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► To cite this version:

Ando Randrianoelina, Maarten Hornikx, Renz van Luxemburg, Stef van Eindhoven, Azar Dastouri. BEM for the prediction of standardized scattering and diffusion coefficients of diffusers. Acoustics 2012, Apr 2012, Nantes, France. hal-00811379

HAL Id: hal-00811379

<https://hal.science/hal-00811379>

Submitted on 23 Apr 2012

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BEM for the prediction of standardized scattering and diffusion coefficients of diffusers

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For the prediction of the scattering and diffusion coefficients of surfaces as defined in standards, a computational method is used. The modelling approach for the diffusion coefficient aims at reproducing the standardized measurement conditions, and is mainly based on geometrical constraints from the standards. The prediction of the scattering coefficient relies on formula of Mommertz to extract the specular component of the pressure field reflected by a surface, by using the correlation with the pressure field reflected by a reference flat surface. The numerical calculations concern a 2D implementation of the Boundary Element Method (BEM) in the frequency domain, by utilizing the open source code OpenBEM. With the simulation method, the scattering and diffusion coefficients of a Schroeder diffuser have been computed. The results indicate the usefulness of OpenBEM for accurately evaluating diffusion and scattering coefficients of surfaces at a reasonable computation time.

1 Introduction

Acoustic scattering properties of surfaces are commonly described in terms of scattering and diffusion coefficients [1]. The diffusion coefficient gives an indication of the uniformity of the energy reflected by a surface into the half space in front of it and the scattering coefficient is the ratio of the diffuse scattered energy relative to the total reflected energy by the surface.

Over the past 30 years, research on the prediction of those coefficient using numerical methods has been conducted. Prediction of the diffusion coefficient is straightforward, as it can be obtained from numerical models reproducing transfer functions with the surface in free field [2,3]. Cox *et al.* [4,5] have demonstrated the suitability of the boundary element method (BEM) for the production of the diffusion coefficient. The prediction of the scattering coefficient according to the measurement standards, on the other hand, is more challenging. It requires calculating the diffuse reflected energy and the total reflected energy from reverberation time measurements with the surface placed on a turntable. This explains why the prediction of the scattering has been subjected to more extensive investigations [5,6]. Mommertz, however, proposed a formula to obtain scattering coefficients from the directional distribution of the scattered pressure in free field (polar responses). This formula has been compared to the Kirchhoff Approximation by Embrechts [9] who also demonstrated that the formula from Mommertz is valid for 1D rough surfaces. Embrechts tested the formula for the case of 1D surfaces with a Gaussian roughness and proved to successfully extract the specular component for this type of surface. Kosaka and Sakuma [10,11] also have investigated a numerical technique using a 3D implementation of BEM based on Mommertz formula. They studied the case of surfaces with a 2D Gaussian roughness. The approach proposed by Mommertz presents a certain advantage, as both the diffusion and scattering coefficients can now be extracted from the polar responses of the surface.

In this paper, we investigate the prediction of the random incidence scattering and diffusion coefficients of 1D diffusers using a 2D implementation of BEM in the frequency domain. The approach consists of reproducing the standardized measurement method for the diffusion coefficients by calculating polar responses and using these responses to determine the diffusion and scattering coefficients of the diffusers. The case of Quadratic Residue Diffuser (QRD) has been investigated.

For the numerical calculation of the sound field, the 2D OpenBEM [12] code has been used. OpenBEM is a library of validated BEM codes developed by Cutanda and Juhl [13]. The codes have been proved to deal well with radiation, diffraction and scattering problems [14-17] and

have also seen many improvements through time [13,18,19]. The codes have extensively been used for noise barrier problems, with few investigations on diffuser problems [16,17,20]. The work presented in this paper illustrates the applicability of the OpenBEM code for the prediction of diffusion and scattering coefficients.

2 Prediction of diffusion and scattering coefficients

2.1 Diffusion coefficient

The ISO normalization of the measurement of the random incidence and directional diffusion coefficient is still under development and is currently based on the existing document AES-4id-2001 [22]. Measurements should be made in free field conditions, under far field or mixed near/far field conditions depending on the future applications of the diffuser and in a semicircular (for 1D diffusers, also referred to as single plane diffusers) or hemispherical transducer set-up.

For each source position, 1/3 octave band reflected sound pressure levels L_i are measured by microphone i , and the directional diffusion coefficient d_θ is calculated as follows (θ is the angle of incidence):

$$d_\theta = \frac{\left(\sum_{i=1}^n \frac{L_i}{10^{10}}\right)^2 - \sum_{i=1}^n \left(\frac{L_i}{10^{10}}\right)^2}{(n-1) \sum_{i=1}^n \left(\frac{L_i}{10^{10}}\right)^2} \quad (1)$$

The random incidence diffusion coefficient d is the average of the directional diffusion coefficients.

