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### ► To cite this version:

Maria Karzova, Edouard Salze, Sébastien Ollivier, Thomas Castelain, Benoît André, et al.. Interaction of weak shocks leading to Mach stem formation in focused beams and reflections from a rigid surface: numerical modeling and experiment. Acoustics 2012, Apr 2012, Nantes, France. hal-00811290

**HAL Id: hal-00811290**

**<https://hal.science/hal-00811290>**

Submitted on 23 Apr 2012

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# ACOUSTICS 2012

## **Interaction of weak shocks leading to Mach stem formation in focused beams and reflections from a rigid surface: numerical modeling and experiment**

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Mach stem is a well-known structure observed in the process of interaction of two strong shock waves (acoustic Mach number  $M_a > 0.1$ ), typically in the case of reflections from a rigid boundary. However, this phenomenon has not been studied in detail for nonlinear acoustic fields that include weak shocks, i.e. for acoustic Mach number within the range from  $10^{-3}$  to  $10^{-2}$ . In acoustics, interaction of shocks can occur in case of reflections or in focusing of high power nonlinear beams. The goal of this paper was to investigate this phenomenon both numerically and experimentally for these representative cases. The KZK nonlinear evolution equation was used to demonstrate the formation of Mach stem in the focal region of periodic and pulsed beams for the Mach number of about  $10^{-3}$ . Experiments were performed in air using spark-generated  $N$ -waves reflecting from a rigid boundary. Shock fronts were visualized using Schlieren method. Regular and irregular reflections were observed and corresponding values of critical parameters for either type of reflection to occur were determined.

## 1 Introduction

Deviation from mirror reflection for strong shocks was first experimentally observed by Mach in 1878 [1]. He had shown that under certain conditions the reflection pattern consists of the incident and reflected shocks, which intersect above the surface and the third one, which connects the intersection point with the surface. This intersection point is called the triple point and the emerging third shock is called the Mach stem [2]. Reflection of strong shocks can be categorized in two types. The first type is a regular reflection, when incident and reflected shocks intersect directly on the rigid surface; the second type is an irregular reflection, when shocks intersect at some distance above the surface. Regular reflection is described by two-shock theory and irregular reflection by three-shock theory [3]. These theories are in a good agreement with experimental observation for acoustic Mach number greater than 0.4, i.e. for strong shocks usually encountered in aerodynamics [4]. However, three-shock theory fails to predict irregular reflection for weak shocks with acoustic Mach number of less than 0.1 [4]. This result, which is in conflict with experimental and numerical evidence, is known as von Neumann paradox [4, 5].

Theoretical and experimental studies on shock-wave reflections have been carried out mostly for step shocks [4, 6, 7]. These step shock waveforms are typical for aerodynamics while  $N$ -waves and sawtooth waves are typical for acoustics. The evidence of existing irregular reflection from rigid boundary for extremely weak ( $M_a = 2 \cdot 10^{-4}$ ) periodic sawtooth waves has been demonstrated experimentally in water, and experimental results were supported by numerical simulations based on the KZ equation [8]. Various regimes of reflections of acoustic shock waves were also studied numerically [2].

The goal of this paper is to demonstrate experimentally that irregular reflection is observed for weak spherically diverging  $N$ -waves in air and to determine values of a critical parameter that separates regular and irregular types of reflection. Since reflection from a rigid boundary is similar to focusing of an axially symmetric beam, the Mach stem formation was also investigated numerically in a focal area of nonlinear periodic and pulsed beams.

## 2 Experimental studies based on optical measurements

The experimental setup for optical visualization of the reflection of shock waves from a rigid boundary is presented in Figure 1 and the scheme of experimental setup is presented in Figure 2. High-amplitude  $N$ -waves were produced by a 15 kV spark source (1) and reflected from a

