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Petr Yuldashev, Edouard Salze, Sébastien Ollivier, Mikhail Averiyarov, Vera Khokhlova, et al.. Propagation of spherically diverging N-waves in a turbulent atmosphere: experiment. Acoustics 2012, Apr 2012, Nantes, France. <hal-00811157>

HAL Id: hal-00811157

<https://hal.science/hal-00811157v1>

Submitted on 23 Apr 2012

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ACOUSTICS 2012

Propagation of spherically diverging N-waves in a turbulent atmosphere: experiment

P. V. Yuldashev^a, E. Salze^a, S. Ollivier^a, M. Averiyarov^b, V. Khokhlova^b and P. Blanc-Benon^a

^aLaboratoire de Mecanique des Fluides et d'Acoustique, 36 Av Guy de Collongue 69134 Ecully Cedex

^bM.V.Lomonosov Moscow State University, Faculty of Physics, M.V.Lomonosov Moscow State, University, Leninskie Gory, 119991 Moscow, Russian Federation
petr@acs366.phys.msu.ru

Studies on propagation of high amplitude N -waves in turbulent atmosphere are relevant to the sonic boom problem which involves high interest due to development of new civil supersonic aircrafts. In this problem it is important to predict the effect of turbulence on the statistical characteristics of random variations of the wave amplitude and shock rise time. In the present work, in relation to the sonic boom problem, the propagation of high amplitude spark-generated N -waves through a layer of thermal turbulence was studied in a model laboratory-scale experiment which is more controlled and reproducible than field measurements. Evolution of statistical distributions of the wave amplitude and shock rise time was investigated at different propagation distances from the spark source. The results were compared to experimental data known from the literature, where N -waves were propagated through a kinematic turbulence. It was shown that for similar propagation distances, turbulent length scales, and index fluctuations in both experiments, kinematic turbulence leads to stronger distortion of the N -wave field. Moreover, with kinematic turbulence the probability to observe intense focusing at caustics is to 2 or 3 times greater than with thermal turbulence.

1 Introduction

Generation and propagation of noise from supersonic aircrafts is an important problem in modern aeroacoustics due to interest in development of new civil supersonic aircrafts [1]. Due to nonlinear effects, a multi-shock wave transforms to a N -wave at some distance from the airplane and propagates through the inhomogeneities of the atmosphere boundary layer [2], which leads to random redistribution of the acoustic field on the ground [3]. The noise level on the ground is therefore a random variable which statistical characteristics depends on properties of atmospheric inhomogeneities. Random variations of the wave amplitude and the shock rise time, which determine subjective loudness [4], can be very significant [5]. To provide ecological security, it is necessary to predict statistical characteristics of random noise of sonic boom on the ground in different atmospheric conditions.

At first, sonic booms were studied experimentally during outdoor flight tests [3, 5]. The main advantage of the outdoor experiments is that the sonic boom problem is studied in its natural conditions. However, the field measurements have important disadvantages: a lack of control on atmosphere conditions; a poor knowledge of turbulence along propagation path of the N -wave; difficulties to acquire sufficiently large statistical database. These inherent drawbacks complicate the analysis of the influence of different physical effects on the statistics of N -wave distortions. As an alternative to outdoor measurements, the study of N -wave propagation through turbulent media can be carried out using laboratory-scale experiments [6–8]. In the model experiments, characteristic scales of the real atmosphere are down-scaled to fit laboratory scale. The downscaling factor is generally between 1:10000 and 1:1000. In such experiments, the turbulence, the acoustic source and the geometry are well controlled.

Distortions of statistics of N -wave parameters (peak positive and negative pressure, rise time, arrival time) depend on type of atmosphere inhomogeneities, which can be scalar or vector type. Scalar-type inhomogeneities of sound speed are the result of temperature fluctuations. Kinematic (vector-type) inhomogeneities are related to fluctuations of mean wind due to vortex formation. Homogeneous isotropic turbulence fields of scalar and vector types have different spectra and properties [9]. Therefore, according to the theoretical analysis of linear wave propagation [10] done using approximation of geometrical acoustic, this difference in spectra leads to different impact of turbulence on the statistics of the wave distortions, even if the acoustical refraction index rms μ_{rms} and the outer scale L_0 are equal for both types of turbulence.

