Improving the acoustic black hole effect for vibration damping in one-dimensional structures

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Flexural vibrations are highly responsible for noise radiation by thin structures. In the transport industry, the damping of such vibrations is often achieved using thick viscoelastic layers, which leads to an undesirable increase of mass. In this regard, the acoustic black hole effect has received recent attention as an efficient and lightweight method for damping flexural vibrations. An acoustic black hole consists of a thin structure presenting a smooth decrease in its wave velocity along a given direction, thus acting as a wave trap where the vibrations can be efficiently damped. Such decrease of the velocity can be achieved by a variation of the mechanical parameters, namely the thickness or the Young's modulus. The present paper summarises recent progress on the enhancement of the acoustic black hole effect in one-dimensional structures. A numerical model of the flexural wave field of a beam with arbitrarily varying properties along its length is developed as a design tool for acoustic black hole structures with optimal performances. A further improvement of the effect is discussed, which consists of a beam subjected to a variation in its Young's modulus obtained by imposing a temperature gradient. The results show an efficient reduction of the vibration level, as well as the suppression of the resonances.

1 Introduction

One of today's major challenges in transportation, aeronautical and aerospace industries is the reduction of fuel consumption. In order to prevent noise radiation and damage due to flexural vibrations, large amounts of foams and viscoelastic layers are usually employed, which are responsible for a significant increase of mass. Therefore, the development of novel lightweight vibration damping solutions has a potentially high impact on environmental and technological issues.

Recent attention has been given to acoustic black holes as lightweight vibration dampers. The acoustic black hole effect is a wave-trapping phenomenon that occurs in a structure with a gradual variation of its properties along a certain direction. If such variation is properly chosen, the wave velocity can be made to decrease with distance, such that waves progressively slow down and stop travelling.

The phase velocity of a flexural wave in a Euler-Bernoulli beam is given by

$$c_\varphi = \left(\frac{Eh^2}{12\rho\,(1-\nu^2)} \right)^{1/4} \sqrt{\omega}, \tag{1}$$

where E is the Young's modulus of the beam, h is the thickness, ρ is the density, ν is the Poisson ratio and ω is the circular frequency. The expression shows that several properties of the structure may be made to vary with distance in order to produce an acoustic black hole. The most viable solution in structural applications consists of a beam with power-law thickness profile, which was at the origin of seminal work in the subject [1–4].

The present paper summarises recent progress in the design and practical implementation of the acoustic black hole in one-dimensional structures, by the present authors and collaborators [5–7]. First, the design of one-dimensional acoustic black holes is discussed, which includes a spatially discrete model of the structure and a method for quantifying uncertainty due to manufacturing imperfections. Second, recent experimental work is reported and discussed.

2 Numerical model

A numerical model of the flexural vibrations of an inhomogeneous beam has been developed and used as a simulation and design tool for acoustic black hole beams [5]. The main aspects of the model are here summarised, and the reader is referred to the original paper for further details.

Consider a beam with a smooth variation of its properties along its length x, as depicted in Fig. 1.

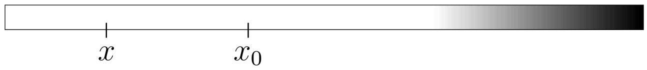

Figure 1: Semi-infinite beam with a smooth variation of its properties near the boundary. x_0, excitation point; x, observation point.

The flexural vibrations of a beam can be written in the matrix form

$$\frac{\partial \mathbf{W}}{\partial x} = \mathbf{H}\mathbf{W}, \tag{2}$$

where

$$\mathbf{W}(x) = \begin{bmatrix} w(x) \\ \theta(x) \\ V(x) \\ M(x) \end{bmatrix} \tag{3}$$

is the state vector, where w is the out-of-plane displacement, θ is the slope, V is the total shear force and M is the bending moment.

The kinematic and force variables are linked together by the impedance matrix of the beam, in the form

$$\begin{bmatrix} V(x) \\ M(x) \end{bmatrix} = j\omega \mathbf{Z}(x) \begin{bmatrix} w(x) \\ \theta(x) \end{bmatrix}. \tag{4}$$

Eqs. (2) and (4) yield a nonlinear relation between the impedance matrix and its spatial variation, in the form of a Riccati equation. The latter is numerically solved by using a Runge-Kutta-Fehlberg method with an adaptive spatial integration step.

The resulting impedance matrix is then used to obtain the reflection matrix of the acoustic black hole boundary as seen from an arbitrary point x of the beam [5] (see Fig. 1).