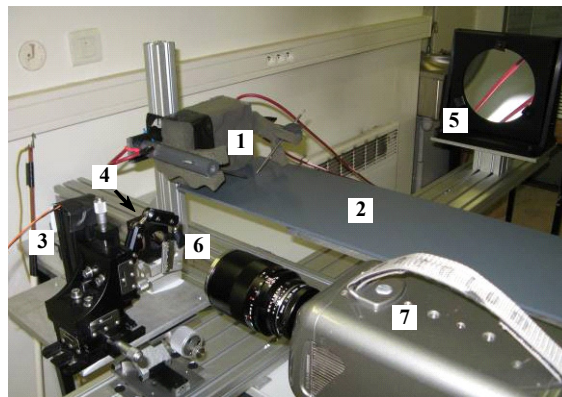


Figure 1: (Color online) Experimental setup. Acoustical equipment: 1 – spark source, 2 – rigid surface. Optical equipment: 3 – continuous light source, 4 – beam splitter, 5 – mirror, 6 – optical knife, 7 – high speed camera.

rigid surface (2). To observe experimentally the types of reflection, Schlieren optical scheme was used [9]. Schlieren system was composed of a QTH continuous light source (3), mounted in the focus of a spherical mirror (5), a beam splitter (4), an optical knife (a razor edge, 6), and a high speed camera Phantom V12 CMOS (7). Light beam was emitted by the source (3), propagated through the beam splitter and then through the area of observation near the rigid surface (2). Then light reflected from the mirror, intersected the observation area once again and propagated back to the beam splitter. Spatial variations of the light refraction index  $n$  caused by the acoustic shock front lead to deflection of a part of light rays from the straight trajectory. This enables us to obtain optical visualization of the reflection pattern using the Schlieren method. For this aim we mounted the optical knife edge at the focal distance from the mirror. Light rays that were not deflected by acoustic pressure inhomogeneity were covered by the optical knife while deflected rays bended the razor edge. Finally, deflected rays were captured by high speed camera and formed shadowgrams. Double passing of the light beam through the observation area provided better resolution of the shadowgrams.

The type of the shock wave reflection depends on the value of critical parameter  $a$ , defined by the acoustic Mach number  $M_a$  and the angle  $\theta$  between the incident front and the perpendicular to the surface [2, 10]:

$$a = \frac{\sin \theta}{\sqrt{2\beta M_a}}. \quad (1)$$

Here  $\beta = 1 + B/2A$ ,  $B/A$  being the nonlinearity parameter [2]. Measurements were carried out at distances

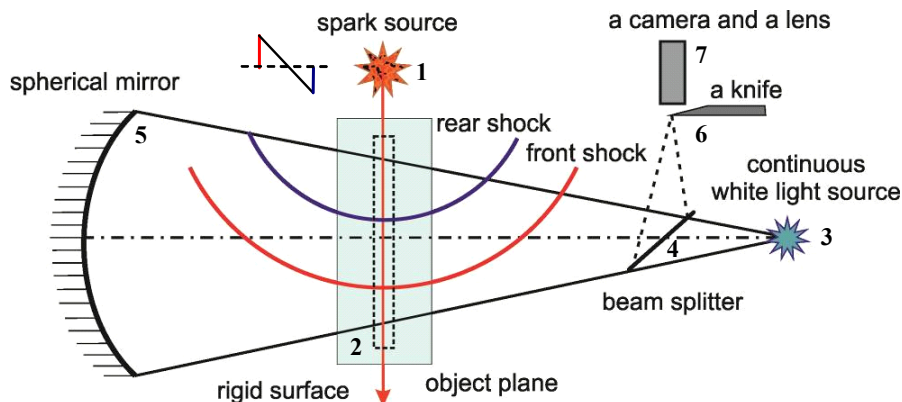


Figure 2: (Color online) Geometry of the experimental setup. Acoustical equipment: 1 – spark source, 2 – rigid surface. Optical equipment: 3 – continuous light source, 4 – beam splitter, 5– mirror, 6 – optical knife, 7 – high speed camera.