In the rest of this paper, the term ‘diffusion coefficient’ refers to the random incidence diffusion coefficient. The measurement is straightforward to implement by BEM calculations as it solely consists of geometrical requirements with the sample placed in free field. The angular spacing of receivers and sources are chosen within the limits given in the standard.

Erreur! Source du renvoi introuvable. below summarizes the requirements from the standards and the modelling approach investigated to fulfil those requirements. Throughout this study, all surfaces were modelled as totally reflective, with infinite impedance.

Table 1: Modelling approach in BEM of reproducing the measurement standard for the diffusion coefficient

	Requirements from standards	Modelling approach
Semicircular measurements	Anechoic room for free field conditions	Model an unbounded domain for free field conditions
	Source positions	Perform separated calculations for 19

	up to 10° resolution	sources position with 10° resolution and investigate the adequacy of using 19 source positions in a single calculation..
	Semicircular measurements (up to 5° resolution)	Use an angular resolution of 2.5° for the receivers. 73 receivers in total.
	Measurements in far field	Use a source and receiver array radii of respectively 30m and 20m.
	Narrow band spectra for results in 1/3 oct. band	Determine an adequate frequency scale to reduce the number of calculation frequencies in the 1/3 octave band range 100 Hz-4000 Hz

2.2 Scattering coefficient

The definition and measurement procedure of the scattering coefficient is normalized by the ISO17497-1 [21].

The calculation of the random incidence scattering coefficient s is according to the definition:

$$s = \frac{E_{diff}}{E_{total}} = 1 - \frac{E_{spec}}{E_{total}} \quad (2)$$

With E_{diff} is the diffuse energy scattered into non-specular directions, E_{spec} is the energy reflected in a specular way and E_{total} is the total reflected energy. As the standard requires measurements of the reverberation time of the (rotating) test sample in diffuse sound field conditions, reproducing these requirements by a frequency domain prediction method is difficult.

Mommertz proposed the following formula for the scattering coefficient δ for the considered frequencies and angles of incidence:

$$\delta = 1 - \frac{|\sum_{i=1}^n p_1(\theta_i) p_0^*|^2}{\sum_{i=1}^n |p_1(\theta_i)|^2 \sum_{i=1}^n |p_0(\theta_i)|^2} \quad (3)$$

where p_l is the complex pressure reflected by the surface from an angle of incidence θ_i , and p_0 is the complex pressure reflected by a flat perfectly reflective surface of the same size. This formula, which is only valid for the case of 1D surfaces as those investigated in this paper since the azimuthal dependency has been neglected, also suggests that the effect of edge diffraction in the specular direction is included in the specular component.

The total and incident pressure at a receiver position, respectively p_{tot} and p_{inc} , are direct output from a BEM calculation. The reflected pressure p_{refl} is calculated as:

$$p_{refl} = p_{tot} - p_{inc} \quad (4)$$

By determining the reflected pressures from the surface of interest and corresponding reference surface, scattering coefficients can be calculated using formula (3).

In the study presented in this paper, it is assumed that the scattering coefficients calculated using the source and receiver positions described in Table 1 can be regarded as the random incidence scattering coefficient and will be referred as s in the rest of this text.

1/3 octave band values are obtained from averaging scattering coefficient values within bands.

3 Validity of numerical modelling

For accurately computing the reflected sound from objects, appropriate calculation parameters within

OpenBEM have been investigated, namely the element types, mesh densities and the calculation frequencies for broadband results.

3.1 Element type and mesh density

This study focuses on the polar responses of the scattered levels from one period of a 1D QRD diffuser (N=7, i.e. 7 wells in a period) with a design frequency of the diffuser is $f_0 = 500$ Hz. f_0 is the frequency above which the diffuser is supposed to scatter sound with even energy in all directions.

It was found that with quadratic elements, convergence is generally achieved at the frequencies considered (250 Hz, 1 kHz, 2 kHz and 4 kHz) with 4 to 6 elements per wavelength, however, at the design frequency, 8 elements per wavelength were required.

As for the choice of the element type to use for the numerical integration, linear elements present the advantage of shorter computation times, as the same geometry presents less nodes than with using higher order elements. However, calculations might require a higher mesh density for the linear elements for similar accuracy. Comparison between results of calculations using linear and quadratic elements with a mesh density of 8 elements per wavelength confirmed this: discrepancies were found between results from the two element types, and they became larger with increasing frequency, especially in the near field case. In the far field case, standard deviations in reflected SPLs (over all receiver angles) of up to 2 dB were found.

