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A dynamical model for simulating a debate outcome

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Abstract. A group of agents is faced with a collective decisional problem. The corresponding debate is viewed as a dynamic process. An initial theoretical model based on a multicriteria decision framework was proposed in [7] yet without semantic justifications and any explicit dynamic representation. A second descriptive model has been proposed in [6], according to which social influences and argumentation strategy govern the dynamics of the debate. This paper seeks to justify the equations introduced in [7], using the semantic concepts discussed in [6], in order to provide a model for a debate held within the framework of control theory that explicitly lays out the dynamic aspects and moreover offers additional perspectives for purposes of controlling the debate.

Keywords: Debate, influence, decisional power, Choquet integral, control, collective decision, social network.

1 Introduction

A group of agents is faced with a collective decision; in response, a debate has been organized to identify which alternative appears to be the most relevant following a deliberation. This study will be limited to the binary, albeit common, situation involving two options denoted $+1$ and -1 . It is assumed that each agent has an inclination to choose one of the alternatives $+1$ and -1 , though due to the influence of other agents this inclination may differ from the agent's actual decision [3]. In general terms, it can be considered that each time a speaker intervenes in the debate, agents may change their preference due to social influences taking place within the group. Once agents' preferences reach a point of no longer changing, then the deliberation process ends and a group decision is made. The aim of this debate is for every agent to hear the arguments of all other agents by the end of the deliberation process and then to make a final decision based on full knowledge of the facts.

This deliberation is viewed as a dynamic process with its own dynamics and where agents' beliefs and preferences evolve as arguments are exchanged. The deliberation outcome thus depends both on the order in which agents intervene in the debate to explain their opinion and on the influence a given agent may exert on a social network.

In this context, social influence is related to the statistical notion of the decisional power held by an individual within a social network, as proposed in [5] and [3].

One of the conclusions drawn in [3] concerns the integration of dynamic aspects into the influence model. The authors' framework is indeed a decision-making process activated after a single step of mutual influence. In reality, such mutual influence does not necessarily stop after just one step but may actually become iterative. This paper proposes a possible extension of the results presented in [3] for use in the dynamic case. The evolution of agents' beliefs throughout the debate either change or reinforce the agents' convictions relative to their initial preference. Intuitively, as well as from other standpoints, an agent's social influence depends on the relative strength of other agents' convictions. The idea for our model therefore is to define influence as a time-dependent variable.

In [7], the concepts of influence and conviction during the simulation of a debate are introduced. This article will follow up on the prior work proposed in [1]. Our main improvement here over previous efforts relates to the fact that in [7], coalitions of agents were modeled using capacities, and the change in conviction during a debate was computed with a symmetric Choquet integral, which is in fact an aggregation function typically introduced in multicriteria decision making [2]. The main drawback in [7] pertains to a lack of semantic justifications.

Reference [3] provides a formal framework to define the notion of influence, while [7] introduces the revision equations relative to agents' convictions and preferences. Moreover, [6] suggests a cybernetic interpretation to merge both of these models. The present paper is intended as a continuation of [7], with [6] also used for guidance. The main contribution of this paper is to propose the state equations of the cybernetic interpretation in order to describe the way agents' convictions may evolve over time. To achieve this goal, a capacity will be introduced to model the relative importance of agents in the debate; such a capacity is based on the decisional power of agents using the generalized Hoede-Bakker index [3, 5]. Consequently, a number of simulations will be proposed to illustrate the collective decision-making process.

The paper will be organized as follows. Section 2 will briefly recall the main concepts of the models presented in [7] and [3]. From this formal framework, Section 3 will establish the state equations that serve to model the dynamic relationships between convictions and influences when a speaker-agent / listener-agent pair is isolated. Following a presentation of preference changes, Section 4 will offer a few illustrations. Lastly, Section 5 will provide the conclusion and outlook for future research.

2 Concepts and notations

2.1 Notion of influence in a debate

The assumption behind our model is that an agent's influence is correlated with his capacity to alter the group decision. It addresses the concept of the "weight" of an agent's choice in a collective voting procedure. This "weight" parameter cannot be static since it needs to evolve with agent preferences, which in turn allow for the formation of certain coalitions that are more likely than others.

It is common experience that during the discussion phase, some agents will change their initial opinions. The reasons for this change, in assuming that it is not a random occurrence, may be of different types. The most natural reason is that they have been swayed by the arguments of a particular agent or group (coalition) of agents, or that they feel somewhat obliged, owing to a hierarchical, political or perhaps even more obscure reason, to follow the opinion of that particular agent or coalition. Another reason may be that they are acting in reaction against a given agent or coalition, by systematically embracing the opposite opinion. We use the generic word "influence" herein to refer to all these types of phenomena [4].

Models have been introduced into game theory in order to represent influence in social networks. The point of departure is the concept of the Hoede-Bakker index, a notion that computes the overall decisional power of an agent within a social network, which in this case is a group of n agents. This index was developed in 1982 [5]; an extended definition of decisional power was proposed in [3] and will now be summarized. The reasons behind the existence of influence phenomena, i.e. why a given individual finally changes his decision, is more a matter of the psychological sciences and lies beyond the scope of such approaches.

Let's start by considering a set of agents $\{a_1, \dots, a_n\}$, denoted $N = \{1, \dots, n\}$ in order to simplify notations along with a power set denoted $2^{\{a_1, \dots, a_n\}}$. Each agent is inclined to choose either $+1$ or -1 . An inclination vector, denoted i , is an n -vector consisting of $+1$ and -1 . The j -th coordinate of i is thus denoted $i_{a_j} \in \{-1, +1\}$ and represents the inclination of agent a_j . Let $I = \{-1, +1\}^n$ be the set of all inclination vectors.

