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# ACOUSTICS 2012

## Power estimation of acoustical sources by an array of microphones

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## Abstract

The problem of estimating the power of signals impinging an array of sensors when the arrival directions are known is presented. It is assumed that the signal field at the array is comprised of  $P$  independent plane-wave arrivals from known directions. In practice the directions of arrival are rarely known exactly, however this difficulty can be overcome by using the standard MUSIC (Multiple Signal Classification) algorithm, which constitutes an angular pseudo-spectrum and an indicator of directions of arrival of different signals. The problem then reduces to the estimation of the signal powers from each of the  $P$  directions. Five estimators are presented in this communication. The conventional beamformer, the Capon estimator, the robust Capon estimator, the covariance vector estimator and the least squares fit estimator of the observed cross-spectral matrix of the array. Numerical and experimental results are presented showing that the performances of the covariance vector estimators are better than other estimators.

## 1 Introduction

Arrays of sensors are used in many fields to detect signals, to resolve closely spaced targets, to estimate the bearing, the position, the strength and other properties of radiating sources whose signals arrive from different directions [1-3]. The emitted source of energy can be acoustic (sound-waves), electro-magnetic (radio-waves), vibrations (signals of geophysical nature), and so on, and the receiving sensors may be any transducers that convert the received energy to electrical signals. The types of sensors used to detect these signals differ accordingly: microphones for acoustic signals, electromagnetic antennas for radio waves, accelerometers/seismometers for the detection of earthquakes. In all that follows, only idealized arrays of receiving sensors are considered. The other assumptions are fairly conventional: the sources are point emitters situated in the far field of the array, the propagation medium is not dispersive and the waves arriving at the array are planar. Furthermore, we assume that both the sources and the sensors are in the same plane and that the signals and noise are random processes with zero mean, stationary and statistically independent.

Processors that are commonly used are the conventional and the adaptive beamformer and the usual method of processing is to form a number of closely spaced receiver beams in fixed directions and to measure the power in each beam, the strength and direction of arrival of a signal may then be estimated by interpolation between beams. The approach taken here is to assume that the signal field at the array is comprised of  $P$  independent plane-wave arrivals from  $P$  known directions and in practice, of course, the directions are rarely known exactly, however this difficulty can be overcome by using the standard MUSIC (Multiple Signal Classification) algorithm [1] which constitutes an angular pseudo-spectrum and an indicator of directions of arrival of different signals. The problem then reduces to estimating the signal powers from each of the  $P$  directions.

This study focuses on developing estimators which are simple to implement on line and which seek to identify the distribution of signal power generated by acoustical sources.

## 2 Signal representation

Consider a receiving array with  $N$  sensors and assume that  $P$  acoustical plane waves at frequency  $f$  impinge upon the array from  $P$  different known directions  $\{\theta_1, \dots, \theta_p\}$ . The signals observed at the outputs of the sensors array are represented by the  $N$ -dimensional vector [1-3]

$$x(t) = \sum_{i=1}^P s_i(t) a(\theta_i) + n(t) \quad (1)$$

where  $s_i(t)$  is the complex amplitude of the  $i^{\text{th}}$  source, it is a zero-mean complex random variable. Its variance, denoted  $p_i$ , characterizes the signal power of the  $i^{\text{th}}$  source which we wish to estimate

$$p_i = \text{Var}[s_i(t)] = E[s_i(t) s_i(t)^*] \quad (2)$$

Here,  $E[\ ]$  is the expectation operator and the superscript  $*$  represents the complex conjugate. The direction of arrival of the  $i^{\text{th}}$  signal source is represented by the  $N$ -dimensional complex vector  $a(\theta_i)$  and  $n(t)$  represents the additive sensor noise vector. This noise does not correspond to a wavefront arrival and is a background noise generated internally in the instrumentation. The noise is assumed to be spatially white (uncorrelated from sensor to sensor) and the same power level is present in each receiver. With these assumptions, the covariance matrix for the noise alone is  $R_n = E[n(t)n(t)^H] = p_n I$ , where  $p_n$  is the noise power,  $I$  the  $(N \times N)$  identity matrix and the superscript  $H$  denotes the Hermitian transpose operation. Equation (1) may be rewritten in the matrix form

