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About the transcendentality of some numbers

Jamel Ghanouchi

Ecole Supérieure des Sciences et Techniques de Tunis

jamel.ghanouchi@topnet.tn

Abstract

Everyone knows that π , e or C_n , the Champernowne number are transcendental, but what about $\pi + e$, $\pi + C_n$ or $e + C_n$? In this paper, we demonstrate a method in order to know if they also are.

The approach

A number is transcendental if it is not the root of a polynomial equation

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_0 = 0$$

where a_i are rational and different from zero, otherwise it would be algebraic. We know that π , e and C_n , the Champernowne number are transcendental, but we do not know anything about $e + \pi$, $\pi + C_n$ or $e + C_n$. Effectively, if A is transcendental : B transcendental, we still do not know the nature of $A+B$ or $A-B$. But if B is algebraic, then $A+B$ and $A-B$ are transcendental. And if A and B are algebraics, their sum as well as their difference are algebraics. Let us try to solve this problem. Let us take

$$C_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 1 & 2 & 3 \\ 2 & 3 & 4 & 5 & 1 \\ 5 & 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 1 & 2 \end{pmatrix}$$

And take

$$C'_1 = \begin{pmatrix} 0 & 5 & 10 & 15 & 20 \\ 10 & 15 & 20 & 0 & 5 \\ 20 & 0 & 5 & 10 & 15 \\ 5 & 10 & 15 & 20 & 0 \\ 15 & 20 & 0 & 5 & 10 \end{pmatrix}$$

Thus $C_1 + C'_1$ is a magic square. It contains all the numbers from 1 to 25.
Let us take

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

And

$$C_2 = \begin{pmatrix} C_1 & C_1 + 25A & C_1 + 50A & C_1 + 75A & C_1 + 100A \\ C_1 + 75A & C_1 + 100A & C_1 & C_1 + 25A & C_1 + 50A \\ C_1 + 25A & C_1 + 50A & C_1 + 75A & C_1 + 100A & C_1 \\ C_1 + 100A & C_1 & C_1 + 25A & C_1 + 50A & C_1 + 75A \\ C_1 + 50A & C_1 + 75A & C_1 + 100A & C_1 & C_1 + 25A \end{pmatrix}$$

And

$$C'_2 = \begin{pmatrix} C'_1 & C'_1 & C'_1 & C'_1 & C'_1 \\ C'_1 & C'_1 & C'_1 & C'_1 & C'_1 \\ C'_1 & C'_1 & C'_1 & C'_1 & C'_1 \\ C'_1 & C'_1 & C'_1 & C'_1 & C'_1 \\ C'_1 & C'_1 & C'_1 & C'_1 & C'_1 \end{pmatrix}$$

The mathematical induction is

$$C_{i+1} = \begin{pmatrix} C_i & C_i + 5^i A & C_i + 2.5^i A & C_i + 3.5^i A & C_i + 4.5^i A \\ C_1 + 3.5^i A & C_1 + 4.5^i A & C_1 & C_1 + 5^i A & C_1 + 2.5^i A \\ C_1 + 5^i A & C_1 + 2.5^i A & C_1 + 3.5^i A & C_1 + 4.5^i A & C_1 \\ C_1 + 4.5^i A & C_1 & C_1 + 5^i A & C_1 + 2.5^i A & C_1 + 3.5^i A \\ C_1 + 2.5^i A & C_1 + 3.5^i A & C_1 + 4.5^i A & C_1 & C_1 + 5^i A \end{pmatrix}$$

And

$$C'_{i+1} = \begin{pmatrix} C'_i & C'_i & C'_i & C'_i & C'_i \\ C'_i & C'_i & C'_i & C'_i & C'_i \\ C'_i & C'_i & C'_i & C'_i & C'_i \\ C'_i & C'_i & C'_i & C'_i & C'_i \\ C'_i & C'_i & C'_i & C'_i & C'_i \end{pmatrix}$$

And $C_i + C'_i$ is a magic square containing all the numbers from 1 to

$$\begin{aligned} 5 + 20(5 + 5^2 + \dots + 5^{i-1}) + 20 &= 5 + 100(1 + 5 + 5^2 + \dots + 5^{i-2}) + 20 \\ &= 5 + 100\left(\frac{5^{i-1} - 1}{4}\right) + 20 = 5 + 25(5^{i-1} - 1) + 20 = 5^{i+1} \end{aligned}$$

