Auxetic transverse isotropic foams: from experimental efficiency to model correlation

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The wide use of porous materials in vibro-acoustics led up to study a novel material that can exhibit interesting behaviours for vibro-acoustics applications, and possibly improve the efficiency of performances. This paper is focused on the analysis of absorbing foams, which are rendered auxetic (Negative Poisson's ratio) thanks to a specific forming process. It first illustrates the efficiency of auxetic foams compared to melamine samples using experimental results. Then a study is conducted in order to improve the identification of mechanical and coupling modelling parameters for the considered auxetic transverse isotropic foam. The method associates a preliminary parameters sensitivity analysis with an optimization study. A global sensitivity analysis of the outputs of interest is performed using the Fast technique in order to estimate the first-order and total effects of the numerous parameters of the model. The results of the analysis are then used to perform the optimal identification of the parameters by readjusting finite elements analyses results over experimental data. The results and benefits of the preliminary use of parameters sensitivity analysis associated with optimization are finally presented.

1 Introduction

Porous materials are widely used in vibro-acoustics for many reasons. Their low cost and good performances in terms of noise attenuation or energy impact dissipation, justify its use in many sectors like transports and building. In this paper we focus on the characterization of an auxetic foam, which exhibits a negative Poisson's ratio. This kind of material raised some interest in the scientific community for potential engineering and sound management applications due to their unconventional mechanical and acoustic properties. Theses unusual acoustic absorption properties can be observed particularity at lower frequencies compared to conventional open cell foams [1]. The particular hinge-like structure of auxetic materials justify theirs negative Poisson's ratio. Such materials are expected to have mechanical properties such as high energy absorption and fracture resistance been able to be used for a variety of applications (personal protection clothing, packing material, robust shock absorbing material, sponge mops and filtration). The isotropic theory allows the use of Poisson's ratio between -1 and 0,5. Some auxetic configurations reach a Poisson's ratio value less than -1.

In this work we try to identify some mechanicals and acoustic foam parameters values including the Poisson's ratio for conventional melamine foam and isotropic transverse auxetic foam. The models used to describe the behaviours of porous materials require knowledge of many parameters. In general, 5 to 9 parameters are used in the most popular models like Johnson-Allard and Biot-Allard[2, 3]. These models have provided to generations of researchers the framework for the constitutive poroelasticity equations, describing in particular the stress wave propagation coupling a fluid phase with a solid phase in poroelastics solids at low and high frequencies [4, 5]. Equations of motion for anisotropic poroelastic media are also available in the litterature [6, 8]. These models can be used in analytical analyses for simple cases or in finite element models for more complex cases.

Therefore, the difficulty to determine experimentally these parameters led us to seek a way to identify their values with satisfactory accuracy to drive some design studies. Our hope is that with a quite simple method of characterization, we could be able to take the necessary information for the practical utilisation of this material in models, noise controls studies and other kind of studies that require knowledge of the parameters.

Based on Johnson-Allard theory [2, 3] we use a model to obtain absorption coefficient and acoustic impedance for foams. With this model, we can compare the values obtained with experimental data measured in a Kundt tube in order to update the parameters' values. A model and some experimental measurements were made for auxetic foam and non-auxetic foam (melamine) to allow us to compare both materials. Then a sensitivity analysis using the FAST method [9] was done with the purpose of facilitating the optimization of parameters identification by readjusting analytical results over experimental data.

2 Open porous media model description:

The porous media are described as the superposition of two media, namely a fluid part and a solid part. Small perturbations and small fluid evolutions are assumed. The solid skeleton forms a continuous network of interconnected pores saturated by the fluid phase and respecting Hook's law of linear-elasticity. An elementary volume is considered and the elementary part occupied by the fluid part is called porosity (ϕ), which is one of the coupling parameters studied here. The other coupling parameters from the Biot theory [4, 5] are resistivity (σ), tortuosity (α_∞), viscous and thermal characteristic length (Λ *and* Λ'), while the mechanical parameters are Young's modulus (E), Poisson's ration (ν), loss factor (η) and frame density (ρ). When the porous structure is anisotropic the description has to take it into acount [7] and a general time domain mixed displacement-pressure formulation for acoustic anisotropic open porous media is available in the literature [8]. In this work, we use a displacement formulation adapted for transversally isotropic poroelastic media [2] for the description of the behaviour of the PU-PE samples.

