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Peculiarities of acoustooptic interaction in media with strong acoustic anisotropy

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Using the method of plane wave angle spectrum the general solution of acoustic beam diffraction in the anisotropic media was obtained. This expression gives a possibility to calculate the structure of acoustic field at the arbitrary distance from the transducer and to solve the problem of acousto-optic interaction in the acoustic field with amplitude and phase inhomogeneity caused by media acoustic anisotropy. Characteristics of acousto-optic collinear interaction in media with and without acoustic anisotropy were calculated and compared. Also influence of acoustic anisotropy on acousto-optic interaction in (1-10) plane of paratellurite crystal was examined Calculations were carried for the acoustic beams with various parameters.

1 Introduction

Actually optoelectronic devices with operating principle based on the acousto-optic (AO) interaction are widely used to control the optical radiation. Such acousto-optic devices as modulators, deflectors and filters are issued by many manufactures and used not only in laser physics and optoelectronics but also in ecology, medicine and warfare. This situation emerged due to the wide functional capabilities, high operation speed, low control voltage, reliability and construction simplicity.

Usually treating the AO interaction acoustic beam is considered to be ideal, this means that magnitude of acoustic wave is equal in all points and wave fronts are plane. But such beams are not realizable in practice. Fundamental reasons for appearance of inhomogeneous acoustic beam are diffraction effects produced by finite size of transducer. Acoustic field inhomogeneity manifests itself in two ways. Inhomogeneity of amplitude only changes the value of acoustic power needed for obtaining given AO diffraction efficiency. Phase inhomogeneity that defines the magnitude of beam wave front distortion has greater influence. In this case such important parameter for acousto-optics as Bragg angle losses sense as it is being counted from the front of acoustic wave. Acoustic field phase inhomogeneity changes magnitude angle and frequency characteristics of AO interaction significantly; off course this influences the AO devices operation [1,2].

Media acoustic anisotropy also influences on the structure of acoustic beam [2-10]. Since nowadays crystals with extra large anisotropy of elastic properties such as tellurium dioxide (TeO_2), calomel (Hg_2Cl_2), tellurium (Te) and others are used in acousto-optics it is obvious that in the process of evaluation of AO devices characteristics it is not possible to neglect media anisotropy.

Problem of ultrasound beams with finite aperture propagation in crystals was treated by many investigators from the beginning of 1960es. It is possible to point out two main methods on the base of scalar diffraction theory: method of Reley-Zommerfeld that uses Green function to evaluate the field produced by plane transducer aperture [2,3], and method of plane waves angle spectrum [5,6]. The last of these two methods was widely spread due to it's physical visibility.

2 Basic relations

Surfaces of slowness play sufficient role in crystal acoustics [9]. Normal **s** to this surface in every point defines the direction of energy flow (Pointing vector). In general case vectors **n** and **s** are not collinear and angle between them characterizes acoustic energy walk off. During the propagation process in the crystal every plane wave component of acoustic beam has unique direction of wave normal and accordingly unique walk off angle. As a consequence beam structure (magnitude and phase) at the adjusted distance z from the transducer differs from the structure of the analogous beam in isotropic media. In the articles mentioned, analysis was carried in the parabolic approximation, when the slowness surface for chosen acoustic mode was approximated by the expressions:

$$S(\varphi) \approx S_0\left(1 + B\varphi^2\right) \text{ or } S(\varphi) \approx S_0\left(1 + A\varphi + B\varphi^2\right) \quad (1)$$

where φ is the angle that defines direction of wave normal **n**, A and B are approximation coefficients. These coefficients are defined by crystal elastic properties. First variant corresponds to pure acoustic modes propagating along crystallographic axes [3-6], and the second one corresponds to the more common case when beams of quasi-longitudinal and quasi-transverse waves propagate in the main planes [7-9]. It was shown that coefficient A is responsible for beam walk off in general, and coefficient B defines increase or reduction of beam divergence. The most important result was the understanding of the fact that media anisotropy produces the effect of acoustic field scaling. This means that field structure on the distance z from the transducer is equal to the structure on the distance $z \cdot w$ in isotropic media, where w – is the coefficient that is defined by the coefficients of expansion Eq.1, this coefficient may be more or less then a unit [7-9]. This is the reason why in the articles [7,9] quantitative calculations were carried without concretization of the crystal treated.

