HAL
open science

A Parallel Thinning Algorithm for Grayscale Images

Michel Couprie, Francisco Nivando Bezerra, Gilles Bertrand

To cite this version:

Michel Couprie, Francisco Nivando Bezerra, Gilles Bertrand. A Parallel Thinning Algorithm for Grayscale Images. Discrete Geometry for Computer Imagery, Mar 2013, Spain. pp.71-82, 10.1007/978-3-642-37067-0_7 . hal-00805682

HAL Id: hal-00805682

https://hal.science/hal-00805682

Submitted on 28 Mar 2013

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

A parallel thinning algorithm for grayscale images

Michel Couprie[1], Nivando Bezerra[2], and Gilles Bertrand[1] *

[1] Université Paris-Est, Laboratoire d'Informatique Gaspard-Monge, Équipe A3SI, ESIEE Paris, France
[2] Instituto Federal do Ceará, IFCE, Maracanaú, Brazil

Abstract. Grayscale skeletonization offers an interesting alternative to traditional skeletonization following a binarization. It is well known that parallel algorithms for skeletonization outperform sequential ones in terms of quality of results, yet no general and well defined framework has been proposed until now for parallel grayscale thinning. We introduce in this paper a parallel thinning algorithm for grayscale images, and prove its topological soundness based on properties of the critical kernels framework. The algorithm and its proof, given here in the 2D case, are also valid in 3D. Some applications are sketched in conclusion.

1 Introduction

Topology-preserving transformations, in particular topology-preserving thinning and skeletonization, are essential tools in many applications of image processing. In the huge litterature dealing with this topic, almost all works are devoted to the case of binary images. Even so, there are cases when thinning directly a grayscale image, instead of a binarization of this one, can be beneficial [23, 1, 9, 13]. First, binarization unsually involves important information loss, and it may be desirable to defer this loss to the latest steps of the processing chain. Second, working with full grayscale information permits to detect and to use specific features, such as crests and valleys, peaks and wells, or saddle points. These features can be precisely defined within the framework exposed in this paper.

Some attention has been given to the development of thinning algorithms acting directly on grayscale images. Dyer and Rosenfeld [11] proposed an algorithm based on a notion of weighted connectedness. The thinning is done directly over the graylevel values of the points but, as pointed out in the same paper [11], the connectivity of objects is not always preserved. Thinning based on a fuzzy framework for image processing has been proposed in [22, 20], but also in this case object connectedness is not ensured in the final skeleton. The more recent works in [25, 2] use an implicit image binarization into a background and a grayscale foreground.

* This work has been partially supported by the "ANR-2010-BLAN-0205 KIDICO" project.

Other approaches for grayscale thinning (that is, thinning of grayscale images without prior segmentation, resulting in either a grayscale or a binary skeleton) are pseudo distance maps [19, 12], pixel superiority index [13], and partial differential equations (see *e.g.* [16]). In all these works, no property relative to topology preservation is claimed.

In this paper, we adopt a topological approach, beginning with a definition of the topological equivalence between two maps. This definition is based on the decomposition of a map into its different sections [8, 7]: let F be a map from $\mathbb{Z}^2$ into $\mathbb{Z}$, the section of F at level k is the set F_k of points x in $\mathbb{Z}^2$ such that $F(x) \geq k$. Following this approach called *cross-section topology*, a transformation is homotopic, i.e. preserves the topology of F, if it preserves the topology in the binary sense of every section F_k. An elementary homotopic transformation consists of lowering the value of a so-called destructible point (a notion introduced in [6], which generalizes the notion of simple point [15] to maps). Based on this elementary operation, sequential thinning algorithms for grayscale images have been proposed in [7, 10], with applications to image segmentation, filtering and restoration.

By nature, all these sequential thinning algorithms have the drawback of producing a result that depends on arbitrary choices that must be done, with regard to the order in which the destructible points are treated.

On the other hand, although parallel thinning of binary images is a quite well developped topic in the image processing community, with thousands of references, very few attempts have been made until now to propose parallel grayscale thinning algorithms. In [18], an algorithm was proposed but no well-stated property and no proof of topological correctness was given. The first (to our best knowledge) approach for parallel grayscale thinning with proved properties was introduced by [17], in the framework of partial orders. Here, the result is a map which is defined on a space which is not the classical pixel grid, but can be seen as a grid with higher resolution. Finally, [21] introduces order-independant thinning for both binary and grayscale images. However their definition is combinatorial in nature, and does not lead to efficient algorithms.