In this formulation, the displacements of interest are the frame displacement $\mathbf{u}^s$ and the fluid-discharged displacement vector $\mathbf{w} = \phi\left(\mathbf{u}^f - \mathbf{u}^s\right)$ where $\mathbf{u}^f$ is the fluid displacement. Using $\zeta = -\mathbf{div}$ w, the total stress components are given by:

$$\sigma^t_{xx} = (2G + A)\, e_{xx} + Ae_{yy} + Fe_{zz} + -K_f \frac{(\theta_s - \zeta)}{\phi} \quad (1)$$

$$\sigma^t_{yy} = Ae_{xx} + (2G + A)\, e_{yy} + Fe_{zz} + -K_f \frac{(\theta_s - \zeta)}{\phi} \quad (2)$$

$$\sigma^t_{zz} = Fe_{xx} + Fe_{yy} + Ce_{zz} + -K_f \frac{(\theta_s - \zeta)}{\phi} \quad (3)$$

$$\sigma^t_{yz} = 2G' e_{yz} \quad (4)$$

$$\sigma^t_{xz} = 2G' e_{xz} \quad (5)$$

$$\sigma^t_{xy} = 2Ge_{xy} \tag{6}$$

where e_{ij} is the strain component defined by

$$e_{ij} = 1/2\left(\partial u^s_i/\partial x_j + \partial u^s_j/\partial x_i\right). \tag{7}$$

The A, F, G, C and G' coefficients are the stiffness coefficients and for the isotropic case we have $G = G'$, $F = A$, $C = (A + G)$. Using engineering notations[7], the Young moduli are denoted by E_x, E_y, E_z and can be rewritten $E_x = E_y = E$, $E_z = E'$ for isotropic transverse foams. The Poisson's ratio are denoted as ν_{yx}, ν_{zy}, ν_{zx} and satisfy the relations $\nu_{yx} = \nu$ et $\nu_{zy} = \nu_{zx} = \nu'$.
The total stress is defined by the relation between the frame stress tensor (σ^s_{ij}) and the fluid phase stress tensor (σ^f_{ij}):

$$\sigma^t_{ij} = \sigma^s_{ij} + \sigma^f_{ij}. \tag{8}$$

The wave stress-strain relations in first representation of the Biot theory can be written with Eq. (9) and Eq. (10) :

$$\frac{\partial\sigma^s_{xi}}{\partial x} + \frac{\partial\sigma^s_{yi}}{\partial y} + \frac{\partial\sigma^s_{zi}}{\partial z} = -\omega^2\left(\tilde{\rho}^i_{ss}u^s_i + \tilde{\rho}^i_{sf}u^f_i\right) \tag{9}$$

$$\frac{\partial\sigma^f_{xi}}{\partial x} + \frac{\partial\sigma^f_{yi}}{\partial y} + \frac{\partial\sigma^f_{zi}}{\partial z} = -\omega^2\left(\tilde{\rho}^i_{ff}u^f_i + \tilde{\rho}^i_{sf}u^s_i\right) \tag{10}$$

where

$$\tilde{\rho}^i_{ff} = \phi\rho_0 - \tilde{\rho}^i_{sf} \tag{11}$$

$$\tilde{\rho}^i_{ss} = \rho_1 - \tilde{\rho}^i_{sf}. \tag{12}$$

In these equations ρ_0 and ρ_1 are respectively the densities of air and frame in the vacuum and $\tilde{\rho}^i_{sf}$ is a parameter depending on the nature and the geometry of the porous medium and the density of the fluid [2].