The reliable experimental comparative data are not yet reported in the literature, except some preliminary results [7].

In this paper a model experiment on propagation of high-amplitude N -wave through a layer of thermal (scalar-type) turbulence is presented. Statistical distributions of amplitude and shock rise time of the N -wave were investigated and broadening of distributions with increasing propagation distance was shown. Statistical data of aforementioned parameters of N -wave were compared to statistics obtained in another experiment known in literature, where N -wave was propagated through kinematic turbulence (M.V. Averiyarov *et al.*, 2011) [8]. It was shown that in the case of turbulent layers with the same parameters (width, characteristic scales, rms values of the fluctuations of refractive index), the kinematic turbulence leads to stronger distortions of the peak pressure and the shock rise time of the N -wave.

2 Experimental setup

The schema of the experiment is shown in Fig. 1. A spherical N -wave generated by a spark source propagates through the field of thermal turbulence created by a special heating grid. The signal is recorded with microphones located at the same height as the source at the distance r from it. As a result of the propagation through the inhomogeneous medium the initial N -shaped wave is distorted. For example, in the focusing zones (caustics) a U -shaped waveform is formed.

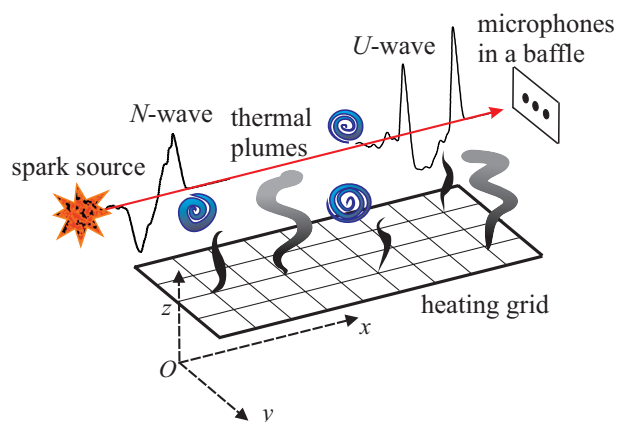


Figure 1: Schematic view of the experiment on the propagation of N -wave in thermal turbulence.

The experimental setup was placed in the anechoic chamber of the Acoustic Center of the Laboratory of Fluid Mechanics and Acoustics (LMFA) in Ecole Centrale de Lyon

(ECL). It was equipped by a three-dimensional positioning system. The thermal turbulence was generated using a heating grid of resistors. The dimensions of the grid, which was located at height of 1.2 m above the floor, were $4.4 \times 1.1 \text{ m}^2$. To measure the temperature two thermal probes Dantec type 55P31 were used. Signals from the thermal probes were recorded during 30 sec.

For the acoustic measurements, two 1/8 inch broadband condenser microphones (Brüel & Kjær, model 4138) were connected to preamplifiers (B & K type 2670) and 4-channel amplifier (B & K Nexus with an extended bandwidth up to 200 kHz). The microphones were mounted in a rectangular baffle to postpone diffractive wave. The microphones were separated by an interval of 2 cm along the direction perpendicular to the source-microphones line. Signals were digitized using a National Instruments digital acquisition card (PXI 1033) with sampling rate of 10 MHz and a digitization of 12 bits. Calibration of the microphones was carried out according to the method described in [11]. During measurements ambient temperature, barometric pressure, and relative humidity were controlled.

3 Characteristics of the thermal turbulence

Temperature fields were measured in the horizontal plane (x, y) , located at the height $z = 1640 \text{ mm}$ above the grid. Spatial scans were performed on different days in three areas; the obtained data were then concatenated into a single map. The mean-square $\mu_{\text{rms}} = \Delta c_{\text{rms}}/c_0 = 0.5T_{\text{rms}}/ <T>$ of the refractive index fluctuations corresponding to the measured temperature data are shown in Fig. 2. Here c_0 is the mean sound speed, Δc_{rms} is the rms of sound speed, T_{rms} is the rms of temperature, and $<T>$ is the mean value of temperature.