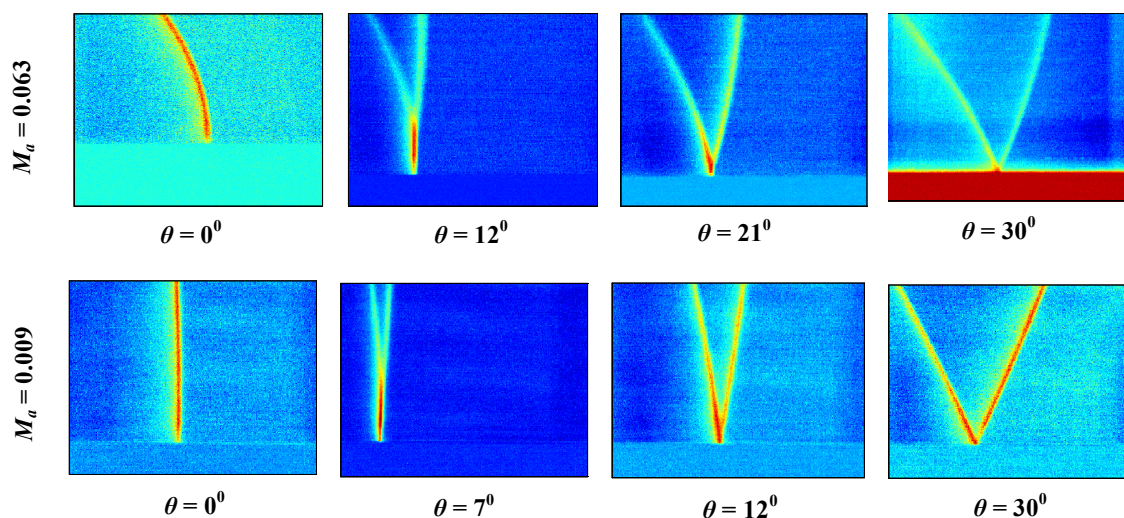


Figure 3: (Color online) Optical visualization of a front shock reflection from the rigid surface positioned 4.6 cm (frames on the top) and 20.7 cm (frames on the bottom) away from the spark source. In each picture the incident shock is on the right and the reflected shock is on the left. For the grazing angle  $\theta = 0^\circ$  the reflected shock is invisible.

4.6 cm and 20.7 cm away from the spark source. In order to determine the value of critical parameter  $a$  at different distances from the source, peak pressure in acoustic waveform was determined using optical methods and acoustic Mach number was calculated in the following way.

Schlieren method made it possible to simultaneously visualize both front and rear shocks of the  $N$ -wave and thus to determine its duration. Based on the change in the  $N$ -wave duration at different distances, caused by amplitude dependent nonlinear propagation effects, the peak pressure was calculated [11]. The distance 4.6 cm corresponds to  $M_a = 0.063$  (the amplitude of pressure  $p_{max} = 6.3$  kPa) and the distance 20.7 cm - to  $M_a = 0.009$  (the amplitude of pressure  $p_{max} = 0.9$  kPa).

Figure 3 shows experimental results for reflection pattern of the front shock from the rigid surface for  $M_a = 0.063$  (top series of frames) and for  $M_a = 0.009$  (bottom series of frames). Every picture was obtained by averaging 20 shadowgrams and subtracting the background.

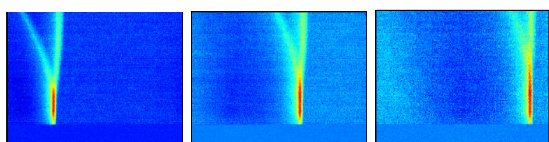


Figure 4: (Color online) Mach stem increasing.

For grazing angle  $\theta = 0^\circ$  the reflected shock is invisible and there is no reflection for both values of acoustic Mach number  $M_a$ . Irregular reflection is observed for angles  $\theta \leq 12^\circ$  when  $M_a = 0.063$  and for  $\theta \leq 7^\circ$  when  $M_a = 0.009$ . As one can see, at these angles the incident and reflected shocks intersect above the surface and Mach stem is formed near the surface. With further increase of the incident angle  $\theta$ , the regime of reflection modifies to the regular reflection when incident and reflected shocks merge right on the surface ( $\theta = 21^\circ$  and  $30^\circ$  for  $M_a = 0.063$ ;  $\theta = 12^\circ$  and  $30^\circ$  for  $M_a = 0.009$ ).

The amplitude of the  $N$ -wave decreases when it propagates along the surface, and, as a result, the value of acoustic Mach number  $M_a$  decreases. But at the same time the angle  $\theta$  decreases because the reflection occurs further from the spark source. For a small distance increase the variation of  $\theta$  has more influence which results in the decrease of the critical parameter value  $a$ . This is confirmed by the three consecutive shadowgrams of irregular reflection in Fig.4. The initial angle is  $\theta = 12^\circ$ . As one can see, the Mach stem increases when it propagates along the surface.

