

# Dual Formulations for Accurate Thin Shell Models in a Finite Element Subproblem Method

Vuong Quoc Dang, Patrick Dular, Ruth V. Sabariego, Laurent Krähenbühl, Christophe Geuzaine

# ▶ To cite this version:

Vuong Quoc Dang, Patrick Dular, Ruth V. Sabariego, Laurent Krähenbühl, Christophe Geuzaine. Dual Formulations for Accurate Thin Shell Models in a Finite Element Subproblem Method. Compumag 2013, Jun 2013, Budapest, Hungary. pp.890. hal-00807079

HAL Id: hal-00807079

https://hal.science/hal-00807079

Submitted on 2 Apr 2013

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Dual Formulations for Accurate Thin Shell Models in a Finite Element Subproblem Method

Vuong Q. Dang<sup>1</sup>, Patrick Dular<sup>1,2</sup>, Ruth V. Sabariego<sup>1</sup>, Laurent Krähenbühl<sup>3</sup> and Christophe Geuzaine<sup>1</sup> University of Liège, Dept. of Electrical Engineering and Computer Science, ACE, B-4000 Liège, Belgium <sup>2</sup>Fonds de la Recherche Scientifique - F.R.S.-.FNRS, B-1000 Brussels, Belgium <sup>3</sup>Université de Lyon, Ampère (CNRS UMR5005), École Centrale de Lyon, F-69134 Écully Cedex, France

Abstract—Subproblem dual finite element magnetostatic and magnetodynamic formulations are developed and compared for correcting the inaccuracies near edges and corners of thin shell models, that replace thin volume regions by surfaces. The surface-to-volume correction problem is defined as one of the multiple subproblems applied to a complete problem, considering successive additions of inductors and magnetic or conducting regions, some of these being thin regions. Each SP requires a proper adapted mesh of its regions, which facilitates meshing and increases computational efficiency.

#### I. Introduction

As proposed in [1], [2], thin shell (TS) finite element (FE) models are used to avoid meshing thin regions (TRs) and lighten the mesh of their surrounds. For that, the volume TRs are replaced by surfaces with interface conditions (ICs) linked to 1-D analytical distributions along their thickness that however neglect end and curvature effects. This leads to inaccuracies of field distributions and associated losses near edges and corners, that increase with the thickness. To overcome these disadvantages, a subproblem method (SPM) based on magnetic flux density formulations, proposing a surface-to-volume local correction, has been proposed in [3].

The SPM for TS correction is herein extended to a dual approach for both magnetic field and magnetic flux density formulations, with generalized mesh projections of solutions between the SPs. Also, the SPM naturally allows parameterized analyses of the TR characteristics: permeability, conductivity and thickness. In the proposed SP strategy, a reduced problem (SP q) with only inductors is first solved on a simplified mesh without thin and volume regions. Its solution gives surface sources (SSs) as ICs for added TS regions (SP p), and volume sources (VSs) for possible added volume regions (SP k). The TS solution is then corrected by a volume correction via SSs and VSs that suppress the TS representation and add the volume model.

# II. THIN SHELL CORRECTION IN THE SUBPROBEM METHOD

## A. Canonical magnetodynamic or static problem

A canonical magnetodynamic or static problem i, to be solved at step i of the SPM, is defined in a domain  $\Omega_i$ , with boundary  $\partial \Omega_i = \Gamma_i = \Gamma_{h,i} \cup \Gamma_{b,i}$ . Material relations and boundary conditions (BCs) are [3]

$$\boldsymbol{h}_i = \mu_i^{-1} \boldsymbol{b}_i + \boldsymbol{h}_{s,i}, \boldsymbol{j}_i = \sigma_i \boldsymbol{e}_i + \boldsymbol{j}_{s,i}$$
 (1a-b)

$$\mathbf{n} \times \mathbf{h}_i|_{\Gamma_{h,i}} = \mathbf{j}_{f,i}, \mathbf{n} \times \mathbf{e}_i|_{\Gamma_{e,i}} = \mathbf{k}_{f,i}$$
 (2a-b)

where  $h_i$  is the magnetic field,  $b_i$  is the magnetic flux density,  $e_i$  is the electric field,  $j_i$  is the electric current density,  $\mu_i$  is the magnetic permeability,  $\sigma_i$  is the electric conductivity and n is the unit normal exterior to  $\Omega_i$ . The notation  $[\cdot]_{\gamma_i} = \cdot|_{\gamma_i^+} - \cdot|_{\gamma_i^-}$  refers to the discontinuity of a quantity through an interface  $\gamma_i$  (with sides  $\gamma_i^+$  and  $\gamma_i^-$ ) in  $\Omega_i$ , defining ICs. The fields  $h_{s,i}$  and  $j_{s,i}$  in (1 a) and (1 b) are VSs that can be used for expressing changes of a material property in a volume region, from  $\mu_p$  and  $\sigma_p$  for SP p to  $\mu_k$  and  $\sigma_k$  for SP k [3], i.e.

