



**HAL**  
open science

# The perturbed low Mach number approximation for the aeroacoustic computation of anisothermal flows

Cyril Nana, David Marx, Christian Prax, Véronique Fortuné

## ► To cite this version:

Cyril Nana, David Marx, Christian Prax, Véronique Fortuné. The perturbed low Mach number approximation for the aeroacoustic computation of anisothermal flows. *Acoustics 2012*, Apr 2012, Nantes, France. hal-00810971

**HAL Id: hal-00810971**

**<https://hal.science/hal-00810971>**

Submitted on 23 Apr 2012

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



# ACOUSTICS 2012

## The perturbed low Mach number approximation for the aeroacoustic computation of anisothermal flows

C. Nana, D. Marx, C. Prax and V. Fortuné

Institut Pprime, CNRS - Université de Poitiers - ENSMA, ENSIP, 6 rue Marcel Doré,  
Batiment B17, BP 633, 86022 Poitiers, France  
[cyril.nana@univ-poitiers.fr](mailto:cyril.nana@univ-poitiers.fr)

We present here a new hybrid method for the computation of the sound emitted by subsonic flows with temperature and density inhomogeneities. This method consists in splitting the flow field into a hydrodynamic part and an acoustic one thanks to a low Mach number approximation of the Navier-Stokes equations. We therefore consider the hydrodynamic part to be quasi incompressible. The acoustic quantities are obtained by a perturbation of the compressible Navier-Stokes equations from which hydrodynamic quantities are subtracted. These become a source term based on the convective derivative of the hydrodynamic pressure. The method has been successfully applied to isothermal and anisothermal excited mixing layers. The validity of the proposed method is assessed by comparison to a compressible direct numerical simulation on the one hand and to LEE computations with different source terms on the other hand.

## 1 Introduction

Aerodynamically generated noise prediction has become a major issue in transport industry. Numerical aeroacoustic computations have established themselves as powerful tools to predict noise radiated by many types of flows.

Two classes of methods are available. The first class of methods consists in performing a direct noise prediction, *e.g.* with a compressible Direct Numerical Simulation (DNS). The compressible Navier-Stokes equations are calculated both in the aerodynamic source region and in the acoustic far field [1, 2, 3]. The connection between the dynamic flow and the sound produced by it is done naturally and requires no model for the sound source. This method requires large computational resources and is inefficient in the low Mach number range. This has motivated the second class of methods, known as hybrid methods [4]. For Mach number less than about 0.3, these methods can lead to a speed-up factor of up to 30 over the DNS [5]. They consist in splitting the full computation into a dynamic flow computation and a sound propagation computation, using a source model in between. The flow computation is typically incompressible [4, 5], but density and temperature inhomogeneities can also be taken into account [6, 7]. The noise computation can be done using some kind of perturbed equations, such as the linearized Euler equations (LEE) [2, 6, 8]. One well known problem with the LEE is that they can sustain unstable vortical modes that can spoil the noise computation. One strategy to avoid this mode is to modify the equations so that they do not support the mode anymore [2, 8, 9]. But a detrimental effect of this is to neglect some sound/flow interactions. Furthermore, density fluctuations are often neglected in the development of the source terms for the LEE [2] whereas these can significantly affect the radiated sound. The question of how density fluctuations contribute to the sound field is still a subject of controversy [10, 11]. It appears crucial to develop a method that can take into account efficiently this phenomena for a better understanding of the processes involved. The Low-Order Low Mach Number Approximation (**LO-LMNA**) flow solver is presented in section 2. The acoustic solver based on a perturbation of the compressible Navier-Stokes equations, the Perturbed Low Mach Number Approximation (**PLMNA**) and its vorticity-filtered version **PLMNA\*** are presented in section 3. In section 4, the LEE are retrieved from the PLMNA with a source term different from the ones classically used. The shear layer configuration and comparisons of the radiated noise obtained with the different strategies are presented in section 5. Conclusions are provided in section 6.

## 2 The Low Mach Number Approximation

The first part of the hybrid approach consists in calculating the flow. Here, a low Mach approximation is used [12, 13, 7, 6] so as to retain temperature and density inhomogeneities, which is necessary for dealing with anisothermal flows. These equations are obtained from the full normalized Navier-Stokes equations that read:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = 0 \quad (1)$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial \tau_{ij}}{\partial x_j} \quad (2)$$

$$\frac{\partial \rho e}{\partial t} + \frac{\partial (\rho e + p) u_j}{\partial x_j} = \frac{\tau_{ij}}{Re} \frac{\partial u_i}{\partial x_j} + \frac{(\gamma - 1)^{-1}}{M^2 Re Pr} \frac{\partial}{\partial x_j} \left( \mu \frac{\partial T}{\partial x_j} \right) \quad (3)$$

$$p = \frac{\rho T}{\gamma M^2} \quad (4)$$

where  $\rho$ ,  $u_{i,j}$ ,  $p$ ,  $T$  are the density, velocity, pressure and temperature respectively.  $Re$ ,  $M$  and  $Pr$  stand respectively for the Reynolds, Mach and Prandtl number,  $\gamma$  is the ratio of specific heats at constant pressure and volume. The internal energy per volume unit  $\rho e$  and the viscid stress tensor  $\tau$  write as follows:

$$\rho e = \frac{p}{\gamma - 1}, \quad \tau_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right). \quad (5)$$

