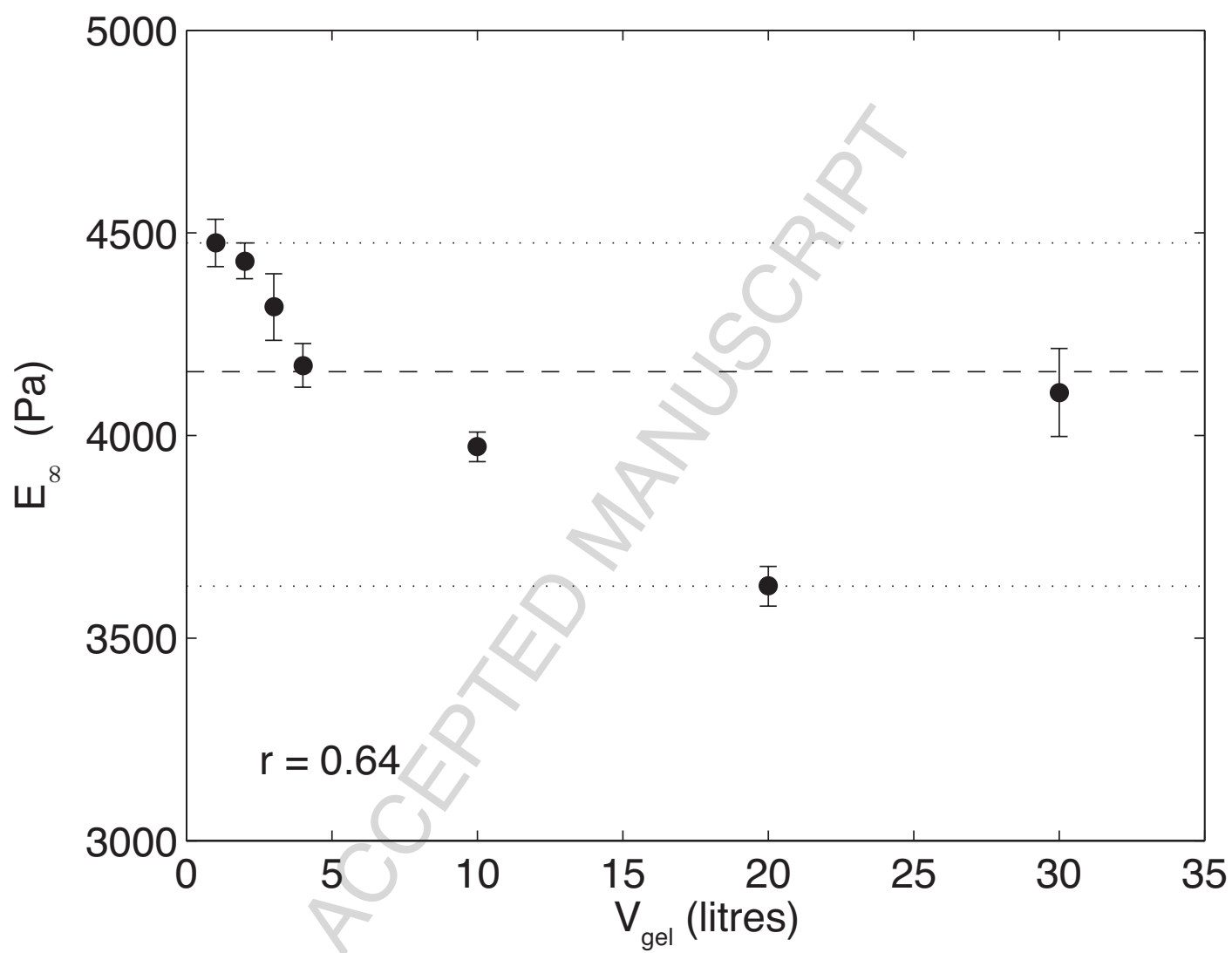


Figure 8