For the fabrication of AO devices of visible and near IR region at the present time paratellurite is mainly used. This crystal is widely used due to its extremely high value of AO figure of merit. Maximal value of figure of merit $M = 1200 \cdot 10^{-18}$ s^3/g is achieved when share acoustic mode propagates along direction [110] in crystal. But this direction is not used on practice due to the great acoustic beam inhomogeneity that is aroused by high acoustic anisotropy near direction [110].

Oblique cuts of paratellurite with lower figure of merit M values are used to obtain homogeneous beam in applied acousto-optics. In this case it is possible to arouse acoustic beam with good homogeneity, but in this case beam walk off appears.

In the acoustical part this paper is the extension of the investigations represented in [8,9]. Original expression that allows to evaluate acoustic beam structure in the near and far diffraction zones for any direction of beam propagation in crystal.

2.1 Propagation of acoustic beams in anisotropic media

Further we will consider that media is linear and homogeneous so it is possible to use superposition principle and decompose acoustic disturbance $u_0(x, y)$ in the input plane $z = 0$ with the help of spatial Fourier transform into the angle spectrum of plane waves $U_0(K_x, K_y)$, then take into consideration spectral components change while they propagate trough arbitrary distance z. After this we will

sum all these components with the help of inverse Fourier transform. For the harmonic field with frequency Ω

$$U_0(S_x, S_y) = \iint_{\infty} u_0(x, y)\exp[j\Omega(S_x x + S_y y)]dxdy \qquad (2)$$

here projections of wave vector $\mathbf{K} = \mathbf{n}K$ are evaluated through the projections of slowness vector $\mathbf{K} = \mathbf{n}\Omega/V = \Omega\mathbf{S}$. Acoustic field $u(x, y, z)$ is also possible to express through its spectrum at the distance z from the transducer $U(S_x, S_y, z)$:

$$u(x, y, z) = \left(\frac{\Omega}{2\pi}\right)^2 \iint_{\infty} U(S_x, S_y, z)\times \times\exp[-j\Omega(S_x x + S_y y)]dS_x dS_y \qquad (3)$$

As far as function $u(x, y, z)$ corresponds to Helmholtz equation

$$\Delta u + K^2 u = 0 \qquad (4)$$

after substitution of Eq.3 into Eq.4 it is possible to obtain differential equation for function $U(S_x, S_y, z)$. Solving this equation with border conditions $U(S_x, S_y, 0) = U_0(S_x, S_y)$, we may find particular solution in the form:

$$U(S_x, S_y, z) = U_0(S_x, S_y)\exp\left[-j\Omega z\sqrt{S^2 - S_x^2 - S_y^2}\right] \qquad (5)$$

In paraxial approximation $S_x, S_x << S$ Eq.5 takes simpler form:

$$U(\alpha S, \beta S, z) \approx \approx U_0(\alpha S, \beta S)\exp\left[-j\Omega S z\left(1 - \frac{\alpha^2 + \beta^2}{2}\right)\right] \equiv \equiv U_0(\alpha S, \beta S)T(\alpha, \beta, S, z) \qquad (6)$$

In this equation direction cosines for vector **S** are: $S_x = \alpha S$ and $S_y = \beta S$.

Basing on the structure of Eq.6 it is possible to conclude that crystal area with length z acts on the acoustic beam as a linear system with transfer function $T(\alpha, \beta, S, z)$. Crystal transfer function has the same view as the optical transfer function of free space, but significantly differs in the moment that slowness S is the functions of direction cosines α and β. From Eq.6 follows that in the beam propagation process its spectral structure doesn't change, but every spectrum component acquire additional phase shift. This shift is defined firstly by distance that component passes from input to output planes, and secondly by the meaning of phase velocity.