Normalization is done using a length scale  $L_{refh}^*$ , a velocity scale  $U_{refh}^*$ , a time scale  $t_{refh}^* = U_{refh}^*/L_{refh}^*$ , a density scale  $\rho_{refh}^*$ , a pressure scale  $p_{refh}^* = \rho_{refh}^* U_{refh}^{*2}$ , and a temperature scale  $T_{refh}^*$ . Then it comes  $Re = \rho_{refh}^* U_{refh}^* L_{refh}^*/\mu$ ,  $M = U_{refh}^*/\sqrt{\gamma r T_{refh}^*}$  and  $Pr = \mu c_p/k$ . A small parameter  $\epsilon = \gamma M^2$  is introduced in the following expansions:

$$\begin{aligned} \rho &= \rho_0 + \epsilon \rho_1 + \dots, & u_i &= u_{i0} + \epsilon u_{i1} + \dots \\ T &= T_0 + \epsilon T_1 + \dots, & p &= \frac{p_0}{\epsilon} + p_1 + \dots \end{aligned} \quad (6)$$

Introducing this expansion into Eq. (1)-(4), an asymptotic expansion of the Navier-Stokes equations is obtained. Keeping the lowest order terms in  $\epsilon$  provides our set of **LO-**

LMNA equations:

$$\frac{\partial \rho_0}{\partial t} + \frac{\partial \rho_0 u_{i_0}}{\partial x_i} = 0 \quad (7)$$

$$\frac{\partial \rho_0 u_{i_0}}{\partial t} + \frac{\partial \rho_0 u_{i_0} u_{j_0}}{\partial x_j} = -\frac{\partial p_1}{\partial x_i} + \frac{1}{R_e} \frac{\partial \tau_{ij_0}}{\partial x_j} \quad (8)$$

$$\rho_0 \frac{\partial u_{i_0}}{\partial x_i} = \frac{1}{R_e P_r T_0} \frac{\partial}{\partial x_j} \left( \mu \frac{\partial T_0}{\partial x_j} \right) \quad (9)$$

$$p_0 = \rho_0 T_0 \quad (10)$$

Density inhomogeneities are not acoustic and the CFL number is doesn't depend on the sound velocity as would be the case for a compressible solver. Thus, the **LO-LMNA** solver is as efficient as an incompressible solver.

### 3 The Perturbed Low Mach Number Approximation

The equations (1)-(4) are perturbed using the decomposition

$$\begin{aligned} \rho &= \rho_0 + \rho'_h, & u_i &= u_{i_0} + u'_{i_h}, \\ T &= T_0 + T'_h, & p &= \epsilon^{-1} p_0 + p_1 + p'_h. \end{aligned} \quad (11)$$

The primed quantities  $\rho'_h$ ,  $u'_{i_h}$ ,  $T'_h$  and  $p'_h$  contain acoustic fields. As a matter of fact, identifying Eq. (11) with Eq. (6) reveals that these are the sum of all the fluctuations of order at least  $\epsilon$ :

$$\begin{aligned} \rho'_h &= \epsilon \rho_1 + \epsilon^2 \rho_2 + \dots, & u'_{i_h} &= \epsilon u_{i_1} + \epsilon^2 u_{i_2} + \dots, \\ T'_h &= \epsilon T_1 + \epsilon^2 T_2 + \dots, & p'_h &= \epsilon p_2 + \epsilon^2 p_3 + \dots. \end{aligned} \quad (12)$$

The continuity equation Eq. (1) becomes:

$$\frac{\partial \rho'_h}{\partial t} + \frac{\partial}{\partial x_j} \left( \rho_0 u'_{j_h} + \rho'_h u_{j_0} \right) = \underbrace{\frac{\partial \rho_0}{\partial t} + \frac{\partial \rho_0 u_{j_0}}{\partial x_j}}_{=0 \text{ (7)}} \quad (13)$$

We proceed in the same way for the momentum equation Eq. (2)

$$\begin{aligned} \frac{\partial \rho_0 u'_{i_h}}{\partial t} + \frac{\partial \rho'_h u_{i_0}}{\partial t} + \frac{\partial}{\partial x_j} \left( \rho_0 u_{i_0} u'_{j_h} + \rho_0 u'_{i_h} u_{j_0} + \rho'_h u_{i_0} u_{j_0} \right) \\ + \frac{\partial p'_h}{\partial x_i} = \frac{1}{R_e} \frac{\partial \tau'_{ij_h}}{\partial x_j} \quad (14) \\ - \left( \underbrace{\epsilon^{-1} \frac{\partial p_0}{\partial x_i}}_{=0 \text{ (10)}} + \underbrace{\frac{\partial \rho_0 u_{i_0}}{\partial t} + \frac{\partial \rho_0 u_{i_0} u_{j_0}}{\partial x_j} + \frac{\partial p_1}{\partial x_i} - \frac{1}{R_e} \frac{\partial \tau_{ij_0}}{\partial x_j}}_{=0 \text{ (8)}} \right) \end{aligned}$$