The approach taken in this paper is based on the framework of critical kernels [3], which is to our knowledge the most general framework to analyze and design parallel homotopic thinning algorithms in discrete spaces, with the guarantee of topology preservation. Our main contribution are algorithm 1, which simultaneously considers all pixels of a grayscale image and lowers some of them in one thinning step, and the proof of its topological soundness (theorem 14). We conclude the paper by an illustration of the algorithm and some applications.

2 Parallel topological transformations of binary images

As we base our notion of topological equivalence for functions on the one for sets (or binary images), we begin by providing some definitions and results for this latter case. The framework of critical kernels, introduced by one of the authors in [3], will serve us to prove the topological soundness of the proposed method. This

framework is established within the context of simplicial or cubical complexes, however the resulting algorithms can be directly implemented in $\mathbb{Z}^2$ thanks to very simple masks. Only a small set of definitions and properties based on cubical complexes are needed to understand the rest of the paper.

Intuitively, a cubical complex may be thought of as a set of elements having various dimensions (*e.g.* squares, edges, vertices) glued together according to certain rules.

Let $\mathbb{Z}$ be the set of integers. We consider the families of sets $\mathbb{F}_0^1$, $\mathbb{F}_1^1$, such that $\mathbb{F}_0^1 = \{\{a\} \mid a \in \mathbb{Z}\}$, $\mathbb{F}_1^1 = \{\{a, a+1\} \mid a \in \mathbb{Z}\}$. A subset f of $\mathbb{Z}^2$, which is the Cartesian product of exactly d elements of $\mathbb{F}_1^1$ and $(2-d)$ elements of $\mathbb{F}_0^1$ is called a *face* or a *d-face* in $\mathbb{Z}^2$, d is the *dimension of* f, we write $\dim(f) = d$.

A d-face is called a *point* if $d = 0$, a *(unit) edge* if $d = 1$, a *(unit) square* or a *pixel* if $d = 2$. We denote by $\mathbb{P}^2$ the set composed of all 2-faces (pixels) in $\mathbb{Z}^2$. We denote by $\mathcal{P}$ the collection of all finite sets which are composed solely of pixels.

Let x, y be two pixels, let $d \in \{0, 1\}$. We say that x and y are *d-adjacent* if there is k, $2 \geqslant k \geqslant d$, such that $x \cap y$ is a k-face. We write $\mathcal{N}_d(x)$ to denote the set of all pixels that are d-adjacent to x. Note that for any pixel x and any d, we have $x \in \mathcal{N}_d(x)$. We set $\mathcal{N}_d^*(x) = \mathcal{N}_d(x) \setminus x$. Remark that we have 4 (resp. 8) pixels in $\mathcal{N}_1^*(x)$ (resp. $\mathcal{N}_0^*(x)$). Let Y be a set of pixels, we say that x and Y are *d-adjacent* if there exists a pixel y in Y such that x and y are d-adjacent.

Let $X \in \mathcal{P}$ and let $Y \subseteq X$, $Y \neq \emptyset$. We say that Y is *d-connected in* X if, for any $x, y \in Y$, there exists a sequence $\langle x_0, \ldots, x_\ell \rangle$ of pixels of X, such that $x_0 = x$, $x_\ell = y$, and for any $i \in \{1, \ldots, \ell\}$, x_i is d-adjacent to x_{i-1}. We say that Y is *a d-connected component of* X if Y is d-connected in X and if it is maximal for the inclusion, that is, we have $Y = Z$ whenever $Y \subseteq Z \subseteq X$ and Z is d-connected in X.

Let $X \in \mathcal{P}$ and let $x \in X$. We denote by $\overline{X}$ the complementary set of X, that is, $\overline{X} = \mathbb{P}^2 \setminus X$. We denote by $T(x, X)$ the number of 0-connected components of $\mathcal{N}_0^*(x) \cap X$. We denote by $\overline{T}(x, X)$ the number of 1-connected components of $\mathcal{N}_0^*(x) \cap \overline{X}$ that are 1-adjacent to x.

Intuitively, a pixel x in a set X of pixels is simple if its removal from X "does not change the topology of X". We recall here a definition of a simple pixel, which is based on the following recursive definition.

Definition 1 ([5])**.** Let $X \in \mathcal{P}$. We say that X is a *reducible set* if either:
i) X is composed of a single pixel, or

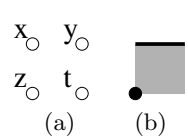


Fig. 1. (a): Four elements x, y, z, t of $\mathbb{Z}^2$. (b): A graphical representation of the set of faces $\{\{x, y, z, t\}, \{x, y\}, \{z\}\}$: a pixel, an edge, and a point.

ii) there exists $x \in X$ such that $\mathcal{N}_0^*(x) \cap X$ is a reducible set and $X \setminus x$ is a reducible set.

