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Consonant Continuous Belief Functions Conflicts Calculation

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Abstract. Much information sources model imprecision by the way of unimodal, consonant and continuous probability density functions (pdfs). We consider here in the framework of belief functions on real numbers, agents of evidence deduced from such pdfs. First are singletons plausibilities in conjunctive and disjunctive combinations proposed to basically merge agents of evidence with consonant focal elements. Second are partial and global conflict calculation methods provided. An application shows the plausibility curves and conflict values obtained in case of combination operations done on Gaussian based agents and at last, an example of conflict management based on an RCR-S adaptive rule of combination is given.

Key words: continuous belief functions. Conjunctive and disjunctive combination. Partial and global conflict calculation.

1 Introduction

The focal elements of agents of evidence obtained from consonant pdfs can be ordered and thus labeled by a continuous index. Under the assumption of cognitive independence between agents, this is useful to simplify expressions of the conjunctive [1] and disjunctive rule of combination [2]. As we will see, this also helps to calculate the partial and global conflict weights when much pieces of evidence are merged.

2 Characteristics of focal intervals

2.1 Focal intervals

Let f be a continuous unimodal and consonant pdf of mode μ and support $\Omega = [\Omega^-, \Omega^+]$ with bounds in \mathcal{R} , the extended set of real numbers [1]. Focal elements of the piece of evidence \mathcal{E} based on f are nested intervals that we label according to their fitting order by an continuous index called z . These intervals correspond to $A^z = [A^{z-}, A^{z+}]$, $A^{z-} \in [\Omega^-, \mu]$, $A^{z+} \in [\mu, \Omega^+]$ and are deduced from the pdf by horizontal cuts such that $f(A^{z-}) = f(A^{z+})$ in case of 'bell shaped' pdfs [1].

2.2 The z index

Suppose that the focal elements bounds A^{z^-} and A^{z^+} are defined according to the distances to the pdf's mode $\Delta^-(z)z$ and $\Delta^+(z)z$ such that:

$$A^z = [\mu - \Delta^-(z)z, \mu + \Delta^+(z)z], \quad z \in [0, \infty]. \quad (1)$$

When Δ^- and Δ^+ differ from 0, the focal elements bounds are linked by a bijective function called γ [1]. This happens in case of symmetrical pdfs as illustrated in figure 1 or for some triangular distributions for instance [2].

When possible, z has to be expressed by a linear relation depending on the r.v $x \in \Omega$ and the pdf's parameters, giving thus constant values for Δ^- and Δ^+ in some occasions. Defining z using the pdfs parameters provides in most cases single belief functions expressions for a same family of pdfs [2]. For Gaussian or Laplace pdfs for instance, Δ equals to the standard deviation σ if the z index is expressed by the absolute value of the standard score:

$$z = \frac{|x - \mu|}{\sigma}, \quad z \in \mathcal{R}^+ = [0, \infty]. \quad (2)$$

Focal intervals A^z correspond in that case to:

$$A^z = [\mu - \sigma z, \mu + \sigma z], \quad z \in [0, \infty]. \quad (3)$$

2.3 Focal set graphical representation

The Grey area in figure 1 illustrates the domain representing the focal intervals set \mathcal{F}_i corresponding to a pdf $Betf_i$ with intervals ordered according to their z label value. For a Gaussian pdf and when z is defined as in (2) like done by Ristic *et al.* [3], \mathcal{F}_i has a triangular shape.

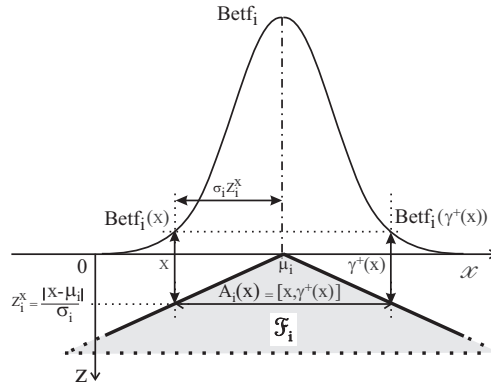


Fig. 1. Focal intervals domain \mathcal{F}_i resulting from a Gaussian pdf.

3 Singleton plausibilities in combination operations of independent consonant basic belief densities

3.1 introduction

We propose to construct the plausibility curve resulting of the combination of cognitively independent consonant pieces of evidence.

