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Multiobjective optimization subject to uncertainty : application to irrigation strategy management

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Abstract

In agricultural systems, multiple objectives and uncertainty are often part of the game when optimization is at stake. Multiobjective dominance rules cannot be simply applied due to this uncertain behavior. We propose some extensions of the well-known Pareto rules to enable the discrimination of multicriteria dominating groups of outcomes. These groups are either the various uncertain outcomes of a decision, or more generally a set of outcomes associated to different decisions and/or different random occurrences. Based on the Pareto dominance rules, we propose definitive, acceptable and undecidable dominance comparisons with regard to two candidate groups. The comparisons of all candidate groups allow to rank them from a multicriteria evaluation perspective. This ranking process is used as the evaluation step of a hierarchical decomposition procedure where the best ranked region is selected as the one to be investigated further. We apply these multicriteria extensions to look for optimal irrigation strategies. The yield, the total amount of water and the number of irrigation rounds are simulated to get economical, environmental and social perspectives simultaneously. Although the computation requires a high amount of simulation runs, the algorithm succeeds in reproducing the front of the non dominated evaluations. The major interest resides in the width of the front achieved. This new information gives direct indication to the decision maker about the reliability of the outcomes with regard to the weather uncertainty, as well as the sensitivity

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of the outcomes with regards to the strategies application.

Key words: Multiobjective optimization, Decision under uncertainty, Irrigation management

1. Introduction

The management of agricultural systems is complex, concerned with conflicting objectives (*e.g.* economical outcomes, resources limitations, sustainability) and subject to uncertain external parameters (*e.g.* climate, crop selling prices). Models such as STICS (Brisson et al., 2003), DSSAT (Jones et al., 2003) or APSIM (Keating et al., 2003) describe, even though partially, the biophysical nature of the soil-crop in accordance to its natural conditions and some of the farmer actions. Simulation of the impact of management practices allows the exploration and assessment of innovative options (*e.g.* Loyce and Wery (2006); Bergez et al. (2010)) in order to look for optimal management (*e.g.* deVoil et al. (2006); Mayer et al. (2008)) or even to assess impacts of potential climate change (*e.g.* Ludwig et al. (2008); Luo et al. (2009)).

Various studies were concerned with multiple objective optimization within an uncertain context. Mebarki and Castagna (2000), Ding et al. (2006), Romero and Rehman (2003) and Ben Abdelaziz et al. (2007) use a more or less complex indicator of the uncertainty which makes the problem deterministic and then optimize the multiple objectives. Another major way of operating is to make a single objective function out of the multiple objectives and deal with it with stochastic dedicated procedures as in Lee et al. (1996), Pukkala (1998), Al-Aomar (2002) or Rosen et al. (2007). In the former, using an indicator of uncertainty is reducing the various information that allows the possible lot, while in the latter objective aggregation requires the consideration of a single perspective of the multiple objectives. Some others proposed approaches that tackle simultaneously multiple objectives and uncertainty but require heavy user preferences such as Klauer et al. (2002) or Lahdelma and Salminen (2006); some even require direct interactions: Urli and Nadeau (2004) or Nowak (2006) for example.

We aim at proposing an optimization approach which requires as little user dependence as possible. We present in this study an optimization algorithm $P2m$ which consists in optimizing multiple objectives in an uncertain context without reducing objectives or uncertain outputs to an aggregated indicator. The challenge is that the efficient decisions have to be chosen with regard to multiple uncertain multicriteria evaluations. We therefore introduce extensions of the usual domination rules to assess the dominance of evaluation groups in this simultaneously multicriteria and uncertain context.

We first give details about the proposed $P2m$ algorithm. It includes basics processes common to the $P2$ decomposition algorithms and the complete definition of the multicriteria dominance rules utilized to rank groups of evaluations. The application we used as a study case is described thereafter. It includes the

40 definition of an irrigation strategy within the bio-decisional crop model used,
 41 the description of the case study and the experiments methodology. Results are
 42 presented through three steps: (1) using *P2m* assuming that any decomposed
 43 region is assessed through one decision evaluation, (2) using *P2m* assuming that
 44 any decomposed region is assessed through multiple decisions and the system
 45 is deterministic, and (3) using *P2m* assuming that any decomposed region is
 46 assessed through multiple decisions and the system is stochastic, *i.e.* multiple
 47 decisions each evaluated through multiple uncertain outcomes. At each step, fo-
 48 cus is given on multiple objectives optimization ability. The results are discussed
 49 from both agricultural and optimization point of view and we finally conclude
 50 highlighting the algorithm limitations and further opening perspectives.

51 2. The *P2m* algorithm

52 2.1. Principles

53 The *P2m* algorithm is an extension of the *P2* algorithm we introduced in
 54 Bergez et al. (2004) and improved in Crespo et al. (2009a) and Crespo et al.
 55 (2009b). It is based on a hierarchical decomposition of the decision space into a
 56 binary tree inspired from the DIRECT algorithm (Jones et al., 1993). It belongs
 57 to the family of stochastic branching methods, like stochastic branch-and-bound
 58 (Norkin et al., 1998) or nested partitions methods (Shi and Ólafsson, 2000). The
 59 decision space $\Theta \in \mathbb{R}$ is a hyper-rectangle that we call a *region*. The *P2m* opti-
 60 mization aims at finding small regions which include the decision vectors that
 61 optimize the system evaluation indicator $J(\theta)$ along multiple objectives. We
 62 assume that a region is small enough, or *unbreakable*, when any decision vec-
 63 tor of this hyper-rectangle is indistinguishable from the others. This is defined
 64 by the user for every dimension of the decision space as the width *step* of the
 65 dimension $d \in D$.

66 *P2m* initialization allocates the initial decision space as the single eligible
 67 region. The first step consists in SELECTING the region which is potentially
 68 optimal: we call it the *promising* region. The second step DIVIDES this promising
 69 region into two offspring regions. During the third step, each of the new offspring
 70 regions is sampled, simulated and EVALUATED. Eventually, the eligible region
 71 list is updated, and the three previous steps are repeated until stopping criteria
 72 are completed. The process stops when the eligible region list is empty, but
 73 additional stopping criteria usually involve time and/or simulation runs limits.
 74 The three main steps of selection, division and evaluation are discussed in Crespo
 75 et al. (2009a) and will be reminded in the description of the case study.