3 Optimisation of the properties of acoustic black holes

The manufacturing of an acoustic black hole in the case of a beam with a power-law thickness profile is subjected to imperfections due to the machining precision. In fact, a perfectly smooth thickness profile is impossible to achieve in practice and always presents a truncation, as illustrated in Fig. 2. A truncated wedge has a non-zero reflection matrix and thus the structure no longer presents the behaviour of an ideal acoustic black hole. However, the undesired effect of the truncation can be compensated by adding a thin damping

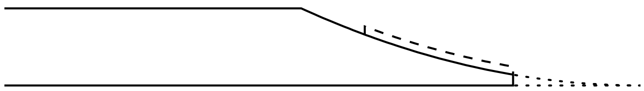

Figure 2: Beam with a power-law thickness profile. $\cdots\cdots$, ideal black hole profile; ——, real profile; - - - -, damping layer.

layer on the thiner region, as shown by Krylov et al. [2–4] (see Fig. 2). In such manner, the different terms of the reflection matrix of the modified wedge can reach values close to zero with realistic properties.

Given a beam with a power-law thickness profile manufactured with the best available precision, the model presented above can be used to find optimal values of the properties of the damping layer. The resulting optimal length of the damping layer is not the total length of the beam [5]. In fact, using a damping layer longer than a certain distance does not significantly improve the global loss factor while linearly increasing the weight. Furthermore, the thickness of the damping layer presents an intermediate optimal value: below the optimal thickness, the damping layer does not provide sufficient damping, and above the optimal thickness it induces an additional discontinuity at which waves reflect back. Fig. 3 shows the input mobility of three different beams: the truncated black hole beam with optimised damping layer, a uniform beam of same length covered with the same amount of damping layer and a uniform beam of same length entirely covered with damping layer. It can be observed that a reduction of up to 20 dB in the maximum vibration level is reached with the acoustic black hole beam.

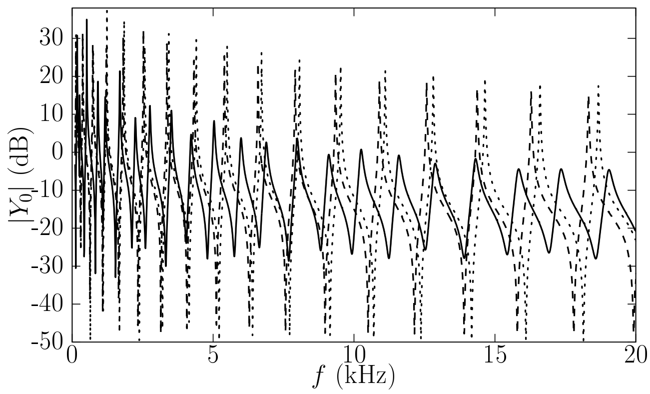

Figure 3: Simulated driving-point mobilities of a beam [5]. ——, black hole beam; $\cdots\cdots$, uniform beam covered with damping layer in the black hole area; - - - - -, uniform beam entirely covered with damping layer.

4 Enhancement of the acoustic black hole effect by a thermal gradient

As shown in Eq. (1), the phase velocity of flexural waves depends on the Young's modulus of the beam, which can be made to vary with distance using a thermal load in order to produce an acoustic black hole. Fig. 4 shows the input impedance of a polymer beam subjected to a thermal load, compared to the case with uniform temperature. It can be observed that the vibration level is significantly reduced and the resonant behaviour of the beam is entirely eliminated.

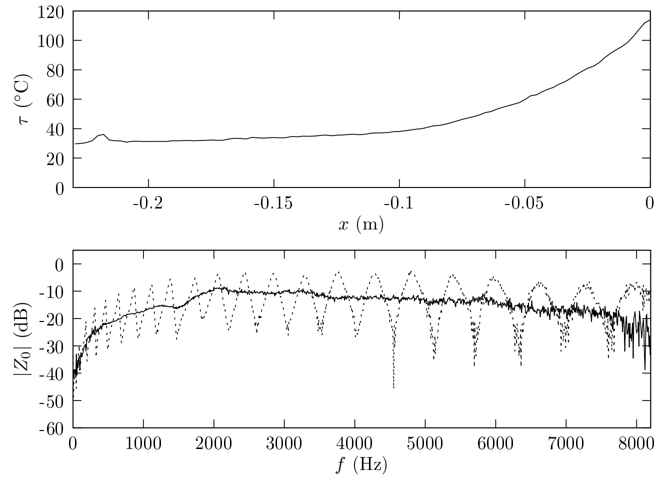

Figure 4: Measured temperature profiles and driving-point impedances for the uniform beam. Top, temperature gradients; bottom, $\cdots\cdots$, reference input impedance with no thermal load; ——, input impedance with thermal load.

5 Summary

In this paper, two tools for improving the acoustic black hole effect in beams have been proposed. First, a numerical model of the flexural vibrations of arbitrarily inhomogeneous beams has been developed and used as a design tool for acoustic black holes. Second, an enhancement of the acoustic black hole effect is achieved by imposing a temperature gradient to a beam, which induces a Young's modulus gradient and thus controls the wave velocity. Both cases show a significant reduction of the vibration level and of the resonant behaviour of the structure.

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