For  $M_a = 0.063$  the regular reflection is observed for  $\theta \geq 18^\circ$ , which corresponds to  $a \geq (0.79 \pm 0.22)$ , otherwise irregular reflection occurs, except when  $a$  is close to zero. For  $M_a = 0.009$  the regular reflection is observed for  $\theta \geq 8^\circ$ ,

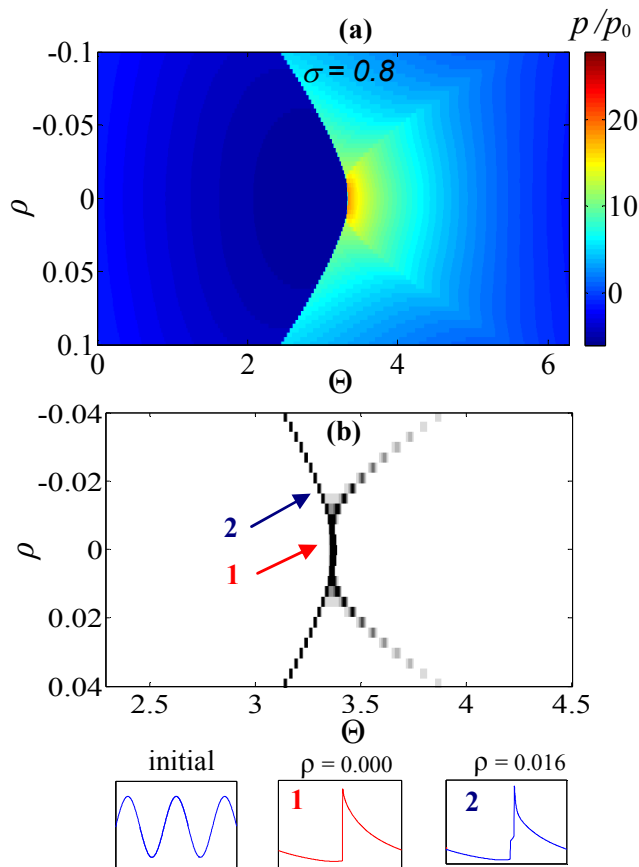


Figure 5: (Color online) Illustration of Mach stem formation in the focused beam of periodic waves: (a) – temporal pressure waveforms at different transverse distances from the beam axis. Different colors correspond to different pressure values. (b) – temporal derivatives of the pressure waveforms shown on (a). Series of frames on the bottom – initial waveform and waveforms at different radial distances from the axis.

i.e. for  $a \geq (0.95 \pm 0.30)$ . These values of  $a$ , when transition from regular to irregular types of  $N$ -waves reflection is observed are in a good agreement with the corresponding value of  $a \approx 0.8$  obtained in [2] with numerical simulation.

### 3 Numerical modelling based on the KZK equation

Note that axisymmetric beams have the same boundary condition on the beam axis as a perfectly rigid surface:

$$\left. \frac{\partial P}{\partial \rho} \right|_{\rho=0} = 0. \quad (2)$$

According to this condition, for axisymmetric focused beams Mach stem formation can take place for a certain range of focusing angles. Periodic sinusoidal wave (Fig. 5) and bipolar pulse (Fig. 6) were used as initial waveforms in numerical simulation and a piston source was used as a boundary condition.

The nonlinear propagation of high amplitude acoustic waves generated by focused sources is modeled here using the KZK equation. The equation takes into account the combined effects of nonlinearity, diffraction, and