and the energy equation Eq. (3)

$$\begin{aligned} \frac{\partial p'_h}{\partial t} + u_{j_0} \frac{\partial p'_h}{\partial x_j} + u'_{j_h} \frac{\partial}{\partial x_j} \left( \epsilon^{-1} p_0 + p_1 \right) + \gamma p'_h \frac{\partial u_{j_0}}{\partial x_j} \\ + \gamma \left( \epsilon^{-1} p_0 + p_1 \right) \frac{\partial u'_{j_h}}{\partial x_j} = F_v - \left[ \frac{\partial p_1}{\partial t} + u_{j_0} \frac{\partial p_1}{\partial x_j} + \gamma p_1 \frac{\partial u_{j_0}}{\partial x_j} \right] \\ - \left[ \underbrace{\frac{\partial \epsilon^{-1} p_0}{\partial t} + u_{j_0} \frac{\partial \epsilon^{-1} p_0}{\partial x_j}}_{=0 \text{ (10)}} + \underbrace{\gamma \epsilon^{-1} p_0 \frac{\partial u_{j_0}}{\partial x_j} - \frac{\gamma \epsilon^{-1}}{R_e P_r} \frac{\partial}{\partial x_j} \left( \mu \frac{\partial T_0}{\partial x_j} \right)}_{=0 \text{ (9)}} \right] \end{aligned} \quad (15)$$

where

$$F_v = \frac{(\gamma - 1)}{R_e} \left( \tau_{ij_0} + \tau'_{ij_h} \right) \frac{\partial (u_{i_0} + u'_{i_h})}{\partial x_j} + \frac{\gamma \epsilon^{-1}}{R_e P_r} \frac{\partial}{\partial x_j} \left( \mu \frac{\partial T'_h}{\partial x_j} \right). \quad (16)$$

For sufficiently high Reynolds numbers, viscous term are small enough to be neglected. The Perturbed Low Mach Number Approximation system (**PLMNA**) is finally:

$$\frac{\partial \rho'}{\partial t} + \frac{\partial}{\partial x_j} \left( \rho_{0_a} u'_j + \rho' u_{j_{0_a}} \right) = 0 \quad (17)$$

$$\begin{aligned} \frac{\partial \rho_{0_a} u'_i}{\partial t} + \rho' \frac{\partial u_{i_{0_a}}}{\partial t} + \left( \rho_{0_a} u'_j + \rho' u_{j_{0_a}} \right) \frac{\partial u_{i_{0_a}}}{\partial x_j} \\ + \frac{\partial}{\partial x_j} \left( \rho_{0_a} u'_i u_{j_{0_a}} \right) + \frac{\partial p'}{\partial x_i} = 0 \end{aligned} \quad (18)$$

$$\begin{aligned} \frac{\partial p'}{\partial t} + (\gamma - 1) \left( p' \frac{\partial u_{j_{0_a}}}{\partial x_j} - u'_j \frac{\partial p_{0_a}}{\partial x_j} \right) \\ + \frac{\partial}{\partial x_j} \left( p' u_{j_{0_a}} + \gamma p_{0_a} u'_j \right) = S_4 \end{aligned} \quad (19)$$

$$S_4 = - \left( \frac{D p_{1_a}}{D t_a} + \gamma p_{1_a} \frac{\partial u_{j_{0_a}}}{\partial x_j} \right) \quad (20)$$

where  $D/D t_a = \partial/\partial t_a + u_{j_0} \partial/\partial x_j$  is the convective derivative. The **PLMNA** equations (17)-(20) are normalized with respect to the acoustic reference quantities  $\rho_{ref_a}^*$ ,  $c_{ref_a}^*$  and  $p_{ref_a}^*$  where  $c_{ref_a}^*$  is the sound velocity and  $t_{ref_a}^* = c_{ref_a}^*/L_{ref_a}^*$ . That leads to the following relations:

$$\begin{aligned} u_{i_{0_a}} &= u_{i_0} M, & p_{0_a} &= \left( \frac{p_0}{\gamma} + p_1 M^2 \right) \frac{\rho_{ref_a}^*}{\rho_{ref_a}^*}, \\ \rho_{0_a} &= \rho_0 \frac{\rho_{ref_a}^*}{\rho_{ref_a}^*}, & p_{1_a} &= p_1 M^2 \frac{\rho_{ref_a}^*}{\rho_{ref_a}^*}, \\ t_a &= t M^{-1}, & L_{ref_a}^* &= L_{ref_a}^* \end{aligned} \quad (21)$$