$$x(t) = A s(t) + n(t) \quad t \in \{t_1, t_2, \dots, t_T\} \quad (3)$$

$A$  is the  $(N \times P)$  array manifold matrix containing the manifold vectors for different sources as its columns,  $A = [a(\theta_1), \dots, a(\theta_p)]$ . At the frequency of interest,  $x_i(t)$  is the complex signal component from the  $i^{\text{th}}$  receiver and  $A$  is a known matrix, each column of which is a source direction vector. For any single plane wave arrival, the outputs from the  $N$  individual receivers will differ in phase by an amount determined by the geometry of the array and the arrival direction. In other words, the elements  $A_{qr}$  of the matrix  $A$  are known functions of the signal arrival angles and the array elements locations. Thus, one has  $A_{qr} = \exp(j\varphi_{qr})$  where  $\varphi_{qr}$  is the phase of the signal at the  $q^{\text{th}}$  receiver from the  $r^{\text{th}}$  source, measured relative to some arbitrary reference point.  $s(t)$  is the  $P$ -dimensional vector, the components of which are the complex amplitudes of the sources. It can readily be seen that the output signal from the  $q^{\text{th}}$  sensor may be written as

$$x_q(t) = \sum_{r=1}^P A_{qr} s_r(t) + n_q(t) \quad (4)$$

Since the  $P$  arrivals are by assumption independent, the source covariance matrix is given by

$$R_s = E[s(t)s(t)^H] = \text{diag}(p_1, \dots, p_p) \quad (5)$$

and the diagonal elements are the powers of the sources from the  $P$  directions which we wish to estimate. The spatial covariance matrix of the receiver outputs can be expressed, for signals uncorrelated of each other and of noise, as

$$R = E[x(t)x(t)^H] = A R_s A^H + R_n \quad (6)$$

In practice, the spatial covariance matrix is estimated by a finite number of time domain samples (snapshots) and an estimate of  $R$  is used. We can now derive a variety of processors to estimate the strengths of  $P$  independent signals arriving at array of  $N$  sensors, when the arrival directions are known.

### 3 Power estimation of acoustical sources

#### 3.1 The conventional beamformer

The conventional beamformer, also called the "time delay and sum" beamformer, consists of a system of delay and sum networks which are designed to make the signals from the beamformer direction in phase at each sensor. The acoustical sources amplitude is estimated from the sensor data which is formed by the output data vector  $x(t)$ . The usual approach is to find a matrix  $M$ , such that  $s(t) = M x(t)$ , and for a conventional beamformer (a conventional phased array) the equation is [3]

$$s_{CB}(t) = \frac{1}{N} A^H x(t) \quad (7)$$

A power estimate for the signals can be found by forming the covariance matrix

$$R_{sCB} = E[s_{CB}(t)s_{CB}(t)^H] = \frac{1}{N^2} A^H R A \quad (8)$$

and the strength of the  $i^{th}$  source estimated by the conventional beamformer is

$$p_{iCB} = \frac{1}{N^2} a(\theta_i)^H R a(\theta_i) \quad (9)$$

However, this leads to a biased estimate as can be seen by substituting (6) into (8)

$$R_{sCB} = \frac{1}{N^2} A^H (A R_s A^H + R_n) A \quad (10)$$

So unless  $A^H A = NI$  and  $R_n = 0$ , neither of which is generally the case, then the estimate will be biased.

