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# ACOUSTICS 2012

## Evolution of intensive acoustical noise pulses (the numerical simulation with Fast Legendre Transform Algorithm)

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We consider the numerical solution of nonlinear equations of hydrodynamic type, which is based on Fast Legendre Transform algorithm, which drastically reduce the number of transactions compared to standard methods. Studied numerically the evolution of intense acoustic noise pulses. We consider two typical types of envelope for each of them obtained the characteristic velocity profiles and the probability distribution of the coordinates of the absolute maximum. It is shown that due to nonlinear interactions are generated non-zero mean velocity field at large distances.

## 1 Introduction

Numerical simulation of stochastic nonlinear acoustic waves using the nonlinear evolution equation of the Burgers [1-2], which describes two physical effects: nonlinearity and absorption. The main difficulty in numerical analysis is the large cost of computer time and a large amount of memory for all numerical models describing the shock wave [4-6]. In connection with this acute problem of optimization algorithms for computing. One of these algorithms is the use of the Fast Legendre transform for the numerical simulation of nonlinear acoustic waves in random stages of the interaction of developed discontinuities

## 2 An analytical description of propagation of intense acoustic noise pulses on the stage of developed discontinuities

Consider the evolution equation of Burgers (dimensionless) for plane acoustic waves with the initial condition [1]:

$$\frac{\partial v}{\partial x} - v \frac{\partial v}{\partial t} = \Gamma \frac{\partial^2 v}{\partial t^2}, \quad v(t, 0) = v_0(t) \quad (1)$$

The solution of this equation on the stage of developed discontinuities (in the case when  $\Gamma \gg 1$ ) can be represented as follows :

$$v(t, x) = \frac{t - \tau^*(t, x)}{x} \quad (2)$$

where  $\tau^*$  - coordinate of the absolute maximum of the function  $\Phi(x, \tau, t)$ :

$$\Phi(\tau, t, x) = s_0(\tau) - \frac{(\tau(t, x) - t)^2}{2x} \quad (3)$$

$s_0(t)$  - the initial capacity of the velocity field:

$$v_0(t) = - \frac{ds_0(t)}{dt} \quad (4)$$

We make the following assumption that  $\Phi(x, \tau, t)$  has two identical absolute maximum,  $H^*$  and  $\tau^*$ , respectively, the value and the coordinate of the absolute maximum being in the interval  $[t_1, t_2]$ ,  $s_0(t_2)$  - the total momentum of the initial field ( $s_0(t_1) = 0$ ).

Consider the evolution of the noise pulses. Assume that they will have two characteristic spatial scale (the scale of the internal filling  $\tau_c$  and scale of the envelope  $T$ ). Then the initial velocity field can be written as follows:

$$v_0(t) = m(t)f(t), \quad (5)$$

where  $m(t)$  - modulating function (regular) and  $f(t)$  - noise (random process). In Fig. 1 shows an implementation of a noise pulse.

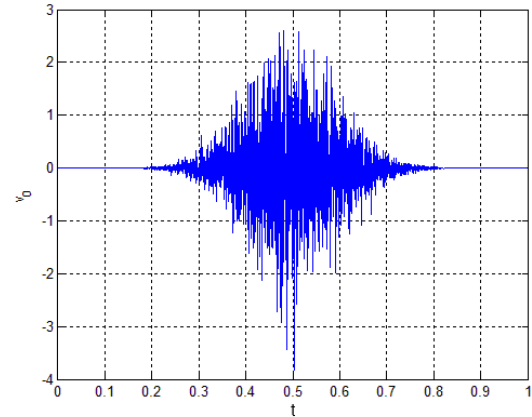


Figure 1: An example of a noise pulse.

For definiteness, in the setting of regular noise pulse envelope, assume that  $m(t)$  has two time scales: the duration of the transition region -  $T_t$  and the duration of the decay function -  $T$ , where  $T_t \ll \tau_c \ll T$ . There are two typical types of modulating function  $m(t)$  (shown in Figure 2): a)  $m_1(t)$  has a sharp jump at the leading and trailing edge; b)  $m_2(t)$  has a smooth function with only one characteristic scale  $T$ .