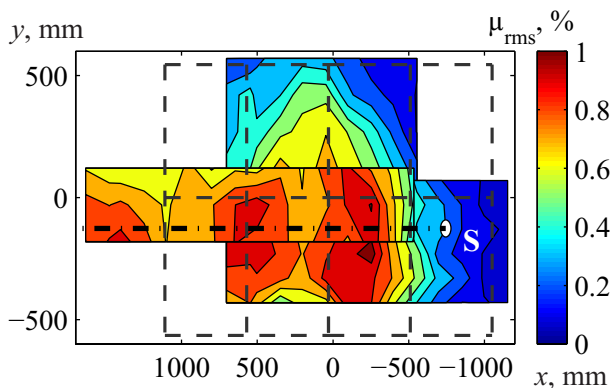


Figure 2: The rms of the refractive index fluctuations μ_{rms} in a horizontal plane. The source position is shown by a white dot S. "Source- microphones" line is traced by a thick dash-dotted line. Contours of two sections of the grid are marked by thin dashed lines.

The spark source was placed in the point with coordinates $(x, y) = (-745, -126) \text{ mm}$, which is marked by a white circle at point S in Fig. 2. The propagation path of the N-wave (source-microphones line) is marked in Fig. 2 by a thick dash-dotted line. Average level of mean-square of the refractive index fluctuations μ_{rms} along the propagation path was estimated to be equal to 0.85%. The "effective" width of turbulent layer for $r = 200 \text{ cm}$ was about 180 cm.

The next step in the characterization of the turbulence was the determination of the outer scale L_0 . For this purpose, correlation measurements were carried out in a horizontal direction along the axis x near the point with coordinates $(x, y) = (175, -30) \text{ mm}$. Evaluating the correlation as $\langle T_1(x + \Delta x)T_2(x) \rangle / T_{1\text{rms}}T_{2\text{rms}}$, where T_1 and T_2 are signals of the first and the second thermal probes, the averaging time was 90 sec. The value of outer scale L_0 was obtained as the best fit of experimental correlation data by a theoretical curve. Theoretical correlation was calculated from von Kàrmàn model spectrum [9]:

$$G(k) = 0.7924 \langle \mu^2 \rangle \frac{k^2}{L_0^{2/3}} \frac{\exp(-k^2/k_m^2)}{(k^2 + k_0^2)^{11/6}}, \quad (1)$$

where $\langle \mu^2 \rangle = 0.25 \langle T_{\text{rms}}^2 \rangle / \langle T \rangle^2$, $k_0 = 1/L_0$, $k_m = 5.92/l_0$, and l_0 is the inner scale.

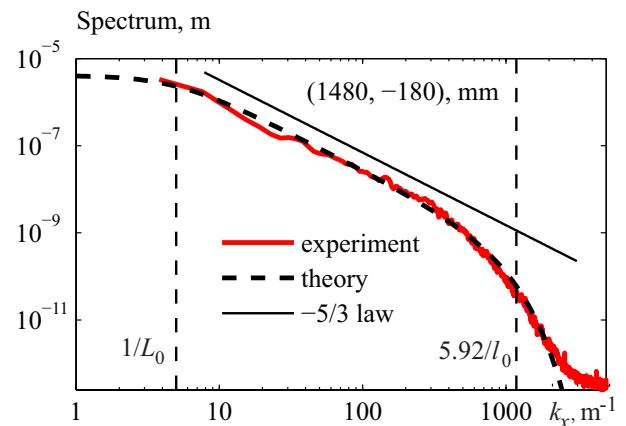


Figure 3: Comparison of one-dimensional experimental spectrum (red line) with theoretical spectrum (dashed line) derived from von Kàrmàn model with $L_0 = 20 \text{ cm}$ and $l_0 = 5 \text{ mm}$. Straight line corresponds to Kormogorv's $k^{-5/3}$ law.