Based on these results, quadratic elements and a mesh density of 8 elements per wavelength for the investigation of the scattering and diffusion coefficient with the OpenBEM have been used.

For the validation of the BEM calculations for the case of diffusers, a calculation was made for the 1D QRD diffuser with 5 periods of the QRD element (N=7) used in the previous study.

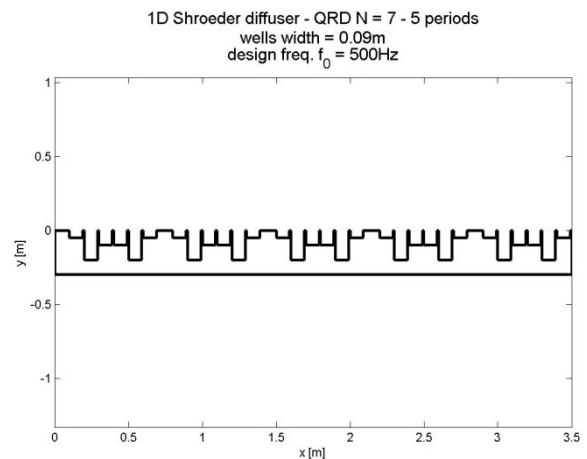


Figure 1: Geometry of 1D QRD diffuser

Calculations were performed at 250 Hz, 1 kHz and 4 kHz. Results in terms of relative scattered levels were compared with pseudospectral time domain calculations (PSTD) according to [23,24].

For the frequencies tested, results of BEM show good agreement with those from the PSTD. Curves of the relative scattered levels show very similar behaviours with coinciding dips. Figure 2 shows an example of the results at 4000 Hz.

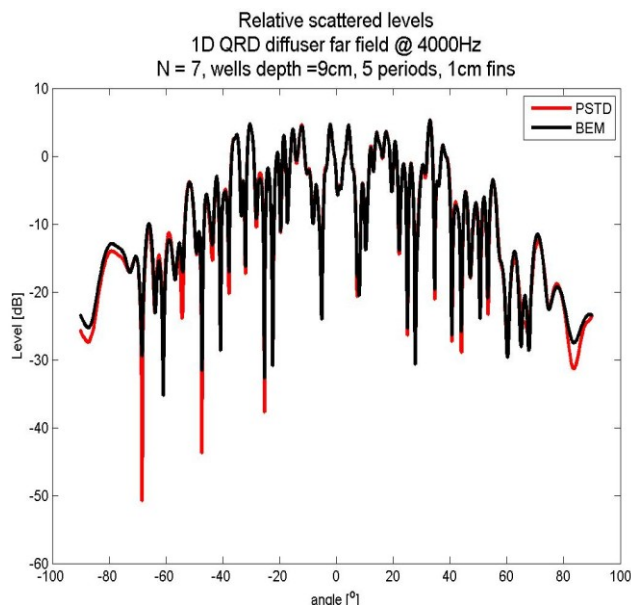


Figure 2: Comparison of BEM results with PSTD results for the case of the 1D QRD diffuser of Fig. 1.

3.2 Optimization of broadband calculations

A solution is here proposed to considerably reduce the number of calculation frequencies for evaluation of scattering and diffusion coefficient in 1/3 octave bands.

For the case of the diffusion coefficient, the solution is based on linear interpolation of SPL between calculation frequencies. Interpolation is made with a frequency resolution of 1 Hz. In this way, a narrow band SPL spectrum is obtained and 1/3 octave band spectrum can be calculated.

This approach cannot be used for the case of the scattering coefficients as its determination is based on the knowledge of complex pressures. In that case, scattering coefficients will be calculated from BEM calculations at certain frequencies and interpolation of scattering coefficients will be made with a frequency resolution of 1 Hz. 1/3 octave band values will be obtained by averaging the scattering coefficient values per band.

3.2.1 Frequency scale

The choice of the calculation frequencies for the BEM calculations has the objective to allow obtaining the same energy levels in 1/3 octave bands from the interpolated spectrum as from the actual narrow band spectrum.

The use of a linear and logarithmic frequency scale was investigated and compared. While the first scale gives better accuracy of broadband SPLs in higher frequency bands, the second scale allows a better approximation of broadband SPLs better in lower frequency bands. This effect is directly related to the number of calculation frequencies in the bands with respect to the bandwidth.