3 Sensitivity analysis

The sensitivities of the parameters on the acoustic performance of porous materials depend on the frequency and the nature of the material. The FAST Method (Fourier Analysis Sensitivity Test) is a global sensitivity analysis method based on variance decomposition [9]. The method estimates the portion of variance due to one parameter alone (first order) or in cooperation with others (other orders) on the absorption coefficient and acoustic impedance which are frequency dependent. The total sensitivity indices are defined as the sum of all partial order indices and represent the total sensitivity including all coupling effects between parameters. Following former studies [10] we have applied an analysis of sensitivity for the melamine foam. The study has a huge importance to decide the strategies for model updating. The results will be shown in section 5.1.

4 Optimisation

The optimization problem consists in minimizing a function G by choosing input parameters values from an allowed set and computing the value of the function. The function G allows us to update the computed values in order to find the best value of parameters associated to the Biot model in order to fit the experimental data obtained in the Kundt tube for (acoustic impedance and absorption coefficient).

The weighted cost function G for n tests on N_f frequency samples is:

$$G = \sum_{i=1}^{N_f} w_i g(w_i) \tag{13}$$

where

$$g(w) = \left|\frac{\alpha(w) - \alpha_e(w)}{\alpha_e(w)}\right| + \left|\frac{Z(w) - Z_e(w)}{Z_e(w)}\right| \tag{14}$$

and

$$\alpha_e = \frac{1}{n}\sum_{i=1}^{n}\alpha_e^{(i)}. \tag{15}$$

The optimization has been performed with the fonction "fmincon" (Find minimum of constrained nonlinear multivariable) from the software Matlab [11]. The optimal identification of the parameters is made by readjusting finite elements analyses results over experimental data.

5 Method implementation

We have first implemented the identification technique for melamine isotropic foam before trying to apply it for auxetic isotropic transverse foam.

5.1 Melamine foam

Based on values found in the literature we have defined the limits for the analysis as shown in Table 1 in order to apply the sensitivity analysis using the FAST method. The analysis has been performed for the mechanicals parameters E, ν, ρ, η and the coupling parameters Φ, σ, α_∞, Λ, Λ'.

Coupling parameters:			
Parameters	Unit	Lower bounds	Upper bounds
Φ	[-]	0.98	0.99
σ	[N.s.m^{-4}]	9000	11000
α_∞	[-]	1	1.03
Λ	[μm]	80	100
Λ'	[μm]	100	300
Mechanicals parameters			
Parameters	Unit	Lower bounds	Upper bounds
E	[kPa]	100	300
ν	[-]	0.14	0.45
ρ	[Kgm$^-$3]	8.5	14.5
η	[%]	5	15

Table 1: Bounds for Melamine poroelastic foam sensitivity analysis

The results of the analysis are presented in Figure 1 for mechanical parameters and in Figure 2 for coupling parameters. The Sensitivity indices are represented by SI for First order indices and STI for Total indices.

These results give the possibility of testing different updating strategies. In a first simulation (Simulation 1) we optimized all parameters together, while in a second updating process (Simulation 2) we optimised only the most sensitive parameters (Λ', ν, E, ρ) assuming for the others parameters an average value considering Table 1. For Simulation 3 some