It can then be assumed that agents influence one another; moreover, due to influences arising in the network, the final decision of an agent may differ from his original inclination. In other words, each inclination vector $i \in I$ is transformed into a decision vector $B(i)$, where $B : I \rightarrow I$, $i \mapsto B(i)$ is the influence function. The coordinates of $B(i)$ are expressed by $(Bi)_{a_j}$, $j \in \{1, \dots, n\}$ and $(Bi)_{a_j}$ is the decision of agent a_j . Lastly, $gd : B(I) \rightarrow \{-1, +1\}$ is a group decision function, assigned the value $+1$ if the group decision is $+1$ and the value -1 for a group decision of -1 .

An influence function B may correspond to a common collective behavior. For example, in [3] a majority influence function $Maj^{[t]}$ parametrized by a real t has been introduced. More precisely, for a given $i \in I$,

$$Maj^{[t]}i = \begin{cases} 1_N & \text{if } |i^+| \geq t \\ -1_N & \text{if } |i^+| < t \end{cases}$$

where $i^+ = \{k \in N \mid i_k = +1\}$ and 1_N (resp. -1_N) is the vector equal to 1 (resp. -1) everywhere.

This set-up corresponds to the intuitive collective human behavior: when a majority of players have an inclination of +1, then all players decide +1. Many classifications of potential collective behavior (polarization, groupthink, mass psychology, etc.) can thus be described mathematically.

An influence function may also be defined as a simple rule. For example, the following rule may be associated with the *Guru* function: “when a_{Guru} thinks +1, then all agents decide +1“. Another example would be the opportunistic behavior, i.e.: “when most of my supervisors decide +1, then I decide +1”.

It can also be anticipated that mapping $B : I \rightarrow I$ is learned from experiment. The identification of B may be perceived as a data-mining step using knowledge bases in which collective decisions have been recorded as minutes of company meetings.

Definition 1. *The Hoede-Bakker index of agent a_j is defined for a given B and a given gd by:*

$$GHB_{a_j} = \frac{1}{2^{n-1}} \sum_{\{i \mid i_{a_j} = +1\}} gd(B(i)).$$

The main drawback with the Hoede-Bakker index is that it blurs the actual role of the influence function, by analyzing the final decision only in terms of success and failure. The decision is successful for an agent once his inclination matches the group decision.

In [3], the authors distinguish the influence component from the group decision component and moreover propose a first modified index of decisional power, whereby the agent’s decision must coincide with the group’s decision to constitute a success for the agent. Lastly, these authors provided a second modified decisional power, allowing the inclination vectors to be assigned unequal probabilities.

Definition 2. *Let $p : I \rightarrow [0, 1]$ be a probability distribution, with $p(i)$ being the probability of an i occurrence. The modified decisional power of agent a_j for given B , gd and p can then be expressed as:*

$$\phi_{a_j}(B, gd, p) = \sum_{\{i \mid (Bi)_{a_j} = +1\}} p(i).gd(B(i)) - \sum_{\{i \mid (Bi)_{a_j} = -1\}} p(i).gd(B(i)).$$

To conclude this summary section, for each agent a_j the probabilities of success and failure are recalled as follows:

$$SUC_{a_j}(B, gd, p) = \sum_{\{b \in I \mid (b)_{a_j} = gd(b)\}} p \circ B^{-1}(b)$$

$$FAIL_{a_j}(B, gd, p) = \sum_{\{b \in I \mid (b)_{a_j} = -gd(b)\}} p \circ B^{-1}(b).$$

$$\text{Note that: } \phi_{a_j}(B, gd, p) = SUC_{a_j}(B, gd, p) - FAIL_{a_j}(B, gd, p).$$

2.2 Convictions and preferences during a debate

This section will present the dynamic model of the debate proposed in [7]. The influence an agent may exert on the others in the debate is modeled by a capacity over $2^{\{a_1, \dots, a_n\}}$.

Definition 3. A capacity v over $2^{\{a_1, \dots, a_n\}}$ is a set function $v : 2^{\{a_1, \dots, a_n\}} \rightarrow [0, 1]$ such that $v(\emptyset) = 0$, $v(\{a_1, \dots, a_n\}) = 1$ and $\forall A, A' \subseteq \{a_1, \dots, a_n\}$, $A \subseteq A' \Rightarrow v(A) \leq v(A')$.

The profile of an agent a_j includes his preference, importance (i.e. his capacity $v(a_j)$) and preference intensity named conviction in the following (it is to be denoted $c_{a_j} \in [0, 1]$).

It is an agreed rule of the debate that agents are to speak in turns. In the proposed model set-up, the agent a_s (speaker-agent) who is currently speaking and any agent a_l (listener-agent) who is listening are formally isolated from the remainder of the group. More precisely, a capacity v_{a_l, a_s} , defined relative to the pair of agents (a_l, a_s) , is introduced as follows:

$$v_{a_l, a_s}(a_l) = \frac{v(a_l)}{v(\{a_l, a_s\})}, v_{a_l, a_s}(a_s) = \frac{v(a_s)}{v(\{a_l, a_s\})} \text{ and } v_{a_l, a_s}(\{a_l, a_s\}) = 1.$$

The change of conviction can then be modeled using the symmetric Choquet integral, which is also called the Sipos integral. The definition of the Choquet and Sipos integrals will now be provided.