Theoretical and experimental one-dimensional spectra of refractive index are compared in Fig. 3. For the theoretical spectrum, the parameters $L_0 = 20 \text{ cm}$ and $l_0 = 5 \text{ mm}$ were set. The mean-square $\langle \mu^2 \rangle$ for the theoretical spectrum is taken from the experimental data. Experimental and theoretical spectra fairly well correspond with each other in a range of scales between outer and inner scale. Decrease of experimental spectra in the inertial range is very well described by Kolmogorov's $k^{-5/3}$ law.

Thus, measurements of temperature fluctuation showed, that thermal turbulence generated in our experiment had $\mu_{\text{rms}} = 0.85\%$ on average along the acoustical propagation path. The spectrum of the turbulence is well described by von Kàrmàn model with an outer scale of 20 cm and an inner scale of 5 mm.

4 Acoustical measurements results

4.1 Propagation of the N-wave in still air

Parameters of the N-wave in homogeneous air were determined according to the method described in [11]. Locations of microphones were the same as in measurements in the presence of turbulence. There were 9 source-microphones distances in the range from $r = 40 \text{ cm}$ up to $r = 200 \text{ cm}$ with an increment of 20 cm. It was found that the amplitude $p_0 =$

430 ± 25 Pa and the half-duration $T_0 = 21.8 \pm 0.2 \mu\text{s}$ at distance $r = 40$ cm from the source. The following Tab. 1 summarizes the experimental data at several distances r : peak positive pressure, half-duration and rise time. The values indicated in the Tab. 1 are obtained by averaging of 200 waveforms at each given distance r .

Table 1: Parameters of the N -wave, measured in homogeneous air at several distances r .

r , cm	40	80	120	140	180	200
$p_{\max 0}$, Pa	430	194	121	100	73	64
T , μs	21.8	23.3	24.3	24.6	25.0	25.1
τ_{sh} , μs , exp.	3.26	3.32	3.41	3.46	3.56	3.59
τ_{sh} , μs , th.	0.48	0.85	1.24	1.43	1.80	1.97

The rise time of the N -wave was defined using the width of the peak of the waveform derivative [8]. A threshold level equal to e^{-1} was set to calculate the width of the derivative peak. For the case of N -wave measurements in homogeneous air, this definition corresponds to the rise time, defined alternatively as the time during which the acoustic pressure on the shock increases from 0.05 to 0.95 of the peak positive pressure. The definition using waveform derivative was applied further to the N -waves distorted after propagation through turbulent layers.

The theoretical rise time (last row of the Tab. 1) was calculated using numerical simulation of spherical N -wave propagation [12]. This theoretical rise time is much smaller than the experimental rise time. Discrepancy between experimental and theoretical rise times is explained by the limited frequency response of the measurement system and was largely discussed in [11, 12].

4.2 Propagation of the N -wave through thermal turbulence

Typical examples of distorted waveforms measured after propagation through a turbulent layer are shown in Fig. 4. The obtained examples are in good agreement with sonic booms, measured in the atmosphere [5] and waveforms from other model experiments [6, 8]. For example, even after the large distance ($r = 2$ m) of propagation through the turbulence the classical N -waveform can be maintained (Fig. 4a). When crossing a caustic a high amplitude U -wave is formed (b). Scattering by inhomogeneities increases the rise time of the shock up to $10 - 15 \mu\text{s}$ (c). In defocusing areas, jagged waves of small amplitude and very long duration can be detected (d).

One of the most important characteristics of the N -wave that determines its noise impact, is the peak positive pressure p_{\max} [4]. Fig. 5 shows the histograms of the peak positive pressure, normalized to the values measured in homogeneous medium, $P_+ = p_{\max}/p_{\max 0}$, corresponding to the four different distances r . On each histogram the mean value m and standard deviation s are given. Histograms show conventional broadening of the statistical distributions with increasing distances r , accompanied by an increase of the standard deviation and a decrease of the mean value. At short distances from the source, the distributions are almost symmetrical, but far from the source the maximum of the distribu-