with the  $M = U_{ref_a}^*/c_{ref_a}^*$  the Mach number of the hybrid calculation. This solver is very similar to the Perturbed Compressible Equations (PCE) developed by Seo and Moon [8]. They proposed to filter the momentum equation Eq. (18) since it can strongly create perturbed vorticity. Indeed, taking the curl of Eq. (18), they derived a transport equation for the perturbed vorticity close to

$$\begin{aligned} \frac{\partial \omega'}{\partial t_a} + (\mathbf{u} \cdot \nabla) \omega' = \underbrace{\left[ (\boldsymbol{\Omega}_{0_a} \cdot \nabla) \mathbf{u}' + (\omega' \cdot \nabla) \mathbf{u}_{0_a} \right]}_I \\ - \underbrace{\left[ (\mathbf{u}' \cdot \nabla) \boldsymbol{\Omega}_{0_a} + \boldsymbol{\Omega}_{0_a} (\nabla \cdot \mathbf{u}') \right]}_{II} \\ - \underbrace{\omega' (\nabla \cdot \mathbf{u}_{0_a})}_{III} + \underbrace{\frac{1}{\rho_{0_a}^2} (\nabla \rho_{0_a} \wedge \nabla p') - \nabla \wedge \left( \frac{\rho'}{\rho_{0_a}} \frac{D \mathbf{u}_{0_a}}{D t_a} \right)}_{IV - V} \end{aligned} \quad (22)$$

where  $\boldsymbol{\Omega}_{0_a} = \nabla \wedge \mathbf{u}_0$  and  $\omega' = \nabla \wedge \mathbf{u}'$  are the hydrodynamic and perturbed vorticity respectively. The term II is the main source of perturbed vorticity but doesn't contribute much to the radiated sound field [8]. The term I is zero for a two-dimensional case. The terms IV – V only cause diffusion of  $\omega'$ . The term is absent in Seo and Moon's analysis since they consider a solenoidal flow. We reasonably neglect the right hand-side of Eq. (22) and obtain the perturbed vorticity free

or vorticity filtered system **PLMNA**\*:

$$\begin{aligned} \frac{\partial \rho'}{\partial t_a} + \frac{\partial}{\partial x_j} (\rho_{0a} u'_j + \rho' u_{j0a}) &= 0 \\ \frac{\partial \rho_{0a} u'_i}{\partial t_a} + \frac{\partial}{\partial x_j} (\rho_{0a} u'_j u_{j0a}) + \rho_{0a} u'_i \frac{\partial u_{j0a}}{\partial x_j} + \frac{\partial \rho'}{\partial x_i} &= 0 \\ \frac{\partial \rho'}{\partial t_a} + (\gamma - 1) \left( p' \frac{\partial u_{j0a}}{\partial x_j} - u'_j \frac{\partial p_{0a}}{\partial x_j} \right) & \\ + \frac{\partial}{\partial x_j} (p' u_{j0a} + \gamma p_{0a} u'_j) &= S_4 \\ S_4 &= - \left( \frac{Dp_{1a}}{Dt_a} + \gamma p_{1a} \frac{\partial u_{j0a}}{\partial x_j} \right). \end{aligned} \quad (23)$$

## 4 From PLMNA to the Linearized Euler Equations

Let us consider the hydrodynamic quantities issued from the **LO-LMNA** solver as the sum of fluctuations  $x''$  about mean temporal averages values  $\bar{x}$  like:

$$u_{i0a} = \bar{u}_{i0a} + u''_{i0a}, \quad p_{0a} = \bar{p}_{0a} + p''_{i0a}, \quad \rho_{0a} = \bar{\rho}_{0a} + \rho''_{0a}. \quad (24)$$

If we now substitute (24) in the **PLMNA** equations (17)-(20), we find:

$$\begin{aligned} \frac{\partial \rho'}{\partial t_a} + \frac{\partial}{\partial x_j} (\bar{\rho}_{0a} u'_j + \rho' \bar{u}_{j0a} + \rho''_{0a} u'_j + \rho' u''_{j0a}) &= 0 \\ \frac{\partial \bar{\rho}_{0a} u'_i}{\partial t_a} + \frac{\partial \rho''_{0a} u'_i}{\partial t_a} + \rho' \frac{\partial u''_{i0a}}{\partial t_a} + \frac{\partial}{\partial x_j} (\bar{\rho}_{0a} u'_i \bar{u}_{j0a} + \rho''_{0a} u'_i u''_{j0a}) & \\ + \frac{\partial}{\partial x_j} (\rho_{0a} u'_i \bar{u}_{j0a} + \rho''_{0a} u'_i u''_{j0a}) + (\bar{\rho}_{0a} u'_j + \rho' \bar{u}_{j0a}) \frac{\partial (\bar{u}_{i0a} + u''_{i0a})}{\partial x_j} & \\ + (\rho''_{0a} u'_j + \rho' u''_{j0a}) \frac{\partial (\bar{u}_{i0a} + u''_{i0a})}{\partial x_j} + \frac{\partial \rho'}{\partial x_i} &= 0 \end{aligned} \quad (26)$$

$$\begin{aligned} \frac{\partial p'}{\partial t_a} + \frac{\partial}{\partial x_j} (p' \bar{u}_{j0a} + \gamma \bar{p}_{0a} u'_j + p' u''_{j0a} + \gamma p''_{0a} u'_j) & \\ + (\gamma - 1) \left( p' \frac{\partial \bar{u}_{j0a}}{\partial x_j} + p' \frac{\partial u''_{j0a}}{\partial x_j} - u'_j \frac{\partial \bar{p}_{0a}}{\partial x_j} - u'_j \frac{\partial p''_{0a}}{\partial x_j} \right) &= S_4 \end{aligned} \quad (27)$$

