Multiobjective optimization subject to uncertainty : application to irrigation strategy management

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Abstract

In agricultural systems, multiple objectives and uncertainty are often part of the game when optimization is at stake. Multiobjective dominance rules cannot be simply applied due to this uncertain behavior. We propose some extensions of the well-known Pareto rules to enable the discrimination of multicriteria dominating groups of outcomes. These groups are either the various uncertain outcomes of a decision, or more generally a set of outcomes associated to different decisions and/or different random occurrences. Based on the Pareto dominance rules, we propose definitive, acceptable and undecidable dominance comparisons with regard to two candidate groups. The comparisons of all candidate groups allow to rank them from a multicriteria evaluation perspective. This ranking process is used as the evaluation step of a hierarchical decomposition procedure where the best ranked region is selected as the one to be investigated further. We apply these multicriteria extensions to look for optimal irrigation strategies. The yield, the total amount of water and the number of irrigation rounds are simulated to get economical, environmental and social perspectives simultaneously. Although the computation requires a high amount of simulation runs, the algorithm succeeds in reproducing the front of the non dominated evaluations. The major interest resides in the width of the front achieved. This new information gives direct indication to the decision maker about the reliability of the outcomes with regard to the weather uncertainty, as well as the sensitivity

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of the outcomes with regards to the strategies application.

Key words: Multiobjective optimization, Decision under uncertainty, Irrigation management

1. Introduction

The management of agricultural systems is complex, concerned with conflicting objectives (*e.g.* economical outcomes, resources limitations, sustainability) and subject to uncertain external parameters (*e.g.* climate, crop selling prices). Models such as Stics (Brisson et al., 2003), Dssat (Jones et al., 2003) or Apsim (Keating et al., 2003) describe, even though partially, the biophysical nature of the soil-crop in accordance to its natural conditions and some of the farmer actions. Simulation of the impact of management practices allows the exploration and assessment of innovative options (*e.g.* Loyce and Wery (2006); Bergez et al. (2010)) in order to look for optimal management (*e.g.* deVoil et al. (2006); Mayer et al. (2008)) or even to assess impacts of potential climate change (*e.g.* Ludwig et al. (2008); Luo et al. (2009)).

Various studies were concerned with multiple objective optimization within an uncertain context. Mebarki and Castagna (2000), Ding et al. (2006), Romero and Rehman (2003) and Ben Abdelaziz et al. (2007) use a more or less complex indicator of the uncertainty which makes the problem deterministic and then optimize the multiple objectives. Another major way of operating is to make a single objective function out of the multiple objectives and deal with it with stochastic dedicated procedures as in Lee et al. (1996), Pukkala (1998), Al-Aomar (2002) or Rosen et al. (2007). In the former, using an indicator of uncertainty is reducing the various information that allows the possible lot, while in the latter objective aggregation requires the consideration of a single perspective of the multiple objectives. Some others proposed approaches that tackle simultaneously multiple objectives and uncertainty but require heavy user preferences such as Klauer et al. (2002) or Lahdelma and Salminen (2006); some even require direct interactions: Urli and Nadeau (2004) or Nowak (2006) for example.

We aim at proposing an optimization approach which requires as little user dependence as possible. We present in this study an optimization algorithm $P2m$ which consists in optimizing multiple objectives in an uncertain context without reducing objectives or uncertain outputs to an aggregated indicator. The challenge is that the efficient decisions have to be chosen with regard to multiple uncertain multicriteria evaluations. We therefore introduce extensions of the usual domination rules to assess the dominance of evaluation groups in this simultaneously multicriteria and uncertain context.

We first give details about the proposed $P2m$ algorithm. It includes basics processes common to the $P2$ decomposition algorithms and the complete definition of the multicriteria dominance rules utilized to rank groups of evaluations. The application we used as a study case is described thereafter. It includes the definition of an irrigation strategy within the bio-decisional crop model used, the description of the case study and the experiments methodology. Results are presented through three steps: (1) using $P2m$ assuming that any decomposed region is assessed through one decision evaluation, (2) using $P2m$ assuming that any decomposed region is assessed through multiple decisions and the system is deterministic, and (3) using $P2m$ assuming that any decomposed region is assessed through multiple decisions and the system is stochastic, *i.e.* multiple decisions each evaluated through multiple uncertain outcomes. At each step, focus is given on multiple objectives optimization ability. The results are discussed from both agricultural and optimization point of view and we finally conclude highlighting the algorithm limitations and further opening perspectives.

2. The $P2m$ algorithm

2.1. Principles

The $P2m$ algorithm is an extension of the $P2$ algorithm we introduced in Bergez et al. (2004) and improved in Crespo et al. (2009a) and Crespo et al. (2009b). It is based on a hierarchical decomposition of the decision space into a binary tree inspired from the DIRECT algorithm (Jones et al., 1993). It belongs to the family of stochastic branching methods, like stochastic branch-and-bound (Norkin et al., 1998) or nested partitions methods (Shi and Ólafsson, 2000). The decision space $\Theta \in \mathbb{R}$ is a hyper-rectangle that we call a *region*. The $P2m$ optimization aims at finding small regions which include the decision vectors that optimize the system evaluation indicator $J(\theta)$ along multiple objectives. We assume that a region is small enough, or *unbreakable*, when any decision vector of this hyper-rectangle is indistinguishable from the others. This is defined by the user for every dimension of the decision space as the width *step* of the dimension $d \in D$.

$P2m$ initialization allocates the initial decision space as the single eligible region. The first step consists in SELECTING the region which is potentially optimal: we call it the *promising* region. The second step DIVIDES this promising region into two offspring regions. During the third step, each of the new offspring regions is sampled, simulated and EVALUATED. Eventually, the eligible region list is updated, and the three previous steps are repeated until stopping criteria are completed. The process stops when the eligible region list is empty, but additional stopping criteria usually involve time and/or simulation runs limits. The three main steps of selection, division and evaluation are discussed in Crespo et al. (2009a) and will be reminded in the description of the case study.