#### 3.2 Adaptive beamformers

The minimum variance distortionless response criterion to estimate the signal power is a class of adaptive beamforming technique widely used in array processing and is also called the standard Capon beamformer [2]. The technique selects a weight vector  $w$  of the array element outputs in such a manner that the power out of the beamformer is minimized, while the response in the direction of the desired signal (the signal coming from the  $i^{th}$  source) is constrained to unity. The constraint ensures that the signal power coming from the  $i^{th}$  source will be reproduced in undistorted form in the processor output. Thus, this adaptive array tries to eliminate as best it can all

the signals received at the sensors except the signal coming from the  $i^{th}$  wanted source. The weight vector  $w$  is selected so as to minimize the output power of the array

$$\min_w E[w^H x x^H w] = w^H R w \quad (11)$$

$$\text{subject to the constraint } w^H a(\theta_i) = 1 \quad (12)$$

Using Lagrange's method we minimise the power output subject to this constraint by defining a cost function

$$g(w) = w^H R w + \lambda (1 - w^H a(\theta_i)) \quad (13)$$

where  $\lambda$  is the Lagrange multiplier. Differentiating (13) with respect to the weight vector  $w$  and equating to zero gives the optimum weight vector as

$$w = R^{-1} a(\theta_i) (a(\theta_i)^H R^{-1} a(\theta_i))^{-1} \quad (14)$$

and the power of the  $i^{th}$  source estimated by the standard Capon beamformer is

$$p_{iSCB} = \frac{1}{a(\theta_i)^H R^{-1} a(\theta_i)} \quad (15)$$

The standard Capon beamformer has better resolution than the conventional beamformer provided that the array steering vector corresponding to the signal of interest is accurately known. However, the performance of this traditional adaptive beamformer can degrade seriously in practice when errors exist in the signal of interest steering vector, which may be due to look direction error, array sensor position error, and small mismatches in the sensor responses. In such cases the signal of interest might be mistaken as an interference signal and might be suppressed. A robust Capon beamforming algorithm, which is a natural extension of the standard Capon algorithm, is presented to overcome this difficulty. In the robust Capon beamforming algorithm we suppose that  $a(\theta_i)$  is the true direction vector of the  $i^{th}$  source,  $\bar{a}(\theta_i)$  is the assumed direction of the  $i^{th}$  source and we consider that  $a(\theta_i)$  is in the vicinity of  $\bar{a}(\theta_i)$ . This can be expressed mathematically by the following inequality :  $\|a(\theta_i) - \bar{a}(\theta_i)\|^2 \leq \epsilon$ , where  $\epsilon$  is a bound controlling the uncertainty in the assumed look direction. To derive the robust Capon beamforming algorithm we use the reformulation of the standard Capon beamforming problem to which we append the previous inequality

$$\min_w w^H R w \quad (16)$$

$$\text{subject to the constraint } \|a(\theta_i) - \bar{a}(\theta_i)\|^2 \leq \epsilon \quad (17)$$

The optimization problem can be rewritten as the following

$$\min_a a(\theta_i)^H R^{-1} a(\theta_i) \quad (18)$$

subject to the constraint (17). We consider the solution on the boundary of the constraint set and we reformulate the optimization problem as the following quadratic form with a quadratic equality constraint

$$\min_a a(\theta_i)^H R^{-1} a(\theta_i) \quad (19)$$

$$\text{subject to} \quad \left\| a(\theta_i) - \bar{a}(\theta_i) \right\|^2 = \varepsilon \quad (20)$$

This problem can be solved by using the Lagrange multiplier method which is based on the cost function

$$f(a) = a(\theta_i)^H R^{-1} a(\theta_i) + \lambda (\left\| a(\theta_i) - \bar{a}(\theta_i) \right\|^2 - \varepsilon) \quad (21)$$

Differentiating (21) with respect to  $a(\theta_i)$  and equating to zero gives the optimal solution

$$a(\theta_i) = \bar{a}(\theta_i) - U (I + \lambda \Gamma)^{-1} U^H \bar{a}(\theta_i) \quad (22)$$

where  $U$  and  $\Gamma$  are  $(N \times N)$  matrices containing the eigenvectors and eigenvalues of the covariance matrix  $R$  and  $\lambda$  is the Lagrange multiplier. Using (22) in the equality constraint of (20) the Lagrange multiplier is obtained as the solution to the constraint equation

$$\left\| U (I + \lambda \Gamma)^{-1} U^H \bar{a}(\theta_i) \right\|^2 = \varepsilon \quad (23)$$