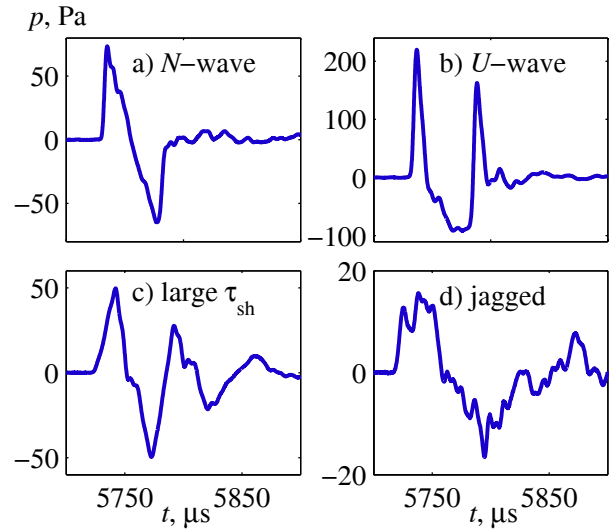


Figure 4: Typical distorted waveforms measured at a distance $r = 200$ cm from the source.

tion is shifted toward smaller values of the amplitudes and at high amplitudes a long "tail" is formed. One can note, that at large distance ($r = 2$ m) there is non-null probability to observe pulses with p_{\max} which is 3-3.5 times higher than the $p_{\max 0}$ measured in homogeneous media. This is the effect of random focusing in the turbulence.

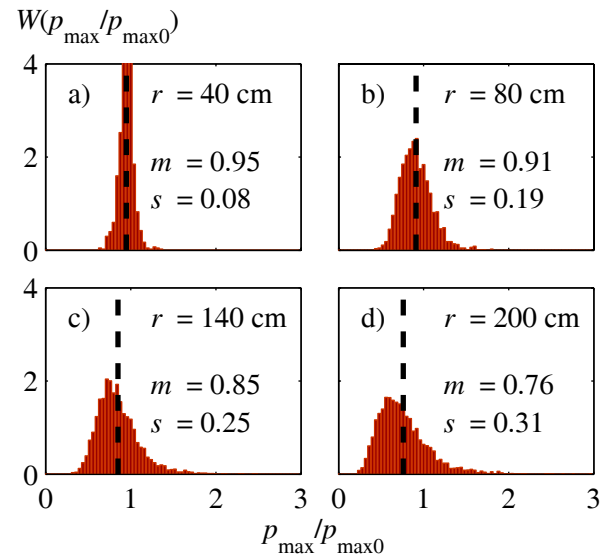


Figure 5: Histograms of the statistical distributions of the normalized peak positive pressure. Black dashed line shows the mean value. The numerical value of the mean is denoted by m and of the standard deviation by s .

Histograms of the rise time τ_{sh} are given in Fig. 6. As for peak positive pressure, conventional broadening of the statistical distributions with increasing distances r is observed. Mean value and standard deviation of τ_{sh} also increase. Due to limited bandwidth of B&K 4138 microphones, shocks with rise time less than $2.7 \mu\text{s}$ are not observed. Statistical distributions for small values of the rise time are distorted: the waveforms with steep shocks are accounted as waveforms with $2.7 \mu\text{s}$ rise time. So, noise effect of sonic boom is underestimated in the given measurements. At large propagation distances (Fig. 6d) the maximum observed rise time is of 15-

20 μs , which is consistent with results obtained in another study [8]. A significant increase of the shock rise time of strongly distorted waveforms is very important for decreasing of noise effects of the sonic boom, since the wave with smoothed shock front causes less discomfort [4].

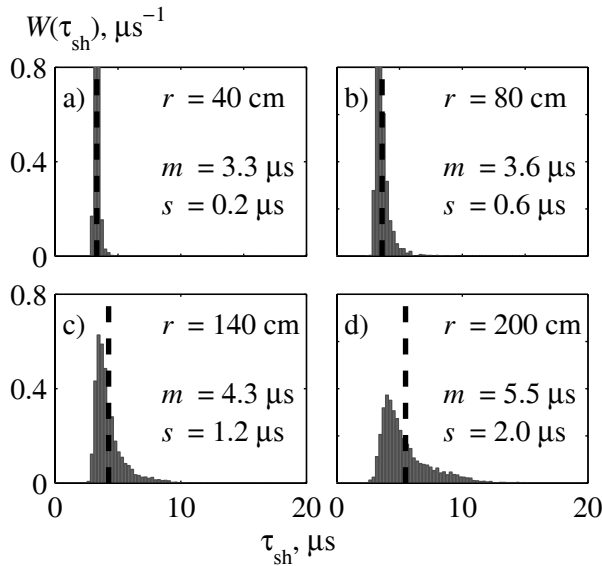


Figure 6: Histograms of the statistical distributions of the shock rise time, given in microseconds. Notations are the same as in Fig. 5.