The sum of C_{i+1} is $S_{i+1} = 5S_i + 10.5^i + 5S'_i$ ($S_1 = 65$, $S'_i = 50.5^{i-1}$). Hence

$$S_{i+1} = 5S_i + 60.5^i$$

$$5S_i = 5^2 S_{i-1} + 60.5^i$$

$$\begin{aligned}
& \dots \\
& 5^{i-1}S_2 = 5^i S_1 + 60.5^i \\
& 5^i S_1 = 5^i(65) = 13.5^{i+1} \\
& S_{i+1} = (60i + 65)5^i = (12i + 13)5^{i+1}
\end{aligned}$$

If M is the greatest number of the square, S the sum, the value of the integer $\pi.10^{p_i}$ that approaches π in the square (example between 1 and 25, $\pi.10^{p_i} = 3$ and between 1 and 125, it is 31). The number $\pi(p_i)$ depends on M and S . This square contains p_i first digit of π without the dot, we note it : $\pi.10^{p_i}$. It contains also $e.10^{p_i}$ and $C_n.10^{p_i}$ (and also $\pi + e$, πe , $\pi + C_n$, $\pi C_n \dots$). For $i + 1$, the greatest number in the square is $M = 5^{i+2}$ which has $E(\frac{(i+2)\log 5}{\log 10}) = i + 1$ ciphers and begins with 1, 2, 3 and then $p_i = i$, or with a greater number and then $p_i = i + 1$. The sum is $S_{i+1} = (12i + 13)5^{i+1}$ which has less than $p_i + 2$ ciphers. The sum $S_i.10^{-p_i}$ tends, in the infinity, to a number S which has less than three ciphers ! But S_i is rational, thus S is rational and algebraic.

Main results

$$\left\{ \begin{array}{l} S = 20 \\ A = S - \pi \\ B = S - e \\ C = S - C_n \\ A_1 = S - \pi - e \\ A_2 = S - \pi + e \\ A_3 = S - \pi e \\ A_4 = S - \pi^2 \end{array} \right.$$

We know that

$$S = \pi + A = e + B = \pi + e + A_1 = \pi - e + A_2 = \pi e + A_3$$

Definition The following numbers can be called bricks or elements, because they constitute the bricks of the numbers. They exist, of course, and we prefer to call them prime numbers because they really generalize the concept of primes. Let us see this : A real number is compound if it is equal to $\pm p_1^{n_1} \dots p_i^{n_i}$ where p_j are integer prime numbers and n_j are rationals. We define other real prime numbers which cannot be expressed like this : π , e , $\ln(2)$. Thus $\sqrt[q]{p} = p^{\frac{1}{q}}$ is compound.

Also $\sqrt[p]{p} + 1$ is prime, with p prime, hence $\sqrt[p]{p} - 1 = (p - 1)(\sqrt[p]{p} + 1)^{-1}$ is compound !

Furthermore π and e are primes instead of π^{n_0} and e^{m_0} with $(n_0 - 1)(m_0 - 1) \neq 0$.

We define the GCD of two numbers as following : If p_1 and p_2 are prime real numbers

$$p_1 \neq p_2 \Rightarrow GCD(p_1, p_2) = 1$$

$$n_1 n_2 < 0 \Rightarrow GCD(p_1^{n_1}, p_1^{n_2}) = 1$$

$$n_1 n_2 > 0 \Rightarrow GCD(p_1^{n_1}, p_1^{n_2}) = p_1^{\min(n_1, n_2)}$$

$$GCD(p_1^{n_1} p_2^{n_2} \dots p_i^{n_i}, p_1^{m_1} p_2^{m_2} \dots p_j^{m_j}) = \prod_{i,j} GCD(p_i^{n_i}, p_j^{m_j})$$

And if $x = p_1^{n_1} p_2^{n_2} \dots p_i^{n_i}$ and $y = p_{l_1}^{m_{l_1}} \dots p_{l_i}^{m_{l_i}}$ then y divides x if $1 < l_k < i$ and for $l_i = j$, $n_j m_{l_i} > 0$, $|m_{l_i}| < |n_j|$.

Thus $\frac{3}{2}$ does not divide the prime 3.