The signal power estimation of the  $i$ th source using the robust Capon beamformer is then [4]

$$p_{iRCB} = \frac{1}{\bar{a}(\theta_i)^H U \Gamma (\lambda^{-2} I + 2 \lambda^{-1} \Gamma + \Gamma^2)^{-1} U^H \bar{a}(\theta_i)} \quad (24)$$

The standard Capon beamformer is an optimal spatial filter that maximizes the signal to noise ratio, provided that the true covariance matrix and the array steering vector are accurately known. However, the covariance matrix can be inaccurately estimated due to limited data samples and the knowledge of the array steering vector can be imprecise due to look direction error, imperfect array calibrations, gain and phase errors in the sensors. The robust Capon beamformer can be used in such situations for signal power estimation and source location as shown in examples given in this communication. Another estimator using the covariance vector of signals is presented in the next section.

### 3.3 Signal power estimation by the covariance vector

Since we are interested in the signal powers, the covariance matrix of the data contains all the information about these signal strengths. We assume that signals from different directions are uncorrelated. From equation (4) the correlation between sensor  $k$  and  $l$  is then

$$r_{kl} = E[x_k x_l^*] = \sum_{i=1}^P p_i A_{ki} A_{li}^* + p_n \delta_{kl} \quad (25)$$

The sensor noise power on each sensor is constant and equals  $p_n$  and  $\delta_{kl} = 1$  for  $k=l$  and zero otherwise. Equation (25) may be split into real and imaginary components as

$$\text{Re}(r_{kl}) = \sum_{i=1}^P \text{Re}(A_{ki} A_{li}^*) p_i + p_n \delta_{kl} \quad (26)$$

$$\text{Im}(r_{kl}) = \sum_{i=1}^P \text{Im}(A_{ki} A_{li}^*) p_i \quad (27)$$

Of the  $2N^2$  equations represented by (26) and (27) only  $(N^2 \sim N + 1)$  equations are independent, since one has :

$$\text{Re}(r_{kl}) = \text{Re}(r_{lk}); \text{Im}(r_{kl}) = -\text{Im}(r_{lk}); \text{Re}(r_{kk}) = \text{Re}(r_{ll}) \text{ for } k \neq l \text{ and } \text{Im}(r_{kk}) = 0.$$

Equations (26) and (27) may then be written in the form

$$r = B p \quad (28)$$

where  $r$ ,  $B$  and  $p$  are reals;  $r$  is the  $(N^2 \sim N + 1)$  vector which contains the real and imaginary components of  $\{r_{kl}\}$  and is called the covariance vector;  $p$  is the  $(P+1)$  vector containing the signal powers  $\{p_i\}$  and sensor noise power  $p_n$ . Note that if the sensor noise is small, it may be desirable to omit the model of sensor noise.  $B$  is the  $(N^2 \sim N + 1) \times (P+1)$  matrix which contains all the array geometry terms and  $\delta_{kl}$  if required

$$B = \begin{bmatrix} \text{Re}(A_{ki} A_{li}^*) \delta_{kl} \\ \text{Im}(A_{ki} A_{li}^*) \delta_{kl} \end{bmatrix} \quad (29)$$

The least squares solution to (28) is given by

$$p_{CV} = (B^T B)^{-1} B^T r \quad (30)$$

$p_{CV}$  is the vector containing the strengths of signals by the covariance vector. Another estimator which is simple to implement on line and uses a least squares fit estimator of the observed covariance matrix is presented in the next section.