76 We define here the notations used for the study. θ is a decision, *i.e.* a vector
 77 of dimension D defining an irrigation strategy in the D -dimensional decision
 78 space. ω is an uncontrollable parameter, translating the uncertainty of our
 79 system so that any evaluation of the decision θ_i subject to ω_j will be unique.
 80 $L(\theta_i, \omega_j)$ is this unique evaluation called performance measure, *i.e.* a vector of
 81 dimension C defining the output of the system given the θ_i decision and the ω_j
 82 uncontrollable parameter in the C -dimensional criteria space. $L(\theta_i, \omega.)$ is the

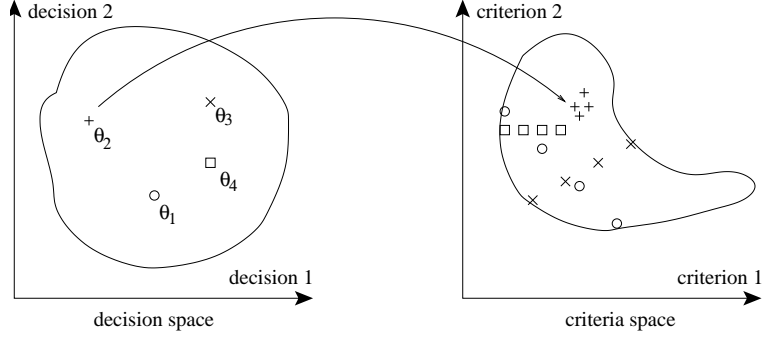


Figure 1: Four decisions evaluated by simulating them subject to four uncertain occurrences and thus leading to four performance measures each. How to decide on efficient decision when considering groups of performance measures?

83 group of all perturbed performance measures given the θ_i decision subject to
 84 every uncontrollable parameter ω , *i.e.* $L(\theta_i, \omega_j)$ for $j \in [1..M]$. With concern
 85 with the $P2m$ hierarchical decomposition procedure we will consider a group
 86 of decisions included in a region. If this group is including N decisions θ_i
 87 ($i \in [1..N]$), then the associated performance measures to a region r are denoted
 88 $rL(\theta, \omega)$, *i.e.* $L(\theta_i, \omega_j)$ for $i \in [1..N]$ so that $\theta_i \in r$ and $j \in [1..M]$. In order to
 89 keep the following formula short, note that L_{ij} stands for $L(\theta_i, \omega_j)$, L_i stands
 90 for $L(\theta_i, \omega)$ and rL stands for $rL(\theta, \omega)$.

91 2.2. The multicriteria evaluation

92 In our case, any decision leads to as many performance measures as the number
 93 of uncontrollable parameters. The multicriteria evaluation is thus designed
 94 such that the $P2m$ algorithm tackles the following problem (minimization of all
 95 objectives).

$$\text{opt}_{\theta \in \Theta} J(\theta) = \begin{cases} \min[L^1(\theta, \omega_j)] \\ \min[L^2(\theta, \omega_j)] \\ \vdots \\ \min[L^C(\theta, \omega_j)] \end{cases}, j \in [1..M] \quad (1)$$

96 Many configurations are possible, including some that are Pareto non dominated
 97 (see figure 1 for example). There is however no obvious way to discriminate non
 98 dominated groups of evaluations with regards to the related decision.

99 We propose three dominance rules to discriminate groups of performance
 100 measures.

- 101 1. Either the dominance is *definitive* when we can directly apply Pareto rules,
 102 *i.e.* there is no overlap between the groups and no ambiguity about the
 103 dominating and dominated groups,
- 104 2. or the dominance is *acceptable* when there is an overlap, but one group
 105 can be preferred according to multicriteria perspectives,

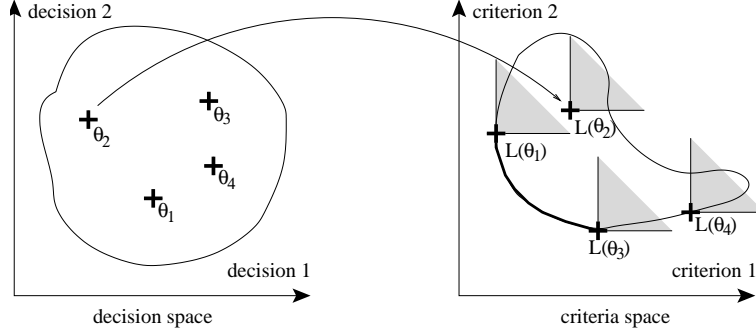


Figure 2: Pareto dominance rules representation by dominance cones, a deterministic example.

106 3. or the dominance is *undecidable* when neither group could be preferred as
 107 multicriteria optimal.

108 These rules are based on the Pareto dominance (Pareto, 1906), restated for
 109 multiple objective optimization in Ramesh and Zionts (2000) or Ehrgott (2005)
 110 for example. The comparison of all candidates according to the Pareto domi-
 111 nance leads to the definition of the non dominated Pareto front, *i.e.* the equally
 112 multicriteria optimal outputs. Assuming $L^c(\theta_i, \omega_j)$ as the $L(\theta_i, \omega_j)$ evaluation
 113 along the criterion $c \in [1..C]$, the Pareto domination and thus non domination
 114 are defined as follow.

115 **Pareto dominance** The performance measure $L_{i'j'}$ is Pareto dominating
 116 $L_{i''j''}$ (denoted $L_{i'j'} \underset{p}{\leq} L_{i''j''}$ for a minimization) if every $L_{i'j'}^c$ measures
 117 are at worst as good as $L_{i''j''}^c$ measures, and that at least one $L_{i'j'}^c$ measure
 118 is better than $L_{i''j''}^c$.

$$L_{i'j'} \underset{p}{\leq} L_{i''j''} \Leftrightarrow \begin{cases} \forall c, & L_{i'j'}^c \leq L_{i''j''}^c \\ \exists c, & L_{i'j'}^c < L_{i''j''}^c \end{cases} \quad (2)$$

119 If either of the previous conditions to the Pareto dominance is unverified, then
 120 the performance measures $L_{i'j'}$ and $L_{i''j''}$ are Pareto non dominated (re-
 121 spectively denoted $L_{i'j'} \not\underset{p}{\leq} L_{i''j''}$ and $L_{i''j''} \not\underset{p}{\leq} L_{i'j'}$ for a minimization).

122 The Pareto dominance equations can be depicted as dominance *cones* such
 123 as those on the figure 2. $L_{i'j'}$ is dominating any performance measure that
 124 would be included in the infinite (on the graph only partly-) shaded cone of
 125 which it is the summit. Assuming a deterministic case where one decision θ_i
 126 leads to one performance measure $L(\theta_i)$, we can observe on the figure 2 that
 127 $L(\theta_1)$ is dominating $L(\theta_2)$ only and $L(\theta_3)$ is dominating $L(\theta_4)$ only, so that
 128 $L(\theta_1)$ and $L(\theta_3)$ are non dominated. The decisions θ_1 and θ_3 are thus defined
 129 as *efficient* while θ_2 and θ_4 are less efficient.