Theorem If T'_1 and T'_2 are transcendental prime real numbers, $T'_1 T'_2$ and $T'_1 + T'_2$ are transcendentals.

Proof of the theorem Let us take T_1 and T_2 two real compound transcendental numbers.

We have 4 possibilities

1)

$T_1 T_2^n$ and $T_1 + m T_2$ are algebraics

2)

$T_1 T_2^n$ is algebraic and $T_1 + m T_2$ is transcendental

3)

$T_1 T_2^n$ is transcendental and $T_1 + m T_2$ is algebraic

4)

$T_1 T_2^n$ and $T_1 + m T_2$ are transcendentals, $\forall m, n$

Thus

1)

$T_1 T_2^n = A_A$ and $T_1 + m T_2 = A'_A$ are algebraics, then

$$T_1 T_2^n + m T_2^{n+1} = A_A + m T_2^{n+1} = A'_A T_2^n$$

And T_2 is supposed to be the fitting solution to this algebraic equation : it is impossible !

2)

If $T_1 T_2^n$ and $T_1 T_2^{n'}$ are algebraics, then

$T_1 T_2^n (T_1 T_2^{n'})^{-1} = T_2^{n-n'}$ is algebraic and it is impossible with $n \neq n'$

There is only one $n = n_0$ for which $T_1 T_2^n$ is algebraic, all the others are transcendentals !

If we suppose that n_0 is not unique, there are three possibilities : $T_1 T_1' (T_2 T_2')^n$ is transcendental for all n or $T_1 T_1'^{-1} (T_2 T_2'^{-1})^n$ is transcendental for all n or

there exists l_0 and l'_0 for which

$$\begin{cases} T_1 T_2^{n_0} = A_A \\ T_1' T_2'^{n'_0} = A'_A \\ T_1 T_1' (T_2 T_2')^{l_0} = A''_A \\ T_1 T_1'^{-1} (T_2 T_2'^{-1})^{l'_0} = A'''_A \end{cases}$$

are algebraics, we have

$$\begin{cases} A_A^2 T_2^{l_0+l'_0-2n_0} T_2'^{l_0-l'_0} = A''_A A'''_A \\ A_A'^2 T_2'^{l_0-l'_0} T_2^{l_0+l'_0-2n'_0} = A''_A A'''_A^{-1} \\ A_A A'_A T_2^{l_0-n_0} T_2'^{l_0-n'_0} = A''_A \\ A_A A_A'^{-1} T_2^{l_0-n_0} T_2'^{n'_0-l'_0} = A''_A \end{cases}$$

$$\Rightarrow \begin{cases} T_2^A = A' T_2'^B \\ A = l_0 + l'_0 - 2n_0 \\ B = l'_0 - l_0 \\ A' = A''_A A'''_A A_A^{-2} \\ T_2^{-B} = B' T_2'^C \\ C = -(l_0 + l_0 - 2n'_0) \\ B' = A_A'^2 A_A'^{-1} A'''_A \\ T_2^D = C' T_2'^E \\ D = l_0 - n_0 \\ E = n'_0 - l_0 \\ C' = A''_A A_A'^{-1} A_A'^{-1} \\ T_2^F = D' T_2'^G \\ F = l'_0 - n_0 \\ G = l'_0 - n'_0 \\ D' = A'''_A A_A'^{-1} A'_A \end{cases}$$