Substituting Eq.6 into Eq.3 we will obtain final equation that gives solution of the problem of acoustic beam propagation in anisotropic media:

$$u(x, y, z) = \left(\frac{\Omega}{2\pi}\right)^2 \iint_{\infty} U_0(\alpha S, \beta S)\left[S^2 + S\left(\alpha\frac{\partial S}{\partial \alpha} + \beta\frac{\partial S}{\partial \beta}\right)\right] \times\exp\left\{-j\Omega S\left[\alpha x + \beta y + \left(1 - \frac{\alpha^2 + \beta^2}{2}\right)z\right]\right\}d\alpha d\beta \qquad (7)$$

Equation 7 gives the opportunity to calculate the structure of acoustic beam for every propagation direction in crystal and at arbitrary length from the transducer in near and far diffraction field. Unlike of the other articles in the process of derivation of equation parabolic approach was not used.

3 Acoustic field structure simulation

With the help of Eq.7 it is possible to calculate acoustic beam structure in media with arbitrary magnitude of acoustic anisotropy. Also the cross-section of the acoustic beam on the media input may be arbitrary in size and structure. We will examine acoustic beams with rectangular cross-section and with Gaussian structure. During the computations we will use spherical system of coordinates with azimuthal angle φ being measured from X axis in the plane XY and polar angle θ, measured from Z axis to the plane XY (X,Y,Z are crystallographic axes).

3.1 Collinear acousto-optic interaction in acoustically isotropic media

At first we will treat the influence of acoustic field structure on the collinear AO diffraction characteristics. One of the most widely used materials for collinear filters fabrication is calcium molybdate ($CaMoO_4$). Shear acoustic mode used in collinear diffraction near X crystallographic axis may be considered isotropic. In this case only diffraction effects, produced by the finite sizes of transducer, influence on the structure of acoustic field.

Fig. 1 represents acoustic field in $CaMoO_4$, when axial component of the beam propagates along X crystallographic axis. Size of the presented area is 2cm in vertical direction and 4cm in horizontal direction.

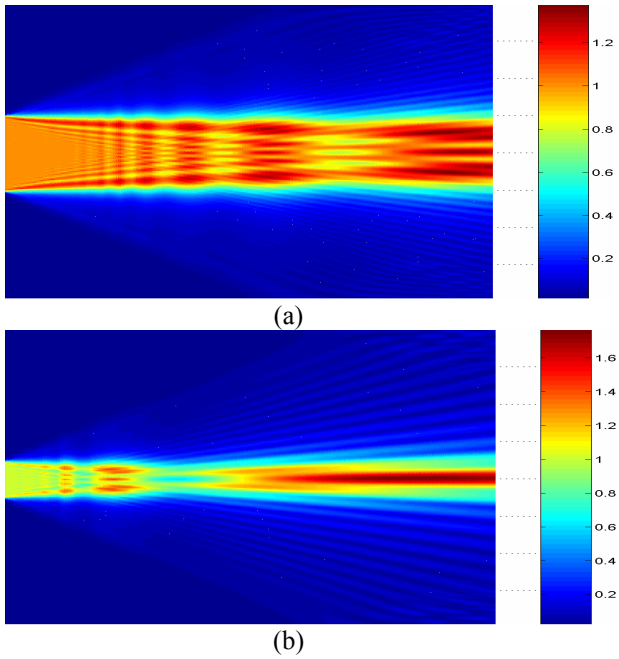


Figure 1: Structure of acoustic field in the CaMoO4 crystal aroused by quadratic transducer with side size 0.5cm (a), and 0.25cm (b). $f = 40$ MHz

After calculation of the acoustic field structure it is possible to examine the characteristics of AO diffraction.

Figure 2 presents the dependences of collinear AO interaction efficiency on the Raman-Nath parameter Γ for the acoustic fields aroused by the transducers of different shapes.