We define here the notations used for the study. θ is a decision, *i.e.* a vector of dimension D defining an irrigation strategy in the D-dimensional decision space. ω is an uncontrollable parameter, translating the uncertainty of our system so that any evaluation of the decision θ_i subject to ω_j will be unique. $L(\theta_i, \omega_j)$ is this unique evaluation called performance measure, *i.e.* a vector of dimension C defining the output of the system given the θ_i decision and the ω_j uncontrollable parameter in the C-dimensional criteria space. $L(\theta_i, \omega_.)$ is the

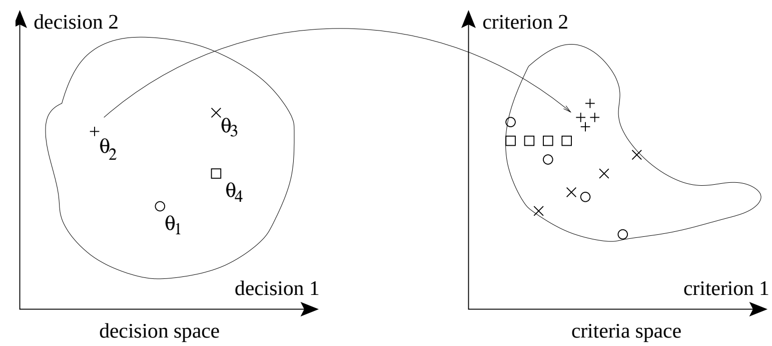

Figure 1: Four decisions evaluated by simulating them subject to four uncertain occurrences and thus leading to four performance measures each. How to decide on efficient decision when considering groups of performance measures?

group of all perturbed performance measures given the θ_i decision subject to every uncontrollable parameter ω, *i.e.* $L(\theta_i, \omega_j)$ for $j \in [1..M]$. With concern with the $P2m$ hierarchical decomposition procedure we will consider a group of decisions included in a region. If this group is including N decisions θ_i ($i \in [1..N]$), then the associated performance measures to a region r are denoted ${}_rL(\theta_., \omega_.)$, *i.e.* $L(\theta_i, \omega_j)$ for $i \in [1..N]$ so that $\theta_i \in r$ and $j \in [1..M]$. In order to keep the following formula short, note that L_{ij} stands for $L(\theta_i, \omega_j)$, $L_{i.}$ stands for $L(\theta_i, \omega_.)$ and ${}_rL$ stands for ${}_rL(\theta_., \omega_.)$.

2.2. The multicriteria evaluation

In our case, any decision leads to as many performance measures as the number of uncontrollable parameters. The multicriteria evaluation is thus designed such that the $P2m$ algorithm tackles the following problem (minimization of all objectives).

$$\underset{\theta \in \Theta}{\text{opt}}\ J(\theta) = \begin{cases} \min[L^1(\theta, \omega_j)] \\ \min[L^2(\theta, \omega_j)] \\ \vdots \\ \min[L^C(\theta, \omega_j)] \end{cases}, j \in [1..M] \qquad (1)$$

Many configurations are possible, including some that are Pareto non dominated (see figure 1 for example). There is however no obvious way to discriminate non dominated groups of evaluations with regards to the related decision.

We propose three dominance rules to discriminate groups of performance measures.

1. Either the dominance is *definitive* when we can directly apply Pareto rules, *i.e.* there is no overlap between the groups and no ambiguity about the dominating and dominated groups,
2. or the dominance is *acceptable* when there is an overlap, but one group can be preferred according to multicriteria perspectives,

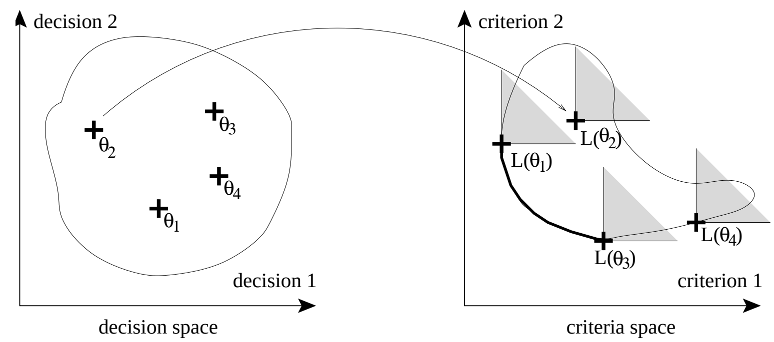

Figure 2: *Pareto dominance rules representation by dominance cones, a deterministic example.*

3. or the dominance is *undecidable* when neither group could be preferred as multicriteria optimal.

These rules are based on the Pareto dominance (Pareto, 1906), restated for multiple objective optimization in Ramesh and Zionts (2000) or Ehrgott (2005) for example. The comparison of all candidates according to the Pareto dominance leads to the definition of the non dominated Pareto front, *i.e.* the equally multicriteria optimal outputs. Assuming $L^c(\theta_i, \omega_j)$ as the $L(\theta_i, \omega_j)$ evaluation along the criterion $c \in [1..C]$, the Pareto domination and thus non domination are defined as follow.

Pareto dominance The performance measure $L_{i'j'}$ is Pareto dominating $L_{i''j''}$ (denoted $L_{i'j'} \underset{p}{\lesssim} L_{i''j''}$ for a minimization) if every $L^c_{i'j'}$ measures are at worst as good as $L^c_{i''j''}$ measures, and that at least one $L^c_{i'j'}$ measure is better than $L^c_{i''j''}$.

$$L_{i'j'} \underset{p}{\lesssim} L_{i''j''} \Leftrightarrow \left\{ \begin{array}{lll} \forall c, & L^c_{i'j'} & \leq & L^c_{i''j''} \\ \exists c, & L^c_{i'j'} & < & L^c_{i''j''} \end{array} \right. \tag{2}$$

If either of the previous conditions to the Pareto dominance is unverified, then the performance measures $L_{i'j'}$ and $L_{i''j''}$ are Pareto non dominated (respectively denoted $L_{i'j'} \underset{p}{\not\lesssim} L_{i''j''}$ and $L_{i''j''} \underset{p}{\not\lesssim} L_{i'j'}$ for a minimization).