$$\Rightarrow \left\{ \begin{array}{l} T_2^{-AB} = A'^{-B} T_2'^{-B^2} \\ = T_2^{-AB} = B'^A T_2'^{AC} \\ T_2'^{BC} = A'^{-C} T_2'^{AC} \\ = T_2'^{BC} = B'^{-B} T_2'^{-B^2} \\ T_2'^{DG} = C'^G T_2'^{EG} \\ T_2'^{EF} = D'^E T_2'^{EG} = D'^E C'^{-G} T_2'^{DG} \\ T_2'^{AE} = A'^E T_2'^{BE} = T_2'^{DB} C'^{-B} \\ A = l_0 + l'_0 - 2n_0 \\ B = l'_0 - l_0 \\ A' = A''_A A'''_A A_A^{-2} \\ C = -(l_0 + l_0 - 2n'_0) \\ B' = A_A'^2 A_A''^{-1} A_A''' \\ D = l_0 - n_0 \\ E = n'_0 - l_0 \\ F = l'_0 - n_0 \\ G = l'_0 - n'_0 \\ C' = A''_A A_A^{-1} A_A'^{-1} \\ D' = A_A''' A_A^{-1} A'_A \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} -B^2 = AC = -(l_0 - l'_0)^2 = -(l_0 + l'_0 - 2n_0)(l_0 + l'_0 - 2n'_0) \\ EF = DG = (n'_0 - l_0)(l'_0 - n_0) = (l_0 - n_0)(l'_0 - n'_0) \\ AE = DB = (l_0 + l'_0 - 2n'_0)(n'_0 - l_0) = (l'_0 - l_0)(l_0 - n_0) \\ B'^{-B^2} = B'^{AC} = A'^{-BC} \\ A = l_0 + l'_0 - 2n_0 \\ B = l'_0 - l_0 \\ A' = A''_A A'''_A A_A^{-2} \\ C = -(l_0 + l_0 - 2n'_0) \\ B' = A_A'^2 A_A''^{-1} A_A''' \\ D = l_0 - n_0 \\ E = n'_0 - l_0 \\ F = l'_0 - n_0 \\ G = l'_0 - n'_0 \\ C' = A''_A A_A^{-1} A_A'^{-1} \\ D' = A_A''' A_A^{-1} A'_A \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} -B^2 = AC = -(l_0 - l'_0)^2 = -(l_0 + l'_0 - 2n_0)(l_0 + l'_0 - 2n'_0) \\ (n_0 - n'_0)^2 = (n_0 + n'_0 - 2l_0)(n_0 + n'_0 - 2l'_0) \\ (n'_0 - l_0)(l'_0 - n_0) = (l_0 - n_0)(l'_0 - n'_0) \\ AE = DB = (l_0 + l'_0 - 2n'_0)(n'_0 - l_0) = (l'_0 - l_0)(l_0 - n_0) \\ (n'_0 - l_0)(l_0 + 2l'_0 - n_0 - 2n'_0) = (l_0 - n_0)(2l'_0 - l_0 - n'_0) \\ = (n'_0 - l_0)(l_0 - n_0) + (n'_0 - l_0)(2l'_0 - 2n'_0) = (l_0 - n_0)(l'_0 - n'_0) + (l_0 - n_0)(l'_0 - l_0) \\ A = l_0 + l'_0 - 2n_0 \\ B = l'_0 - l_0 \\ A' = A''_A A'''_A A_A^{-2} \\ C = -(l_0 + l_0 - 2n'_0) \\ B' = A_A^2 A_A''^{-1} A_A''' \\ D = l_0 - n_0 \\ E = n'_0 - l_0 \\ F = l'_0 - n_0 \\ G = l'_0 - n'_0 \\ C' = A''_A A_A^{-1} A_A'^{-1} \\ D' = A_A''' A_A^{-1} A_A' \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} -B^2 = AC = -(l_0 - l'_0)^2 = -(l_0 + l'_0 - 2n_0)(l_0 + l'_0 - 2n'_0) \\ (n_0 - n'_0)^2 = (n_0 + n'_0 - 2l_0)(n_0 + n'_0 - 2l'_0) \\ (n'_0 - l_0)(l'_0 - n_0) = (l_0 - n_0)(l'_0 - n'_0) \\ AE = DB = (l_0 + l'_0 - 2n'_0)(n'_0 - l_0) = (l'_0 - l_0)(l_0 - n_0) \\ (l_0 - n_0)(n'_0 - l'_0) = (l'_0 - n'_0)(l_0 - n_0 - 2n'_0 + 2l_0) = (l'_0 - n'_0)(3l_0 - n_0 - 2n'_0) \\ A = l_0 + l'_0 - 2n_0 \\ B = l'_0 - l_0 \\ A' = A''_A A'''_A A_A^{-2} \\ C = -(l_0 + l_0 - 2n'_0) \\ B' = A_A^2 A_A''^{-1} A_A''' \\ D = l_0 - n_0 \\ E = n'_0 - l_0 \\ F = l'_0 - n_0 \\ G = l'_0 - n'_0 \\ C' = A''_A A_A^{-1} A_A'^{-1} \\ D' = A_A''' A_A^{-1} A_A' \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} -B^2 = AC = -(l_0 - l'_0)^2 = -(l_0 + l'_0 - 2n_0)(l_0 + l'_0 - 2n'_0) \\ (n_0 - n'_0)^2 = (n_0 + n'_0 - 2l_0)(n_0 + n'_0 - 2l'_0) \\ (n'_0 - l_0)(l'_0 - n_0) = (l_0 - n_0)(l'_0 - n'_0) \\ AE = DB = (l_0 + l'_0 - 2n'_0)(n'_0 - l_0) = (l'_0 - l_0)(l_0 - n_0) \\ (l'_0 - n'_0)(-n_0 + l_0 + 3l_0 - n_0 - 2n'_0) = 0 = (l'_0 - n'_0)(4l_0 - 2n_0 - 2n'_0)A = l_0 + l'_0 - 2n_0 \\ B = l'_0 - l_0 \\ A' = A''_A A'''_A A^{-2}_A \\ C = -(l_0 + l_0 - 2n'_0) \\ B' = A'^2_A A''^{-1}_A A'''_A \\ D = l_0 - n_0 \\ E = n'_0 - l_0 \\ F = l'_0 - n_0 \\ G = l'_0 - n'_0 \\ C' = A''_A A^{-1}_A A'^{-1}_A \\ D' = A'''_A A^{-1}_A A'_A \end{array} \right. \Rightarrow l_0 = l'_0 = n'_0 = n_0$$