Definition 2 ([5])**.** Let $X \in \mathcal{P}$. A pixel $x \in X$ is *simple for* X if $\mathcal{N}_0^*(x) \cap X$ is a reducible set. If x is simple for X, we say that $X \setminus x$ is an *elementary thinning* of X.
Let $X, Y \in \mathcal{P}$. We say that Y *is a thinning of* X if there exists a sequence $\langle X_0, \ldots, X_\ell \rangle$ such that $X_0 = X$, $X_\ell = Y$, and for any $i \in \{1, \ldots, \ell\}$, X_i is an elementary thinning of X_{i-1}.

In [5] it has been shown that the above definition of a simple pixel is equivalent to a definition based on the notion of collapse [24], this operation being a discrete analogue of a continuous deformation (a homotopy). Furthermore, the following proposition, which is a straightforward consequence of Prop. 8 [5], shows that definition 2 leads to a characterization of simple pixels which is equivalent to previously proposed ones (see *e.g.* [14]).

Proposition 3. Let $X \in \mathcal{P}$ and let $x \in X$. The pixel x is simple for X if and only if $T(x, X) = \overline{T}(x, X) = 1$.

Now, we are ready to give a short introduction to the framework of critical kernels [3], which is to our knowledge the most powerful framework to study and design parallel topology-preserving algorithms in discrete spaces. We limit ourselves to a minimal yet sufficient set of notions, interested readers may refer to [3–5] for a more complete presentation.

Let $C \in \mathcal{P}$, let $d \in \{0, 1, 2\}$. We say that C is a *d-clique*, or simply a *clique*, if $\cap\{x \in C\}$, the intersection of all pixels in C, is a d-face.

Let $X \in \mathcal{P}$ and let $C \subseteq X$ be a clique. We say that C is *essential for* X if we have $D = C$ whenever D is a clique such that:
i) $C \subseteq D \subseteq X$, and
ii) $\cap\{x \in D\} = \cap\{x \in C\}$.

Remark that, if C is composed of a single pixel (*i.e.* C is a 2-clique), then C is necessarily essential.

Definition 4 ([5])**.** Let $S \in \mathcal{P}$. The $\mathcal{K}$*-neighborhood* of S, written $\mathcal{K}(S)$, is the set made of all pixels that are 0-adjacent to each pixel in S. We set $\mathcal{K}^*(S) = \mathcal{K}(S) \setminus S$.

Notice that we have $\mathcal{K}(S) = \mathcal{N}_0(x)$ if and only if S is made of a single pixel x.

Definition 5 ([5])**.** Let $X \in \mathcal{P}$ and let C be a clique that is essential for X. We say that the clique C is *regular for* X if $\mathcal{K}^*(C) \cap X$ is a reducible set. We say that C is *critical for* X whenever C is not regular for X.

Remark that, if C is a singleton $\{x\}$, the clique C is regular whenever x is simple.

The following result is a consequence of a general theorem which holds for complexes of arbitrary dimension (see [3], Th. 4.2).

Theorem 6 ([5])**.** Let $X \in \mathcal{P}$ and let $Y \subseteq X$. If any clique that is critical for X contains at least one pixel of Y, then Y is a thinning of X.

Our goal is to define a subset of an object X that contains at least one pixel of each critical clique. We also want this subset to be as small as possible, in order to obtain an efficient thinning procedure. This motivates the following definition, where the set K plays the role of a constraint set (that is, a set of pixels that must be preserved from deletion, for other reasons than topology preservation).

Definition 7 ([5])**.** Let $X \in \mathcal{P}$, let $K \in \mathcal{P}$, and let $C \subseteq X \setminus K$ be a d-clique that is critical for X, $d \in \{0, 1, 2\}$. We say that the clique C is *d-crucial (or crucial) for* $\langle X, K \rangle$ if
i) $d = 2$, or
ii) $d = 1$ and C does not contain any non-simple pixel, or
iii) $d = 0$ and C does not contain any non-simple pixel, nor any pixel belonging to a 1-clique which is crucial for $\langle X, K \rangle$.

The following corollary directly follows from theorem 6.

Corollary 8 ([5])**.** Let $X \in \mathcal{P}$ and let $Y \subseteq X$. If any clique that is crucial for X contains at least one pixel of Y, then Y is a thinning of X.

The following proposition allows us to characterize crucial cliques by the use of only two masks, which apply directly to any object represented by a set of pixels (there is no need to consider the underlying cubical complex, nor to check the condition of definition 5.)

a	b
C	D
e	f

M_1

A	B
C	D

M_0

Fig. 2. Masks for 1-crucial (M_1) and 0-crucial (M_0) pixels.