130 In order to tackle both uncertainty and multiple objectives, we concentrate
 131 on ranking performance measure groups *i.e.* the dominance of a group of multi-
 132 ple performance measures over another group of multiple performance measures.
 133 These groups can either be the multiple uncertain performance measures of one
 134 decision, *i.e.* $L(\theta_i, \omega)$, or the multiple uncertain performance measures of multi-
 135 ple decisions included in a region r , *i.e.* $rL(\theta, \omega)$. The rules hereafter deal with
 136 both of these cases, yet due to our algorithm procedure we assume from now
 137 on that a group is representative of a region and made of the performance mea-
 138 sures $rL(\theta, \omega)$ (or rL). The figures included in the following sections represent
 139 envelopes of multiple performance measures in the 2-dimensional criteria space
 140 in such configurations that they help to depict the multicriteria dominance rules
 141 proposed. We assume that every objective has to be minimized.

142 *2.2.1. Definitive group dominance*

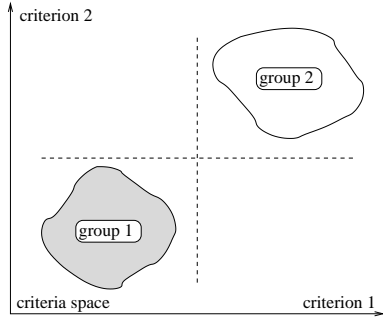


Figure 3: *The group 1 is definitively dominating the group 2.*

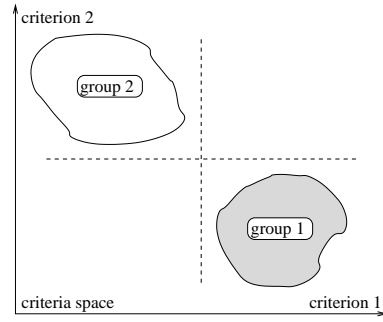


Figure 4: *Groups 1 and 2 are definitively non-dominated.*

143 The first dominance rule describes the configuration where groups do not
 144 overlap and there is no doubt about the dominance. Either one group is domi-
 145 nating the other one (figure 3), or both are non dominated (figure 4). In either
 146 case there is no ambiguity and we thus call the dominance *definitive*. The
 147 definitive dominance is assessed relying on the Pareto dominance (equation 2).

148 **Definitive group dominance** The group r_1L is definitively dominating the
 149 group r_2L ($r_1L \ll r_2L$) if each performance measure $r_1L_{i'j'}$ is Pareto
 150 dominating every performance measure $r_2L_{i''j''}$.

$$r_1L \ll r_2L \Leftrightarrow \forall(i', j'), \forall(i'', j'') : r_1L_{i'j'} \underset{p}{\prec} r_2L_{i''j''} \quad (3)$$

151 *2.2.2. Acceptable group dominance*

152 The second dominance rule describes the configuration where groups overlap
 153 each other so that the dominance is not obvious. Both compared groups are
 154 including performance measures which are Pareto dominated. According to
 155 the overlap configurations we define an *acceptable* dominance rule made of two
 156 parts. The first part allows discriminating acceptable non dominance from other

157 configurations. The configurations left apart involve acceptable domination or
 158 undecidability, which are discriminate with the second part of the acceptable
 159 dominance rule.

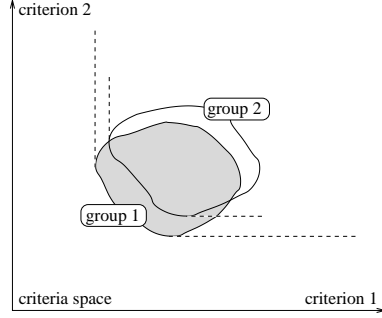


Figure 5: The group 1 is acceptably dominating the group 2.

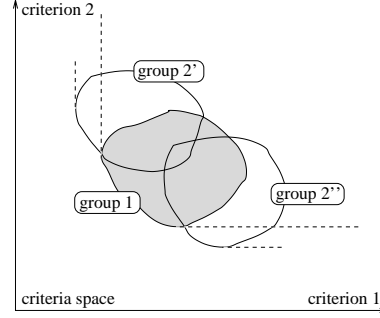


Figure 6: Groups 1, 2' and 2'' are acceptably non dominated.

160 We define a group as acceptably non dominated as soon as it includes at least
 161 one Pareto non dominated performance measure (figure 6). Limbourg (2005)
 162 proposed a similar rule that relies on the two worst and best ideal corners, yet
 163 they do not make any further discrimination.

164 **Acceptable group dominance part 1 : non dominance discrimination**

165 r_2L is acceptably non dominated by r_1L ($r_1L \not\prec_g r_2L$) as soon as it exists
 166 one performance measure $r_2L(\theta_{i''}, \omega_{j''})$ Pareto non dominated in front of
 167 any r_1L performance measure (figure 6 configuration).

$$r_1L \not\prec_g r_2L \Leftrightarrow \exists(i'', j''), \forall(i', j') : r_1L_{i'j'} \not\prec_p r_2L_{i''j''} \quad (4)$$

168 Granting that the previous equation 4 is untrue, it means that r_1L is po-
 169 tentially acceptably dominating while r_2L is potentially acceptably dominated.
 170 The classification of these remaining configurations can be processed with re-
 171 gards to the worst performance measures. We accept the domination of the
 172 potentially dominating group over the potentially dominated group if one of
 173 these situations occurs.

174 **Acceptable group dominance part 2: domination discrimination** The

175 equation 4 being untrue means that every performance measures of the po-
 176 tentially dominated group r_2L is Pareto dominated by at least one perfor-
 177 mance measure of the potentially dominating group r_1L , i.e. $\forall(i'', j''), \exists(i', j') :$
 178 $r_1L_{i'j'} \prec_p r_2L_{i''j''}$. Then r_1L is acceptably dominating r_2L ($r_1L \prec_g r_2L$)
 179 as soon as one of the following rules is verified.