### 3.4 Signal power estimation by the least squares fit estimator

We know the observed or true covariance matrix  $R$  of the array output vector. A possible approach to estimate signal strengths is to select the  $P$  diagonal elements of the diagonal matrix  $\tilde{R}_s$  which matches the unknown matrix  $M = A \tilde{R}_s A^H$  to the observed covariance matrix  $R$  in some sense. We can match  $M$  to  $R$  in various ways. One which is naturally suggested in the communication is to minimize the mean square difference between the corresponding elements of  $M$  and  $R$ . We minimize

$$\xi = \|M - R\|_F^2 = \text{tr}[(M-R)^H (M-R)] = \sum_{i=1}^N \sum_{j=1}^N |M_{ij} - R_{ij}|^2 \quad (31)$$

with respect to the elements of  $\tilde{R}_s$ , where  $\|\cdot\|_F$  denotes the Frobenius matrix norm. Noting that the partial derivatives are  $\partial(A \tilde{R}_s A^H) / \partial p_i = a(\theta_i) a(\theta_i)^H$  and setting  $\partial \xi / \partial p_i = 0$ ,  $i=1, \dots, P$ , we obtain the set of equations

$$a(\theta_i)^H A \tilde{R}_s A^H a(\theta_i) = a(\theta_i)^H R a(\theta_i) \quad (32)$$

The explicit solution to (32) is given by

$$p_{LS} = N^2 [ (A^H A) \quad (A^H A)^* ]^{-1} p_{CB} \quad (33)$$

where  $p_{LS}$  is a vector whose entries are the least squares fit estimates of signal strengths,  $p_{CB}$  is a vector whose entries are the outputs of the conventional beamformer  $p_{CBi} = a(\theta_i)^H R a(\theta_i) / N^2$  and  $\odot$  denotes the Hadamard product of two matrices [5], also called elementwise multiplication; we have  $A \odot B = [A_{ij} B_{ij}]$ . Note that the least squares fit estimator comprises the conventional beamformer followed by further processing.

### 4 Numerical and experimental tests

Numerical simulations and experimental tests were designed to evaluate the performances of the estimators presented in the communication. The conventional beamformer (CB), the standard Capon beamformer (SCB), the robust Capon beamformer (RCB), the covariance vector estimator (CV) and the least squares fit estimator (LS) of the observed covariance matrix are employed to estimate the strengths of signals arriving at an array of receivers. In our simulations, we assume a uniform linear array with  $N=6$  omnidirectional sensors and half-wavelength sensor spacing. Four point sources are located at bearings of  $-30^\circ$ ,  $0^\circ$ ,  $22^\circ$  and  $45^\circ$ . The source powers are respectively 60 dB, 55 dB, 80 dB and 70 dB and the number of snapshots is  $T=4096$ . The signal to noise ratio is  $SNR = 20$  dB. Figure 1 shows the power estimates as a function of the direction angle in the case where there are no gain and phase errors in the acoustical sensors. The small circles denote the true direction of arrival and the true power of the four sources. The SCB and RCB estimators provide excellent power estimates of the incident sources and can also be used to determine their directions of arrival based on the peak power locations. The CB estimator has much poorer resolution than both SCB and RCB.

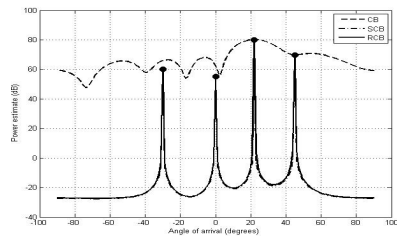


Figure 1: Power estimates versus the steering direction using CB, SCB and RCB without gain and phase errors

Figure 2 shows the power estimates in the case where there are a gain error of 0.02 and a phase error  $0.2^\circ$  in each acoustical sensor. We note that SCB and RCB can still give good direction of arrival estimates for the incident signals based on the peak locations, however, the SCB estimates of the incident signal powers are way off. In this case, only the RCB algorithm gives good power estimates of the incident sources and can also be used to determine their directions of arrival based on the peak locations.

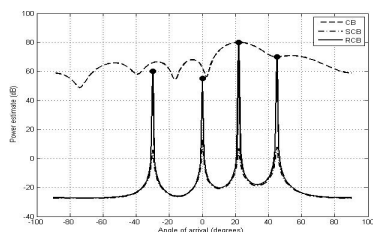


Figure 2: Power estimates versus the steering direction using CB, SCB and RCB with gain and phase errors

Figure 3 shows the effect of SNR on source power estimation using the RCB, the CV and the LS fit estimator. A gain error of 0.02 and a phase error  $0.2^\circ$  in each sensor have been considered. We note that these three estimators give good power estimates, only a maximum bias of 1 dB is obtained for the source of 55 dB situated at  $0^\circ$ . The source of 70 dB is slightly underestimated by the CV and LS algorithms which give the same result.