The masks M_1, M_0 are given in figure 2. For the mask M_1, we also consider the mask obtained from it by applying a $\pi/2$ rotation: we get 3 masks (2 for M_1, and 1 for M_0).

Definition 9. Let $X \in \mathcal{P}$, and let M be a set of pixels of X.
1) The set M *matches the mask* M_1 if:
i) $M = \{C, D\}$; and
ii) the pixels C, D are simple for X; and
iii) the sets $\{a, b\} \cap X$ and $\{e, f\} \cap X$ are either both empty or both non-empty.
2) The set M *matches the mask* M_0 if:
i) $M = \{A, B, C, D\} \cap X$; and
ii) the pixels in M are simple and not matched by M_1; and
iii) at least one of the sets $\{A, D\}, \{B, C\}$ is a subset of M.

Proposition 10. Let $X \in \mathcal{P}$, $K \subseteq X$, and let M be a set of pixels in $X \setminus K$ that are simple for X.
Then, M is a crucial clique for $\langle X, K \rangle$ if and only if M matches the mask M_0 or the mask M_1.

This proposition was proved with the help of a computer program, by examination of all possible configurations (see also [5] for similar characterizations in 3D).

3 Parallel thinning for grayscale images

In this section, topological notions such as those of simple pixel, thinning, crucial clique, are extended to the case of grayscale images. Then, we introduce our parallel thinning algorithm and prove its topological properties.

A 2D grayscale image can be seen as a function F from $\mathbb{P}^2$ into $\mathbb{Z}$. For each pixel x of $\mathbb{P}^2$, $F(x)$ is the gray level, or the luminosity of x. The *support of* F is the set of pixels x such that $F(x) > 0$, denoted by $\mathrm{Supp}(F)$. We denote by $\mathcal{F}$ the set of all functions from $\mathbb{P}^2$ into $\mathbb{Z}$ that have a finite support.

Let $F \in \mathcal{F}$ and $k \in \mathbb{Z}$, the *cross-section (or threshold) of* F *at level* k is the set F_k composed of all pixels $x \in \mathbb{P}^2$ such that $F(x) \geqslant k$. Observe that a cross-section is a set of pixels, that is, a binary image.

Intuitively, we say that a transformation of F preserves topology if topology of all cross-sections of F is preserved. Hence, the "cross-section topology" of a function (*i.e.*, of a grayscale image) directly derives from the topology of binary images [7]. Based on this idea, the following notion generalize the notion of simple pixel to the case of functions.

Definition 11 ([7])**.** Let $F \in \mathcal{F}$, $x \in \mathbb{P}^2$, and $k = F(x)$. The pixel x is *destructible (for* F*)* if x is simple for F_k. If x is destructible for F, we say that the map F' defined by:

$$F'(y) = \begin{cases} F(x) - 1 & \text{if } y = x, \\ F(y) & \text{otherwise} \end{cases}$$

is an *elementary thinning* of F.
Let $F, G \in \mathcal{F}$. We say that G *is a thinning of* F if there exists a sequence $\langle F_0, \ldots, F_\ell \rangle$ such that $F_0 = F$, $F_\ell = G$, and for any $i \in \{1, \ldots, \ell\}$, F_i is an elementary thinning of F_{i-1}.

Intuitively, the gray level of a destructible pixel may be lowered of one unit, while preserving the topology of F.

We define also:

$$\mathcal{N}^{--}(x) = \{y \in \mathcal{N}_0^*(x); F(y) < F(x)\}$$

$$F^-(x) = \begin{cases} \max\{F(y); y \in \mathcal{N}^{--}(x)\}, & \text{if } \mathcal{N}^{--}(x) \neq \emptyset \\ F(x) & \text{otherwise.} \end{cases}$$

It is easy to see that lowering a destructible pixel x down to the value $F^-(x)$ is a topology-preserving transformation. Informally, it is due to the fact that in all the cross-sections from the value $F(x)$ down to the value $F^-(x) + 1$, the neighborhood of x is the same. The following proposition shows that a more

general property holds for cliques that contain x. Let C be a clique and $k \in \mathbb{Z}$, we denote by $\mathcal{K}_k(C)$ the $\mathcal{K}$-neighborhood of C in F_k. In addition, we set $\mathcal{K}_k^*(C) = \mathcal{K}_k(C) \setminus C$.

Proposition 12. Let $F \in \mathcal{F}$, let $x \in \mathrm{Supp}(F)$, let $\ell = F(x)$. Let C be a clique of F_ℓ such that $x \in C$ (possibly $C = \{x\}$). Let $k = F^-(x)$.