In the same manner as done by Béchara *et al.* [14], we will neglect each term involving products of hydrodynamic  $x''$  and acoustic fluctuations  $x'$ . We retrieve then the **LEE** with a source term  $\mathbf{S_E} = (0, 0, 0, S_4)^T$  in the energy equation :

$$\begin{aligned} \frac{\partial \rho'}{\partial t_a} + \frac{\partial}{\partial x_j} (\bar{\rho} u'_j + \rho' \bar{u}_j) &= 0 \\ \frac{\partial \bar{\rho} u'_i}{\partial t_a} + \frac{\partial}{\partial x_j} (\bar{\rho} u'_i \bar{u}_j) + (\bar{\rho} u'_j + \rho' \bar{u}_j) \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \rho'}{\partial x_i} &= 0 \\ \frac{\partial \rho'}{\partial t_a} + \frac{\partial}{\partial x_j} (p' \bar{u}_j + \gamma \bar{p} u'_j) + (\gamma - 1) \left( p' \frac{\partial \bar{u}_j}{\partial x_j} - u'_j \frac{\partial \bar{p}}{\partial x_j} \right) &= S_4 \\ S_4 &= - \left( \frac{Dp_{1a}}{Dt_a} + \gamma p_{1a} \frac{\partial u_{j0a}}{\partial x_j} \right) \end{aligned} \quad (28)$$

where the mean hydrodynamic flow fields are

$$\bar{u}_i = \bar{u}_{i0a} = \bar{u}_{i0} M, \quad \bar{\rho} = \bar{\rho}_{0a} = \rho_0^* \frac{\rho_{refd}^*}{\rho_{refa}^*}, \quad (29)$$

$$\bar{p} = \bar{p}_{0a} = \left( \frac{p_0}{\gamma} + p_1 M^2 \right) \frac{\rho_{refd}^*}{\rho_{refa}^*}.$$

The **LEE** are numerically solved using the same matrix formulation as the one employed by [2, 6] :

$$\frac{\partial \mathbf{U}}{\partial t_a} + \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{F}}{\partial y} + \mathbf{H} = \mathbf{S} \quad (30)$$

where  $\mathbf{S}$  is the source term used to form the analogy. In our new method,

$$\mathbf{S} = \mathbf{S_E} = (0, 0, 0, S_4). \quad (31)$$

### 4.1 Classical formulation of the source terms

Bogey *et al.* [2] developed a different source term by deriving Lilley's wave equation from the **LEE** and consequently making the hypothesis of a strictly parallel mean flow with  $\bar{u} = \bar{u}(y)$  and  $\bar{v} = 0$ . They also considered the mean pressure  $\bar{p}$  constant and the mean density and sound velocity only as function of the transverse coordinate  $y$ ,  $\bar{\rho} = \bar{\rho}(y)$  and  $\bar{c} = \bar{c}(y)$ . The source term is given by

$$\mathbf{S} = (0, S2 - \bar{S2}, S3 - \bar{S3}, 0) \quad (32)$$

where

$$S2 = - \frac{\partial \rho u'_i u'_j}{\partial x_i}, \quad S3 = - \frac{\partial \rho u'_i u'_j}{\partial x_j} \quad (33)$$

and  $\bar{S2}, \bar{S3}$  are time averaged quantities. This expression of the source term was successfully used by [15, 2] to calculate noise radiated by mixing layers arisen from DNS and LES computations however, Bogey *et al.* neglected fluctuations of density, arguing that the triple product of fluctuations  $\rho'' u'_i u'_j$  would be very small. Furthermore, they needed to nullify the term  $\mathbf{H}$  to cancel instabilities.

Comparing the expression of the source term Eq.(33) with Eq. (8) leads to

$$- \frac{\partial \rho_{0a} u''_{i0a} u''_{j0a}}{\partial x_j} = \frac{\partial p''_{1a}}{\partial x_i} - \frac{1}{R_e} \frac{\partial \tau''_{ij0a}}{\partial x_j} + \frac{\partial \rho_{0a} u''_{i0a}}{\partial t_a}. \quad (34)$$

We approximate mean velocity, pressure and dilatation fields of the **LEE** as temporal averages of the ones given by the **LO-LMNA** system of equations Eq. (7)-(10). The viscous term can be neglected, providing that the Reynolds number is sufficiently high. The source terms become:

$$- \frac{\partial \rho_{0a} u''_{i0a} u''_{j0a}}{\partial x_j} = \frac{\partial p''_{1a}}{\partial x_i} + \frac{\partial \rho_{0a} u''_{i0a}}{\partial t_a}. \quad (35)$$