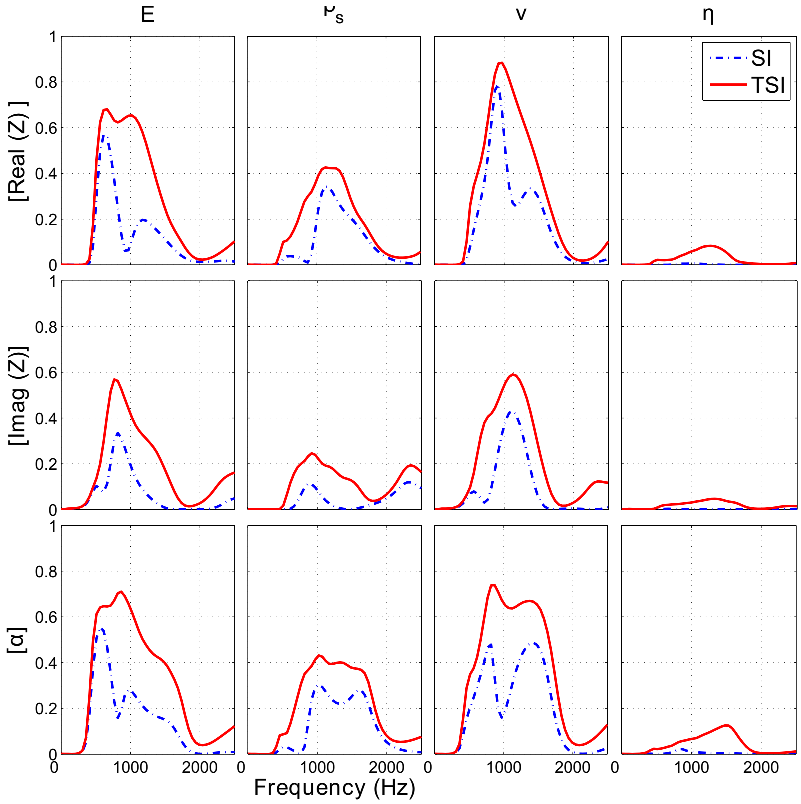

Figure 1: Mechanicals parameters sensitivity analysis results for Melamine foam.

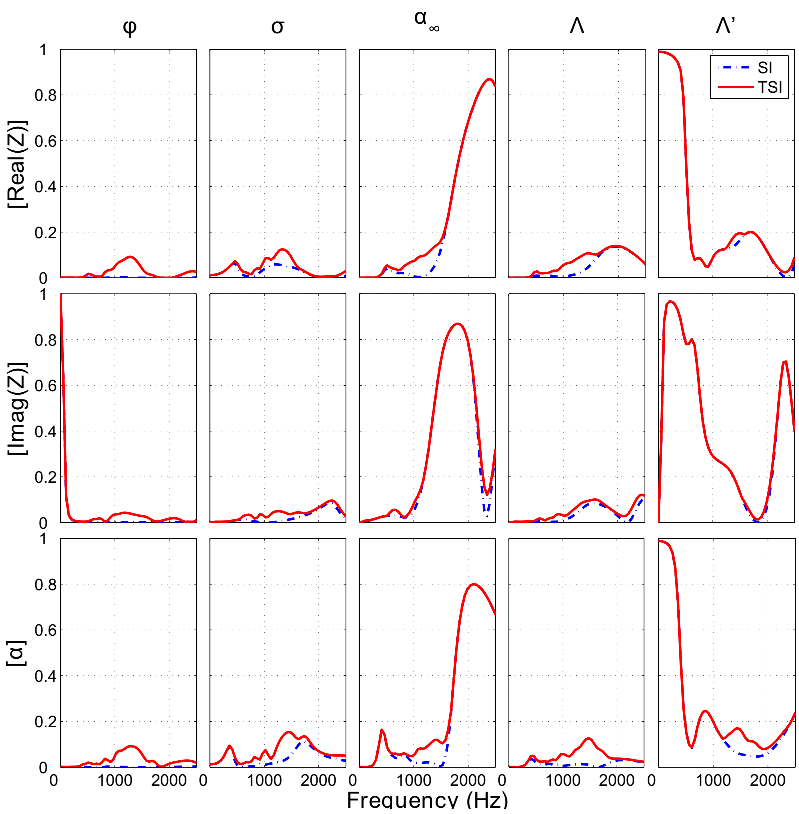

Figure 2: Coupling parameters sensitivity analysis results for Melamine foam.

less sensitive parameters (α_∞, σ, Λ) are added in the optimisation after the optimisation of the more important parameters maintaining only the less sensitive parameters (ϕ, η) defined by average values. Finally in the last updating strategy (Simulation 4) we optimized all parameters in groups of equivalent sensitivity, starting from the higher to the lowest values of indices.