For any $j \in \{k+1, \ldots, \ell\}$, we have $\mathcal{K}_j^*(C) = \mathcal{K}_\ell^*(C)$.

The proof is quite easy and left to the reader as an exercice.

Thinning a grayscale image is a useful operation, with applications to image segmentation, filtering, and restoration [7, 10]. Intuitively, this operation extends the minima of an image while reducing its crests to thin lines. In [10], several sequential algorithms to perform this operation have been proposed and studied. Basically, these algorithms consider one destructible point at a time and lower it. Their common drawback lies in the fact that arbitrary choices have to be made concerning the order in which destructible points are considered. In consequence, notions such as the result of a thinning step can hardly be defined with this approach.

Here, we introduce a new thinning algorithm for grayscale images, that lowers points in parallel. Then, we prove that the result of this thinning, which is uniquely defined, can also be obtained through a process that lowers one destructible point at a time: this guarantees the topological soundness of our algorithm.

The following algorithm consitutes one step of parallel thinning. This operation may be repeated a certain number of times, depending on the application, or until stability if one wants to thin an image as much as possible. Furthermore, we introduce as a parameter of the algorithm, a secondary grayscale image K that plays the role of a constraint: whatever a point x, it cannot be lowered below the level $K(x)$.

Algorithm 1: ParGrayThinStep(F, K)

Data : $F \in \mathcal{F}$, $K \in \mathcal{F}$ such that $K \leqslant F$
$D = \{x \in \mathrm{Supp}(F) \mid x$ is destructible for F and $F(x) \neq K(x)\}$;
$R = \{x \in D \mid x$ is crucial for $\langle F_k, K_k \rangle$, with $k = F(x)\}$;
foreach $x \in \mathit{Supp}(F)$ **do**
  if $x \in D \setminus R$ **then** $G(x) = \max\{F^-(x), K(x)\}$; **else** $G(x) = F(x)$;
return G

The next proposition is an essential step for proving the topological soundness of this algorithm (theorem 14).

Proposition 13. Let $F \in \mathcal{F}$, let $K \in \mathcal{F}$ such that $K \leqslant F$.

Let $G =$ParGrayThinStep(F, K).

For any $k \in \mathbb{Z}$, $k > 0$, if C is a critical clique of F_k, then G_k contains at least one pixel of C.

Proof: Let C be a critical clique of F_k, note that C may be composed of only one, non-simple pixel. If there exist two pixels x and y of C that are such that

$F(x) > F(y)$, then $G(x) \geqslant F^-(x) \geqslant F(y)$. As $F(y) \geqslant k$, we have $x \in G(k)$, thus G_k contains at least one pixel of C.

Now suppose that, for any $x, y \in C$, $F(x) = F(y) = \ell$, thus $\ell \geqslant k$ (for C is a clique of F_k), and $C \subseteq F_\ell$.

Suppose that $C \cap K_\ell \neq \emptyset$. Since $G(x) = \max\{F^-(x), K(x)\}$ (line 8), we have $G(x) \geqslant \ell$, for any $x \in C \cap K_\ell$. We have $C \cap K_\ell \subseteq G_\ell$ and $C \cap K_\ell \subseteq G_k$ (since $G_\ell \subseteq G_k$, for $\ell \geqslant k$): G_k contains at least one pixel of C.

In the sequel, we suppose that $C \cap K_\ell = \emptyset$.

The set $\mathcal{K}_k^*(C)$ is not reducible, for C is a critical clique of F_k. We also remark that C is necessarily an essential clique for F_ℓ.

1) Suppose that $\mathcal{K}_\ell^*(C)$ is not reducible. This implies that C is a critical clique of F_ℓ. By definition of a crucial pixel, there exists at least one pixel x of C that is crucial for F_ℓ (and, by consequence, for $\langle F_\ell, K_\ell \rangle$). In this case, we have $x \in R$ (line 4), hence $G(x) = F(x)$ (line 8), and we have $x \in G_\ell$ and $x \in G_k$.

2) Suppose that $\mathcal{K}_\ell^*(C)$ is reducible, thus $C \subseteq D \backslash R$ (line 4). This implies that $\mathcal{K}_\ell^*(C) \neq \mathcal{K}_k^*(C)$, and that there exists $x \in \mathcal{K}_k^*(C)$, $x \notin \mathcal{K}_\ell^*(C)$. Thus, we have $F(x) \geqslant k$ and $F(x) < \ell$. If $y \in C$, then $F^-(y) \geqslant F(x) \geqslant k$. Hence $G(y) \geqslant k$, and $C \subseteq G_k$. $\square$

Based on the above property, we can now prove the following theorem, which is the main result of this article. Intuitively, it assesses that algorithm ParGray-ThinStep is topology-preserving, in the sense of cross-section topology.