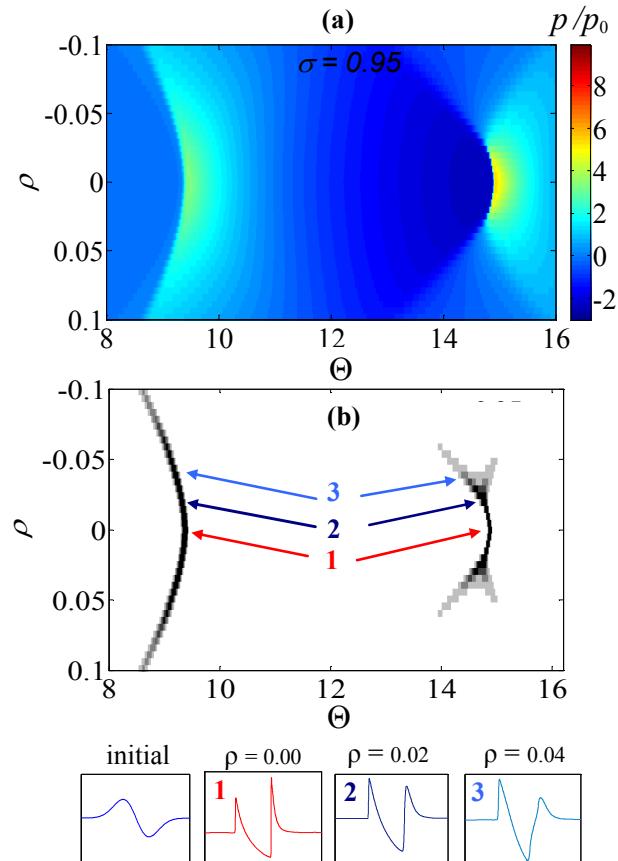


Figure 6: (Color online) Illustration of Mach stem formation in the focused beam of shock pulses: (a) – temporal pressure waveforms at different transverse distances from the beam axis. Different colors correspond to different pressure values. (b) – temporal derivatives of the pressure waveforms shown on (a). Series of frames on the bottom – initial waveform and waveforms at different radial distances from the axis.

absorption. For axisymmetric beams the equation can be written in dimensionless form as:

$$\frac{\partial}{\partial \Theta} \left[ \frac{\partial P}{\partial \sigma} - NP \frac{\partial P}{\partial \Theta} - A \frac{\partial^2 P}{\partial \Theta^2} \right] = \frac{1}{4G} \left( \frac{\partial^2 P}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial P}{\partial \rho} \right). \quad (3)$$

Here  $P = p/p_0$  is the acoustic pressure normalized by the initial amplitude  $p_0$  at the transducer;  $\sigma = x/F$  is the propagation distance normalized by the transducer focal length  $F$ ;  $\rho = r/a_0$  is the lateral distance normalized by the transducer radius  $a_0$ ;  $\Theta = 2\pi\tau/T_0$  is the dimensionless time;  $\tau = t - x/c_0$  is the retarded time;  $c_0$  is the sound speed in water;  $T_0$  is the signal duration (for the harmonic wave it is equal to one period). Equation (2) has three dimensionless parameters:  $N = 2\pi F \varepsilon p_0 / \rho_0 c_0^3 T_0$  is the nonlinear parameter, where  $\varepsilon$  is the nonlinearity coefficient,  $\rho_0$  is the medium density;  $G = \pi a_0^2 / c_0 F T_0$  is the diffraction parameter, and  $A$  is the absorption parameter [12].

Figure 5 shows nonlinear pressure fields of a periodic acoustic beam. The initial profile of the wave is shown as a blue curve titled initial. The frame (a) of Figure 5 demonstrates temporal field of pressure at different dimensionless transverse distances  $\rho$  from the beam axis. Different colors correspond to different pressure values; the

scale of the pressure is shown on the right side of the frame (a). This distribution of the pressure is presented in the focal region of the transducer at dimensionless distance along the axis  $\sigma = 0.95$ . The frame (b) of this figure shows the distribution of temporal derivatives of the pressure field (a). Grey levels correspond to the value of the derivative, it is darker for higher values. Thus, the frame (b) is the equivalent picture that we have in shadowgrams. One can clearly observe the spatial structure in (b) similar to the Mach stem. The formation of Mach stem in this case occurs because the edge wave and the central wave from the transducer merge in the focal area and form two shock fronts in one period of the periodic wave [13]. Since the velocity of the shock front depends on its pressure, the shock front with higher pressure propagates faster. Blue curve 2 in figure 5 shows one cycle of the temporal profile of the periodic wave. It contains two shocks – one higher and one lower. The first one corresponds to the edge wave and the second one - the central wave. Since the pressure of the edge wave is higher, it propagates faster and merge with the central wave [13]. This is shown as a waveform 1 of this figure. Thus, for transverse distance  $\rho = 0$ , i.e. on the beam axis, shock fronts of the edge and central waves merge and one shock is observed (red profile 1). This shock corresponds to Mach stem structure. For transverse distance  $\rho = 0.016$ , the central and the edge waves have not merged yet and two shocks structure is observed.