Definition 4. Let $c = (c_{a_1}, \dots, c_{a_n}) \in [0, 1]^n$ be a vector of convictions, $()$ be a permutation on $\{1, \dots, n\}$ such that $c_{a_{(1)}} \leq \dots \leq c_{a_{(n)}}$ and v a capacity on $2^{\{a_1, \dots, a_n\}}$.

The Choquet integral of c with respect to v is expressed as:

$$C_v(c) = \sum_{i=1}^n [c_{a_{(i)}} - c_{a_{(i-1)}}] v(\{(i), \dots, (n)\}) \quad \text{with } c_{a_{(0)}} = 0.$$

Definition 5. Let $c = (c_{a_1}, \dots, c_{a_n}) \in [-1, 1]^n$ be a vector capable of assuming negative values, $()$ be the permutation on $\{1, \dots, n\}$ such that $c_{a_{(1)}} \leq c_{a_{(p)}} < 0 \leq c_{a_{(p+1)}} \leq \dots \leq c_{a_{(n)}}$ and v a capacity on $2^{\{a_1, \dots, a_n\}}$.

The symmetric Choquet Integral of c with respect to v is given by:

$$\begin{aligned} \check{C}_v(c) = & \sum_{i=1}^{p-1} [c_{a_{(i)}} - c_{a_{(i+1)}}] v(\{(1), \dots, (i)\}) + c_{a_{(p)}} v(\{(i), \dots, (p)\}) \\ & + c_{a_{(p+1)}} v(\{(p+1), \dots, (n)\}) + \sum_{i=p+2}^n [c_{a_{(i)}} - c_{a_{(i-1)}}] v(\{(i), \dots, (n)\}). \end{aligned}$$

In [7] the Sipos integral is defined on the set of agents $\{a_l, a_s\}$ and denoted $\check{C}_{v_{a_l, a_s}}$. The changes of conviction proposed can then be summarized as follows:

If agents a_l and a_s have the same preference, then one of them is more convinced, and this situation entails two possible cases.

– If $c_{a_s} > c_{a_l}$ then the new conviction of agent a_l becomes:

$$\check{C}_{v_{a_l, a_s}}(c_{a_s}, c_{a_l}) = c_{a_l} + (c_{a_s} - c_{a_l})v_{a_l, a_s}(a_s).$$

– If $c_{a_l} > c_{a_s}$ then the new conviction of agent a_l becomes:

$$\check{C}_{v_{a_l, a_s}}(c_{a_s}, c_{a_l}) = c_{a_s} + (c_{a_l} - c_{a_s})v_{a_l, a_s}(a_l).$$

If agents a_l and a_s have different preferences, then the new conviction of agent a_l is:

$$-\check{C}_{v_{a_l, a_s}}(c_{a_s}, c_{a_l}) = -c_{a_s}v_{a_l, a_s}(a_s) + c_{a_l}v_{a_l, a_s}(a_l).$$

The main drawback to this model is its lack of semantic justification with regard to capacity v (i.e. influence is merely a normalized relative importance); in addition, the concept of conviction has not been formally defined and the revision equations are not provided in an appropriate formalism, in which time would appear explicitly (i.e. dynamic aspects).

3 Presentation of our dynamic model

This section presents our dynamic model for simulating a debate outcome. To begin, let's note that within the framework of this paper, the influence function used in [3] is perceived as a disturbance function applied to the set of all possible inclination vectors.

3.1 Decisional power and capacities

One of the new ideas presented in this paper is the ability to design a capacity based on the decisional power included in the above model.

For any inclination vector i in I , Bi is the decision vector obtained from i whose influence is modeled by B . $gd(Bi)$ is the final decision of the group, whereby the group decision function is modeled by gd . For any $i \in I$, $gd(Bi)$ belongs to $\{-1, +1\}$, which implies that the modified decisional power for any agent a_j as denoted $\phi_{a_j}(B, gd, p)$ lies in the interval $[-1, 1]$.

Note that if the decisional power of an agent is close to -1 , this means that the agent only rarely chooses the alternative to what the collective body ultimately chooses: he fails most of the time (*FAIL*). On the other hand, when his decisional power is close to 1, the agent is most often successful (*SUCC*); his decisional power therefore is strong. Hence, for any agent a_j , we can normalize $\phi_{a_j}(B, gd, p)$ in order to obtain his importance.

As an example, without any further information, the importance of agent a_j , i.e. his capacity $v(a_j)$, can be defined as follows:

Definition 6. *The importance of agent a_j for a given B , gd and p is $v_\phi(a_j) = \frac{1}{2}\phi_{a_j}(B, gd, p) + \frac{1}{2}$.*

Note that for any agent a_j , $v(a_j) \in [0, 1]$ with $v(a_j) = 0$ if and only if $\phi_{a_j}(B, gd, p) = -1$ and $v(a_j) = 1$ if and only if $\phi_{a_j}(B, gd, p) = 1$.

A capacity v_ϕ can then be generated over $2^{\{a_1, \dots, a_n\}}$, with constraints, $\forall A, A' \subseteq \{a_1, \dots, a_n\}$, $A \subseteq A' \Rightarrow v(A) \leq v(A')$. Without any further knowledge, it may

be stated: $v_\phi(A) = \max_{a_j \in A} v(a_j)$, $\forall A \subset \{a_1, \dots, a_n\}$ and $v_\phi(\{a_1, \dots, a_n\}) = 1$. This last condition is necessary because it is uncertain that an agent can be found whose capacity is equal to 1.