Theorem 14. Let $F \in \mathcal{F}$, let $K \in \mathcal{F}$ such that $K \leqslant F$.

Let $G =$ParGrayThinStep(F, K).

Then, G is a thinning of F.

Proof: Let $M = \max\{F(x) \mid x \in \mathrm{Supp}(F)\}$, $m = \min\{F(x) \mid x \in \mathrm{Supp}(F)\}$. For any $k \in \{m, \ldots, M\}$, we define the map $H^{(k)}$ as follows:

$$\text{For any } x \in \mathrm{Supp}(F), H^{(k)}(x) = \begin{cases} G(x) & \text{if } G(x) \geqslant k, \\ \min\{F(x), k\} & \text{otherwise.} \end{cases}$$

By construction, we have $H^{(M)} = F$, $H^{(m)} = G$, and for any $k \in \{m, \ldots, M\}$, we have $H_k^{(k)} = F_k$.

Let C be any critical clique of $H_k^{(k)}$. By proposition 13, G_k contains at least one pixel of C. We can see that $G \leqslant H^{(k-1)}$ (indeed $G \leqslant H^{(j)}$, for any j), hence $G_k \subseteq H_k^{(k-1)}$ and $H_k^{(k-1)}$ contains at least one pixel of C. Thus by theorem 6, $H_k^{(k-1)}$ is a thinning of $H_k^{(k)}$. In other words, there exists a sequence of elementary (binary) thinnings from $H_k^{(k)}$ to $H_k^{(k-1)}$. By construction, to this sequence corresponds a sequence of elementary (grayscale) thinnings from $H^{(k)}$ to $H^{(k-1)}$. Thus $H^{(k-1)}$ is a thinning of $H^{(k)}$ for any $k \in \{m+1, \ldots, M\}$, hence $G = H^{(m)}$ is a thinning of $F = H^{(M)}$. $\square$

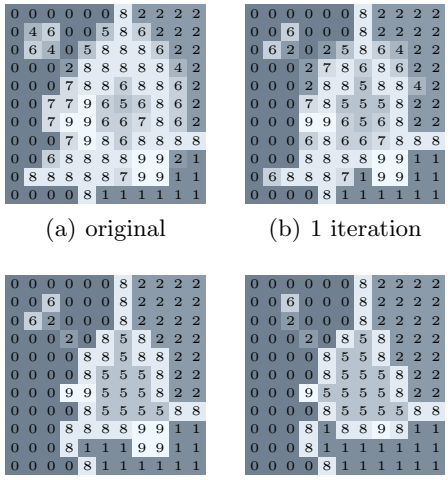
Remark: proposition 13, theorem 14 and their proofs hold whatever the (finite) dimension of the space.

4 Illustration and applications

Figure 3 presents an example of gray level thinning. We have a gray level image with 4 dark minima separated by lighter borders, as well as 3 maxima in (a). After one iteration of symmetric parallel thinning, we see in (b) that the "width of the borders" has been reduced. The image in (c) is obtained after 3 iterations, when stability is achieved. We note that all the 4 minima and the 3 maxima are preserved at their original height. The minimal height of the borders separating the minima is also preserved but these borders are thinner and the minima are larger.

However, the borders and the maxima can be further thinned by a variant of our algorithm, called asymmetric parallel thinning. The three maxima in (c), for example, correspond to crucial cliques and are completely preserved by the symmetric thinning algorithm. The variant consists of lowering, in such a configuration, all the points but one. A precise statement and validation of this algorithm will appear in an extended version of this article.

The result of asymmetric parallel thinning applied to (c) is shown in (d). We see that the borders are now even thinner, and each maximum is now reduced to a peak point.