The optimization analysis results are presented in Table 2. The four simulations gave satisfactory results compared with

Parameters	Simulation			
	1	2	3	4
$\Lambda'[\mu m]$	168	177	177	177
E[kPa]	213	207	207	207
ν[-]	0,302	0,299	0.299	0.299
ρ[Kgm⁻3]	14,5	14,5	14,5	14,5
α_∞[-]	1	1	1	1
σ[N.s.m^{-4}]	11000	10000	11000	11000
$\Lambda[\mu m]$	99.9	89,9	91,4	91,4
η[%]	0,138	0,100	0,100	0,056
Φ[-]	0,990	0,985	0,985	0,990

Table 2: Results of model updating for Melamine foam

experimental data. We had better results for Simulations 1 and 4. Comparing the results of Simulation 1 given in Figures 3 and 5 with those of Simulation 4 (Figures 4 and 6) we can see that the performances are very close, therefore, the Simulation 4 took about 40% less computing time than Simulation 1.

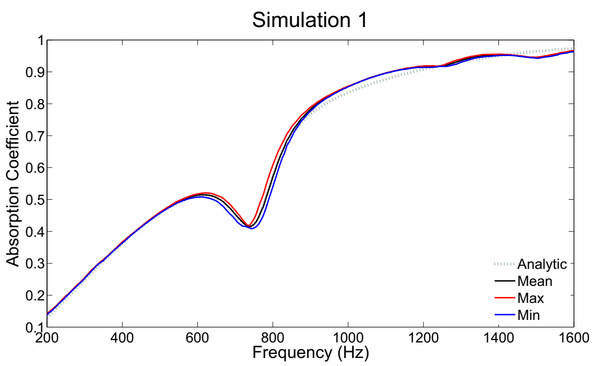

Figure 3: Absorption coefficient for a melamine sample (Simulation 1)

5.2 Auxetic foam

The transversally isotropic auxetic poroelastic media studied is obtained from a conventional open cell polyurethane-polyethylene (PU-PE) foam which acquires auxetics proper-

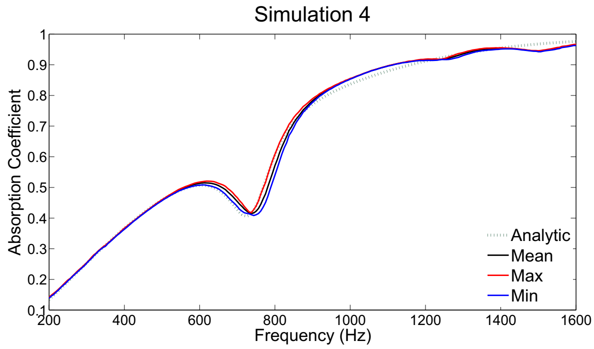

Figure 4: Absorption coefficient for a melamine sample (Simulation 4)

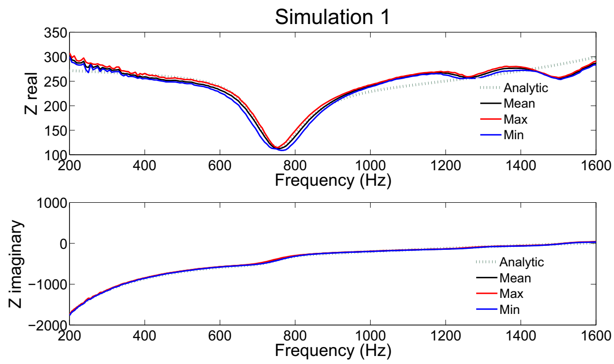

Figure 5: Acoustic impedance for a melamine sample (Simulation 1)

ties thanks to a specific process. It presents interesting properties compared with a melamine foam like we can see in Figure 7, in particular in the low frequency range.