The Pareto dominance equations can be depicted as dominance *cones* such as those on the figure 2. $L_{i'j'}$ is dominating any performance measure that would be included in the infinite (on the graph only partly-) shaded cone of which it is the summit. Assuming a deterministic case where one decision θ_i leads to one performance measure $L(\theta_i)$, we can observe on the figure 2 that $L(\theta_1)$ is dominating $L(\theta_2)$ only and $L(\theta_3)$ is dominating $L(\theta_4)$ only, so that $L(\theta_1)$ and $L(\theta_3)$ are non dominated. The decisions θ_1 and θ_3 are thus defined as *efficient* while θ_2 and θ_4 are less efficient.

In order to tackle both uncertainty and multiple objectives, we concentrate on ranking performance measure groups *i.e.* the dominance of a group of multiple performance measures over another group of multiple performance measures. These groups can either be the multiple uncertain performance measures of one decision, *i.e.* $L(\theta_i, \omega_.)$, or the multiple uncertain performance measures of multiple decisions included in a region r, *i.e.* $_rL(\theta_., \omega_.)$. The rules hereafter deal with both of these cases, yet due to our algorithm procedure we assume from now on that a group is representative of a region and made of the performance measures $_rL(\theta_., \omega_.)$ (or $_rL$). The figures included in the following sections represent envelops of multiple performance measures in the 2-dimensional criteria space in such configurations that they help to depict the multicriteria dominance rules proposed. We assume that every objective has to be minimized.

2.2.1. *Definitive group dominance*

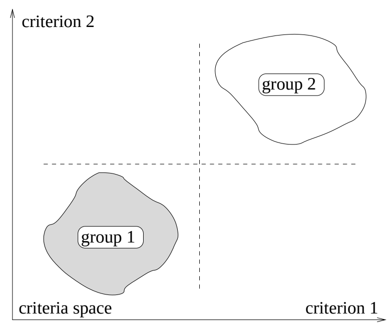

Figure 3: *The group* 1 *is definitively dominating the group* 2.

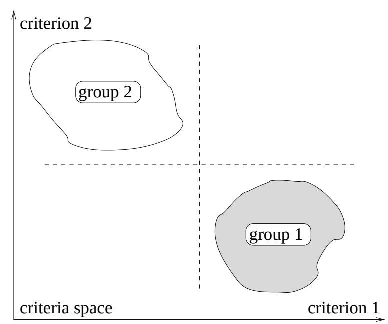

Figure 4: *Groups* 1 *and* 2 *are definitively non-dominated.*

The first dominance rule describes the configuration where groups do not overlap and there is no doubt about the dominance. Either one group is dominating the other one (figure 3), or both are non dominated (figure 4). In either case there is no ambiguity and we thus call the dominance *definitive*. The definitive dominance is assessed relying on the Pareto dominance (equation 2).

Definitive group dominance The group $_{r1}L$ is definitely dominating the group $_{r2}L$ ($_{r1}L \underset{g}{\lll} {}_{r2}L$) if each performance measure $_{r1}L_{i'j'}$ is Pareto dominating every performance measure $_{r2}L_{i''j''}$.

$$_{r1}L \underset{g}{\lll} {}_{r2}L \Leftrightarrow \forall(i', j'), \forall(i'', j'') \ : \ {}_{r1}L_{i'j'} \underset{p}{<} {}_{r2}L_{i''j''} \tag{3}$$

2.2.2. *Acceptable group dominance*

The second dominance rule describes the configuration where groups overlap each other so that the dominance is not obvious. Both compared groups are including performance measures which are Pareto dominated. According to the overlap configurations we define an *acceptable* dominance rule made of two parts. The first part allows discriminating acceptable non dominance from other configurations. The configurations left apart involve acceptable domination or undecidability, which are discriminate with the second part of the acceptable dominance rule.

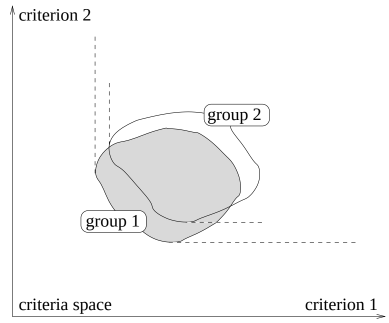

Figure 5: *The group* 1 *is acceptably dominating the group* 2.

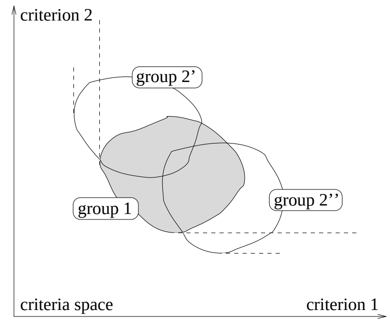

Figure 6: *Groups* 1, $2'$ *and* $2''$ *are acceptably non dominated.*

We define a group as acceptably non dominated as soon as it includes at least one Pareto non dominated performance measure (figure 6). Limbourg (2005) proposed a similar rule that relies on the two worst and best ideal corners, yet they do not make any further discrimination.

Acceptable group dominance part 1 : non dominance discrimination ${}_{r2}L$ is acceptably non dominated by ${}_{r1}L$ (${}_{r1}L \underset{g}{\not\lesssim} {}_{r2}L$) as soon as it exists one performance measure ${}_{r2}L(\theta_{i''}, \omega_{j''})$ Pareto non dominated in front of any ${}_{r1}L$ performance measure (figure 6 configuration).

$$ {}_{r1}L \underset{g}{\not\lesssim} {}_{r2}L \Leftrightarrow \exists(i'', j''), \forall(i', j') \; : \; {}_{r1}L_{i'j'} \underset{p}{\not\lesssim} {}_{r2}L_{i''j''} \qquad (4) $$

Granting that the previous equation 4 is untrue, it means that ${}_{r1}L$ is potentially acceptably dominating while ${}_{r2}L$ is potentially acceptably dominated. The classification of these remaining configurations can be processed with regards to the worst performance measures. We accept the domination of the potentially dominating group over the potentially dominated group if one of these situations occurs.

Acceptable group dominance part 2: domination discrimination The equation 4 being untrue means that every performance measures of the potentially dominated group ${}_{r2}L$ is Pareto dominated by at least one performance measure of the potentially dominating group ${}_{r1}L$, *i.e.* $\forall(i'', j''), \exists(i', j') \; : \; {}_{r1}L_{i'j'} \underset{p}{\lesssim} {}_{r2}L_{i''j''}$. Then ${}_{r1}L$ is acceptably dominating ${}_{r2}L$ (${}_{r1}L \underset{g}{\lesssim} {}_{r2}L$) as soon as one of the following rules is verified.