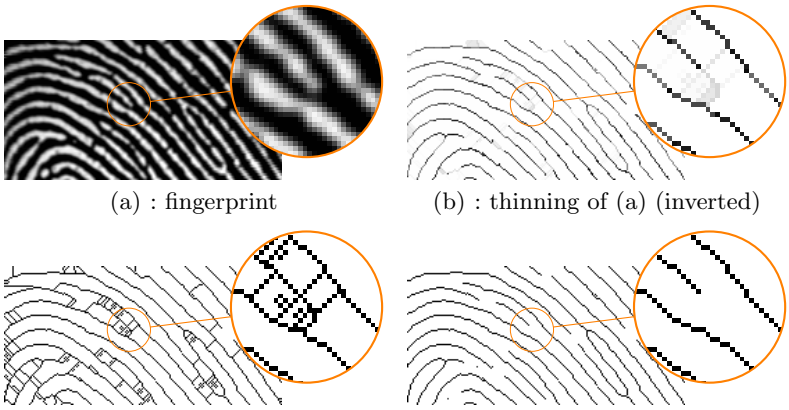


(a) original (b) 1 iteration

(c) symmetric thinning (d) asymmetric thinning

Fig. 3. Gray scale thinning

The gray scale thinning can be used to postpone the binarization process necessary in many applications to obtain a skeleton. This approach allows further processing steps in the richer gray scale space before transforming the image to the more constrained binary image space. In the rest of this section, we show



(a) : fingerprint (b) : thinning of (a) (inverted)

(c) : crests of (b) with contrast 0 (binary) (d) : crests of (b) with contrast 50 (binary)

Fig. 4. Fingerprint grayscale thinning and skeleton extraction.

three examples of applications where grayscale skeletonization can be preferred to binary skeletonization [23, 1, 9, 13]: fingerprint analysis, medical image processing and optical character recognition.

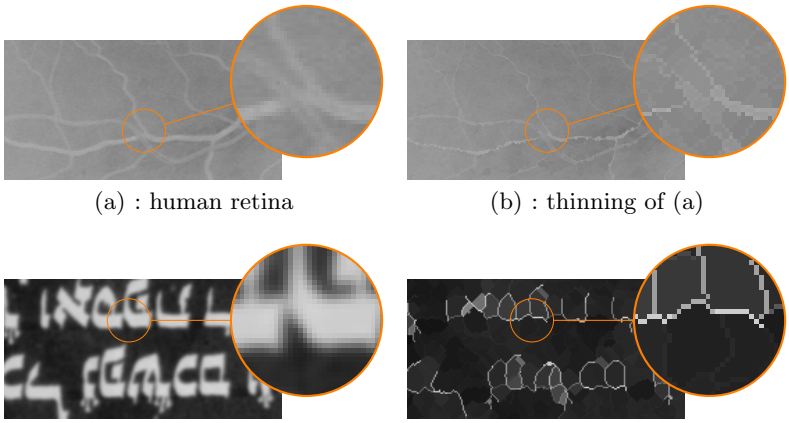
Many fingerprint analysis systems use skeletonization as an essential step. Usually, the fingerprint image is binarized before skeletonization. Here, we present a way to obain a (binary) skeleton without a prior binarization of the image (see figure 4). After a grayscale thinning (b), we use the remaining gray scale information to select robust crest points (c) having high contrast with their background (d).

The crest points are formally defined as follows. Let α be an integer, we say that a point x is a *crest point with contrast* α for an image F if there exists a level k such that $\overline{T}(x, F_k) \geqslant 2$, and such that $k - \max\{F(y), y \in \overline{F_k} \cap \mathcal{N}_0^*(x)\} \geqslant \alpha$. For example in figure 3(d), the points at levels 8 and 9 are not crest points with contrast $\alpha = 10$ for example, but they are crest points with contrast $\alpha = 2$. In figure 5(d) we show the crest points with contrast $\alpha = 50$. As we can see the resulting skeleton is free of spurious branches and is well centered.

We illustrate two other applications in figure 5. The first is the thinning of a vascular network in an image of a human retina. The vessels correspond to the lighter pixels in figure 5(a). After the gray scale thinning, we obtain the image in (b). A second application is the thinning of scanned characters shown in figures 5(c) and (d).

5 Conclusion

In this paper, we introduced a parallel thinning algorithm and proved its topological soundness, using some properties issued from the framework of critical



(a) : human retina (b) : thinning of (a)

(c) : characters (d) : thinning of (c)

Fig. 5. Gray scale thinning applications.

kernels. We also sketched some possible applications, in areas where the benefits of avoiding segmentation prior to skeletonization have been pointed out by several authors. The perspectives of this work include: the analysis of the computational cost of our algorithm, both in theory and in practice; the introduction and study of an asymmetric parallel thinning algorithm, evoked in the previous section; the introduction and study of a faster algorithm dedicated to the case of ultimate thinning; the validation of this approach by its evaluation in the context of a real-world application. These items will be developped in a forthcoming paper.