5 Comparison of statistical distributions of N -wave parameters in thermal and in kinematic turbulence

The authors of the study [8] provided the database of the experiment on the propagation of an N -wave in kinematic turbulence. In this experiment, measurements were made at different levels of turbulent fluctuations of the refraction index, namely at $\mu_{rms} = 0.33, 0.47, 0.56, 0.74, 0.89$ and 1% . In the main series of measurements, microphones were placed at a distance of 220 cm from the spark source, and the "effective" width of the fully developed turbulence layer was about 180 cm. Thus, in both experiments the width of the turbulent layers was approximately the same. The outer scale of kinematic turbulence was also $L_0 = 20$ cm. The inner scales were different: 5 mm for the thermal turbulence and 1.3 mm for the kinematic turbulence. However, this difference of inner scales is not critical, since the amplitude of the corresponding inhomogeneities is very small in comparison to the most energetic large-scale inhomogeneities.

The statistical distribution functions for the normalized peak pressure $P_+ = p_{max}/p_{max0}$ (a) and the shock rise time τ_{sh} (b) are compared in Fig. 7. Additionally, in Tab. 2 the mean and standard deviation for these wave parameter, as well as the probability of exceeding thresholds of P_+ equal to 1, 1.5 and 2 are presented.

First we consider series of measurements, where the rms values of the refractive index μ_{rms} in both experiments are approximately the same (0.85% for thermal and 0.89% for kinematic turbulence). Presented results show, that the distribution function of P_+ in the case of kinematic turbulence

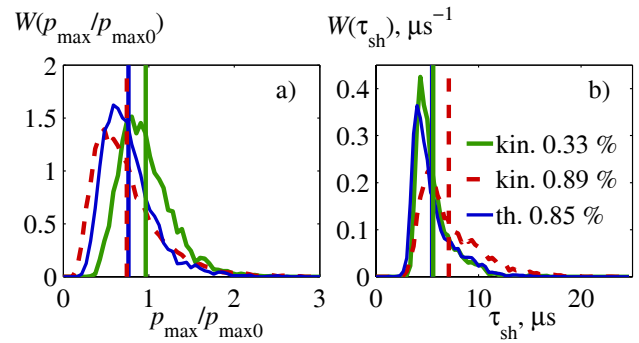


Figure 7: Comparison of the probability distributions of the normalized peak positive pressure $P_+ = p_{max}/p_{max0}$ (a) and the shock rise time τ_{sh} (b) between the cases of the N -wave propagation in turbulent air flow (red line, $\mu_{rms} = 0.89\%$, green line, $\mu_{rms} = 0.33\%$) and in thermal turbulence (blue line, $\mu_{rms} = 0.85\%$).

Table 2: Comparison of the mean value (noted by bar) and standard deviation (δ) of the normalized peak positive pressure P_+ and the shock rise time τ_{sh} . Cumulative probability $W(P_+ > \alpha)$, % of exceeding some value α is denoted here as P_+^α .