Let's conclude this section with the following remark. The decisional power of individuals a_j on which $v_\phi : 2^{\{a_1, \dots, a_n\}} \rightarrow [0, 1]$ is based, measures those cases where the final decision of a_j matches the group decision. An agent with considerable decisional power is expected to sway several other agents; thus, decisional power is construed as an estimation of his influence within the group, although this is not an influence index in the sense of [3].

3.2 Time-varying probabilities

This subsection focuses on the design of probability p as a time-varying function, to be denoted $p[k]$ at time k . Along with this time-varying probability, a time-varying extended decisional power, as presented in [3], can be computed. The following method proposes basing the probability computation on the convictions of agents with respect to the available alternatives. In this part therefore, the conviction vectors are assumed to be known. $c(k)$ (resp. $c'(k)$) denotes the conviction vector of agents w.r.t. alternative +1 (resp. -1) at time k :

$c(k) = (c_{a_1}(k), \dots, c_{a_j}(k), \dots, c_{a_n}(k))$, where $c_{a_j}(k)$ is the conviction of agent a_j w.r.t alternative +1 at time k .

$c'(k) = (c'_{a_1}(k), \dots, c'_{a_j}(k), \dots, c'_{a_n}(k))$, where $c'_{a_j}(k)$ is the conviction of agent a_j w.r.t alternative -1 at time k .

Their respective computations will be provided in the next section.

The conviction of an agent concerning a given alternative is correlated with the probability that this particular agent chooses this alternative, i.e. the probability of his inclination as defined in [3].

Let $i \in I$ be an inclination vector. Each coordinate i_{a_j} is the preference of agent a_j and constitutes one of the two alternatives.

Definition 7. *Let $i \in I$ be an inclination vector. The conviction vector of i at time k is $c(i, k) = (\bar{c}_{a_1}(k), \dots, \bar{c}_{a_n}(k))$, where for any j , $\bar{c}_{a_j}(k)$ is $c_{a_j}(k)$ if $i_{a_j} = 1$ and is $c'_{a_j}(k)$ if $i_{a_j} = -1$.*

Let $i \in I$ be an inclination vector and let's define $c^i(k) \in [0, 1]$ as an average conviction at time k for i . This value summarizes the distributions of agents' convictions in i at time k . $c^i(k)$ is an "aggregated conviction" of the group of agents for i . This aggregation should take into account the relative importance of agents and their interactions. Consequently, it seems only natural to state the following definition.

Definition 8. *Let $i \in I$ be an inclination vector and $v[k]$ be a capacity defined at time k on $2^{\{a_1, \dots, a_n\}}$, then $c^i(k+1) = C_{v[k]}(\bar{c}_{a_1}(k), \dots, \bar{c}_{a_n}(k))$, where $C_{v[k]}$ is the Choquet integral with respect to $v[k]$.*

The time-varying probability is built by recurrence on k . We start at time $k = 0$ and will proceed by presenting how to compute $p[k+1]$ using $p[k]$.

At time $k = 0$:

Each agent assigns a score to each alternative in the interval $[0, 1]$. For each agent, if we were to denote n_{+1} (resp. n_{-1}) as the score of $+1$ (resp. -1), then the convictions could be computed by $c_{a_j}(0) = \frac{n_{+1}}{n_{+1}+n_{-1}}$ and $c'_{a_j}(0) = \frac{n_{-1}}{n_{+1}+n_{-1}}$. We then have $c_{a_j}(0) + c'_{a_j}(0) = 1$. Initially, at time $k = 0$, if i_{a_j} is the preference of a_j then the probabilities of the agent a_j regarding his preference and the other alternative would be: $p_{a_j}(i_{a_j})[k = 0] = c_{a_j}(0)$ and $p_{a_j}(-i_{a_j})[k = 0] = 1 - c_{a_j}(0)$.

We assume that before the debate starts, the inclination of each agent does not depend on the social network. The probability distribution associated with a *priori* probabilities is thus the product of the individual probabilities p_{a_j} at $k = 0$, leading to the following probability:

$$\forall i \in I, p(i)[0] = \prod_{j=1}^n p_{a_j}(i_{a_j})[0].$$

It is thus possible to compute the following

- the decisional power for any agent a_j at $k = 0$: $\phi_{a_j}(B, gd, p[0])$;
- the capacity $v_\phi[0]$ over $2^{\{a_1, \dots, a_n\}}$, for $k = 0$, as proposed in Subsection 3.1: $v_\phi[0](a_j) = \frac{1}{2}\phi_{a_j}(B, gd, p[0]) + \frac{1}{2}$, and the capacity on a set A is the maximum of the capacity of agents present in the considered coalition.

How to compute $p[k+1]$, $\phi_{a_j}(B, gd, p[k+1])$, $v_\phi[k+1]$ using $p[k]$, $\phi_{a_j}(B, gd, p[k])$ and $v_\phi[k]$

The capacity $v_\phi[k]$ is used to compute $c^i(k+1)$, i.e. the aggregation conviction for the inclination vector i at time $k+1$: $c^i(k+1) = C_{v_\phi[k]}(\bar{c}_{a_1}(k), \dots, \bar{c}_{a_n}(k))$. The time-varying probability $p[k+1]$ can then be defined as follows:

$$\forall i \in I, \quad p(i)[k+1] = \frac{c^i(k+1)}{\sum_{j \in I} c^j(k+1)}.$$

It then becomes possible to compute:

- the decisional power for any agent a_j at $k+1$: $\phi_{a_j}(B, gd, p[k+1])$;
- the capacity $v_\phi[k+1]$ over $2^{\{a_1, \dots, a_n\}}$, at time $k+1$, as proposed in Subsection 3.1.