1. Either every performance measures belonging to the potentially dominating group ${}_{r1}L$ is Pareto dominating at least one performance measure of the potentially dominated group ${}_{r2}L$ (figure 5).

$$ {}_{r1}L \underset{g}{\lesssim} {}_{r2}L \Leftrightarrow \forall(i', j'), \exists(i'', j'') \; : \; {}_{r1}L_{i'j'} \underset{p}{\lesssim} {}_{r2}L_{i''j''} \qquad (5) $$

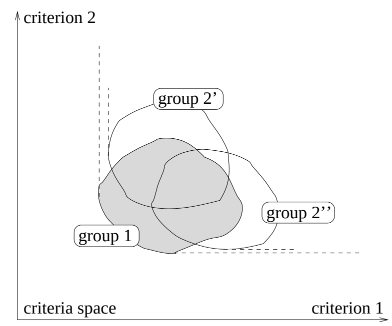

Figure 7: *The group* 1 *is acceptably dominating groups* $2'$ *and* $2''$.

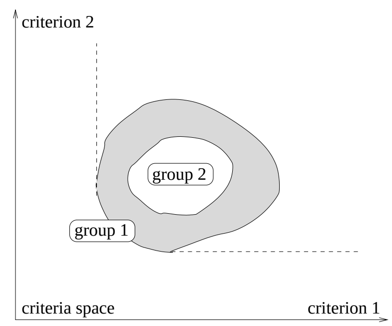

Figure 8: *The group* 1 *is acceptably dominating group* 2.

2. Or it exists Pareto non dominating performance measures belonging to the potentially dominating group ${}_{r1}L$, and at least one of them is Pareto non dominated by any performance measure of the potentially dominated group ${}_{r2}L$ (figures 7 and 8).

$$ {}_{r1}L \underset{g}{<} {}_{r2}L \Leftrightarrow \exists(i',j'),\forall(i'',j'')\ : \begin{cases} {}_{r1}L_{i'j'} \underset{p}{\nless} {}_{r2}L_{i''j''} \\ {}_{r1}L_{i'j'} \underset{p}{\ngtr} {}_{r2}L_{i''j''} \end{cases} \tag{6} $$

The figure 8 configuration is the most ambiguous. With regard to our hierarchical decomposition approach it is natural to decide that the wide spread *group* 1 is acceptably dominating the small dense *group* 2. We do make this choice first because the point of the $P2m$ approach is to divide wide regions into smaller sub regions that would be discriminate as potentially optimal ones, and secondly because doing this choice the approach will tend to produce comparable spread sized group and thus face less ambiguous configurations. We concede that the first reason given is arguable when considering a group of performance measures as representative of one decision ($L(\theta_i, \omega_.)$) and not as representative of many decisions included in a region (${}_rL(\theta_., \omega_.)$). In that case the spread is representative of the uncertainty variability associated to a decision, and thus a smaller spread group will show robustness. Yet the choice is still fair as non selected decision would keep being eligible.

2.2.3. Undecidable group dominance cases

The third dominance rule is defining the configuration where dominance or non dominance is undecidable. Because these dominance configurations can not be define as definitive, it means that there exists a potentially dominating group and a potentially dominated one. Because they do not verify the acceptable dominance definition part 1, it means that every performance measures of the potentially dominated group is dominated by at least one performance measure of the potentially dominating group. Because they do not verify any of the acceptable dominance definition part 2 rules, it means that it exists non dominating performance measures belonging to the potentially dominating group,

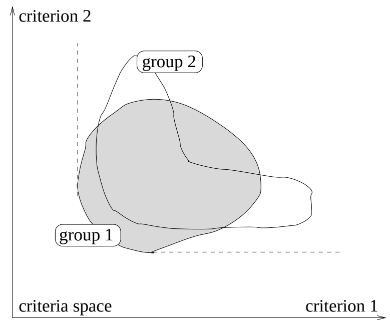

Figure 9: *Undecidable multicriteria groups comparison.*

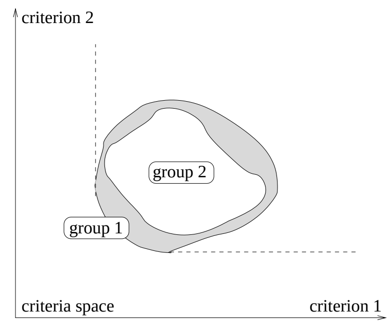

Figure 10: *Undecidable multicriteria groups comparison.*

and that all of them are dominated by at least one performance measure of the potentially dominated group (figures 9 and 10). All these cases are undecidable. The non validation of previous dominance rules is sufficient to discriminate the undecidable cases. However we formalize undecidability as follow.

Undecidability Dominance is undecidable (${}_{r1}L \underset{g}{\sim} {}_{r2}L$) even though every performance measures of the potentially dominated group ${}_{r2}L$ is Pareto dominated by at least one performance measure of the potentially dominating group ${}_{r1}L$, if it exists Pareto non dominating performance measures belonging to ${}_{r1}L$ and that all of these are Pareto dominated by at least one performance measure of ${}_{r2}L$ (figures 9 and 10).

$$ {}_{r1}L \underset{g}{\sim} {}_{r2}L \Leftrightarrow \begin{cases} \forall(i'',j''), \exists(i',j') : {}_{r1}L_{i'j'} \underset{p}{<} {}_{r2}L_{i''j''} \\ \exists(i',j'), \begin{cases} \forall(i'',j'') : {}_{r1}L_{i'j'} \underset{p}{\not<} {}_{r2}L_{i''j''} \\ \exists(i'',j'') : {}_{r1}L_{i'j'} \underset{p}{>} {}_{r2}L_{i''j''} \end{cases} \end{cases} \qquad (7)$$