In order to compromise between both scales, a scale is proposed with a number of calculation frequencies in the n^{th} 1/3 octave band as function of the number of calculation frequencies in the first 1/3 octave band considered and on the logarithm of the ratios of 1/3 octave band sizes, such that:

$$Nf_{n,1/3oct} = Nf_{1,1/3oct} \cdot \log_{10} \left(\frac{w_n}{w_1} \right) \quad (5)$$

$Nf_{n,1/3oct}$ is the number of calculation frequencies in the n^{th} band. $Nf_{1,1/3oct}$ is the number of calculation frequencies in the first band. w_n is the width of the n^{th} band in hertz. w_1 is the width of the first band in hertz. In each band, calculation frequencies are linearly spaced.

Initial tests made on artificially generated random signals and showed that a mean error less than 1 dB and standard deviations of less than 2 dB in all bands is achieved for 4 frequencies in the first 1/3 octave band, 100 Hz. This corresponds to a total of 164 calculation frequencies. The number of calculation frequencies is reduced by 3.6 compared to a linear scale for a similar degree of accuracy, also the standard deviations vary less between 1/3 octave bands than with the traditional linear and logarithmic scales.

3.2.2 Testing in the case of a diffuser

The proposed frequency scale was subsequently tested for the case of sound scattered from one period of the 1D QRD 734 diffuser manufactured by RPG inc. (www.rpginc.com) (See Figure 4) in order to assess its suitability for the case of a spectrum of sound scattered by a diffuser.

BEM calculations of the sound pressure were made in narrow band at a frequency resolution of 1 Hz in the 1/3 octave bands 250 Hz and 1 kHz, as well as at specific frequencies given by the designed frequency scale in those bands. This corresponded to 4 frequencies and 12 frequencies per band respectively.

Sources and receivers were positioned in semicircular configuration (far field) in accordance to the measurement setup of AES-4id-2001.

Figure 3 and shows the difference in the 1/3 octave bands levels obtained from narrow band calculation and from interpolation in the 250 Hz band for all source and receivers positions. Errors of less than 0.8 dB, with mean error of less than 0.01 dB and standard deviations of less than 0.20 dB for all results. Results in the 1 kHz were even better with a mean error of 0.00 dB and a standard deviation of 0.12 dB.

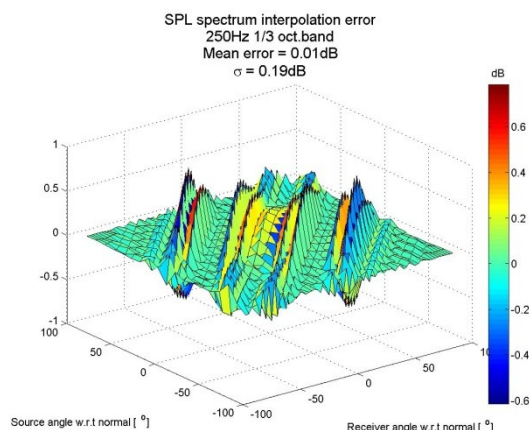


Figure 3: Estimation error of SPLs from a diffuser in the 250 Hz 1/3 octave band.

3.2.3 Study of the diffusion and scattering coefficients

In order to understand the impact of using interpolated SPLs on the calculation of the diffusion and scattering coefficients, instead of SPL from narrow band BEM

calculations, we compared those coefficients for calculations made in the 250 Hz, 500 Hz (diffuser's design frequency) and 1 kHz 1/3 octave bands.

Diffusion coefficient

Table 2 summarizes the results for the diffusion coefficients obtained from narrow band calculation and interpolation of the SPLs in the two 1/3 octave bands considered. d is the diffusion coefficient.

In terms of diffusion coefficient, AES-4id-2001 sets the significant differences are greater than 0.1. It can be seen from the results that the difference between the diffusion coefficients obtained from narrow band calculations and interpolation of SPL is negligible.

Table 2: Comparison of results of diffusion coefficients

d	250 Hz 1/3 oct. band	500 Hz 1/3 oct. band	1 kHz 1/3 oct. band
Narrow band	0.7384	0.7488	0.6853
Interpolation	0.7383	0.7598	0.6836

Scattering coefficient

Table 3 summarizes the results for the scattering coefficients obtained from narrow band calculation and interpolation of the scattering coefficients between calculation frequencies in the two 1/3 octave bands considered. s is the scattering coefficient, (s is the mean of scattering coefficient calculated from directional coefficients obtained for each source position).

Table 3: Comparison of results of scattering coefficients

s	250 Hz 1/3 oct. band	500 Hz 1/3 oct. band	1 kHz 1/3 oct. band
Narrow band	0.1901	0.8832	0.9668
Interpolation	0.1926	0.9291	0.9670

In both cases of the diffusion and scattering coefficients, influence of the estimation error of the scattered SPL is negligible in the bands below and above the diffuser's design frequency. In the band of the diffuser's design frequency, this estimated error does not achieve significant levels either.