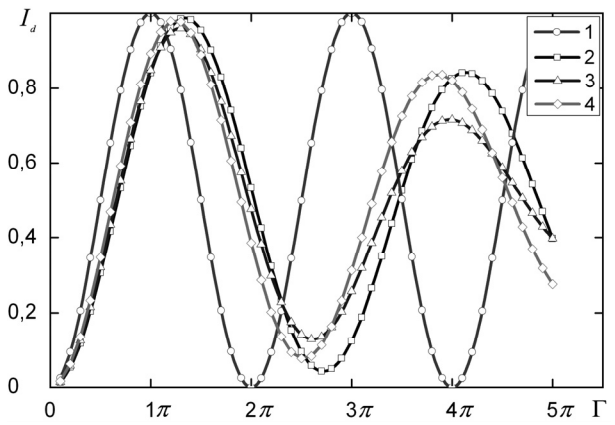
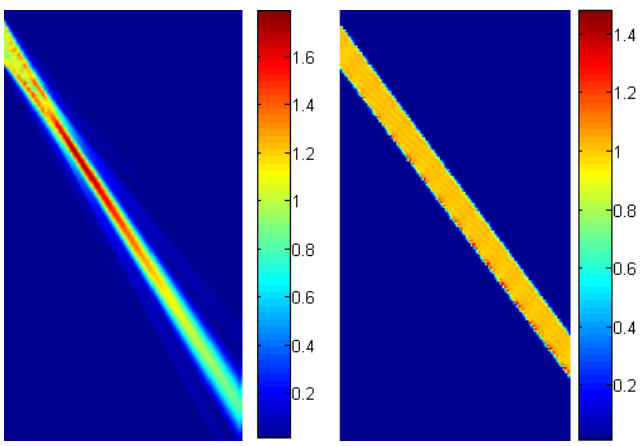


Figure 2: Diffraction efficiency dependences on the Raman-Nath parameter for the transducers with side size 0.25cm. 1 – ideal acoustic field, 2 – quadratic transducer, 3 – diamond-shaped transducer, 4 – elliptical transducer.

All calculations were carried for the transducer with size 0.25cm, as in this case acoustic files is more inhomogeneous. Examining these curves it is possible to conclude that acoustic field inhomogeneity leads to the higher acoustic power needed for obtaining maximal diffraction efficiency. It is important that in the case of transducers of finite sizes AO diffraction never achieves 100%. Also diffraction efficiency doesn't depend on the shape of transducer significantly when acoustic power is very high - $\Gamma \le 2\pi$.

3.2 Acousto-optic interaction in acoustically anisotropic media

Nowadays most part of AO devices is made on the base of paratellurite crystals. This material is so popular due to the extremely high value of figure of merit. Also it posses extremely high acoustic anisotropy in the *XY* plane. Acoustic anisotropy coefficient $\kappa_\varphi(\varphi_0)$ [9] may achieve 52



(a) (b)

Figure 3: Structure of acoustic field in XY plane of TeO_2 crystal. a - $\varphi_0 = 43°$, b - $\varphi_0 = 12.64°$, $f = 60$ MHz.

and walk off angle χ_φ may be so high as 74°.

Figure 3 presents the structure of acoustic field aroused in *XY* plane of TeO_2 crystal by 0.2cm transducer. The size of presented area is 1cm in vertical direction and 2cm in horizontal. Case *a* take place when angle $\varphi_0 = 43°$, then $\kappa_\varphi \approx 20$ and $\chi_\varphi = 59°$. Case *b* takes place when $\varphi_0 = 12.64°$ that is the direction of autocollimation in *XY* plane.

The same calculations were carried for the acoustic beams with Gaussian structure. Acoustic field structures that take place for the directions $\varphi_0 = 43°$ and $\varphi_0 = 12.64°$ are shown at Fig.4. The main feature of Gaussian beams is the absence of thin structure of the beam, only widening takes place.

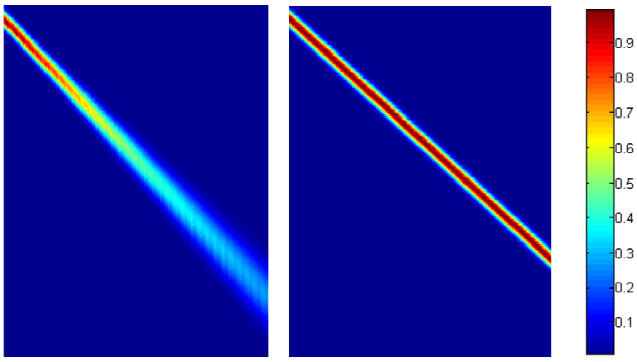


Figure 4: Structure of acoustic field in XY plane of TeO_2 crystal. a - $\varphi_0 = 43°$, b - $\varphi_0 = 12.64°$, $f = 60$ MHz.