3. Application to irrigation strategies design

3.1. Irrigation strategy definition

Moderato (Bergez et al., 2001) is a model aimed at evaluating current irrigation strategies for corn and at proposing improved strategies. It combines a dynamic and biophysical corn crop model with a dynamic decision model. The crop model is described in Wallach et al. (2001). The decision model consists of a set of decision rules for different management decisions, in particular irrigation ones (the full decision model is given in Bergez et al. (2001)). A decision rule is a function linking states of the system (indicator) and action (see Bergez et al. (2006) for a lengthily description of such models). It can be written as:

$$\textbf{if } (Indicator\ \textbf{OPERATOR}\ Threshold)\ \textbf{then } \textit{Action} \qquad (8)$$

A simple action is described by a decision rule. A complex action is described by a set of decision rules (a block of rules). The overall management is described by the decision model. MODERATO is a deterministic model which growth simulation is taking into account a strategy θ and is subject to an uncertain and uncontrollable weather series ω. Though the latter is not the only source of uncertainty impacting the simulated crop growth (*e.g.* input data, model), it is one of the major, especially considering irrigation and it is the only uncertain parameter used in this study. As the weather series cannot be known *a priori*, ω is a randomly chosen weather series. We aim at optimizing irrigation management through the optimization of these controllable inputs (*i.e.* the decision rules) given that the model outputs are simulated subject to uncontrollable inputs (*i.e.* the weather). The weather series are unknown prior to the decision making and thus make the optimization process stochastic. From a practical point of view weather series are randomly selected within an historical data set.

In MODERATO one can manage sowing, fertilization, irrigation and harvest by using different set of decisions. The crop model updates the state variables by taking into account the every day applied actions and passes their values to the decision model together with the explanatory variables of that day. Within that collection of variables are the indicators of the decision rules. The decision model then evaluates the rules to decide if a management action is to be taken. According to the weather, soil and plant status and some other constraints (*e.g.* resource availability, day of the year) a decision will be taken. This information is passed back to the crop model (for example the amount of water or the sowing density). For instance, the timing of irrigation includes the following rules.

Starting irrigation This rule determines the starting day to begin irrigation during the growing season and the water amount for the first irrigation round.

Next irrigation round This rule is invoked after a round of irrigation has been completed. It determines when to start the next round and the irrigation amount for rounds after the first.

Stopping irrigation This rule is invoked at the end of an irrigation round. It has one of these three conclusions: either (1) the previous round of irrigation was the last, or (2) another round of irrigation is to be performed and will be the last, or (3) we will re-invoke this rule after another round of irrigation. Granting that the next round is the last, the amount of irrigation is given.

3.2. Case study

The comparison between the developed optimization options was performed on an eight-parameter strategy (*i.e.* 8-decision space) as follows.

> The main irrigation period starts from **T1** ($°C.day$) as soon as the soil water deficit reaches **D1** (mm). An amount **I1** (mm) is applied. Once an irrigation round ends, a new round starts when the soil water deficit reaches **D2** (mm). An amount **I2** (mm) is applied. For the irrigation round following **T3** ($°C.day$), if the soil water

deficit is greater than **D3** (mm) before this irrigation round starts, a last irrigation round is performed; otherwise the irrigation campaign ends. An amount **I3** (mm) is applied.

Operation	Rules
Sowing	Sowing is between 20 April and 30 May as soon as the cumulative rainfall during the previous 3 days is less than 15 mm. Variety Cécilia is sown at 80 000 plants/ha. Cécilia is a late growing variety requiring 1045 accumulated thermal units (ATU) from sowing to flowering and 1990 ATU from sowing to maturity (35% grain humidity).
Fertilization	A single application of 200 kg/ha of nitrogen is made at sowing.
Harvest	The crop is harvested when grain moisture content reaches 20% or accumulated thermal units from sowing reach 2100 ATU and if the cumulative rainfall during the previous 3 days is less than 15 mm. In any case, the crop must be harvested before 15 October.
Irrigation	***Sowing*** Irrigation to facilitate plant emergence (caused either by dryness or crust created by heavy rainfall on silty soil) is not simulated, nor irrigation to dissolve fertilizer. ***Starting irrigation*** Part of the optimization process. ***Next irrigation round*** Part of the optimization process. ***Delay irrigation*** Precipitation delays irrigation. When the cumulative rainfall over the 5 previous days is more than 10 mm, one day delay is applied for every 4 mm. The delay cannot exceed 7 consecutive days. ***Stopping irrigation*** Part of the optimization process.

Table 1: *General description of the strategy simulated.*

The other cultural operations are given in table 1. The irrigation equipment used for the study allows a 3.5 mm/day maximum flow rate. A 180 mm limitation of available water is applied. No flow rate restrictions during summer (except those due to the equipment) are imposed.

All simulations were performed using a medium clay-silt soil : 0.8m deep, with clay accumulation at depth, locally called "Boulbènes moyennes" (fluvisol). This type of soil is representative of a large area of the Midi-Pyrénées and has a 150 mm cumulative available water capacity. The soil was assumed to be at field capacity at the beginning of the simulation, namely the 1^{st} of January. The climate used is from the weather records of Toulouse-Blagnac from 1949 to 1997. On average, July and August receive a total of 92 mm rainfall and the cumulative potential evapotranspiration (ET_0) is 290 mm. The average climatic moisture deficit (ET_0 minus rainfall) for this two-month period is around 200 mm. However, there is a large variation in rainfall during the two summer months as it ranges from 30 to 240 mm, underlining the unpredictable nature of rainfall in the area. Cumulative ET_0 is less variable, ranging from 235 to

372 mm.

3.3. Experiments methodology

The $P2$ division and selection techniques have been discussed in Crespo et al. (2009a) and only the evaluation phase is analyzed here. N decisions are sampled in the promising region following a uniform distribution and M climate dependent performance measures are simulated for each decision. In $P2m$, the criterion of selection is the multicriteria dominance rank computed thanks to the new multicriteria dominance rules we introduced in section 2. Only one region has to be selected as promising. The selection process includes a probability of selecting this region randomly (usually a low probability set to 20% here), and otherwise the one with the lowest multicriteria rank is selected. If it occurs that multiple regions have been attributed an equal lowest rank, the one among those with the highest expected harvest will be chosen. This choice does not disturb the final result made of multiple non dominated performance measures, but prioritize the exploration of the non dominated performance measures with nearly optimal harvest compromises. The algorithm stops when the list of eligible region is empty, or when the simulation run amount reaches 2 000 000.