Suppose two such pieces of evidence \mathcal{E}_i and \mathcal{E}_j related to unimodal ($\mu_i \leq \mu_j$) and consonant basic belief densities (bbd) m_i and m_j and at last a r.v x on $\Omega = [-\infty, \infty]$. Focal sets are called \mathcal{F}_i and \mathcal{F}_j . According to x , ordering indexes are z_i^x and z_j^x . We note \mathcal{F}_i^x and \mathcal{F}_j^x the subsets of \mathcal{F}_i and \mathcal{F}_j that intersect with x , $\overline{\mathcal{F}_i^x}$ and $\overline{\mathcal{F}_j^x}$ their complements such that:

$$\begin{cases} \mathcal{F}_i = \mathcal{F}_i^x \cup \overline{\mathcal{F}_i^x}, \mathcal{F}_i^x = \{A \in \mathcal{F}_i, x \in \Omega, A \cap x \neq \emptyset\}, \\ \mathcal{F}_j = \mathcal{F}_j^x \cup \overline{\mathcal{F}_j^x}, \mathcal{F}_j^x = \{A \in \mathcal{F}_j, x \in \Omega, A \cap x \neq \emptyset\} \end{cases} \quad (4)$$

3.2 Graphical representation of combined focal sets

Figure 2 illustrates a Venn diagram of the intervals of $\mathcal{F}_i \times \mathcal{F}_j$ concerned in a combination operation of \mathcal{E}_i and \mathcal{E}_j relatively to x . The whole domain represents $\mathcal{F}_i \times \mathcal{F}_j$ into a Cartesian coordinate system that becomes a n -dimensional space when n agents have to be combined. Axes correspond to the focal sets \mathcal{F}_i and \mathcal{F}_j , ordered in accordance to their respective z labels. $\mathcal{F}_i^x, \mathcal{F}_j^x$ sets and their complements $\overline{\mathcal{F}_i^x}$ and $\overline{\mathcal{F}_j^x}$ are thus separated on axes at locations z_i^x and z_j^x .

When focal domains have a triangular shape as illustrated in figure 1, linear relations link pairs $(z_i^x, z_j^x), x \in \Omega$ and draw the lines ①, ② and ③ shown in figure 2. According to z_i^x and z_j^x expressions given for Gaussian pdfs by relation (2) and the inter-modal distance $|\mu_i - \mu_j|$, these line relations correspond to:

$$\begin{cases} \textcircled{1} : z_j^x = \frac{|\mu_i - \mu_j| + \sigma_i z_i^x}{\sigma_j}, x \in [-\infty, \mu_i], \\ \textcircled{2} : z_j^x = \frac{|\mu_i - \mu_j| - \sigma_i z_i^x}{\sigma_j}, x \in [\mu_i, \mu_j], \\ \textcircled{3} : z_j^x = \frac{-|\mu_i - \mu_j| + \sigma_i z_i^x}{\sigma_j}, x \in [\mu_j, +\infty] \end{cases} \quad (5)$$

The line segment called ② corresponds to the inter-modal interval $[\mu_i, \mu_j]$ and pairs (z_i^x, z_j^x) satisfying the inequality (11) correspond to disjoint focal intervals A_i^z and A_j^z . Relatively to the partial conflict $k_{i,j}$ existing between the agents \mathcal{E}_i and \mathcal{E}_j , the values of the z indexes $z_i^{\mu_j}$ and $z_j^{\mu_i}$ are called $K_{i,j}$ and $K_{j,i}$ such as:

$$\begin{cases} K_{i,j} = z_i^{\mu_j} = \frac{|\mu_j - \mu_i|}{\sigma_i}, \\ K_{j,i} = z_j^{\mu_i} = \frac{|\mu_j - \mu_i|}{\sigma_j} \end{cases} \quad (6)$$

And we note $K_{i,j}$ and $K_{j,i}$ the bounds of the line segment ② such as:

$$\begin{cases} K_{i,j} = (K_{i,j}, 0), \\ K_{j,i} = (0, K_{j,i}) \end{cases} \quad (7)$$

At last, as we can see in figure 2, each pair (z_i^x, z_j^x) separates the domain $\mathcal{F}_i \times \mathcal{F}_j$ in four subsets of interest as in a Karnauth map. These subsets correspond to $\overline{\mathcal{F}_i^x} \times \overline{\mathcal{F}_j^x}, \overline{\mathcal{F}_i^x} \times \mathcal{F}_j^x, \mathcal{F}_i^x \times \overline{\mathcal{F}_j^x}, \mathcal{F}_i^x \times \mathcal{F}_j^x$.