- 180 1. Either every performance measures belonging to the potentially dom-
 181 inating group r_1L is Pareto dominating at least one performance
 182 measure of the potentially dominated group r_2L (figure 5).

$$r_1L \prec_g r_2L \Leftrightarrow \forall(i', j'), \exists(i'', j'') : r_1L_{i'j'} \prec_p r_2L_{i''j''} \quad (5)$$

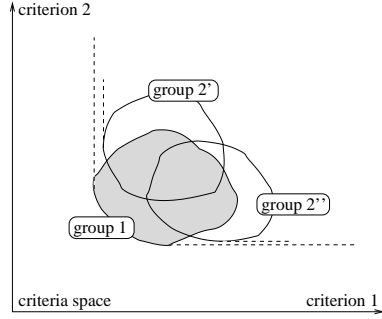


Figure 7: *The group 1 is acceptably dominating groups 2' and 2''.*

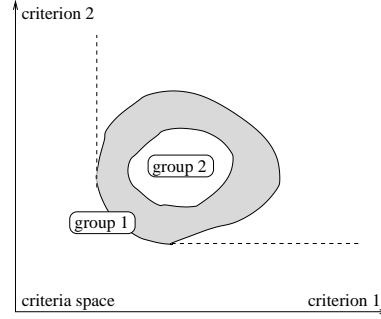


Figure 8: *The group 1 is acceptably dominating group 2.*

- 183 2. Or it exists Pareto non dominating performance measures belonging to
 184 the potentially dominating group r_1L , and at least one of them is
 185 Pareto non dominated by any performance measure of the potentially
 186 dominated group r_2L (figures 7 and 8).

$$r_1L \underset{g}{\lesssim} r_2L \Leftrightarrow \exists(i', j'), \forall(i'', j'') : \begin{cases} r_1L_{i'j'} \not\underset{p}{\lesssim} r_2L_{i''j''} \\ r_1L_{i'j'} \underset{p}{\lesssim} r_2L_{i''j''} \end{cases} \quad (6)$$

187 The figure 8 configuration is the most ambiguous. With regard to our hi-
 188 erarchical decomposition approach it is natural to decide that the wide spread
 189 *group 1* is acceptably dominating the small dense *group 2*. We do make this
 190 choice first because the point of the *P2m* approach is to divide wide regions into
 191 smaller sub regions that would be discriminate as potentially optimal ones, and
 192 secondly because doing this choice the approach will tend to produce compar-
 193 able spread sized group and thus face less ambiguous configurations. We concede
 194 that the first reason given is arguable when considering a group of performance
 195 measures as representative of one decision ($L(\theta_i, \omega.)$) and not as representative
 196 of many decisions included in a region ($rL(\theta, \omega.)$). In that case the spread is
 197 representative of the uncertainty variability associated to a decision, and thus
 198 a smaller spread group will show robustness. Yet the choice is still fair as non
 199 selected decision would keep being eligible.

200 *2.2.3. Undecidable group dominance cases*

201 The third dominance rule is defining the configuration where dominance or
 202 non dominance is undecidable. Because these dominance configurations can
 203 not be define as definitive, it means that there exists a potentially dominating
 204 group and a potentially dominated one. Because they do not verify the accept-
 205 able dominance definition part 1, it means that every performance measures of
 206 the potentially dominated group is dominated by at least one performance mea-
 207 sure of the potentially dominating group. Because they do not verify any of the
 208 acceptable dominance definition part 2 rules, it means that it exists non dom-
 209 inating performance measures belonging to the potentially dominating group,

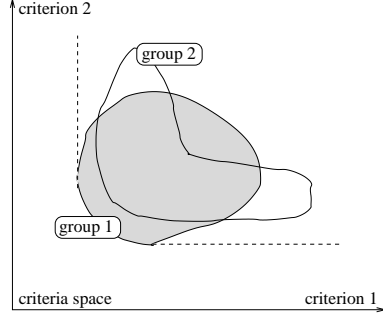


Figure 9: *Undecidable multicriteria groups comparison.*

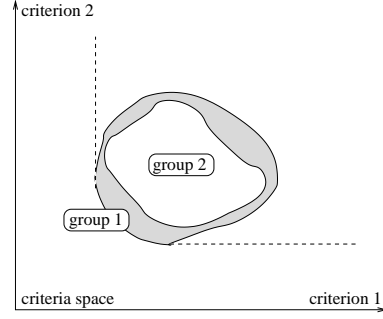


Figure 10: *Undecidable multicriteria groups comparison.*

210 and that all of them are dominated by at least one performance measure of the
 211 potentially dominated group (figures 9 and 10). All these cases are undecidable.
 212 The non validation of previous dominance rules is sufficient to discriminate the
 213 undecidable cases. However we formalize undecidability as follow.

214 **Undecidability** Dominance is undecidable ($r_1L \overset{g}{\sim} r_2L$) even though every
 215 performance measures of the potentially dominated group r_2L is Pareto
 216 dominated by at least one performance measure of the potentially domi-
 217 nating group r_1L , if it exists Pareto non dominating performance measures
 218 belonging to r_1L and that all of these are Pareto dominated by at least
 219 one performance measure of r_2L (figures 9 and 10).

$$r_1L \overset{g}{\sim} r_2L \Leftrightarrow \begin{cases} \forall(i'', j''), \exists(i', j') : r_1L_{i'j'} \overset{p}{\leq} r_2L_{i''j''} \\ \exists(i', j'), \left\{ \begin{array}{l} \forall(i'', j'') : r_1L_{i'j'} \not\overset{p}{\leq} r_2L_{i''j''} \\ \exists(i'', j'') : r_1L_{i'j'} \overset{p}{\geq} r_2L_{i''j''} \end{array} \right. \end{cases} \quad (7)$$

220 3. Application to irrigation strategies design

221 3.1. Irrigation strategy definition

222 MODERATO (Bergez et al., 2001) is a model aimed at evaluating current
 223 irrigation strategies for corn and at proposing improved strategies. It combines
 224 a dynamic and biophysical corn crop model with a dynamic decision model. The
 225 crop model is described in Wallach et al. (2001). The decision model consists of
 226 a set of decision rules for different management decisions, in particular irrigation
 227 ones (the full decision model is given in Bergez et al. (2001)). A decision rule is
 228 a function linking states of the system (indicator) and action (see Bergez et al.
 229 (2006) for a lengthily description of such models). It can be written as:

$$\text{if } (Indicator \text{ OPERATOR } Threshold) \text{ then Action} \quad (8)$$

230 A simple action is described by a decision rule. A complex action is described
 231 by a set of decision rules (a block of rules). The overall management is described

232 by the decision model. MODERATO is a deterministic model which growth sim-
 233 ulation is taking into account a strategy θ and is subject to an uncertain and
 234 uncontrollable weather series ω . Though the latter is not the only source of
 235 uncertainty impacting the simulated crop growth (*e.g.* input data, model), it is
 236 one of the major, especially considering irrigation and it is the only uncertain
 237 parameter used in this study. As the weather series cannot be known *a priori*,
 238 ω is a randomly chosen weather series. We aim at optimizing irrigation man-
 239 agement through the optimization of these controllable inputs (*i.e.* the decision
 240 rules) given that the model outputs are simulated subject to uncontrollable in-
 241 puts (*i.e.* the weather). The weather series are unknown prior to the decision
 242 making and thus make the optimization process stochastic. From a practical
 243 point of view weather series are randomly selected within an historical data set.