We have thus defined a time-varying probability. Note that the proposed method seems to be rather intuitive since it corresponds to the notion that an agent's social influence depends on the degree of assurance in the convictions of the other agents when he speaks.

3.3 Conviction state equations

The aim of this section is to establish the state equations that serve to model the dynamic relationship between convictions and influences. Let's consider a_l to be any listener-agent and a_s a speaker-agent. Their convictions at time k for the alternative $+1$ (resp. -1) are then $c_{a_l}(k)$ and $c_{a_s}(k)$ (resp. $c'_{a_l}(k)$ and $c'_{a_s}(k)$).

Two variables are necessary to model the rhetorical quantity exchanged between the two agents a_l and a_s , namely: the difference in their conviction and their relative importance at time k , as modeled by the capacities $v_\phi[k](a_s)$ and $v_\phi[k](a_l)$.

Four rhetorical exchanges can be distinguished. These four situations are presented in the case when the agent a_l prefers alternative +1. Two sub-cases can then be identified for agent a_s : his preferred alternative is either the same as a_l 's or the other one. Each case can be divided once again into two sub-cases: a_s 's conviction is either greater or less than a_l 's conviction. When agent a_l prefers alternative -1 , convictions c' replace convictions c in the formula. More precisely, the equations appearing in the computation of $c_{a_l}(k+1)$ when both agents express the same preference are the same as those used to compute $c'_{a_l}(k+1)$ in the case of opposite preferences and *viceversa*. Hence, the rhetorical exchanges can be summarized by the following exchanges: synergistic exchange, revisionist exchange, and antagonistic exchange. Let's take a closer look at each of them.

Synergistic exchange

In this case, the preference of agent a_l is reinforced by the intervention of agent a_s , who resolutely looks favorably upon the same alternative.

The conviction of agent a_l then increases, to an extent proportional to the difference between both convictions as well as to the capacity of speaker-agent a_s .

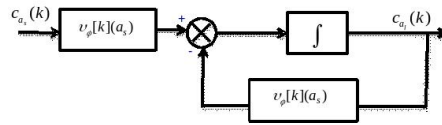


Fig. 1. Synergistic Exchange

This situation, as represented in figure 1, corresponds to the case when a_l and a_s have the same preference and moreover $c_{a_s} > c_{a_l}$. The intuitive difference equation is then written:

$$c_{a_l}(k+1) - c_{a_l}(k) = (c_{a_s}(k) - c_{a_l}(k))v_\phi[k](a_s), \text{ which is equivalent to:}$$

$$c_{a_l}(k+1) = c_{a_l}(k) + (c_{a_s}(k) - c_{a_l}(k))v_\phi[k](a_s).$$

Revisionist exchange

In this situation, agent a_l understands the argument of agent a_s , who has the same preference but a more moderate support. Agent a_s appears to speak with restraint relative to a_l 's point of view, and this exposes a_l 's doubt. a_l 's conviction is thus mitigated by a_s 's intervention. This situation, which is depicted in figure 2, corresponds to the case when a_l and a_s have the same preference with $c_{a_l} > c_{a_s}$. The intuitive difference equation is then written as:

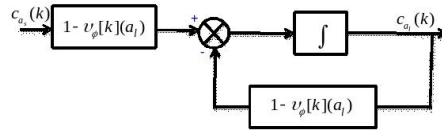


Fig. 2. Revisionist Exchange

$c_{a_l}(k+1) - c_{a_l}(k) = (c_{a_l}(k) - c_{a_s}(k))(1 - v_\phi[k](a_l))$ which is equivalent to:
 $c_{a_l}(k+1) = c_{a_s}(k) + (c_{a_l}(k) - c_{a_s}(k))v_\phi[k](a_l)$.

Agent a_l observes the indecision on the part of agent a_s who nevertheless shares his opinion: a_s contributes to a_l 's doubt. The level of conviction decreases due to a_s 's intervention. which is proportional on the one hand to $1 - v_\phi[k](a_l)$ (resulting from a_l 's lack of assurance relative to his social position within the group) and on the other hand to the difference between both agents' convictions.

Antagonistic exchange

In this situation, the two agents do not share the same preference: agent a_l nevertheless understands the advantages of a_s preference. A convincing intervention from a_s may contribute to making a_l dubious, whereas an unperceptive intervention might on the contrary strengthen a_l 's preference.

$(1 - c'_{a_s}(k))$ is a measure of a_s 's hesitation and provides a_l with an estimation of the strength of a_s 's opposition. Depending on the strength of this hesitation, the previous difference equations can again be used with $(1 - c'_{a_s}(k))$, yielding two situations to be distinguished (see figure 3).

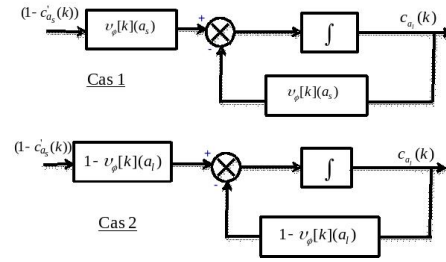


Fig. 3. Antagonistic Exchange

An overly weak preference expressed by a_s implies weak opposition from a_l 's point of view and reinforces a_l 's opinion, resulting in a likely strengthening of a_l 's conviction.