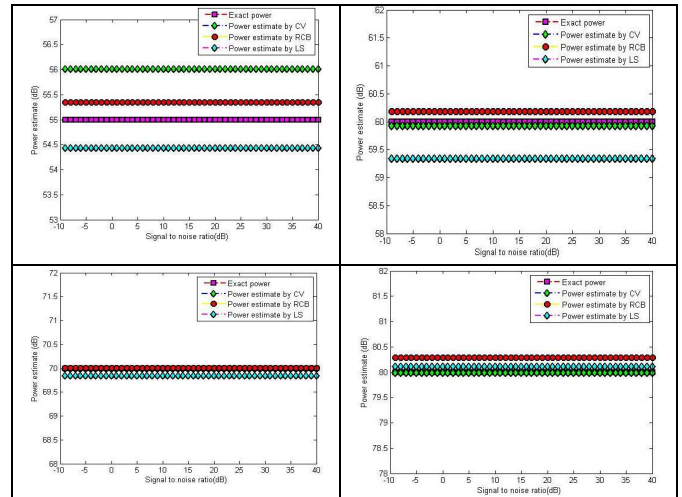


Figure 3: Power estimates versus the SNR

Figure 4 shows the source power estimates versus the number of snapshots using the RCB and the CV algorithm with 0.02 gain error and  $0.2^\circ$  phase error in each sensor and  $SNR = 20$  dB. The RCB algorithm overestimates slightly the true source powers with a maximum bias of 1 dB. For the simulated data, the RCB, the CV and the LS power estimation algorithms presented in the paper exhibit only a very slight bias.

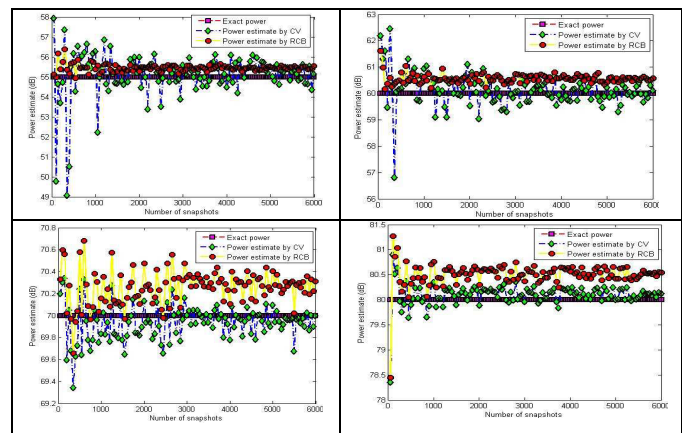


Figure 4: Comparison of the power estimates versus the number of snapshots

The remaining part of the section is focused on the application of the developed algorithms to the experimental identification of noise sources generated by two loudspeakers. The experimental setup is schematically shown in the block diagram of Figure 5 where an acoustical array and two sources (the loudspeakers) are placed in the anechoic chamber. The receiving acoustical array is linear and formed with six omnidirectional microphones equally spaced, with interelement spacing of  $d = 4.5$  cm. The two sources and the acoustical array are in the same horizontal plane. The transmitting loudspeakers generate two typical audio signals at a frequency of 3800 Hz corresponding to a microphone separation distance of one-half wavelength. The number of snapshots is  $T=4096$ . We are able to find the direction of the two sources by using the MUSIC algorithm, however, unlike the methods mentioned earlier, MUSIC does not physically correspond to the signal power. The MUSIC algorithm is only an indicator of directions of arrival of different signals.