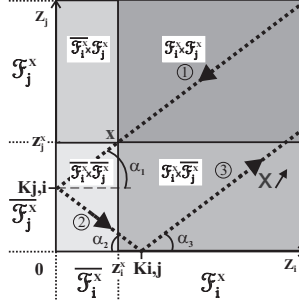


Fig. 2. Venn diagram of two Gaussian pdfs combination ($\mu_i < \mu_j$).

3.3 Singleton's plausibility in conjunctive and disjunctive combination of consonant bbds

The plausibility of a singleton relatively to a piece of evidence \mathcal{E} with focal elements indexed by z based on a consonant and unimodal pdf is:

$$Pl(x) = Pl(z^x) = \int_{z=z^x}^{z=z^xmax} m(z)dz = 1 - M(z^x) \quad (8)$$

with M the integral of the bbd m and z^xmax the upper bound of z 's domain.

The singletons plausibility after conjunctive combination of \mathcal{E}_i and \mathcal{E}_j ($\mathcal{E}_i \perp \mathcal{E}_j$) corresponds to:

$$Pl_{i \odot j}(x) = Pl_i(z_i^x)Pl_j(z_j^x) \quad . \quad (9)$$

In case of the disjunctive combination of \mathcal{E}_i and \mathcal{E}_j [2] and as for Smets Disjunctive Rule of Combination (DRC) [5], the plausibility $Pl_{i \odot j}(x)$ is given by:

$$\begin{aligned} Pl_{i \odot j}(x) &= 1 - (1 - Pl_i(z_i^x))(1 - Pl_j(z_j^x)), \\ &= 1 - M_i(z_i^x)M_j(z_j^x) \quad . \end{aligned} \quad (10)$$

Relations (9) and (10) can be generalized to merge n agents [2].

4 Conflict calculation in case of consonant bbds

4.1 Partial conflict

The domain of the partial conflict $k_{i,j}$ existing between two cognitively independent pieces of evidence \mathcal{E}_i and \mathcal{E}_j based on unimodal and consonant pdfs can be observed graphically. It corresponds in figure 2 to the triangle $(0; K_i, j; K_j, i)$. The intervals concerned by this area are disjoint and in case of Gaussian pdfs for instance, satisfy:

$$0 \leq z_j^x \leq \frac{|\mu_i - \mu_j| - \sigma_i z_i^x}{\sigma_j} \quad . \quad (11)$$

Calculation of $k_{i,j}$ following this is expressed here according to the z labels and the inter-modal distance.

Consider modes $\mu_i < \mu_j$, bounded or infinite supports $\Omega_i = [\Omega_i^-, \Omega_i^+]$ and $\Omega_j = [\Omega_j^-, \Omega_j^+]$, labels $z_i \in [0, z_{max_i}]$, $z_j \in [0, z_{max_j}]$ satisfying (1). If $\Omega_i \cap \Omega_j \neq \emptyset$, the partial conflict $k_{i,j}$ differs from 1 and corresponds to:

$$k_{i,j} = m_{i \odot j}(\emptyset) = \int_{z_i=0}^{z_i=z_{Max_i}} \int_{z_j=0}^{z_j=z_{Max_j}} m_i(z_i) m_j(z_j) dz_j dz_i \quad (12)$$

where:

$$\begin{cases} z_{Max_i} = \min\left(\frac{|\mu_i - \mu_j|}{\Delta_i^+}, z_{max_i}\right), \\ z_{Max_j} = \min\left(\frac{|\mu_i - \mu_j| - \Delta_i^+ z_i}{\Delta_j^-}, z_{max_j}\right) \end{cases} \quad (13)$$

Partial conflict $k_{j,i} = k_{i,j}$ can also be calculated from the variable z_j . Note that in case of pdfs with infinite support, relations (13) reduce to:

$$\begin{cases} z_{Max_i} = \frac{|\mu_i - \mu_j|}{\Delta_i^+}, \\ z_{Max_j} = \frac{|\mu_i - \mu_j| - \Delta_i^+ z_i}{\Delta_j^-} \end{cases} \quad (14)$$