For isothermal cases, we expect the last term of the RHS of Eq. (35) not to radiate as stated by [16] since it is divergence free. Instead of acting on  $\mathbf{H}$ , we can use a simplified form of the source term  $\mathbf{S_P}$  and compare the results with classical  $\mathbf{S_L}$  terms where  $\mathbf{S_P}$  and  $\mathbf{S_L}$  are:

$$\mathbf{S_P} = \frac{\partial p''_{1a}}{\partial x_i}, \quad \mathbf{S_L} = - \frac{\partial \rho_{0a} u''_{i0a} u''_{j0a}}{\partial x_j}. \quad (36)$$

## 5 Application to a shear layer with density gradient

### 5.1 Flow configuration

We consider the spatial development of a bidimensional mixing layer between two streams of velocity, temperature

and density ( $U_u, T_u, \rho_u$ ) and ( $U_d, T_d, \rho_d$ ) respectively. The initial mean velocity field is given by hyperbolic-tangent profile

$$U(y) = \frac{U_u + U_d}{2} + \frac{U_u - U_d}{2} \tanh\left(\frac{2y}{\delta_{\omega_0}}\right) \quad (37)$$

where  $\delta_{\omega_0}$  is the initial vorticity thickness. The temperature profile is defined by the Crocco-Buseman relation

$$T(y) = \frac{1}{2C_p} \left[ U(y)(U_u + U_d) - U_u U_d - U^2(y) \right] + (T_u - T_d) \frac{U(y)}{U_u - U_d} + \frac{T_d U_u - T_u U_d}{U_u - U_d} \quad (38)$$

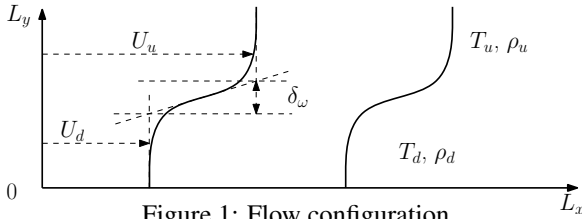


Figure 1: Flow configuration

This mean flow is forced with two subharmonics  $f_0/2$  and  $f_0/4$  where  $f_0$  is the frequency of the most unstable mode as found by [17]. From now on, velocity and length are scaled with respect to the sound velocity  $c_0$  and the initial vorticity thickness  $\delta_{\omega_0}$ . Compressible DNS calculation schemes used for comparison with the hybrid method are extensively explicated in [3]. The **LO-LMNA** solver is detailed in [6].

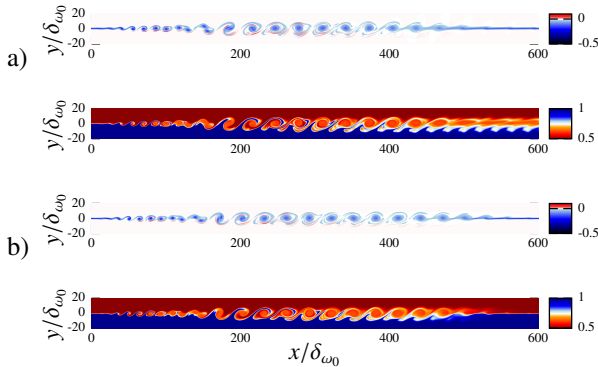


Figure 2: Up vorticity plot, down density plot:  
a) compressible DNS, b) **LO-LMNA**

Initially, the upper and lower velocity  $U_u$  and  $U_d$  (Figure 1) are set to 0.50 and 0.25 respectively with a Reynolds number  $Re = 400$ . In the following, two setups are used: an isothermal configuration (case1) with  $T_u = T_d$  and  $\rho_u = \rho_d$  and an anisothermal one (case2) with  $T_u = 2T_d$  and  $\rho_u = 0.5\rho_d$ . Figure 2 shows a good agreement between compressible DNS and **LO-LMNA** calculations for the anisothermal case. The computational domain extends to  $L_x = 600$ ,  $L_y = 180$  with  $1001 \times 601$  points for the case1 and  $L_x = 600$ ,  $L_y = 40$  with  $1537 \times 289$  points for the case2. The  $x$ -direction is stretched from  $x/\delta_{\omega_0} = 350$  where a buffer zone is also applied. Because of the gradient of density, the case2 needs to be more refined in the  $y$ -direction compared to case1. The acoustic computational domain is for both cases  $L_x = 600$  and  $L_y = 800$ . Hydrodynamic fields are mapped on the acoustic grid and interpolated in time using a cubic spline scheme to match with the acoustic time  $t_a$ .