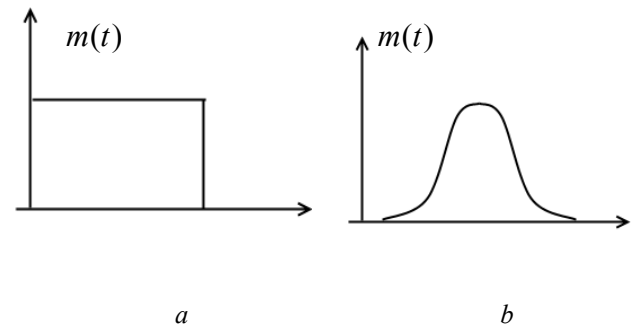


Figure 2: The qualitative form of the modulating functions (a)  $m_1(t)$ , (b)  $m_2(t)$ .

In [3] conducted a statistical analysis of intense acoustic noise pulses of various shapes at the stage of developed discontinuities, is formed when the N-wave, which is completely determined by specifying the coordinates of the position of discontinuities and zero field. For the probability density of zero coordinates of N-wave  $\tau^*$  for two different types of envelope are the analytical values at the stage of developed discontinuities of interaction when it is self-similar regime of evolution of intense acoustic noise.

### 3 Fast Legendre Transform for the numerical simulation of the evolution of noise pulse

To confirm the analytical results for the dissemination and transformation of the profile and the statistical characteristics of intense noise pulses on the stage of developed discontinuities in this section we present results of numerical simulation.

The main objective of the numerical analysis of the evolution of random waves is the task of finding the coordinates of the absolute maximum of the solution (2). The scheme of numerical solution of nonlinear equations of hydrodynamic type are well studied and described in detail in [1,2,4]. In our case, to find the coordinates of the absolute maximum of the pulses of noise filling the algorithm we propose to use the Fast Legendre Transforms (FLT), which will significantly reduce the number of required operations in comparison with standard methods. If  $N$  - duration of the initial implementation process, with a standard method for sorting the number of operations will be  $N^2$ , in the case of FLT number of operations will be significantly reduced and  $N \cdot \log_2 N$ . Another important feature of this algorithm is the fact that he had no fundamental changes can be applied to multidimensional cases.

Evolution of the noise pulses seen at a distance of interaction developed discontinuities and these characteristic distances indicated in the figures by the dimensionless parameter  $Z$ . The initial noise field (at  $Z = 0$ ) was implemented numerically using the random number generator, which is created with the implementation of the Gaussian distribution, zero mean and unit variance.

#### 3.1 Evolution of a rectangular pulse

In Figure 3 shows the initial profile (at  $Z = 0$ ) and the evolution of the noise pulse with the envelope of a rectangular form at a distance of formation and interaction of developed discontinuities.

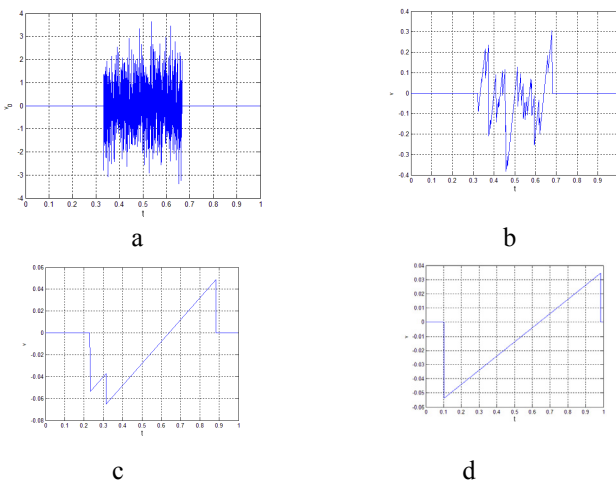


Figure 3: Evolution of a rectangular pulse with noise. The characteristic length (a)  $Z = 0$ , (b)  $Z = 0,05$ ; (c)  $Z = 0,1$ ; (d)  $Z = 10$ . The length of the implementation of  $N = 4000$  points.