For symmetrical pdfs as Gaussian or Laplace ones, relations (13) become:

$$\begin{cases} z_{Max_i} = \frac{|\mu_i - \mu_j|}{\sigma_i}, \\ z_{Max_j} = \frac{|\mu_i - \mu_j| - \sigma_i z_i}{\sigma_j} \end{cases} \quad (15)$$

Relation (12) can be reduced to a single integral by using the bbd's cumulative expression M (or equivalently $1 - Pl$) [2]. The weight of conflict becomes thus:

$$k_{i,j} = m_{i \odot j}(\emptyset) = \int_{z_i=0}^{z_i=z_{Max_i}} m_i(z_i) M_j(z_{Max_j}) dz_i \quad (16)$$

Since bbds resulting from pdfs are normalized, it is possible to calculate the partial conflict from intersecting intervals instead of those that do not. When $\mu_i \leq \mu_j$, $k_{i,j}$ corresponds to:

$$\begin{aligned} k_{i,j} &= 1 - \int_{z_i=0}^{z_i=z_{max_i}} \int_{z_j=z_{Min_j}}^{z_j=z_{Max_j}} m_i(z_i) m_j(z_j) dz_j dz_i \\ &= 1 - \int_{z_i=0}^{z_i=z_{max_i}} m_i(z_i) (Pl_j(z_{Min_j}) - Pl_j(z_{Max_j})) dz_i \end{aligned} \quad (17)$$

with:

$$\begin{cases} z_{Min_j} = \max\left(0, \frac{|\mu_i - \mu_j| - \Delta_i^+ z_i}{\Delta_j^-}\right), \\ z_{Max_j} = \max\left(z_{max_j}, \frac{|\mu_i - \mu_j| - \Delta_i^+ z_i}{\Delta_j^-}\right) \end{cases} \quad (18)$$

Relations (18) are justified for z_{Min_j} by the fact that $\frac{|\mu_i - \mu_j| - \Delta_i^+ z_i}{\Delta_j^-} < 0$ when $A_i^{z_i} > \mu_j$ and means in that case, that all the focal elements of \mathcal{E}_j have to be considered ($z_{Min_j} = 0$). z_{Max_j} 's relation takes into account the case of a bounded support pdf for \mathcal{E}_j leading to a total conflicting situation when $A_i^{z_i} \cap \Omega_j = \emptyset$, corresponding thus to $z_{Min_j} = z_{Max_j}$.

Relations (17) and (18) can be simplified in case of infinite supports and symmetrical pdfs.

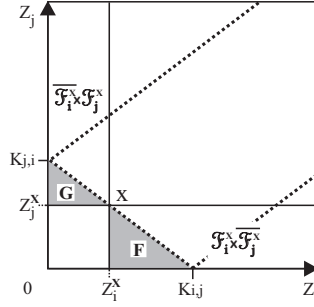


Fig. 3. Conflicting domains F and G included in the $\mathcal{F}_i^x \times \overline{\mathcal{F}_j^x}$ and $\overline{\mathcal{F}_i^x} \times \mathcal{F}_j^x$ sets.

4.2 Conflict abacus for Least Committed bbd's based on Gaussian pdfs

Figure 4 presents an abacus of the partial conflict for bbd's deduced from two Gaussian pdfs ($\mathcal{N}_1(x; \mu_1, \sigma_1^2)$, $\mathcal{N}_2(x; \mu_2, \sigma_2^2)$). Parameters $K_{i,j}$ and $K_{j,i}$ are defined using relations (6) and correspond to the bounds of the line segment ② showed in figure 2.

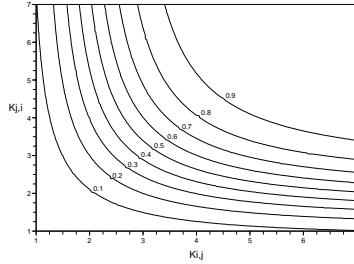


Fig. 4. Conflict's map in case of two Gaussian based agents of evidence combination.