turb., μ_{rms}	\bar{P}_+	δP_+	$\bar{\tau}_{sh}$	$\delta \tau_{sh}$	P_+^1	$P_+^{1.5}$	P_+^2
1 t., 0.85%	0.76	0.3	5.5	2.0	18	2.9	0.4
2 k., 0.89%	0.74	0.4	7.1	2.8	20	5.1	1.3
3 k., 0.33%	0.97	0.3	5.6	1.8	39	5.8	0.7

is wider (Fig. 7a) and has higher standard deviation for the same mean value $\langle P_+ \rangle$ (compare lines no. 1 and no. 2 in Tab. 2). The most important differences can be noted by comparing the cumulative probability P_+^α of exceeding threshold α . For example, the probability of exceeding the amplitude in homogeneous medium, P_+^1 , is about the same in both cases (18% and 20%). However, for higher thresholds $\alpha = 1.5$ and $\alpha = 2$ the kinematic turbulence gives a 2-3 times greater probability (see $P_+^{1.5}$ and P_+^2) than the thermal turbulence. Thus, kinematic turbulence is much more effective in terms of the appearance of intense focusing. Meanwhile the kinematic turbulence also leads to more efficient smoothening of the shock front (Fig. 7b). The corresponding distribution function (red line) is shifted toward larger values of τ_{sh} , has larger mean value and standard deviation than the distribution function in the case of the thermal turbulence (blue line).

Also we consider here two cases with comparable degree of distortion of the N -wave parameters in turbulence fields of two different types. For example, it is the case of kinematic turbulence for $\mu_{rms} = 0.33\%$, which is shown in Fig. 7 by green line. In Tab. 2 relevant data are shown in line no. 3. Although the mean peak pressure differs, the width of the distributions, defined by the standard deviation, is the same ($\delta P_+ = 0.3$). Nevertheless, as previously, the probabilities of observing N -waves with large amplitudes, $P_+^{1.5}$ and P_+^2 , in the kinematic turbulence case are 2 times greater than in the case of thermal turbulence. For the shock rise time, the distribution functions were found to be very close each to another (Fig. 7b). Thus, comparable level of distortions of the N -

wave propagating in the kinematic turbulence is achieved at significantly lower turbulence intensity than in thermal turbulence (μ_{rms} is smaller by 2-3 times).

6 Conclusion

Propagation of high amplitude N -pulses (430 Pa at 40 cm from the source) in thermal turbulence was investigated experimentally. Measurements of turbulent field showed that the spectra of refraction index fluctuations are well described by the von Kàrmàn model spectrum with the following parameters: outer scale $L_0 = 20$ cm, inner scale $l_0 = 5$ mm, and mean-square value of $\mu_{\text{rms}} = 0.85\%$ on average.

Statistical distributions of the peak positive pressure and rise time of the N -wave were obtained at different propagation distances from the source in the range from 40 cm to 200 cm. Classical effects of broadening of the probability distributions of the N -wave parameters and trends for the mean and standard deviation with increasing propagation distance were demonstrated [6,8,13,14]. The appearance of strong focusing in the caustics, where the waveform is U -shaped and its amplitude is more than 3 times greater than the N -wave amplitude in homogeneous media, was also observed.

The results of the present experiment were compared to the results of the experiment on N -wave propagation in kinematic turbulence [8]. For similar conditions in terms of acoustic source and turbulence parameters (characteristic scale L_0 , turbulence intensity μ_{rms} , propagation distance) it is shown that kinematic turbulence leads to much stronger distortions of the peak pressure and the shock rise time of the N -wave. In addition, the kinematic turbulence was shown to be much more efficient in terms of appearance of strong focusing: in comparison to the case of thermal turbulence, it produces 2-3 times greater cumulative probability for the peak positive pressure to exceed two times the amplitude of the N -wave propagated in homogeneous air. Kinematic turbulence also leads to stronger smearing of the shock front. Comparable level of distortions of the peak pressure and the shock rise time is achieved in the kinematic turbulence at much lower values of turbulence intensity – for μ_{rms} 2.5 times lower than in the thermal turbulence.

In perspective, the obtained comparative results of the two experiments should be validated using numerical modeling of N -wave propagation at least in 2D geometry. Preliminary numerical simulations have been done using computational resources provided by MSU supercomputer center.

Acknowledgments

This work was partially supported by grants of Projet International de Coopération Scientifique (PICS RFBR 10-02-91062/CNRS 5603), and by special stipend of the French Government for the preparation of the dissertation under the co-supervision. The authors would like to thank Jean-Michel Perrin and Emmanuel Jondeau for their help in setting up the experiment.

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