244 In MODERATO one can manage sowing, fertilization, irrigation and harvest
 245 by using different set of decisions. The crop model updates the state variables
 246 by taking into account the every day applied actions and passes their values to
 247 the decision model together with the explanatory variables of that day. Within
 248 that collection of variables are the indicators of the decision rules. The decision
 249 model then evaluates the rules to decide if a management action is to be taken.
 250 According to the weather, soil and plant status and some other constraints (*e.g.*
 251 resource availability, day of the year) a decision will be taken. This information
 252 is passed back to the crop model (for example the amount of water or the sowing
 253 density). For instance, the timing of irrigation includes the following rules.

254 ***Starting irrigation*** This rule determines the starting day to begin irrigation
 255 during the growing season and the water amount for the first irrigation
 256 round.

257 ***Next irrigation round*** This rule is invoked after a round of irrigation has
 258 been completed. It determines when to start the next round and the
 259 irrigation amount for rounds after the first.

260 ***Stopping irrigation*** This rule is invoked at the end of an irrigation round.
 261 It has one of these three conclusions: either (1) the previous round of
 262 irrigation was the last, or (2) another round of irrigation is to be performed
 263 and will be the last, or (3) we will re-invoke this rule after another round
 264 of irrigation. Granting that the next round is the last, the amount of
 265 irrigation is given.

266 3.2. Case study

267 The comparison between the developed optimization options was performed
 268 on an eight-parameter strategy (*i.e.* 8-decision space) as follows.

269 The main irrigation period starts from **T1** ($^{\circ}C.day$) as soon as the
 270 soil water deficit reaches **D1** (mm). An amount **I1** (mm) is applied.
 271 Once an irrigation round ends, a new round starts when the soil
 272 water deficit reaches **D2** (mm). An amount **I2** (mm) is applied.
 273 For the irrigation round following **T3** ($^{\circ}C.day$), if the soil water

274 deficit is greater than **D3** (*mm*) before this irrigation round starts, a
 275 last irrigation round is performed; otherwise the irrigation campaign
 276 ends. An amount **I3** (*mm*) is applied.

Operation	Rules
Sowing	Sowing is between 20 April and 30 May as soon as the cumulative rainfall during the previous 3 days is less than 15 mm. Variety Cécilia is sown at 80 000 plants/ha. Cécilia is a late growing variety requiring 1045 accumulated thermal units (ATU) from sowing to flowering and 1990 ATU from sowing to maturity (35% grain humidity).
Fertilization	A single application of 200 kg/ha of nitrogen is made at sowing.
Harvest	The crop is harvested when grain moisture content reaches 20% or accumulated thermal units from sowing reach 2100 ATU and if the cumulative rainfall during the previous 3 days is less than 15 mm. In any case, the crop must be harvested before 15 October.
Irrigation	Sowing Irrigation to facilitate plant emergence (caused either by dryness or crust created by heavy rainfall on silty soil) is not simulated, nor irrigation to dissolve fertilizer. Starting irrigation Part of the optimization process. Next irrigation round Part of the optimization process. Delay irrigation Precipitation delays irrigation. When the cumulative rainfall over the 5 previous days is more than 10 mm, one day delay is applied for every 4 mm. The delay cannot exceed 7 consecutive days. Stopping irrigation Part of the optimization process.

Table 1: *General description of the strategy simulated.*

277 The other cultural operations are given in table 1. The irrigation equipment used
 278 for the study allows a 3.5 *mm/day* maximum flow rate. A 180 mm limitation
 279 of available water is applied. No flow rate restrictions during summer (except
 280 those due to the equipment) are imposed.

281 All simulations were performed using a medium clay-silt soil : 0.8m deep,
 282 with clay accumulation at depth, locally called “Boulbènes moyennes” (fluvi-
 283 sol). This type of soil is representative of a large area of the Midi-Pyrénées and
 284 has a 150 *mm* cumulative available water capacity. The soil was assumed to be
 285 at field capacity at the beginning of the simulation, namely the 1st of January.
 286 The climate used is from the weather records of Toulouse-Blagnac from 1949
 287 to 1997. On average, July and August receive a total of 92 *mm* rainfall and
 288 the cumulative potential evapotranspiration (ET_0) is 290 *mm*. The average cli-
 289 matic moisture deficit (ET_0 minus rainfall) for this two-month period is around
 290 200 *mm*. However, there is a large variation in rainfall during the two summer
 291 months as it ranges from 30 to 240 *mm*, underlining the unpredictable nature
 292 of rainfall in the area. Cumulative ET_0 is less variable, ranging from 235 to

294 *3.3. Experiments methodology*

295 The *P2* division and selection techniques have been discussed in Crespo et al.
 296 (2009a) and only the evaluation phase is analyzed here. N decisions are sam-
 297 pled in the promising region following a uniform distribution and M climate
 298 dependent performance measures are simulated for each decision. In *P2m*, the
 299 criterion of selection is the multicriteria dominance rank computed thanks to
 300 the new multicriteria dominance rules we introduced in section 2. Only one
 301 region has to be selected as promising. The selection process includes a prob-
 302 ability of selecting this region randomly (usually a low probability set to 20%
 303 here), and otherwise the one with the lowest multicriteria rank is selected. If it
 304 occurs that multiple regions have been attributed an equal lowest rank, the one
 305 among those with the highest expected harvest will be chosen. This choice does
 306 not disturb the final result made of multiple non dominated performance mea-
 307 sures, but prioritize the exploration of the non dominated performance measures
 308 with nearly optimal harvest compromises. The algorithm stops when the list of
 309 eligible region is empty, or when the simulation run amount reaches 2 000 000.

310 The region evaluations are based on $N \times M$ performance measures simu-
 311 lated for the N decisions θ_i included in the region subject to M uncontrollable
 312 parameters ω_j . We present the results through three steps. First *P2m* is used
 313 assuming that any decomposed region is assessed through one evaluation. This
 314 unique evaluation is the average of the $N \times M$ performance measures simulated.
 315 Then *P2m* is used assuming that any decomposed region is assessed through
 316 multiple decisions without uncertainty. The region assessment is thus based on
 317 a group of N evaluations related to the N decisions included in the concerned
 318 region and computed as the averages of the M performance measures simulated
 319 per decision. Eventually *P2m* is used assuming that any decomposed region
 320 is assessed through multiple decisions with uncertainty. This final region as-
 321 sessment is based on the group of $N \times M$ performance measures simulated per
 322 region of interest.