## 5.2 Results

For the case1, Figure 3, the reference compressible DNS solution a) is computed using  $1035 \times 431$  grid points. Source terms are damped in the  $x$ -direction for the **PLMNA** b), the **PLMNA\*** c) and the **LEE** with the source term in the energy equation **LEE+S<sub>E</sub>** d). Only a very small buffer zone amounting to 1% of the calculated quantities was necessary to dissipate high frequency fluctuations. Pressure field b), c) and d) show a good agreement with the reference solution a). The filtering applied on the **PLMNA** is very efficient since the fluctuations appearing downstream of the pairing process on b) disappeared on c). For the **LEE** with the **S<sub>L</sub>** and **S<sub>P</sub>** terms respectively e) and f), damping in the  $x$ -direction isn't possible anymore since it creates additional radiation. The buffer zone needs to be more efficient and amount to 10% of the calculated quantities. This treatment also creates additional radiation that can change the directivity pattern. It is noticeable on e).

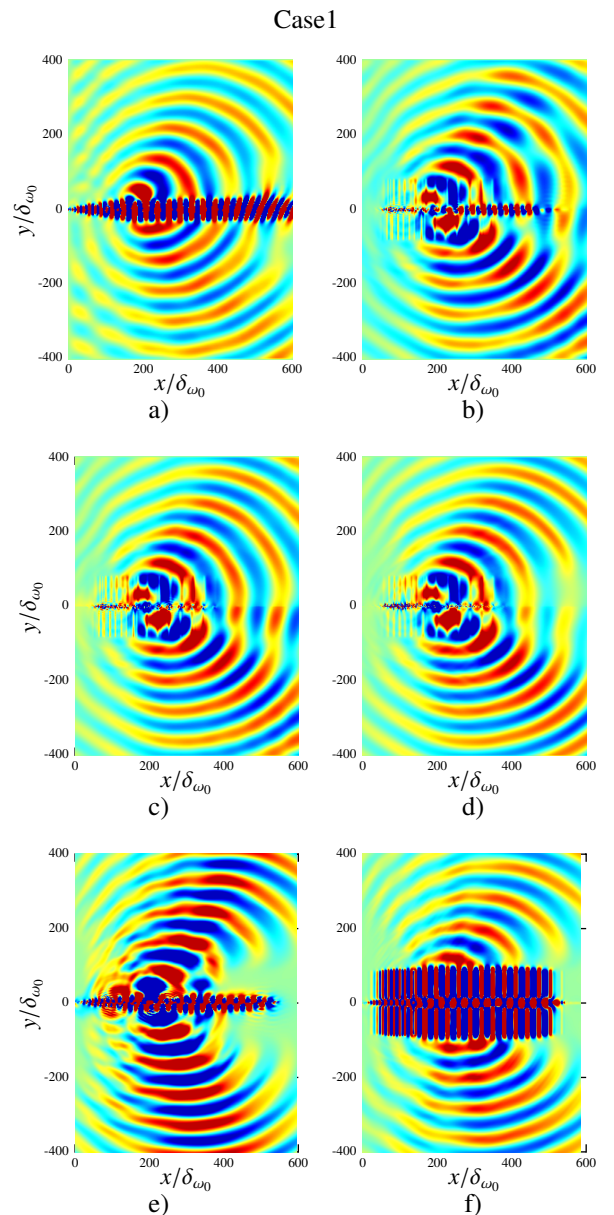


Figure 3: Pressure: a) compressible DNS, b) **PLMNA**, c) **PLMNA\***, d) **LEE+S<sub>E</sub>**, e) **LEE+S<sub>L</sub>**, f) **LEE+S<sub>P</sub>**, levels from  $-5.10^{-5}$  to  $5.10^{-5}$ .

In the anisothermal flow of the case2 on Figure 4, the **LEE+S<sub>L</sub>** and **LEE+S<sub>P</sub>** were not able to predict a pressure

field similar to the reference solution a). The latter is computed using  $2071 \times 785$  grid points. The **PLMNA**, **PLMNA\***, gave similar results respectively b) and c). The directivity pattern is in great agreement with a). Therefore, the amplitude of pressure fluctuations is smaller by a factor about 4. That may be due to the size of the hydrodynamic domain in the  $y$ -direction that could be too small. Indeed, for the **LEE+S<sub>E</sub>** on d), a greater hydrodynamic domain was used with  $L_y = 80$  and the factor is only about 2.