The results show that the frequency scale introduced to perform the BEM calculations for the prediction of the diffusion and scattering coefficients by interpolation of SPLs and s respectively, provide values very close to those of the coefficients obtained from narrow band calculations in the bands considered. This approach has been used for prediction of 1/3 octave band diffusion and scattering coefficients of two types of diffusers. Results are presented in the following section.

4 Prediction of scattering using 2D BEM

This section presents results regarding the scattering and diffusion coefficients of a diffuser, predicted using the modelling approaches defined in previous sections studies. The diffuser tested is a 1D diffuser manufacturer by RPG inc.

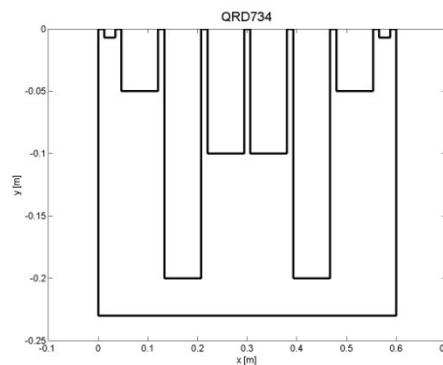


Figure 4: Geometry of QRD 734 from RPG Inc.

Figure 5 show the results obtained from the BEM calculations. Prediction results of diffusion coefficients of a reference surface are also shown.

As information is missing about the test conditions, and sample size, it should be noted that manufacturer's data as shown here are rather informative, making a comparison between the BEM results and manufacturer's data is difficult.

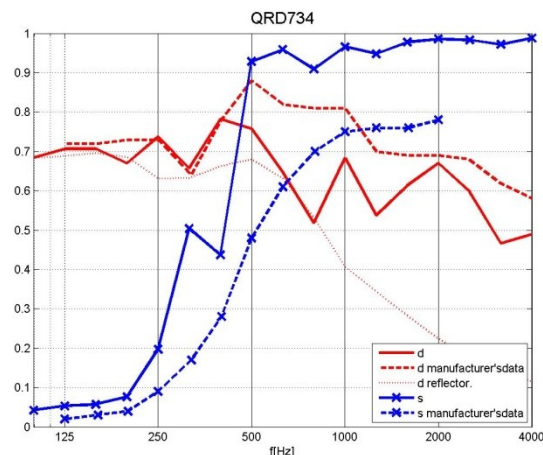


Figure 5: Random incidence scattering and diffusion coefficients of QRD734

5 Conclusions

For the prediction of scattering and diffusion coefficients using 2D OpenBEM codes, it was found that for scattering problems focusing on diffusers, it is recommended to use a higher mesh density than the usual 4 or 6 elements per wavelength. 8 elements per wavelength, and quadratic elements were used in this study. Prediction of the diffusion coefficients of 1D diffusers was based on the measurement method described in AES-4id-2001, using a semicircular set-up. Results from the calculations were used for the prediction of scattering coefficients based on Mommertz formula.

A frequency scale was proposed to efficiently compute 1/3 octave band results, while keeping accurately computed coefficients.

Diffusion and scattering coefficients for the case of a 1D QRD diffuser were computed and results were compared to manufacturer's data. Predicted diffusion

coefficient values below and above the diffuser's design frequency (500 Hz) are close to the manufacturer's data, with larger discrepancies observable in the bands 500 Hz to 1 kHz. These are however not explained by the need of a finer frequency scale in the design frequency region. Predicted scattering coefficient curves show similar characteristics but higher values than the manufacturer's data. Apart from possible differences in the sample sizes and edge diffraction effects, higher predicted values can be explained by the fact that coefficients were computed in the azimuthal direction with most scattering, i.e. perpendicular to the irregularities, while measurements in a reverberant room include scattered energy in all directions, including the direction parallel to the diffusers wells and fins, where scattering is minimal. Thus, when measuring a 2D element in a laboratory, the scattering coefficients are likely to be lower than the coefficients obtained with the prediction method presented here. A similar prediction method using a 3D BEM scheme should be investigated to improve the match with measurement data, hence improve the prediction accuracy.

Acknowledgments

The work presented in this paper resulted from a common interest in combining acoustics and mathematics to explore ways of using numerical methods in the field architectural acoustics.

We are grateful to Vincente Cutanda and Peter Juhl, authors of the OpenBEM codes, who accepted to share their library with us for carrying out our work and for providing support and guidance on the BEM and OpenBEM files.

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