The intuitive difference equation is then:

Case 1: $1 - c'_{a_s} \geq c_{a_l}$

$c_{a_l}(k+1) - c_{a_l}(k) = ((1 - c'_{a_s}(k)) - c_{a_l}(k))(v_\phi[k](a_s))$, which is equivalent to $c_{a_l}(k+1) = c_{a_l}(k) + (1 - c_{a_s}(k) - c_{a_l}(k))(v_\phi[k](a_s))$.

Case 2: $1 - c'_{a_s} < c_{a_l}$.

In this case a_l 's conviction weakens following a_s 's intervention.

$c_{a_l}(k+1) - c_{a_l}(k) = -(c_{a_l}(k) - (1 - c'_{a_s}(k)))(1 - v_\phi[k](a_l))$, which is equivalent to $c_{a_l}(k+1) = (1 - c'_{a_s}(k)) + (c_{a_l}(k) + c'_{a_s}(k) - 1)(v_\phi[k](a_l))$.

All these various types of exchanges can be synthesized using a Sipos integral.

Proposition 1. *If agents a_s and a_l express the same preference, then:*

$$c_{a_l}(k+1) = \check{C}_{v_\phi}[k](c_{a_s}(k), c_{a_l}(k));$$

If agents a_s and a_l do not share the same preference, then:

$$c_{a_l}(k+1) = \check{C}_{v_\phi}[k](1 - c'_{a_s}(k), c_{a_l}(k)).$$

As a conclusion to this section of the paper, the decisional power ϕ provides a semantic interpretation for the capacity v in the recurrence equations presented in [7], with conviction here being related to the probability an agent will choose one alternative over the other (i.e. probability distribution over inclination vectors). The model in [7] thus becomes interpretable within a game theory framework [3]. The revision equations for conviction appear as input-output balances according to the alternatives assessment. Introducing time into the equations in [7] implies that revision equations of conviction are now seen as state equations of agents' mental perception. This new interpretation then provides a semantics for the debate model in [7]: it incorporates the notions of influence and decisional power, as proposed in [3], with a formalism close to that of dynamic models found in control theory, as suggested in [6].

4 Illustration

4.1 Preference calculus

This section discusses how to compute preferences during the debate. Initially, each agent a_j assesses both alternatives $+1$ and -1 lying in the interval $[0, 1]$. These assessments are denoted $n_{a_j}^{+1}$ and $n_{a_j}^{-1}$, respectively. It is then possible to build initial preferences and convictions as follows: Let a in $\{-1, 1\}$

- a_j prefers alternative a with the highest score,
- a_j 's conviction relative to alternative a equals $\frac{n_{a_j}^a}{n_{a_j}^a + n_{a_j}^{-a}}$.

Preference changes depend on how convictions evolve over time. For any agent a_j , it is assumed that a threshold $\epsilon_{a_j} > 0$ exists such that when the difference between two convictions lies below this threshold, then agent a_j cannot have a preference. This threshold value may be characteristic of each agent. To summarize: if $|c_{a_j} - c'_{a_j}| < \epsilon_{a_j}$, then a_j has no preference; if $|c_{a_j} - c'_{a_j}| \geq \epsilon_{a_j}$, a_j prefers the alternative with the highest conviction.

An agent without a preference cannot intervene, which is stated as one of the debate rules.

4.2 Simulations of debates outcome

In order to illustrate the principle of the above dynamic representation of a debate, the four following elementary models for influence function B have been implemented:

- B is the identity, i.e. for any inclination vector i , it can be stated that: $Bi = i$;
- B is the opposite of identity, i.e. for any inclination vector i , it is stated that: $Bi = -i$;
- B is a mass psychology effect function. More precisely, let's denote $i^\epsilon = \{k \in N | i_k = \epsilon\}$, where B satisfies the following: for each $i \in I$: $|i^\epsilon| > t$, then $i^\epsilon \subseteq (Bi)^\epsilon$, where $t \in [1, n]$ and $\epsilon = +1$ or -1 ;

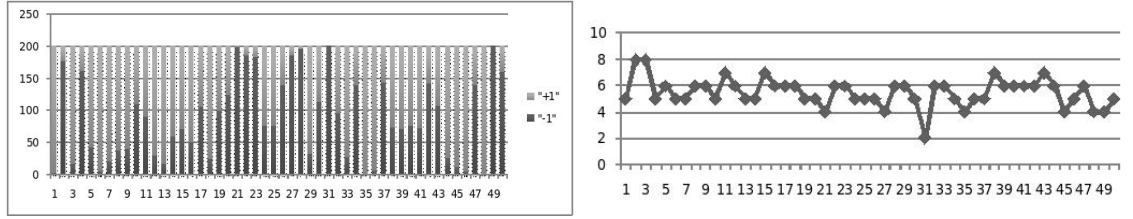


Fig. 4. Identity

- B is a majority influence function that models behavior of the following types: if a majority of agents have an inclination +1, then all agents decide +1; if not, all agents decide -1.