323 Simulations were run with a dual processor of 3 GHz each, and 2 GB of
 324 RAM with Windows XP operating system. Optimization took about 3 hours
 325 and 40 minutes for 2 million of simulation runs within a few minutes for the
 326 *P2m* procedure. We replicated the optimization process 10 times. The initial
 327 feasible region is defined in table 2 as the ranges of the different parameters of
 328 the strategy described previously.

329 We ran the crop model focusing on the optimization of the three followings.

$$\begin{cases} \max(L_{ij}^1) & \text{the crop harvest,} \\ \min(L_{ij}^2) & \text{the total water consumption,} \\ \min(L_{ij}^3) & \text{the irrigation account number.} \end{cases} \quad (9)$$

330 **4. Results**

331 We ran the *P2m* algorithm for the simultaneous optimization of the three
 332 previous objectives. However, in order to keep figures easily readable, we present

Names	Meaning	unit	min	max	step
T1	Accumulated thermal unit to start the irrigation campaign	$^{\circ}C.day$	200	1250	5
D1	Soil water deficit to start the irrigation	mm	20	150	3
I1	Irrigation applied at the first irrigation	mm	5	50	2
D2	Soil water deficit to start a new irrigation round	mm	20	150	3
I2	Irrigation depth applied	mm	5	50	2
T3	Accumulated thermal units to stop the irrigation	$^{\circ}C.day$	1250	2000	5
D3	Soil water deficit to stop irrigation	mm	20	150	3
I3	Irrigation applied at the last irrigation round	mm	5	50	2

Table 2: The eight parameters of the irrigation strategy to be optimized. Min and max show the range of each parameter within which the optimal is sought. A step is the minimum discernible range of the according parameter.

333 the results in the 2-criteria space made of the total water consumption (*criterion*
334 *1*) and the crop harvest (*criterion 2*) respectively related to the water resource
335 management and the economic outcome. They are strongly conflicting and show
336 distinctly the pros and cons of the multiobjective optimization approach.

337 4.1. Single evaluation per eligible region

338 Figure 11 shows 10 final region ensembles achieved by 10 replications of the
339 algorithm. One dot of the graph is the average, for one replication, of the $N \times M$
340 performance measures simulated in one region. The multicriteria ranking of the
341 regions has been processed according to this unique average evaluation.

342 Except for one replication which final sub regions ensemble is far from the
343 expected front, the nine others final ensembles describe nine clear and continu-
344 ous fronts made of Pareto non dominated evaluations. Regions are assessed with
345 no uncertainty so that the multicriteria dominance rules proposed behave as the
346 common Pareto dominance rules. It can be verified noticing that all shown re-
347 gion evaluations belonging to the same replication are non-dominated by any
348 other. The Pareto fronts achieved are uniformly and frequently represented and
349 thus are considered as satisfying discrimination of the efficient decisions while
350 considering one evaluation per considered region.

351 The front is convex spreading from low water amount compromises (close to
352 no irrigation) until high harvest compromises (up to $9.75 t/ha$). Considering the
353 line joining these extremes as a baseline, the front achieved is a curve reaching
354 the highest difference from this baseline for criterion 1 in 100 until 140 mm and
355 criterion 2 in 8.5 until $9.5 t/ha$.

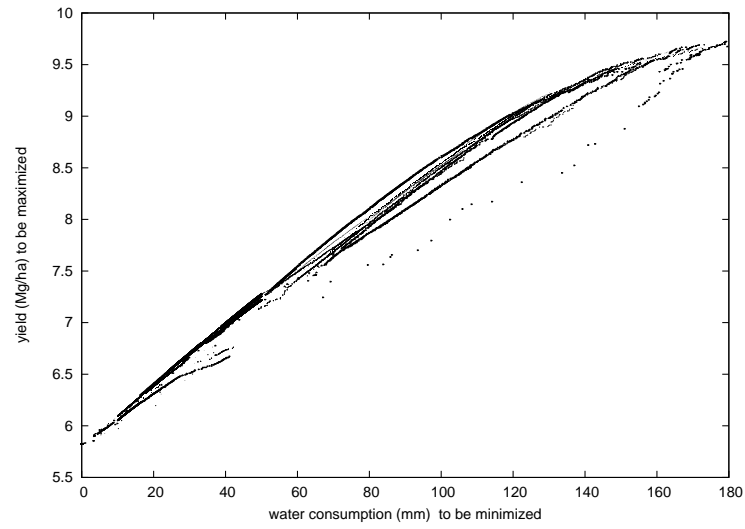


Figure 11: Averages of regions achieved within 2 000 000 simulations for 10 repetitions of the P2m algorithm : the multicriteria ranking rely on 1 evaluation per region.

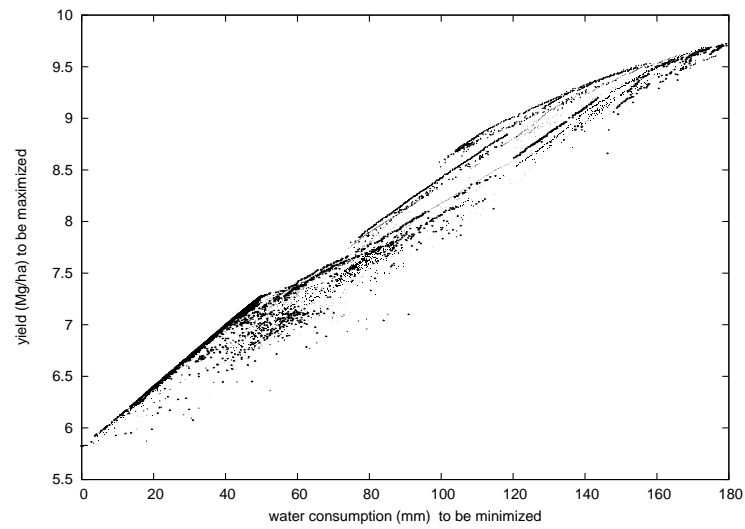


Figure 12: Averages of regions achieved within 2 000 000 simulations for 10 repetitions of the P2m algorithm : the multicriteria ranking rely on N evaluations per region.

356 *4.2. As many evaluations as decisions included in the eligible region*

357 The fronts shown on the figure 12 have been achieved while ranking one
358 region against the other ones according to N evaluations consisting in the N
359 averages of the M performance measures simulated per decision.