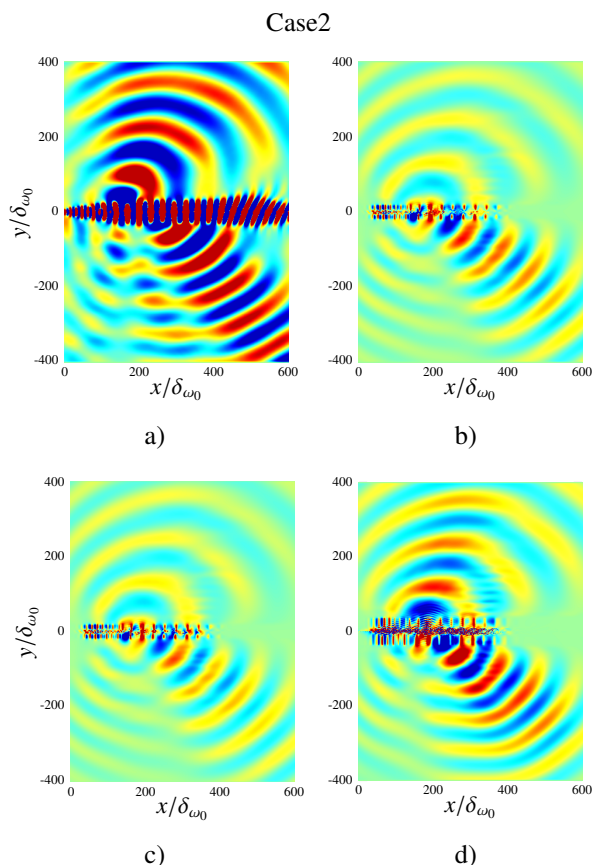


Figure 4: Pressure: a) compressible DNS, b) **PLMNA**, c) **PLMNA\***, d) **LEE+S<sub>E</sub>**, levels from  $-2 \cdot 10^{-5}$  to  $2 \cdot 10^{-5}$ .

## 6 Conclusion

In this study, a hybrid method based on a low Mach number approximation and a perturbation of the Navier-Stokes equation was successfully used to compute the acoustic field caused by fluctuations of anisothermal flows. Source terms for the classical linearized Euler equations were straightforwardly developed from the perturbation equations giving a single term in the energy equation. The main advantage of the proposed method is the absence of any hypothesis on the shape of the background hydrodynamic flow field.

## Acknowledgments

Part of the calculation resources were provided by the **IDRIS** (Institut du Développement et des Ressources en Informatique Scientifique, CNRS).

## References

- [1] T. Colonius, S. K. Lele, and P. Moin. Sound generation in a mixing layer. *J. Fluid Mech.*, **330**:375–409, 1997.
- [2] C. Bogey, C. Bailly, and D. Juvé. Computation of flow noise using source terms in linearized Euler's equations. *AIAA Journal*, **40**(2):235–243, 2002.
- [3] E. Lamballais, V. Fortuné, and S. Laizet. Straightforward high-order numerical dissipation via the viscous term for direct and large eddy simulation. *J. Comp. Phys.*, **230**:3270–3275, 2011.
- [4] J.C. Hardin and D.S. Pope. An acoustic/viscous splitting technique for computational aeroacoustics. *Theoret. Comput. Fluid Dynamics*, **6**:323–340, 1994.
- [5] J.-H. Seo and Y. J. Moon. Perturbed compressible equations for aeroacoustic noise prediction at low Mach numbers. *AIAA Journal*, **43**:1716–1724, 2005.
- [6] C. Prax, F. Golanski, and L. Nadal. Control of the vorticity mode in the linearized Euler equations for hybrid aeroacoustic prediction. *J. Comp. Phys.*, **227**:6044–6057, 2008.
- [7] C.-D. Munz, M. Dumbser, and S. Roller. Linearized acoustic perturbation equations for low Mach number flow with variable density and temperature. *J. Comp. Phys.*, **224**:352–364, 2007.
- [8] J.-H. Seo and Y. J. Moon. Linearized perturbed compressible equations for low Mach numbers aeroacoustics. *J. Comp. Phys.*, **218**:702–719, 2006.
- [9] R. Ewert and W. Schröder. Acoustic perturbation equations based on flow decomposition via source filtering. *J. Comp. Phys.*, **188**:365–398, 2003.
- [10] K. Viswanathan. Jet aeroacoustic testing: Issues and implications. *AIAA Journal*, **41**(9):1674–1689, 2003.
- [11] K. Viswanathan. Aeroacoustic of hot jets. *J. Fluid Mech.*, **516**:39–82, 2004.
- [12] P. A. McMurtry, W.-H. Jou, J. J. Riley, and R. W. Metcalfe. Direct numerical simulations of a reacting mixing layer with chemical heat release. *AIAA Journal*, **24**(6):962–970, 1986.
- [13] R. B. Pember, L. H. Howell, J. B. Bell, P. Colella, C. Y. Crutchfield, W. A. Fiveland, and J.P. Jessee. An adaptive projection method for unsteady, low Mach number combustion. *Combustion Science and Technology*, 1998.
- [14] W. Béchara, C. Bailly, P. Lafon, and S. Candel. Stochastic approach to noise modeling for free turbulent flows. *AIAA Journal*, **32**(3):455–463, 1994.
- [15] M. Billson, L.-E. Erikson, and L. Davidson. Acoustic source terms for the linear Euler equations on conservative form. *AIAA paper*, **02-2582**, 2002.
- [16] B. T. Chu and L. S. G. Kovaszny. Non-linear interactions in a viscous heat-conducting compressible gas. *J. Fluid Mech.*, **3**(5):494–514, 1958.
- [17] A. Michalke. On spatially growing disturbances in an inviscid shear layer. *J. Fluid Mech.*, **23**:521–544, 1965.