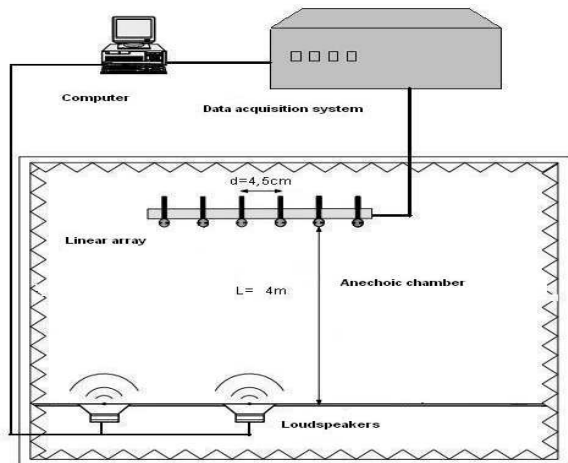


Figure 5: Block diagram experimental system

Figure 6 shows the normalized angular spectrum function obtained from MUSIC where important peaks appear at the signal directions. We obtain the angular position of the sources  $\theta_1 = 10^\circ$  and  $\theta_2 = 19^\circ$ . Once the arrival angles have been determined we can estimate the power of the two acoustical sources by the proposed algorithms.

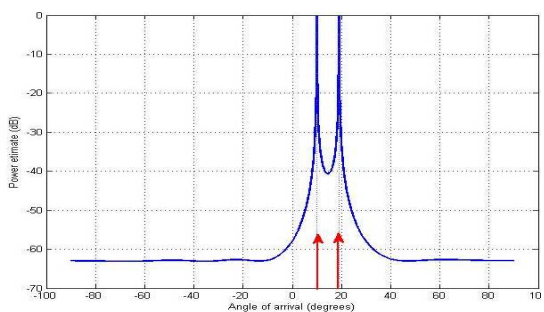


Figure 6: Experimental MUSIC spatial spectra

Table 1 shows the results obtained by our algorithms. The experimental results confirm that the CV and the LS estimators give very similar results and the RCB estimator overestimates very slightly the source powers.

Table 1: Power estimation using different estimators

| Estimator                       | RCB          | CV          | LS           |
|---------------------------------|--------------|-------------|--------------|
| Source 1, $\theta_1 = 10^\circ$ | 72,0<br>8 dB | 70,95 dB    | 70,94<br>dB  |
| Source 2, $\theta_2 = 19^\circ$ | 73,1<br>1 dB | 71,81<br>dB | 71,8<br>0 dB |

Figure 7 shows the power estimates versus the steering direction using the RCB algorithm. From this plot we obtain simultaneously an estimation of the directions of arrival and an estimation (slightly overestimated) of the power estimates based on the peak locations.

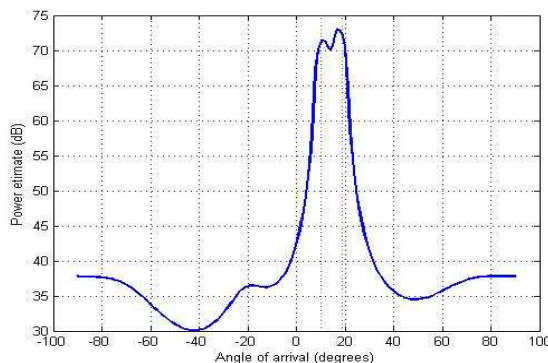


Figure 6: Experimental robust Capon beamformer

## 5 Conclusion

Five signal processors to estimate the strengths of signals arriving at an array of sensors have been studied. The conventional beamformer and the standard Capon beamformer provide in general poor power estimates of the incident sources. The robust Capon beamformer gives good power estimates and can also be used to determine the directions of arrival of incident sources. The covariance vector and the least squares fit estimator give excellent power estimates. From numerical simulations and field tests, the RCB, CV and LS algorithms exhibit remarkable effectiveness in finding the strengths of signals.

These techniques have been developed for the estimation of signal strengths using an array of receivers. However, the principles can be applied to a wide range of other estimation problems, of which the spectrum analysis of a time series is an example.

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