n_0 is unique

3)

$(T_1 + nT_2) - (T_1 + n'T_2) = (n - n')T_2 = A_A$ is algebraic, T_2 could not be a solution to this algebraic equation, this can not be possible while $n = n'$.

There is only one possibility, it is when $n = m_0$ making $T_1 + nT_2$ algebraic, since all the others are transcendentals!

If m_0 is not unique, there are three possibilities : $T_1 + T'_1 + m(T_2 + T'_2)$ is transcendental for all m or $T_1 - T'_1 + m(T_2 - T'_2)$ is transcendental for all m or there exists l_0, l'_0 for which

$$\left\{ \begin{array}{l} T_1 + m_0T_2 = A_A \\ T'_1 + m'_0T'_2 = A'_A \\ T_1 + T'_1 + l_0(T_2 + T'_2) = A''_A \\ T_1 - T'_1 + l'_0(T_2 - T'_2) = A'''_A \end{array} \right.$$

algebraics, then

$$\left\{ \begin{array}{l} 2T_1 + l_0(T_2 + T'_2) + l'_0(T_2 - T'_2) = A''_A + A'''_A \\ = 2T_1 + 2m_0T_2 + l_0(T_2 + T'_2) + l'_0(T_2 - T'_2) - 2m_0T_2 \\ = 2A_A + (l_0 + l'_0 - 2m_0)T_2 + (l_0 - l'_0)T'_2 \\ 2T'_1 + l_0(T_2 + T'_2) - l'_0(T_2 - T'_2) = A''_A - A'''_A \\ 2A'_A + (l_0 - l'_0)T_2 + (l_0 + l'_0 - 2m'_0)T'_2 \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} AT_2 + BT'_2 = C \\ A = l_0 + l'_0 - 2m_0 \\ B = l_0 - l'_0 \\ C = A''_A + A'''_A - 2A_A \\ BT_2 + B'T'_2 = C' \\ B' = l_0 + l'_0 - 2m'_0 \\ C' = A''_A - A'''_A - 2A'_A \end{array} \right.$$

$$\Rightarrow \begin{cases} ABT_2 + B^2T_2' = BC \\ B'AT_2 + BB'T_2' = B'C \\ ABT_2 + AB'T_2' = AC' \\ B^2T_2 + BB'T_2' = B'C' \\ A = l_0 + l_0' - 2m_0 \\ B = l_0' - l_0 \\ C = A_A'' + A_A''' - 2A_A \\ B' = l_0 + l_0' - 2m_0' \\ C' = A_A'' - A_A''' - 2A_A' \end{cases}$$

$$\Rightarrow \begin{cases} B^2 = AB' \\ ABT_2 + AB'T_2' = BC = AC' \\ BC = AC' \\ B'C' = B'C \\ A = l_0 + l_0' - 2m_0 \\ B = l_0' - l_0 \\ C = A_A'' + A_A''' - 2A_A \\ B' = l_0 + l_0' - 2m_0' \\ C' = A_A'' - A_A''' - 2A_A' \end{cases}$$