360 In comparison to the previous figure 11, three major differences appear. First
361 we clearly notice that the fronts are not made only of non dominated dots from
362 a Pareto perspective. The presented region averages are indeed non dominated
363 according to our group dominance rules, including acceptable dominance which
364 can give dominance to a group of measures even though some of the involved
365 measures are Pareto dominated. It makes the front wider, but still depicts a
366 solid and frequently represented front of non dominated groups. Though close
367 from the expected Pareto front achieved with single evaluations, the second
368 observation is that out of 10 replications, only a few achieved the previous non-
369 dominated front. This is particularly visible on the harvest extreme, while low
370 water amount compromises are correctly defined. We notice that none of the
371 replication fronts are reaching the whole non dominated front achieved on figure
372 11. Some achieve distinctly the low water amount compromises but struggle to
373 reach high harvest compromises, and some do the opposite. The combination
374 of these two kinds of front gives a precise definition of the non dominated front,
375 yet the third major difference is the less accurate definition of the central part
376 of this front.

377 Though one replication might not be enough to draw it, the front envelop
378 is similar to the one achieved with one evaluation per region and thus the dis-
379 crimination gives satisfying results.

380 *4.3. As many evaluations as performance measures simulated per eligible region*

381 The fronts shown on figure 13 have been achieved while ranking one region
382 against the other ones according to the groups of $N \times M$ performance measures
383 simulated for the N decisions within the regions and subject to M disturbance
384 parameters.

385 In comparison with the first result (figure 11), the previous major differences
386 appear more significantly. First the non dominated regions are shown on the
387 figure 13 as wide areas. Though the envelop of the non dominated group of
388 performance measures is similar to the one pictured on figure 11, not one repli-
389 cation is reaching it all along. In addition to these observations already noticed
390 with N evaluations per region, the number of compromises achieved is high for
391 high harvest compromises, while there are few of them at the low water amount
392 extreme. Though the density of non dominated compromise regions is already
393 significantly different comparing the two extremes of the front, optimal regions
394 defining the middle part are scarcer.

395 Though not shown, the exact same results have been achieved in the 3-
396 criteria space representing the 3 objectives optimized : harvest, water consump-
397 tion and irrigation round number. The front achieved (a surface in 3D) is highly
398 defined for the high harvest compromises, regularly defined along a linear sec-
399 tion for the low water amount compromises and the central part of the front is
400 irregularly represented.

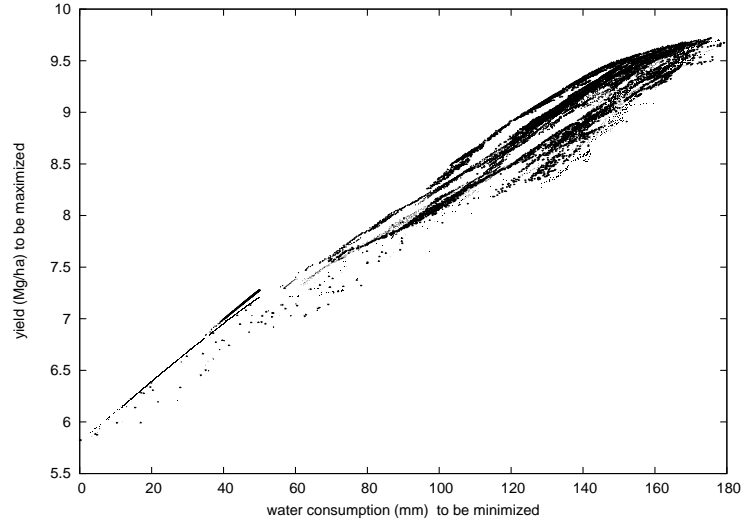


Figure 13: Averages of regions achieved within 2 000 000 simulations for 10 repetitions of the *P2m* algorithm : the multicriteria ranking rely on $N \times M$ evaluations per region.

401 5. Discussions

402 The *P2m* algorithm is based on the loop repetition of three main steps
 403 described in section 2 : (1) selection of one promising decision's region to be
 404 explored further, (2) division of this region and (3) evaluation of the produced
 405 eligible regions. The multicriteria group dominance proposed is involved in the
 406 evaluation process and strongly related to the selection of the accurate promising
 407 region. The efficiency of the dominance rules proposed is assessed through its
 408 capacity to produce the front of the non dominated region of performance mea-
 409 sures which directly leads to the efficient decisions. We discuss multiobjective
 410 optimization interests for agricultural decision purposes.

411 5.1. Multicriteria group dominance efficiency

412 The simulations reached a three sections non dominated front. A first sec-
 413 tion from the lowest water amount compromises up until the medium harvest
 414 compromises translating a high priority to minimizing the water amount and a
 415 low concern of harvest yield. The middle curved section translates a relatively
 416 sensitive compromise reaching a yield included in $[8.5, 9.5] t/ha$ while demand-
 417 ing in between $[100, 140] mm$ of irrigation water. The third section translates
 418 a high priority to the maximization of the harvest yield and a low interest for
 419 the water amount. The extreme sections relate to decisions linearly altered one
 420 from the other by the decision maker preference regarding either the amount of
 421 water or the expected yield. On the other hand the middle section is concerned
 422 with heterogeneous decisions and simulations give the decision maker valuable
 423 information to make a decision. Even under uncertainty the approach is able

424 to reach satisfyingly the front of the non dominated performances, which front
425 shows variability concerns through its width. This variability represents both
426 the decisions variability within the region and the uncertainty of the system
427 evaluation of the decisions simulated.

428 Though it is not related to the multicriteria approach, we first would like
429 to emphasize the realism of dealing with decision regions instead of decisions
430 vectors leading to single measures. This approach allows flexibility in the appli-
431 cation of the decision, making application easier in the field. It is for instance,
432 less perilous to apply a decision defined by a lower and upper boundaries, *e.g.*
433 34 to 36.5 *mm*, rather than a value, *e.g.* 35.52 *mm*. However, the decision
434 maker accepts that he might miss the very optimal decision which would have
435 prescribes hardly applicable recommendations and would be arguable in such
436 an uncertain context as ours.

437 The first additional information is concerned with variability : the variabil-
438 ity due to the decisions inside a region and the variability due to the system
439 uncertainty. Though the *P2m* algorithm includes 2 theoretical techniques to
440 separate these variability which are mentioned in Crespo et al. (2009a), the re-
441 sults shown in the previous section 4 rely on all performance measures included
442 in region and thus mix decision and uncertainty variability. They however state
443 clearly that the variability is translated through the front width and thus gives
444 indication to the decision maker about outcomes reliability. As an example and
445 according to figure 12 where climate uncertainty is averaged, the compromises
446 including high harvest yields are much more sensitive to the decisions than the
447 low water ones. It thus translates a higher outcome variation from one to a
448 neighbor decision than it would be for low water amount compromises. Con-
449 sidering a problem that do not involve a unique optimal solution as it is often
450 true considering conflicting objectives, overseeing different alternative costs and
451 benefits allow a better understanding of the eventual decision to take. Consid-
452 ering our study case, a decision maker can directly observe that all the efficient
453 decisions leading to the non dominated regions belonging to the straight lines
454 are equally satisfying the preferences defining these lines, so that changing for
455 one of these decisions is linearly related to the expected outcome. On the other
456 hand, decisions associated with the optimal regions belonging to the curved sec-
457 tion will be more or less efficient according to the decisions maker preferences.
458 Benefits and costs could then be estimated and guide the best decision to make.