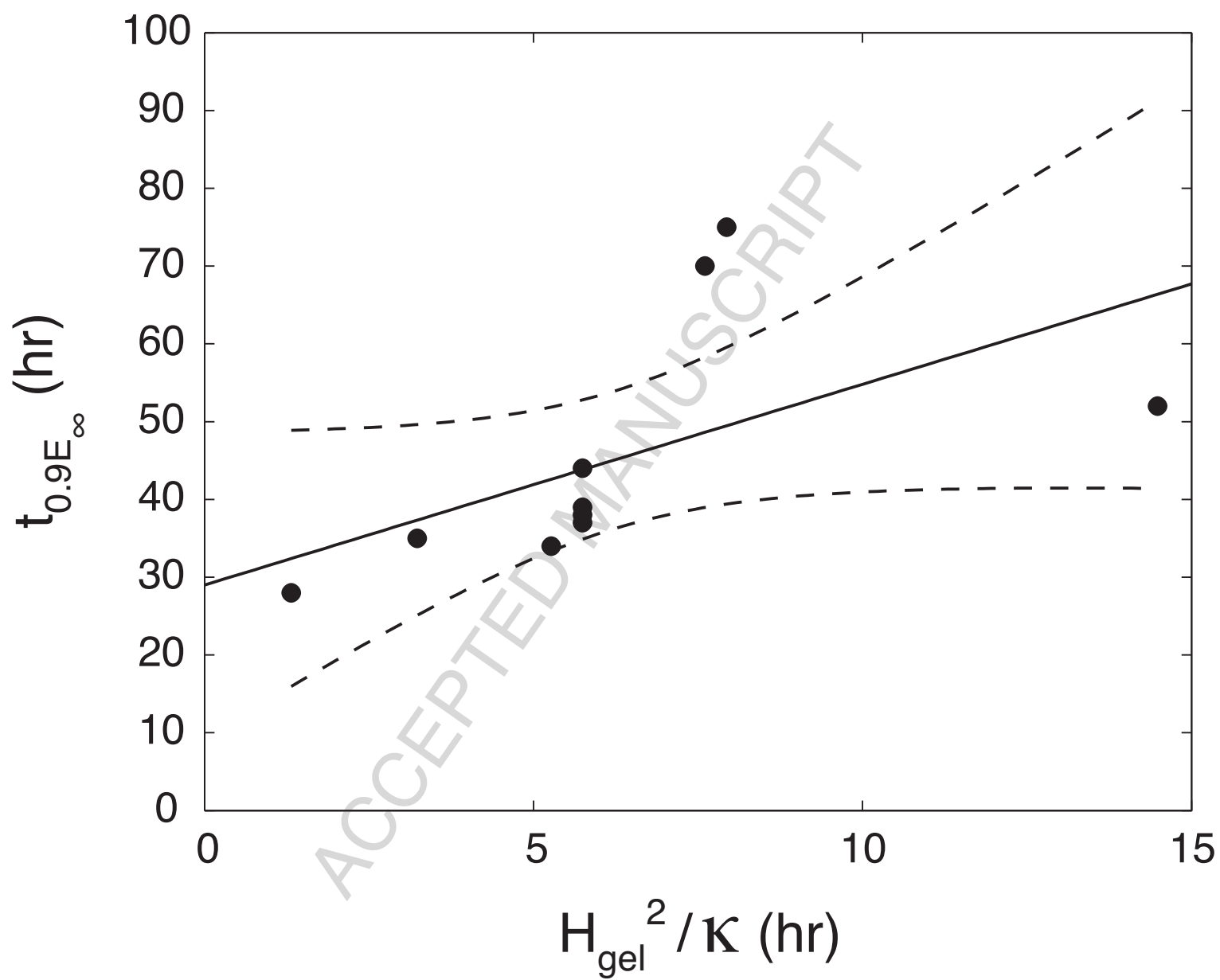


Figure 9