Figure 6 presents pictures similar to the Figure 5, but for the pulsed wave. The initial profile of the pulse is shown in blue and titled as initial. One can see the front and the rear shocks simultaneously in the pictures (a) and (b). The Mach stem structure is observed only for the rear shocks, which are on the right side. Rear shocks in the profile 2 and 3 of the edge wave are smoother than in the periodic fields. They provide less values of the derivative and the picture of the Mach stem is blurred.

## 4 Conclusion

In acoustics, interaction of shocks can occur in case of reflection over a surface or in focusing of high power nonlinear beams. Although Mach stem structure is well-known for interaction of two strong shocks, this phenomenon has not been studied in detail for nonlinear acoustic fields, i.e. for acoustic Mach number within the range from  $10^{-3}$  to  $10^{-2}$ . In this paper we studied experimentally the reflection of spherically diverging  $N$ -waves grazing at a small angle over a rigid surface in air. The irregular reflection was observed and values of critical parameter  $a$  that correspond to regular and irregular types of reflection were determined. Since the reflection from a rigid boundary is similar to focusing of an axially symmetric beam, the Mach stem formation was also investigated numerically in a focal area of nonlinear periodic and pulsed beams. It was shown that KZK equation makes it possible to describe effects related to Mach stem formation.

## Acknowledgments

This work was partially supported by PICS CNRS 5603/RFBR 10-02-91062. The numerical simulations were done using the resources of MSU supercomputer center.

## References

- [1] E. Mach, Über den Verlauf von Funkenwellen in der Ebene und im Raume, *Sitzungsbr. Akad. Wiss. Wien* 78, 819–838, (1878).
- [2] E. Mach, Über den Verlauf von Funkenwellen in der Ebene und im Raume, *Sitzungsbr. Akad. Wiss. Wien* 78, 819–838, (1878).
- [3] S. Baskar, F. Coulouvrat, R. Marchiano, “Nonlinear reflection of grazing acoustic shock waves: unsteady transition from von Neumann to Mach to Snell-Descartes reflections”, *J. Fluid Mech.*, Vol. 575, 27–55, (2007).
- [4] J. von Neumann, “Oblique reflection of shocks”, *In John von Neumann Collected Work (ed. A. H. Taub)*, Vol 6, 238–299, Pergamon Press (1963).
- [5] P. Colella, L.F. Henderson., “The von Neumann paradox for the diffraction of weak shock waves”, *J. Fluid Mech.*, Vol. 213, 71-94, (1990).
- [6] G. Birkhoff, “Hydrodynamics. A Study in Logic. Fact and Similitude”, *Princeton University Press*, (1950).
- [7] V. Kulkarny, “An experimental investigation on focussing of weak shock waves in air”, PhD Thesis *California Institute of Technology*, (1975).
- [8] B. Skews, J. Ashworth, “The physical nature of weak shock wave reflection”, *J. Fluid Mech.*, Vol. 542, 105-114, (2005).
- [9] R. Marchiano, F. Coulouvrat, S. Baskar, “Experimental evidence of deviation from mirror reflection for acoustical shock waves”, *Physical Review E* 76, 056602 (2007).
- [10] G.S. Settles, “Schlieren and shadowgraph techniques”, *Springer*, (2001).
- [11] M. Brio, J.K. Hunter, “Mach reflection for the two-dimensional Burgers equation”, *Physica D*, 60, 194-207, (1992).
- [12] P. Yuldashev, S. Ollivier, M.V. Averiyarov, O.A. Sapozhnikov, V.A. Khokhlova, and P. Blanc-Benon, “Nonlinear propagation of spark-generated  $N$ -waves in air: modeling and measurements using acoustical and optical methods”, *J. Acoust. Soc. Am.*, 128(6), 3321-3333, (2010).
- [13] M.M. Karzova, M.V. Averiyarov, O.A. Sapozhnikov, V.A. Khokhlova, “Mechanisms for saturation of nonlinear pulsed and periodic signals in focused acoustic beams”, *Acoustical physics*, 58 (1), 81-89, (2012).
- [14] V.A. Khokhlova, R. Souchon, J. Tavakkoli, O.A. Sapozhnikov, and D. Cathignol, “Numerical modeling of finite amplitude sound beams: Shock formation in the near field of a cw plane piston source”, *J. Acoust. Soc. Am.*, 110(1), 95-108, (2001).