459 5.2. Agricultural decision interests

460 Though the theoretical justification of multicriteria definition is not directly
461 related to the application, we propose here some interpretation of the multicri-
462 teria group evaluation rules given above. As for an example we consider two
463 objectives as depicted in the result figures. The irrigation water amount is to
464 be minimized while the harvest yield is to be maximized. These objectives are
465 conflicting. According to the irrigation strategy applied, different outcomes will
466 be reached, each defined by the combination : water amount used and harvest
467 yield achieved accordingly. In these conditions, an efficient decision, defined as

468 by usual Pareto dominance rules, is a decision such that its outcomes is non
469 dominated. The results achieved with one evaluation per region are showing
470 a front made of Pareto non dominated regions for which either the combina-
471 tion water-yield is multicriteria optimal. As soon as the group dominance rules
472 that we proposed are involved, a region is definitively dominating granting that
473 all the decisions subject to all the climates required less water while reached
474 higher yield than any decision subject to any climate simulated in the alterna-
475 tive region. When this strong relationship is not verified, then the acceptable
476 dominance is considered. A region will be acceptably dominating if (1) its best
477 outcomes are requiring less water while reaching higher yield than all outcomes
478 of the acceptably dominated one and (2) its worst outcomes are either requiring
479 less water or reaching higher yield than any outcomes of the acceptably domi-
480 nated one. Though some strategies subject to some climate might require less
481 water and reach more yield than any other from the alternative candidate, the
482 dominance is said undecidable if it also exists other strategy-climate combina-
483 tion requiring more water to reach poorer yield than the alternative candidate.

484 The front envelop achieved, directly gives an interpretation of the variability
485 of the region considered. Low variability gives comfort about applying the ac-
486 cording efficient decision by predicting a most likely realization of the expected
487 outcomes, while high variability translates the uncertain climate and its impact
488 on the related efficient decision. For example on the figure 11, an amount of
489 100 *mm* of irrigation water is expected to reach a yield included in 8.5 *t/ha* up
490 to 9.4 *t/ha*, while 40 *mm* of irrigation water predicts a yield of 7 *t/ha*. Though
491 oppositely extreme these examples give two sides of the variability representa-
492 tion.

493 In addition to this variability, the shape of the front gives the decision maker
494 new information about potential alternatives. The first linear part from no
495 irrigation up to 100 *mm* translates that the harvest reached will be highly
496 responsive to the water amount added : an increase of 0.6*t/ha* per 20 *mm*.
497 While considering the top linear section above 140*mm*, the yield response to
498 water is also proportional, yet the same amount of water will increase only
499 slightly the yield : 0.1*t/ha* per 20 *mm*. The curved section translates the
500 junction in between the bottom high responsive yield to water, and the top
501 low responsive yield to water sections. Without limitation (either maximum
502 water available, or minimum living income) a sensitive compromise would be in
503 that section. The final decision is up to the decision maker and could be help
504 with secondary objectives or decision making approaches that would help him
505 to clarify his preferences, see for example Saaty (1980), Steuer and Choo (1983),
506 Roy (1985) or Vincke (1988).

507 5.3. Potential extensions

508 The methodology could be extended to the computation of probability ac-
509 cording to the different uncertain scenarios (*e.g.* extremely dry or wet weathers
510 occur with a lower probability). In which case it would requires a significant
511 number of scenarios in order to represent the range of possible. The method-
512 ology is however already stressed with simulation number, and we choose not

513 to do so and deal with a global representation of the uncertainty. Though it is
514 not shown here, the methodology could indeed be used with a single uncertain
515 occurrence, which does not translate the range of possible for the considered
516 decision input, but does translate the global uncertainty when decision inputs
517 are regarded as groups.

518 Extension of the Pareto dominance rules to group dominance can include
519 the use of percentage of Pareto dominating outcomes. It would however impose
520 to the decision maker to express a new preference. Thus we did not explore
521 further these directions as we sought to keep the approach generic and with as
522 little as possible user preferences.

523 Regarding the base front, the major disadvantage of using multicriteria eval-
524 uation granting the same simulation run number, is the loss of robustness in
525 reaching the front of the non dominated region and the resolution loss for lower
526 water amount compromises. We could expect a better robustness and resolution
527 achievement granting a higher amount of simulation runs. It would however
528 require higher computational capacities and a fast enough evaluation process
529 (Crespo et al., 2009a). As for an example, the results shown in the previous
530 section were achieved within a limit of 2 000 000 of simulation runs, and pri-
531 ority was given to high harvest compromises when equally multicriteria ranked
532 regions were eligible for the promising region. The simulation run number limit
533 explains the global loss of robustness and resolution, while the use of yield as
534 secondary objective explains the definition of high harvest compromises first.

535 6. Conclusion

536 Our contribution consisted in presenting an optimization procedure that si-
537 multaneously tackles multiple conflicting objectives and uncertainty while not
538 aggregating either one or the other into indicators. We propose extensions of
539 the usual multicriteria dominance rules in order to evaluate groups of outcomes
540 rather than the outcomes themselves. These rules are based on the widely used
541 Pareto dominance and used as the evaluation step of a simulation-based opti-
542 mization procedure. The resulting *P2m* algorithm is used to optimize irrigation
543 strategies that are evaluated by crop model simulation.

544 The efficient decision ensemble reached includes the traditional strategies
545 optimizing the yield outcome plus additional strategies demanding less water
546 yet reaching lower yields. These additional strategies are new strategies that
547 are multicriteria optimal and could fit better specific conditions such as limited
548 resources or new global concerns (*e.g.* share of resources, biodiversity). The
549 multicriteria simulation requirements have to be taken into account according
550 to the system evaluation speed. However in our case, the explored strategies
551 have been simulated with a crop model that requires significantly higher compu-
552 tational resources than the multicriteria comparison rules proposed. Thus, from
553 our perspectives, the satisfying achievement of the Pareto front while providing
554 new valuable information that help the decision maker towards an appropriate
555 decision, justify the extend.

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