$$\Rightarrow \begin{cases} (l_0 - l_0')^2 = (l_0 + l_0' - 2m_0)(l_0 + l_0' - 2m_0') \\ (l_0' - l_0)C = (l_0 + l_0' - 2m_0)C' \\ (l_0 + l_0' - 2m_0')(C' - C) = 0 \\ A = l_0 + l_0' - 2m_0 \\ B = l_0' - l_0 \\ C = A_A'' + A_A''' - 2A_A \\ B' = l_0 + l_0' - 2m_0' \\ C' = A_A'' - A_A''' - 2A_A' \end{cases}$$

$$\Rightarrow l_0 = l_0' = m_0 = m_0'$$

m_0 is unique !

Thus there are finally two possibilities :

I)

There are three sub-possibilities :

$T_1T_1''(T_2T_2'')^n$ is transcendental for all n , for all T_1', T_2' transcendental prime numbers, there exists T_1, T_1'', T_2, T_2'' for which $T_1 = T_1'T_1''^{-1}$ and $T_2 = T_2'T_2''^{-1}$, in this case $T_1'T_2'$ is always transcendental and of course $T_1' + T_2'$ too.

If $T_1T_1''^{-1}(T_2T_2''^{-1})^n$ is transcendental for all n , for all T_1', T_2' transcendental prime numbers, there exist T_1, T_1'', T_2, T_2'' for which $T_1 = T_1'T_1''$ and $T_2 = T_2'T_2''$, in this case $T_1'T_2'$ is always transcendental and of course $T_1' + T_2'$ too.

$T_1T_2^n$ is transcendental $\forall n \neq n_0$ and $T_1 + mT_2$ is transcendental $\forall m$, in this case $T_1^2T_2^{n_0}$ is transcendental, and $T_1^2T_2^{n_0}$ and $T_1 + mT_2$ are always trans-

dentals. For all T_1', T_2' prime real numbers exist $T_1 = T_1'^{\frac{1}{2}}$ and $T_2 = T_2'^{\frac{1}{n_0}}$ and $m = 1$, thus $T_1'T_2'$ and $T_1' + T_2'$ are transcendentals !

II)

There are three sub-possibilities :

$T_1 + T_1'' + m(T_2 + T_2'')$ is transcendental for all m , for all T_1', T_2' transcendental prime numbers, there exists T_1, T_1'', T_2, T_2'' for which $T_1 = T_1' - T_1''$ and $T_2 = T_2' - T_2''$, in this case $T_1' + T_2'$ is always transcendental and of course $T_1'T_2'$ too.

$T_1 - T_1'' + m(T_2 - T_2'')$ is transcendental for all m , for all T_1', T_2' transcendental prime numbers, there exists T_1, T_1'', T_2, T_2'' for which $T_1 = T_1' + T_1''$ and $T_2 = T_2' + T_2''$, in this case $T_1' + T_2'$ is always transcendental and of course $T_1'T_2'$ too.

$T_1T_2^n$ is transcendental $\forall n$ and $T_1 + mT_2, \forall m \neq m_0$, in this case $2T_1 + mT_2$ is always transcendental, for all T_1', T_2' prime real numbers exist $T_1 = \frac{T_1'}{2}$ and $T_2 = \frac{T_2'}{m_0}$ and $n = 1$, thus $T_1' + T_2'$ and $T_1'T_2'$ are transcendentals!

The theorem application $T_1' = \pi$ and $T_2' = e$ are transcendental prime real numbers which implies that $e\pi, e + \pi$ and $e - \pi$, are transcendentals for example!

$\pi^m e^n$ and $\pi^m + e^n$ are transcendental for all m, n .

Also, by the same way, we prove that $\pi^n, e^m, C_n^r, \pi^n e^m, C_n^r \pi^n$ and $C_n^r e^m$ are transcendentals for every n, m, r !

Conclusion

Through this exposé, we have given a method to find the nature of several numbers, we have shown the nature of $\pi^n e^m, \pi^n C_n^p, e^m C_n^p, \pi^n, e^m$ and $C_n^p \dots$

Références

- [1] Alan Baker, Transcendental number theory *Cambridge University Press*, (1975).