$$\boldsymbol{h}_{s,k} = (\mu_k^{-1} - \mu_p^{-1})\boldsymbol{b}_p, \ \boldsymbol{j}_{s,k} = (\sigma_k^{-1} - \sigma_p^{-1})\boldsymbol{e}_p$$
 (3)

with  $\mu_p = \mu_0$ ,  $\mu_k = \mu_{r,volume}$ ,  $\sigma_p = 0$  and  $\sigma_k = \sigma_{volume}$ . The fields  $j_{f,i}$  and  $k_{f,i}$  in (2 a) and (2 b) are SSs. They define possible SSs that account for particular phenomena occuring in the idealized TR between  $\gamma_i^+$  and  $\gamma_i^-$  [3]. This is the case when some field traces in SP p are forced to be discontinuous (e.g. in TS model), whereas their continuity must be recovered via a SP k, which is done via a SS in SP k fixing the opposite of the trace discontinuity solution of SP p.

### B. SSs via ICs for subproblems

The constraints for the SPs are defined via SSs and VSs. The TS problem [2] is defined in SP p (Fig. 1, right) following the already computed inductor source field from SP q (Fig. 1, left). Thus, SSs for SP p are to be defined via BCs and ICs (Fig. 1, right) of impedance type BCs (IBC) given by the TS model [2] associated with contributions from SP q. The

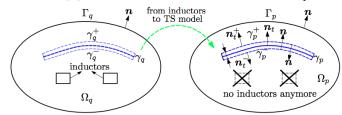


Figure 1: Interface condition between SP q and SP p.

**b**-formulation uses a magnetic vector potential  $a_i$  (such that curl  $a_i = b_i$ ), split as  $a = a_{i,c} + a_{i,d}$  [2]. The **h**-formulation uses a similar splitting for the magnetic field,  $h_i = h_{i,c} + h_{i,d}$ . The fields  $a_{i,c}$ ,  $h_{i,c}$  and  $a_{i,d}$ ,  $h_{i,d}$  are continuous and discontinuous respectively through the TS. There is no TR in SP q (Fig. 1, left), but to get a relative constraint between SP q and SP

p via the corresponding ICs with  $\gamma_t = \gamma_t^{\pm} = \gamma_q^{\pm} = \gamma_p^{\pm}$  and  $\mathbf{n}_t = -\mathbf{n}$  for the TS, one has to imagine that a TR appears in SP q, and therefore there is no such discontinuity in SP q, i.e.  $[\mathbf{n} \times \mathbf{h}_q]_{\gamma_q} = \mathbf{n} \times \mathbf{h}_q|_{\gamma_q^+} - \mathbf{n} \times \mathbf{h}_q|_{\gamma_q^-} = 0$  for  $\mathbf{b}$ -formulation and  $\mathbf{n} \times \mathbf{e}_q|_{\gamma_q^+} - \mathbf{n} \times \mathbf{e}_q|_{\gamma_q^-} = 0$  for  $\mathbf{h}$ -formulation. In addition, one has TS-ICs in both formulations [2]

$$[\mathbf{n} \times (\mathbf{h}_q + \mathbf{h}_q)]_{\gamma_p} = [\mathbf{n} \times \mathbf{h}_q]_{\gamma_p} + [\mathbf{n} \times \mathbf{h}_p]_{\gamma_p} = \mu_p \beta_p \, \partial_t (2\mathbf{a}_c + \mathbf{a}_d)$$
(4)  
$$[\mathbf{n} \times (\mathbf{e}_q + \mathbf{e}_q)]_{\gamma_p} = [\mathbf{n} \times \mathbf{e}_q]_{\gamma_p} + [\mathbf{n} \times \mathbf{e}_p]_{\gamma_p} = \mu_p \beta_p \, \partial_t (2\mathbf{h}_c + \mathbf{h}_d)$$
(5)

with  $\beta_p$  given in [2]. The resulting FE formulations are then written for SPs q, p and k, which will be developed in the full paper.

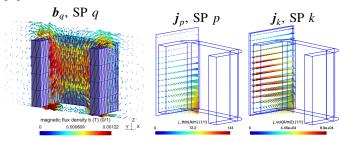
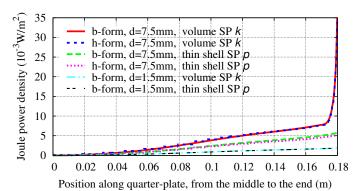


Figure 2: TEAM problem 21 (1/4th of the geometry): magnetic flux density  $b_q$  (in a cut plane) generated by a stranded inductor (*left*), eddy current density  $j_p$  on TS model (*middle*) and its volume correction  $j_k$  (*right*) (thickness d = 10 mm).