3.3 Acousto-optic interaction in tellurium dioxide

XY plane is not used in AO devices due to the extremely high acoustic anisotropy. Plane (1-10) is mostly used for technical applications. It has comparatively low acoustic anisotropy. But still it is enough to influence on the acoustic field. This influence leads to curvature of acoustic wave fronts. We will examine AO interaction with the following parameters $\vartheta_i = 39°$, $l = 2$ mm, $f = 30$ MHz. Ultrasound wave vector is directed at $\varphi_0 = 45°$ $\theta_0 = 88°$, in this case walk off angle $\chi_\theta = 18.3°$ $\kappa_\theta = 10$. The influence of acoustic field structure on AO interaction characteristics we will examine calculating them for the cases when optical beam passes through the AO cell at different distances L_0 from the transducer.

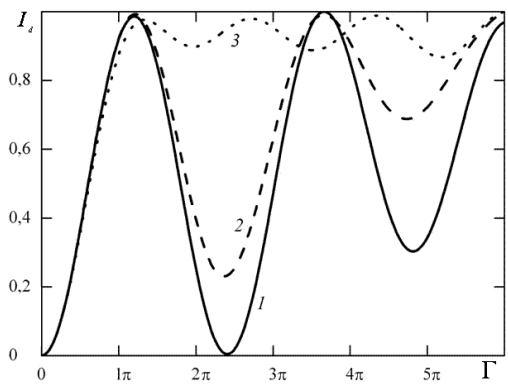


Figure 5: Dependences of AO diffraction efficiency on Raman-Nath parameter. 1 - $L_0 = 0.25$ cm, 2 - $L_0 = 0.75$ cm, 3 - $L_0 = 2$ cm.

Dependences of AO diffraction efficiency on Raman-Nath parameter for the acoustic beam with initially rectangular cross section are at Fig. 5. Curve 1 takes place when $L_0 = 0.25$ cm. Its shape is close to the $\sin^2$ law that is valid for ideal acoustic field. This means that at the small distances from transducer media does not manage to influence the acoustic field significantly. But with the growth of L_0 wave front twists this causes the growing difference between ideal case and real. At the distance $L_0 = 2$ cm acoustic wave front is curved so strongly that after the achieving of maximal value it becomes almost independent on the Raman-Nath parameter Γ, that is proportional to the acoustic power. This situation was described in [2].

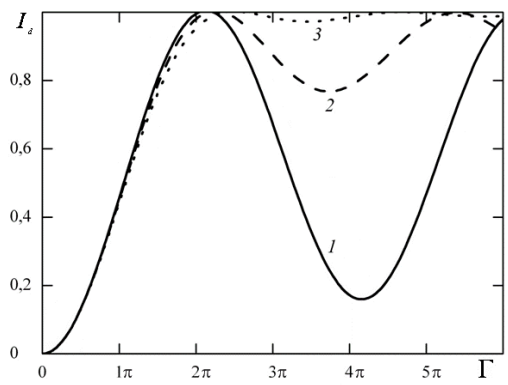


Figure 6: Dependences of AO diffraction efficiency on Raman-Nath parameter for Gaussian acoustic beam. 1 - $L_0 = 0.25$ cm, 2 - $L_0 = 0.75$ cm, 3 - $L_0 = 2$ cm.

The same calculations were carried for the acoustic beam with Gaussian cross section (Fig.6). The curves behave in the same manner as those presented at Fig.5. The only difference is the higher Raman-Nath parameter needed for achieving maximal diffraction efficiency. This difference may be explained by the lower real interaction length.