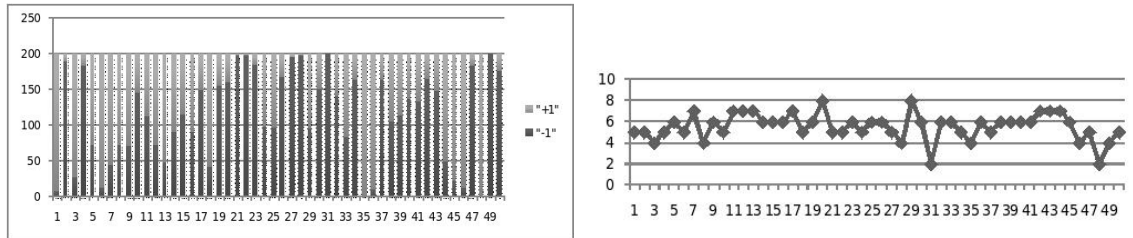


Fig. 5. Opposite of identity

For these four cases, the group decision function gd is, a mere majority and a basic capacity is designated, as proposed in Section 1.3.

Let's now consider a group of $n = 8$ agents. The initial convictions of agents relative to both alternatives are considered as variates: 50 random drawings of these 8 initial probabilities have been carried out. For each of these 50 initial conviction vectors, the order of agents' intervention in the debate can then be considered: 200 permutations are randomly selected (among the $8!$ possible rankings) for each initial conviction vector.

Each of the four elementary illustrations has been plotted in the following figures (i.e. one for each B function). For each of the 50 initial conviction vectors selected randomly, a bar represents the number of +1 and -1 outcomes (light gray for +1 and dark gray for -1)

Each figure is to be associated with the maximum number of rounds required to achieve the ground decision for each initial conviction vector. In the proposed simulations, this number does not exceed 8 rounds in any of the cases chosen for B .

The indifference threshold is $\epsilon = 0.01$ for any agent. Agents speak in turn according to the order generated by the 200 permutations, upon the condition

that they are able to express a clear opinion, specifically: agent a_j can speak if $|c_{a_j} - c'_{a_j}| \geq \epsilon$.

For the same initial conviction vector, it can be observed that for each function B , the outcome of the debate may depend on the order the agents intervene in the debate. This type of situation can be interpreted as a weak expression of preferential contexts, whereby any perturbation is able to change the debate outcome. From this point of view, influence function B is a disturbance function for this dynamic model of a debate. As a consequence, simulations allow verifying that the order the agents intervene in the debate and their influence are both decisive variables with regard to the convergence of conviction state equations.

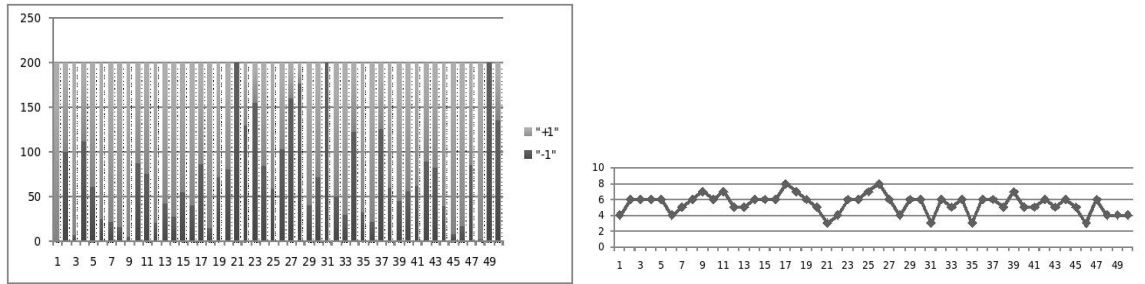


Fig. 6. Majority

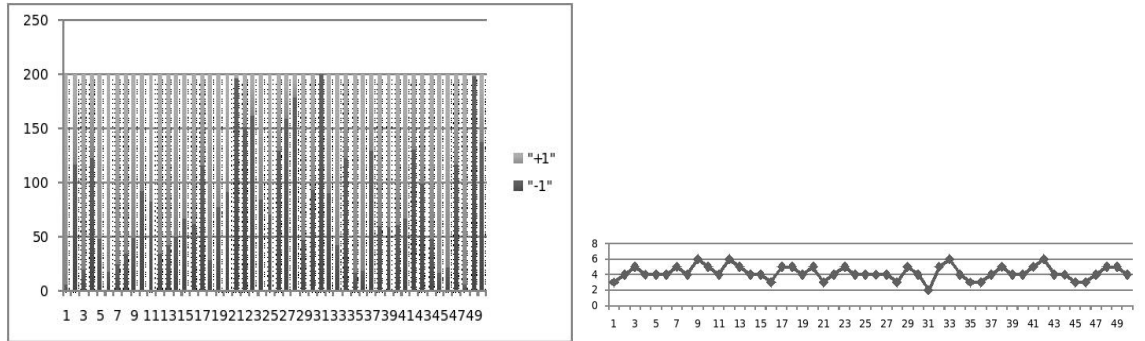


Fig. 7. Mass Psychology Effect

The social influence of an agent may thus be considered as a disturbance in the deliberation process, except if it is relevantly used by the debate manager to guide the discussion. In this latter case, social influence can be viewed as an actuator that enables controlling the debate outcome or at least accelerating its convergence. For example, when the debate outcome is practically certain

(i.e. the bar is almost completely light or dark gray), then the simplest control might consist of choosing the order of agents intervention that minimizes the maximum number of rounds. More complex controls could clearly be foreseen, yet the aim of this paper has merely been to propose a dynamic model of the debate within a framework close to control theory representations, making for a natural implementation of control techniques in the future.