The region evaluations are based on $N \times M$ performance measures simulated for the N decisions θ_i included in the region subject to M uncontrollable parameters ω_j. We present the results through three steps. First $P2m$ is used assuming that any decomposed region is assessed through one evaluation. This unique evaluation is the average of the $N \times M$ performance measures simulated. Then $P2m$ is used assuming that any decomposed region is assessed through multiple decisions without uncertainty. The region assessment is thus based on a group of N evaluations related to the N decisions included in the concerned region and computed as the averages of the M performance measures simulated per decision. Eventually $P2m$ is used assuming that any decomposed region is assessed through multiple decisions with uncertainty. This final region assessment is based on the group of $N \times M$ performance measures simulated per region of interest.

Simulations were run with a dual processor of 3 GHz each, and 2 GB of RAM with Windows XP operating system. Optimization took about 3 hours and 40 minutes for 2 million of simulation runs within a few minutes for the $P2m$ procedure. We replicated the optimization process 10 times. The initial feasible region is defined in table 2 as the ranges of the different parameters of the strategy described previously.

We ran the crop model focusing on the optimization of the three followings.

$$\begin{cases} \max(L_{ij}^1) & \text{the crop harvest,} \\ \min(L_{ij}^2) & \text{the total water consumption,} \\ \min(L_{ij}^3) & \text{the irrigation account number.} \end{cases} \tag{9}$$

4. Results

We ran the $P2m$ algorithm for the simultaneous optimization of the three previous objectives. However, in order to keep figures easily readable, we present

Names	Meaning	unit	min	max	step
T1	Accumulated thermal unit to start the irrigation campaign	$^{\circ}C$.day	200	1250	5
D1	Soil water deficit to start the irrigation	mm	20	150	3
I1	Irrigation applied at the first irrigation	mm	5	50	2
D2	Soil water deficit to start a new irrigation round	mm	20	150	3
I2	Irrigation depth applied	mm	5	50	2
T3	Accumulated thermal units to stop the irrigation	$^{\circ}C$.day	1250	2000	5
D3	Soil water deficit to stop irrigation	mm	20	150	3
I3	Irrigation applied at the last irrigation round	mm	5	50	2

Table 2: *The eight parameters of the irrigation strategy to be optimized. Min and max show the range of each parameter within which the optimal is sought. A step is the minimum discernible range of the according parameter.*

the results in the 2-criteria space made of the total water consumption (*criterion 1*) and the crop harvest (*criterion 2*) respectively related to the water resource management and the economic outcome. They are strongly conflicting and show distinctly the pros and cons of the multiobjective optimization approach.

4.1. Single evaluation per eligible region

Figure 11 shows 10 final region ensembles achieved by 10 replications of the algorithm. One dot of the graph is the average, for one replication, of the $N \times M$ performance measures simulated in one region. The multicriteria ranking of the regions has been processed according to this unique average evaluation.

Except for one replication which final sub regions ensemble is far from the expected front, the nine others final ensembles describe nine clear and continuous fronts made of Pareto non dominated evaluations. Regions are assessed with no uncertainty so that the multicriteria dominance rules proposed behave as the common Pareto dominance rules. It can be verified noticing that all shown region evaluations belonging to the same replication are non-dominated by any other. The Pareto fronts achieved are uniformly and frequently represented and thus are considered as satisfying discrimination of the efficient decisions while considering one evaluation per considered region.

The front is convex spreading from low water amount compromises (close to no irrigation) until high harvest compromises (up to 9.75 t/ha). Considering the line joining these extremes as a baseline, the front achieved is a curve reaching the highest difference from this baseline for criterion 1 in 100 until 140 mm and criterion 2 in 8.5 until 9.5 t/ha.

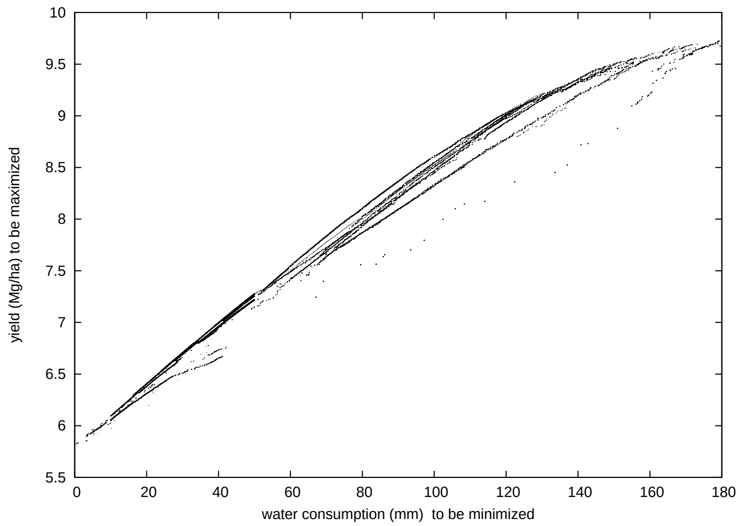

Figure 11: *Averages of regions achieved within 2 000 000 simulations for 10 repetitions of the P2m algorithm : the multicriteria ranking rely on 1 evaluation per region.*

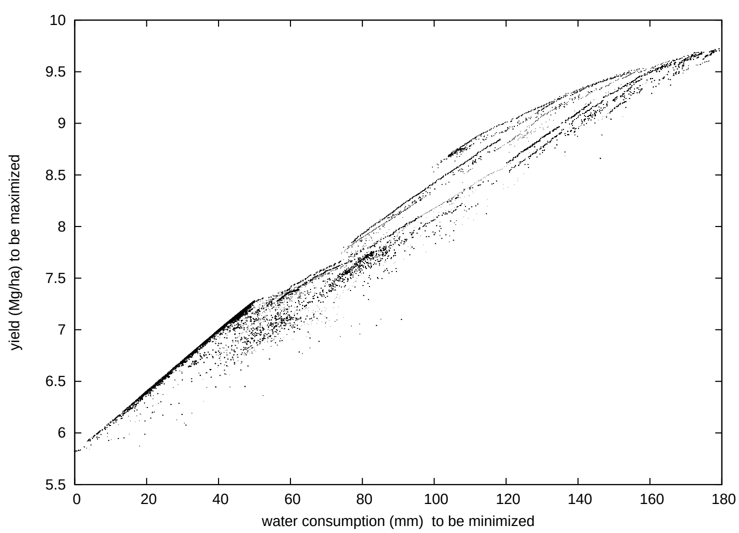

Figure 12: *Averages of regions achieved within 2 000 000 simulations for 10 repetitions of the P2m algorithm : the multicriteria ranking rely on N evaluations per region.*

4.2. As many evaluations as decisions included in the eligible region

The fronts shown on the figure 12 have been achieved while ranking one region against the other ones according to N evaluations consisting in the N averages of the M performance measures simulated per decision.