From the implementations can be clearly seen as primarily a noise pulse in the interaction and merging of discontinuities as it propagates in a nonlinear medium is converted into  $N$ -wave. As a result of this interaction is a loss of information about the structure of the initial signal. Thus, at some distance from the source of two different implementations of noise exposure are qualitatively in distinguishable.

Consider the evolution of the statistical characteristics of a rectangular pulse with the noise on the stage of developed discontinuities of interaction. In Figure 4 shows the probability distribution of the coordinates of the absolute maximum of the function  $\Phi(x, \tau, t)$  obtained by numerical simulation.

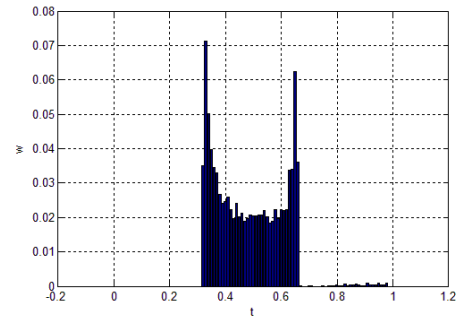


Figure 4: Probability distribution of the coordinates of the absolute maximum of  $\tau^*$  for the rectangular pulse to the characteristic distance  $Z = 10$ .

#### 3.2 Evolution of a Gaussian pulse

This section presents the results of numerical modeling of the evolution of intense acoustic pulses with noise, the envelope in the case of a Gaussian form. This is the most interesting case for the use of FLT algorithm for numerical simulation of solutions of the Burgers equation. In Figure 5 shows the results of transformation of a single Gaussian noise pulse in the  $N$ -wave.

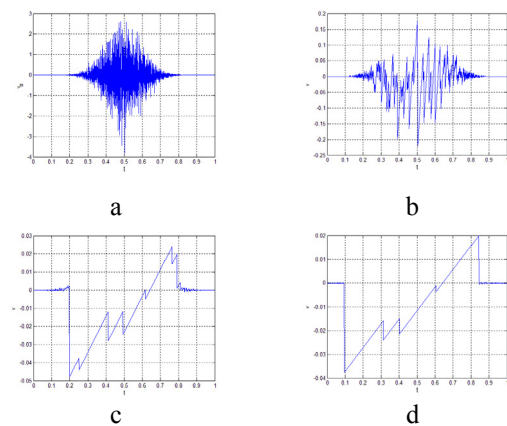


Figure 5: Evolution of a Gaussian pulse with noise. The characteristic length (a)  $Z = 0$ , (b)  $Z = 0,05$ ; (c)  $Z = 0,1$ ; (d)  $Z = 10$ . The length of the implementation of  $N = 4000$  points.

From the implementations can be clearly seen that small-scale structure of the pulse with Gaussian envelope in contrast to previous cases where the envelope of the pulse had sharp boundaries, remain quite at the stage of developed discontinuities interaction at sufficiently large distances characteristic. And only when it is phase out the acoustic field on the self-similar stage, there is a rupture of the absorption maximum amplitude, which leads to the formation of N-wave.

Model", *Journal of Scientific Computing*. 9(3), 259-281 (1994)

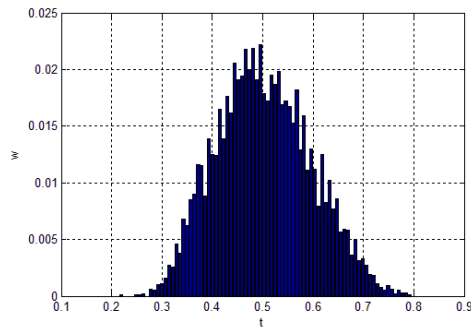


Figure 6: Probability distribution of the coordinates of the absolute maximum of  $\tau^*$  for the Gaussian pulse to the characteristic distance  $Z = 10$ .

Good agreement between the results of numerical simulations and analytical predictions for the observed probability distribution of the coordinates of the absolute maximum of  $\Phi(x, \tau, t)$  in the case of the Gaussian envelope of the initial pulse. Numerical probability distribution consistent with a Gaussian form, which confirmed the theoretical analysis.

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