A dynamic mechanical analysis (DMA) technique has been used to obtain reference values of Young's moduli E, E' of the material. An optical method based on Digital Image Correlation (DIC) that employs tracking and image registration techniques for accurate 2D measurements of changes in images gave us also a approximate value of Poisson coefficients ν and ν'. Adding this information with those presents in literature we could define bounds limits for our sensitivity analysis and optimization identification study. The considered values are shown in Table 3.

The study will be concluded in a few weeks with a sensitivity analysis and a FE model updating of parameters in order to identify the parameters' value for the auxetic foam.

6 Conclusion

Considering differences between the various experimental data from the Kundt tube, we can conclude that the results obtained in Simulation 1 and 4 are acceptable for the melamine sample.

We have identified parameters from simple measures without implementing methodology dedicated to the identification for each individual parameter.

A preliminary parameters sensitivity analysis can be used to pre-organize the optimization in order to reduce calculation time and consider the cross influence of the parameters.

Auxetic foams present high efficiency and their parameters identification could be done using FE analysis.

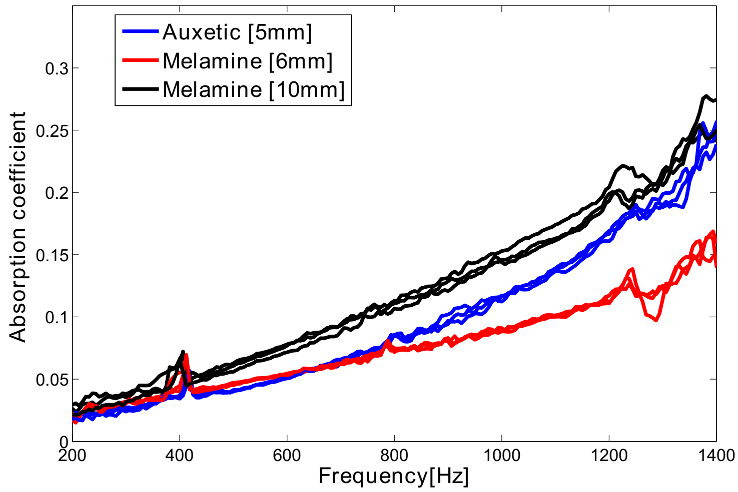

Figure 7: Comparison: Auxetic foam and Melamine foam

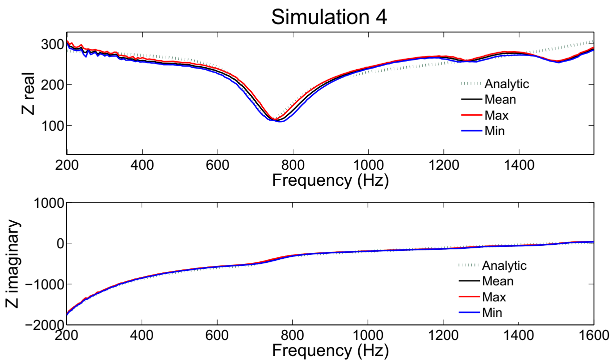

Figure 6: Acoustic impedance for a melamine sample (Simulation 4)

Coupling parameters:			
Parameters	Unit	Lower bounds	Upper bounds
Φ	[-]	0.70	0.99
σ	[N.s.m^{-4}]	1500	200000
α_∞	[-]	1	2
Λ	[μm]	5	200
Λ'	[μm]	5	400
Mechanicals parameters			
Parameters	Unit	Lower bounds	Upper bounds
E	[kPa]	600	1400
E'	[kPa]	5	20
ν	[-]	0.25	0.35
ν'	[-]	-0,82	-0.1
ρ	[Kgm$^-$3]	20	34
η	[%]	0	25

Table 3: Bounds for isotropic tranverse auxetic poroelastic media sensitivity analysis

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