In comparison to the previous figure 11, three major differences appear. First we clearly notice that the fronts are not made only of non dominated dots from a Pareto perspective. The presented region averages are indeed non dominated according to our group dominance rules, including acceptable dominance which can give dominance to a group of measures even though some of the involved measures are Pareto dominated. It makes the front wider, but still depicts a solid and frequently represented front of non dominated groups. Though close from the expected Pareto front achieved with single evaluations, the second observation is that out of 10 replications, only a few achieved the previous non-dominated front. This is particularly visible on the harvest extreme, while low water amount compromises are correctly defined. We notice that none of the replication fronts are reaching the whole non dominated front achieved on figure 11. Some achieve distinctly the low water amount compromises but struggle to reach high harvest compromises, and some do the opposite. The combination of these two kinds of front gives a precise definition of the non dominated front, yet the third major difference is the less accurate definition of the central part of this front.

Though one replication might not be enough to draw it, the front envelop is similar to the one achieved with one evaluation per region and thus the discrimination gives satisfying results.

4.3. As many evaluations as performance measures simulated per eligible region

The fronts shown on figure 13 have been achieved while ranking one region against the other ones according to the groups of $N \times M$ performance measures simulated for the N decisions within the regions and subject to M disturbance parameters.

In comparison with the first result (figure 11), the previous major differences appear more significantly. First the non dominated regions are shown on the figure 13 as wide areas. Though the envelop of the non dominated group of performance measures is similar to the one pictured on figure 11, not one replication is reaching it all along. In addition to these observations already noticed with N evaluations per region, the number of compromises achieved is high for high harvest compromises, while there are few of them at the low water amount extreme. Though the density of non dominated compromise regions is already significantly different comparing the two extremes of the front, optimal regions defining the middle part are scarcer.

Though not shown, the exact same results have been achieved in the 3-criteria space representing the 3 objectives optimized : harvest, water consumption and irrigation round number. The front achieved (a surface in 3D) is highly defined for the high harvest compromises, regularly defined along a linear section for the low water amount compromises and the central part of the front is irregularly represented.

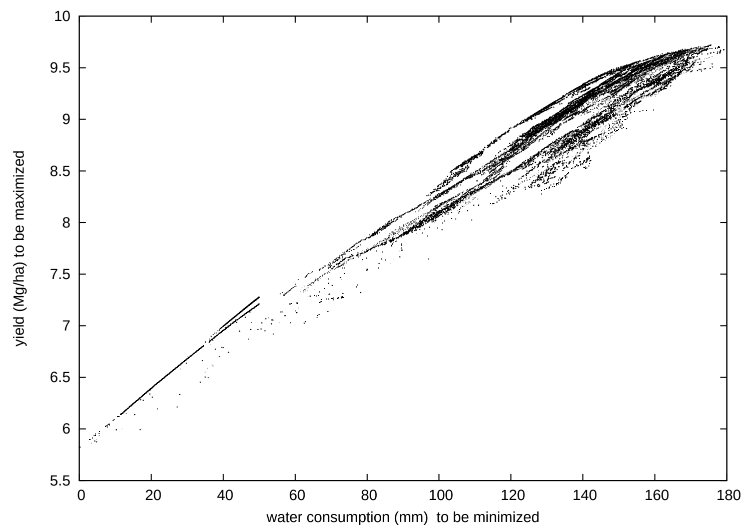

Figure 13: *Averages of regions achieved within 2 000 000 simulations for 10 repetitions of the P2m algorithm : the multicriteria ranking rely on* $N \times M$ *evaluations per region.*

5. Discussions

The $P2m$ algorithm is based on the loop repetition of three main steps described in section 2 : (1) selection of one promising decision's region to be explored further, (2) division of this region and (3) evaluation of the produced eligible regions. The multicriteria group dominance proposed is involved in the evaluation process and strongly related to the selection of the accurate promising region. The efficiency of the dominance rules proposed is assessed through its capacity to produce the front of the non dominated region of performance measures which directly leads to the efficient decisions. We discuss multiobjective optimization interests for agricultural decision purposes.

5.1. Multicriteria group dominance efficiency

The simulations reached a three sections non dominated front. A first section from the lowest water amount compromises up until the medium harvest compromises translating a high priority to minimizing the water amount and a low concern of harvest yield. The middle curved section translates a relatively sensitive compromise reaching a yield included in $[8.5, 9.5]$ t/ha while demanding in between $[100, 140]$ mm of irrigation water. The third section translates a high priority to the maximization of the harvest yield and a low interest for the water amount. The extreme sections relate to decisions linearly altered one from the other by the decision maker preference regarding either the amount of water or the expected yield. On the other hand the middle section is concerned with heterogeneous decisions and simulations give the decision maker valuable information to make a decision. Even under uncertainty the approach is able to reach satisfyingly the front of the non dominated performances, which front shows variability concerns through its width. This variability represents both the decisions variability within the region and the uncertainty of the system evaluation of the decisions simulated.

Though it is not related to the multicriteria approach, we first would like to emphasize the realism of dealing with decision regions instead of decisions vectors leading to single measures. This approach allows flexibility in the application of the decision, making application easier in the field. It is for instance, less perilous to apply a decision defined by a lower and upper boundaries, *e.g.* 34 to 36.5 mm, rather than a value, *e.g.* 35.52 mm. However, the decision maker accepts that he might miss the very optimal decision which would have prescribes hardly applicable recommendations and would be arguable in such an uncertain context as ours.