#### III. APPLICATION EXAMPLE

A 3-D test problem is based on TEAM problem 21 (model B, coil and plate, Fig. 2). An SP scheme considering three procedures is developed. A first FE SP q with the stranded inductors alone is solved on a simplified mesh without any TR (Fig. 2, left). Then a SP p is solved with the added TR via a TS FE model (Fig. 2, middle). At last, a SP k replaces the TS FEs with the actual volume FEs (Fig. 2, right). The TS error on  $j_p$  locally reaches 83% (Fig. 2, middle), with f = 50Hz,  $\mu = 200$  and  $\sigma = 6.484$  MS/m (skin depth  $\delta = 6.15$  mm). The inccuracies on the Joule power loss densities of TS SP p are pointed out by the importance of the correction SP k (Fig. 3, top). Significant error on TS SP p along the horizontal half inner width (y-direction) reach 85% near the plate ends (Fig. 3, top), with  $\delta = 2.1 \,\mathrm{mm}$  and thickness  $d = 7.5 \,\mathrm{mm}$ . For d = 1.5 mm, it is reduced to below 10%. In particular, accurate local corrections with volume correction SP k are checked to be close to the solution of the complete problem, with errors lower than 0.1% (Fig. 3, bottom).

Table I shows the Joule losses in the plate with an approximate BC for SP k. The exterior boundary of SP k is first chosen at a distance  $D_{bound} = 200d$  from the TR, with thickness d = 10 mm. The inaccuracies on Joule losses for TS SP p reach 76.4%, or 1.2% for volume correction SP k, with f = 50 Hz,  $\mu = 100$  and  $\sigma = 6.484$  MS/m in both cases. The proposed SP strategy allows to locally focus on the mesh of volume correction SP k and its neighborhood. It is shown that even if  $D_{bound}$  is reduced to 2d, the error on SP k is 1.53%, which is still very accurate. For d = 2 mm, the errors on Joule losses for SP p are reduced to 6.09%, or 0.05% for SP k.



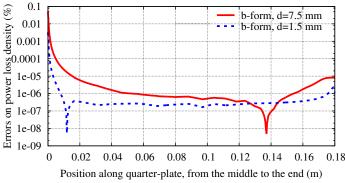


Figure 3: Power loss density with TS and volume solutions (top); errors on the power loss density between volume correction SP k and complete problem (bottom), along horizontal half inner width (y-direction), with effects of d  $(f = 50 \text{ Hz}, \mu_r = 200, \sigma = 6.484 \text{ MS/m})$ .

The SPM allows to correct the inaccuracies proper to the TS model. Accurate eddy current and power loss densities are obtained, especially along the edges and corners of the TRs, also for significant thicknesses. The refined mesh for volume correction can be reduced to a close neighborhood of the TR.

Table I: Joule losses in the plate with approximate BCs (f = 50 Hz,  $\mu = 100$ ,  $\sigma = 6.484 \text{ MS/m}$ ), with **b**-formulation.

	d = 10  mm (thickness of the plate)			Errors %	
$D_{bound}$	Thin shell	Volume	Reference	Between P <sub>thin</sub>	Bet P <sub>vol</sub>
	$P_{thin}$ (W)	$P_{vol}$ (W)	$P_{ref}$ (W)	and $P_{ref}$	and $P_{ref}$
200d	0.0114	0.0477	0.0483	76.4	1.2
20 <i>d</i>	0.0114	0.0476	0.0483	76.4	1.35
2 <i>d</i>	0.0114	0.0475	0.0483	76.4	1.53
	d = 1  mm (thickness of the plate)				
200d	0.0108	0.0115	0.0115	6.09	0.00
2 <i>d</i>	0.0108	0.011506	0.0115	6.09	0.05

#### REFERENCES

- L. Krähenbühl and D. Muller, "Thin layers in electrical engineering. Examples of shell models in analyzing eddy- currents by boundary and finite element methods," IEEE Trans. Magn., vol. 29, no. 2, pp. 1450– 1455, 1993.
- [2] C. Geuzaine, P. Dular, and W. Legros, "Dual formulations for the modeling of thin electromagnetic shells using edge elements," IEEE Trans. Magn., vol. 36, no. 4, pp. 799–802, 2000.
- [3] P. Dular, Vuong Q. Dang, R. V. Sabariego, L. Krähenbühl and C. Geuzaine, "Correction of thin shell finite element magnetic models via a subproblem method," IEEE Trans. Magn., vol. 47, no. 5, pp. 158 –1161, 2011.