4.3 Debate as a decision-making process

This part of the paper will present a potential application of the dynamic model discussed herein. The aim is to apply the model like a voting system. For this example in particular, both alternatives -1 and $+1$ are not considered to be equivalent: $+1$ is the right decision, while -1 is associated with an erroneous decision. This situation could occur in classification problems when the agents are competitive classification algorithms.

The agents are expected to provide the correct answer most of the time, but they typically disagree on individual cases. One common solution is to employ a voting process in order to yield a group decision, i.e. let $d_1, d_2 \dots d_n$ be the respective decision of the various agents, then the group decision is written as:

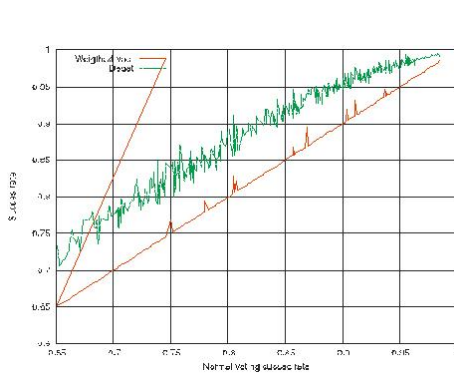


Fig. 8. Simulation-weighted vote and debate

$$\begin{cases} 1 & \text{if } \sum_{i=1}^n d_i > 0 \\ -1 & \text{else} \end{cases}$$

For example, let the agents be 7 different classification algorithms whose success rates equal respectively: 0.6, 0.7, 0.8, 0.8, 0.6, 0.7, and 0.6. The group success rate according to a normal vote would thus be 0.86. A better aggregation process will achieve a higher success rate.

The first idea here is to use a weighted vote, i.e. let $\alpha_1, \dots, \alpha_n \in [0, 1]^n$:

$$\begin{cases} 1 & \text{if } \sum_{i=1}^n \alpha_i d_i > 0 \\ -1 & \text{else} \end{cases}$$

One possible set of weights is the individual success rate of each agent; however, it is possible to compute the Shapley-Shubik power index [9], and our example delivers a value of $\frac{1}{7}$ for each agent. This is exactly the same value achieved in a normal vote. Since the weights do not differ considerably for a small number of agents, the sign of the weighted sum is the same as that produced during

the normal vote. This finding indicates that even if some agents possess a more powerful vote, the final decision is always shared by at least 4 agents.

If we were to use our debate model as the voting process, such an outcome would not occur. The least agents also happen to be those who most readily change their point of view. More precisely, we can run the debate with identity as the B function and as success rates for convictions. It is assumed that 7 competitive classification algorithms are available and moreover that the correct solution is supposed to be alternative +1.

As said above, the initial probability of the 7 algorithms to choose alternative +1 are: 0.6, 0.7, 0.8, 0.8, 0.6, 0.7, and 0.6. The debate stops when all classification algorithms are in agreement. We will assume then that their answers are independent random variables and that 10,000 cases are studied by each agent. For each case, the agent's answer is inferred according to his probability of being correct.

Next, for each of these 10,000 cases, we compute the group decision according to 3 methods:

- the choice with a majority vote procedure,
- the choice with a weighted majority vote procedure,
- the decision derived using our debate model.

While simple and weighted majorities yield the correct answer at a rate of 86 %, our method produced a 94 % rate. Hence, the aggregation by a debate significantly increases success rate.

In order to verify this good result, we tried using different situations of the same model. For 7 agents, several values for the probability of making the right decision were randomly generated, and the 3 corresponding rates computed (results are presented in Figure 8). In this figure, both the weighted voting rate and our debate output vs. this rate are plotted. Note that the same rate for the simple vote can be obtained with very different sets of probabilities. The debate always yields a better rate, although its preferences change according to the specific probability profile. The weighted vote success rate is quite close to that of the simple vote, except for very unique probability sets where several agents (algorithms) perform much better than the others.

5 Conclusion and outlook

The state equations derived in this paper allow simulating macroscopically the outcome of a debate according to the initial inclinations of agents and the social influences taking place within the group (whereby the influence function is a priori known). The deliberation outcome depends not only on the order in which the agents intervene in the debate to explain their opinions, but also on the influence an agent is able to exert on a social network.

The model formalism proposed in this paper is close to the one used in control theory to model the dynamic behavior of technical systems. Guiding a debate might then be seen as a control problem, whose aim could, for example, be how

to reach a consensus as quickly as possible or how to reinforce one alternative over the other, etc.

A debate is thus seen as a continuous dynamic system: a state equation representation has been preferred to the multicriteria decision-making framework in [7] given that time explicitly appears in the revision of convictions. The model semantic has also been inspired from the game theory concepts proposed in [3]: influence and decisional power in a social network. In our dynamic extension, decisional power is a time-varying variable itself and may be used as the actuator signal in the debate control loop. The system of state equations established in this paper allows stochastically simulating the outcome of a debate and effects of a control strategy on this particular issue.

One possible application of this model would obviously be to simulate a debate outcome in order to obtain certain indications regarding the final collective decision. When simulations are performed for a large number of initial agent convictions and speaker intervention rankings, the probability that outcome is ± 1 can be estimated. Hence, the dynamic influence model can be applied to either make the debate outcome more certain (this may appear to be a dishonest method when agents are actual human beings, yet remains a relevant technique when agents are artificial, such as sensors or classifiers) or modify the convergence dynamics of the debate.

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