References

1. S.S. Abeysinghe, M. Baker, W. Chiu, and T. Ju. Segmentation-free skeletonization of grayscale volumes for shape understanding. In *Shape Modeling and Applications, 2008. SMI 2008. IEEE International Conference on*, pages 63 –71, 2008.
2. C. Arcelli and G. Ramella. Finding grey-skeletons by iterated pixel removal. *Image and Vision Computing*, 13(3):159–167, 1995.
3. G. Bertrand. On critical kernels. *Comptes Rendus de l'Académie des Sciences, Série Math.*, I(345):363–367, 2007.
4. G. Bertrand and M. Couprie. Two-dimensional thinning algorithms based on critical kernels. *J. of Mathematical Imaging and Vision*, 31(1):35–56, 2008.
5. G. Bertrand and M. Couprie. Powerful parallel and symmetric 3d thinning schemes based on critical kernels. *J. of Mathematical Imaging and Vision*, 2012. to appear, DOI 10.1007/s10851-012-0402-7.
6. G. Bertrand, J.C. Everat, and M. Couprie. Topological approach to image segmentation. In *SPIE Vision Geometry V*, volume 2826, pages 65–76, 1996.
7. G. Bertrand, J.C. Everat, and M. Couprie. Image segmentation through operators based upon topology. *J. of Electronic Imaging*, 6(4):395–405, 1997.

8. S. Beucher. *Segmentation d'images et morphologie mathématique*. PhD thesis, École des Mines de Paris, France, 1990.
9. C. Costes, R. Garello, G. Mercier, J.-P. Artis, and N. Bon. Convective clouds modelling and tracking by an airborne radar. In *OCEANS 2008*, pages 1 –5, sept. 2008.
10. M. Couprie, F.N. Bezerra, and G. Bertrand. Topological operators for grayscale image processing. *J. of Electronic Imaging*, 10(4):1003–1015, 2001.
11. C. Dyer and A. Rosenfeld. Thinning algorithms for gray-scale pictures. *IEEE Trans. on Pattern An. and Machine Int.*, 1(1):88–89, 1979.
12. Jeong-Hun Jang and Ki-Sang Hong. A pseudo-distance map for the segmentation-free skeletonization of gray-scale images. In *Computer Vision, 2001. ICCV 2001. Proceedings. Eighth IEEE International Conference on*, volume 2, pages 18–23, 2001.
13. K.W. Kang, J.W. Suh, and J.H. Kim. Skeletonization of grayscale character images using pixel superiority index. In *IAPR Workshop on Document Analysis Systems*, 1998.
14. T.Y. Kong. On topology preservation in 2D and 3D thinning. *Int. J. on Pattern Recognition and Artificial Intelligence*, 9:813–844, 1995.
15. T.Y. Kong and A. Rosenfeld. Digital topology: introduction and survey. *Computer Vision, Graphics and Image Processing*, 48:357–393, 1989.
16. F. Le Bourgeois and H. Emptoz. Skeletonization by gradient diffusion and regularization. In *Image Processing, 2007. ICIP 2007. IEEE International Conference on*, volume 3, pages 33–36, 2007.
17. C. Lohou and G. Bertrand. New parallel thinning algorithms for 2d grayscale images. In L. J. Latecki, D. M. Mount, and A. Y. Wu, editors, *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series*, volume 4117 of *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series*, pages 58–69, 2000.
18. S.S. Mersa and A.M. Darwish. A new parallel thinning algorithm for gray scale images. In *IEEE Nonlinear Signal and Image Proc. Conf*, pages 409–413, 1999.
19. A. Nedzved, S. Uchida, and S. Ablameyko. Gray-scale thinning by using a pseudo-distance map. In *Pattern Recognition, 2006. ICPR 2006. 18th International Conference on*, volume 2, pages 239 –242, 2006.
20. S.K. Pal. Fuzzy skeletonization of an image. *Pattern Recognition Letters*, 10:17–23, 1989.
21. Vincent Ranwez and Pierre Soille. Order independent homotopic thinning for binary and grey tone anchored skeletons. *Pattern Recognition Letters*, 23(6):687 – 702, 2002. Discrete Geometry for Computer Imagery.
22. A. Rosenfeld. The fuzzy geometry of image subsets. *Pattern Recognition Letters*, 2:311–317, 1984.
23. A.M. Saleh, A.M. Bahaa Eldin, and A.-M.A. Wahdan. A modified thinning algorithm for fingerprint identification systems. In *Computer Engineering Systems, 2009. ICCES 2009. International Conference on*, pages 371 –376, 2009.
24. J.H.C. Whitehead. Simplicial spaces, nuclei and m-groups. *Proceedings of the London Mathematical Society*, 45(2):243–327, 1939.
25. S-S. Yu and W-H. Tsai. A new thinning algorithm for gray-scale images by the relaxation technique. *Pattern Recognition*, 23(10):1067–1076, 1990.