The first additional information is concerned with variability : the variability due to the decisions inside a region and the variability due to the system uncertainty. Though the $P2m$ algorithm includes 2 theoretical techniques to separate these variability which are mentioned in Crespo et al. (2009a), the results shown in the previous section 4 rely on all performance measures included in region and thus mix decision and uncertainty variability. They however state clearly that the variability is translated through the front width and thus gives indication to the decision maker about outcomes reliability. As an example and according to figure 12 where climate uncertainty is averaged, the compromises including high harvest yields are much more sensitive to the decisions than the low water ones. It thus translates a higher outcome variation from one to a neighbor decision than it would be for low water amount compromises. Considering a problem that do not involve a unique optimal solution as it is often true considering conflicting objectives, overseeing different alternative costs and benefits allow a better understanding of the eventual decision to take. Considering our study case, a decision maker can directly observe that all the efficient decisions leading to the non dominated regions belonging to the straight lines are equally satisfying the preferences defining these lines, so that changing for one of these decisions is linearly related to the expected outcome. On the other hand, decisions associated with the optimal regions belonging to the curved section will be more or less efficient according to the decisions maker preferences. Benefits and costs could then be estimated and guide the best decision to make.

5.2. Agricultural decision interests

Though the theoretical justification of multicriteria definition is not directly related to the application, we propose here some interpretation of the multicriteria group evaluation rules given above. As for an example we consider two objectives as depicted in the result figures. The irrigation water amount is to be minimized while the harvest yield is to be maximized. These objectives are conflicting. According to the irrigation strategy applied, different outcomes will be reached, each defined by the combination : water amount used and harvest yield achieved accordingly. In these conditions, an efficient decision, defined as by usual Pareto dominance rules, is a decision such that its outcomes is non dominated. The results achieved with one evaluation per region are showing a front made of Pareto non dominated regions for which either the combination water-yield is multicriteria optimal. As soon as the group dominance rules that we proposed are involved, a region is definitively dominating granting that all the decisions subject to all the climates required less water while reached higher yield than any decision subject to any climate simulated in the alternative region. When this strong relationship is not verified, then the acceptable dominance is considered. A region will be acceptably dominating if (1) its best outcomes are requiring less water while reaching higher yield than all outcomes of the acceptably dominated one and (2) its worst outcomes are either requiring less water or reaching higher yield than any outcomes of the acceptably dominated one. Though some strategies subject to some climate might require less water and reach more yield than any other from the alternative candidate, the dominance is said undecidable if it also exists other strategy-climate combination requiring more water to reach poorer yield than the alternative candidate.

The front envelop achieved, directly gives an interpretation of the variability of the region considered. Low variability gives comfort about applying the according efficient decision by predicting a most likely realization of the expected outcomes, while high variability translates the uncertain climate and its impact on the related efficient decision. For example on the figure 11, an amount of 100 mm of irrigation water is expected to reach a yield included in 8.5 t/ha up to 9.4 t/ha, while 40 mm of irrigation water predicts a yield of 7 t/ha. Though oppositely extreme these examples give two sides of the variability representation.

In addition to this variability, the shape of the front gives the decision maker new information about potential alternatives. The first linear part from no irrigation up to 100 mm translates that the harvest reached will be highly responsive to the water amount added : an increase of $0.6t/ha$ per 20 mm. While considering the top linear section above $140mm$, the yield response to water is also proportional, yet the same amount of water will increase only slightly the yield : $0.1t/ha$ per 20 mm. The curved section translates the junction in between the bottom high responsive yield to water, and the top low responsive yield to water sections. Without limitation (either maximum water available, or minimum living income) a sensitive compromise would be in that section. The final decision is up to the decision maker and could be help with secondary objectives or decision making approaches that would help him to clarify his preferences, see for example Saaty (1980), Steuer and Choo (1983), Roy (1985) or Vincke (1988).

5.3. Potential extensions

The methodology could be extended to the computation of probability according to the different uncertain scenarios (*e.g.* extremely dry or wet weathers occur with a lower probability). In which case it would requires a significant number of scenarios in order to represent the range of possible. The methodology is however already stressed with simulation number, and we choose not to do so and deal with a global representation of the uncertainty. Though it is not shown here, the methodology could indeed be used with a single uncertain occurrence, which does not translate the range of possible for the considered decision input, but does translate the global uncertainty when decision inputs are regarded as groups.

Extension of the Pareto dominance rules to group dominance can include the use of percentage of Pareto dominating outcomes. It would however impose to the decision maker to express a new preference. Thus we did not explore further these directions as we sought to keep the approach generic and with as little as possible user preferences.

Regarding the base front, the major disadvantage of using multicriteria evaluation granting the same simulation run number, is the loss of robustness in reaching the front of the non dominated region and the resolution loss for lower water amount compromises. We could expect a better robustness and resolution achievement granting a higher amount of simulation runs. It would however require higher computational capacities and a fast enough evaluation process (Crespo et al., 2009a). As for an example, the results shown in the previous section were achieved within a limit of 2 000 000 of simulation runs, and priority was given to high harvest compromises when equally multicriteria ranked regions were eligible for the promising region. The simulation run number limit explains the global loss of robustness and resolution, while the use of yield as secondary objective explains the definition of high harvest compromises first.

6. Conclusion

Our contribution consisted in presenting an optimization procedure that simultaneously tackles multiple conflicting objectives and uncertainty while not aggregating either one or the other into indicators. We propose extensions of the usual multicriteria dominance rules in order to evaluate groups of outcomes rather that the outcomes themselves. These rules are based on the widely used Pareto dominance and used as the evaluation step of a simulation-based optimization procedure. The resulting $P2m$ algorithm is used to optimize irrigation strategies that are evaluated by crop model simulation.

The efficient decision ensemble reached includes the traditional strategies optimizing the yield outcome plus additional strategies demanding less water yet reaching lower yields. These additional strategies are new strategies that are multicriteria optimal and could fit better specific conditions such as limited resources or new global concerns (*e.g.* share of resources, biodiversity). The multicriteria simulation requirements have to be taken into account according to the system evaluation speed. However in our case, the explored strategies have been simulated with a crop model that requires significantly higher computational resources than the multicriteria comparison rules proposed. Thus, from our perspectives, the satisfying achievement of the Pareto front while providing new valuable information that help the decision maker towards an appropriate decision, justify the extend.

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