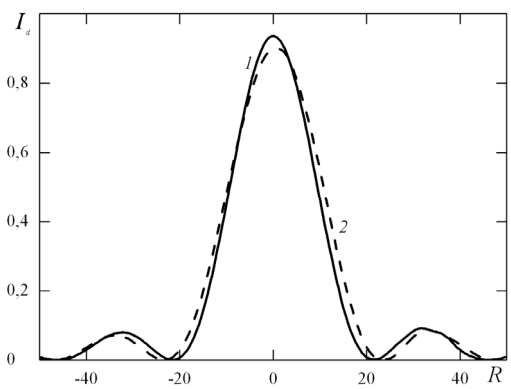


Figure 7: Transmission functions of AO interaction for $\Gamma = \pi$. a - cm $L_0 = 0.25$ cm, b - $L_0 = 2$ cm.

Strong media acoustic anisotropy also may affect the AO diffraction transmission functions. This effect is illustrated at Fig. 7. Here curve *1* takes place when $L_0 = 0.25$ cm and curve *2* is calculated for the case when $L_0 = 2$ cm. Analyzing presented functions it is possible to conclude that acoustic wave front curvature may lead not only to the reduction of AO diffraction efficiency and filter pass band broadening, but also for the shift of the transmission band.

Maximal value of diffraction efficiency for curve *1* is 0.96, and for curve *2* – 0.90. Transmission band for case *2* is 11.6% higher then for case *1*. The shift of transmission band reaches 3% of the transmission band. One more feature is the different shape and magnitude of side lobes.

5 Conclusion

The examination of acoustic anisotropy influence on the structure of acoustic beam was carried in this paper. Presented theory gives an opportunity to calculate the structure of acoustic beam propagating in arbitrary direction at ay distance form the transducer. Examination of amplitude and phase acoustic inhomogeneity enables to explore the influence of acoustic field structure on the AO interaction characteristics.

References

[1] Balakshy V.I., Linde B.B., Vostrikova A.N. “Acousto-optic interaction in a non-homogeneous acoustic field excited by a wedge-shaped transducer”, *Ultrasonics* 2008. V. 48. № 5. P. 351-356.

[2] S.N. Antonov, A.V. Vainer, V.V. Proklov, Yu.G. Rezvov, “New acoustooptic effect: Constant high diffraction efficiency in wide range of acoustic power”, *Technical Physics,* 2009. V. 54. № 6. P. 882-887.

[3] Papadakis E.P., “Diffraction of ultrasound radiating into an elastically anisotropic medium”, *J. Acoust. Soc. Amer.*, 1964. V. 36. № 3. P. 414-422.

[4] Cohen M.C. “Optical study of ultrasonic diffraction and focusing in anisotropic media”, *J. Appl. Phys*. 1967. V. 38. № 10. P. 3821-3828.

[5] Kharusi M.S., Farnell G.W. “On diffraction and focusing in anisotropic crystals”, *Proc. IEEE.* 1972. V. 60. № 8. P. 945-956.

[6] Szabo T.L. Generalized Fourier transform theory for parabolically anisotropic media”, *J. Acoust. Soc. Amer*. 1978. V. 63. № 1. P. 28-34.

[7] Newberry B.P., Thomson R.B. “A paraxial theory for the propagation of ultrasonic beams in anisotropic solids”, *J. Acoust. Soc. Amer.* 1989. V. 85. № 6. P. 2290-2300.

[8] Kastelik J.S., Gazalet M.J., Bruneel C., Bridoux E. “Acoustic shear wave propagation in paratellurite with reduced spreading”, *J. Appl. Phys.* 1993. V. 74. № 4. P. 2813-2817.

[9] V.N. Belyi, N.S. Kazak, V.K. Pavlenko, E.G. Katranji, S.N. Kurilkina. “Propagation of ultrasonic beams in paratellurite crystal”, *Ultrasonics.* 1999. V.37, pp.377-383.

[10] Voloshinov V.B., Polikarpova N.V. “Acousto-optic investigation of propagation and reflection of acoustic waves in paratellurite crystal”, *Appl. Opt.* 2009. V. 48, № 7, P. C55-C66