4.3 Conflict's part in disjunctive combination

The disjunctive combination of two agents includes the XOR combination of sets \mathcal{F}_i^x and \mathcal{F}_j^x . This may take into account non convex focal intervals located in the partial conflict's domain as we can see in figures 2 and 3. Singletons x concerned take values in $[\mu_i - \frac{\Delta_i^-}{\Delta_i^+} |\mu_i - \mu_j|, \mu_j + \frac{\Delta_j^+}{\Delta_j^-} |\mu_i - \mu_j|]$ when $\mu_i \leq \mu_j$ and figure 3 illustrates this for $x \in [\mu_i, \mu_j]$ with the grey areas F and G . In case of infinite

support pdfs, F and G are defined as:

$$\begin{cases} F(x) = \int_{z_i=z_i^x}^{z_i=K_{i,j}} M_i(z_i)m_j\left(\frac{|\mu_i-\mu_j|-\Delta_i^+ z_i}{\Delta_j^-}\right)dz_i & \text{if } 0 \leq z_i^x \leq K_{i,j} \\ = 0 & \text{otherwise,} \\ G(x) = \int_{z_j=z_j^x}^{z_j=K_{j,i}} M_j(z_j)m_i\left(\frac{|\mu_i-\mu_j|-\Delta_j^- z_j}{\Delta_i^+}\right)dz_j & \text{if } 0 \leq z_j^x \leq K_{j,i} \\ = 0 & \text{otherwise.} \end{cases} \quad (19)$$

Figure 5 shows the degrees of conflict F and G taken into account in disjunctive combination of two bbds based on Gaussian pdfs. The maximum conflicting situations correspond as an evidence to singletons μ_i and μ_j and thus respectively to index pairs (z_i, z_j) equal to $(0, K_{j,i})$ and $(K_{i,j}, 0)$.

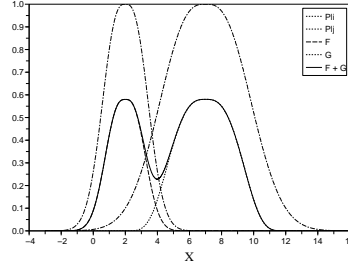


Fig. 5. Weight of conflicting intervals in the disjunctive combination.

4.4 Global conflict

When n pieces of evidence have to be merged, the global conflict do not correspond to the sum of the partial conflicts because the domains of intervals on which they are based intersect. Consequently, the global conflict K can never be lower than the most important partial conflict $k_{i,j}$ value existing between the n pieces of evidence.

Depending on the pdfs characteristics, the hyper-volume's shape of conflicting elements (z_1, \dots, z_n) is complex. It is thus easier to calculate the global conflict from intervals combinations providing an non empty intersection as done in relation (17). When all supports of unimodal and consonant pdfs related to n agents of evidence intersect, and if we consider $\mu_i \leq \mu_j, \forall i, j \in \{1, \dots, n\}, i \leq j$ and to simplify, integrals infinite upper bounds assuming that $m_i(z_i) = 0$ if $z_i > z_{max_i}$, the global conflict K existing between these agents corresponds thus to:

$$K = 1 - \int_{z_1=0}^{z_1=\infty} \int_{z_2=z_{Min_2}}^{z_2=\infty} \dots \int_{z_n=z_{Min_n}}^{z_n=\infty} m_1(z_1)m_2(z_2) \dots m_n(z_n)dz_n \dots dz_2 dz_1 \quad (20)$$

with:

$$z_{Min_j} = \max\left(0, \frac{|\mu_j - \mu_i| - \Delta_i^+ z_i}{\Delta_j^-} \forall i \in \{1, \dots, j-1\}\right) \forall j \in \{2, \dots, n\}. \quad (21)$$

Relatively to the bounds of the considered intervals, relation (20) is equivalent to:

$$A_j^{zMin_j^-} \leq (\min(A_1^{z1^+}, \dots, A_i^{zi^+}) \forall i \in \{1, \dots, j-1\}) \forall j \in \{2, \dots, n\} . \quad (22)$$

5 Applications

5.1 Conjunctive and disjunction combinations of Gaussian based agents of evidence

Under the assumption of cognitive independence, we consider three agents of evidence $\mathcal{E}_1, \mathcal{E}_2$ and \mathcal{E}_3 based on Gaussian pdfs $\mathcal{N}_1(x; \mu_1 = 8, \sigma_1^2 = 4)$, $\mathcal{N}_2(x; \mu_2, \sigma_2^2 = 0.5)$ and $\mathcal{N}_3(x; \mu_3, \sigma_3^2 = 1)$. μ_3 and μ_2 are supposed to decrease until to reach μ_1 's value. Partial and global conflicts calculations are given in table 5.1 when figure 6 illustrates the plausibility curves resulting of the conjunctive and disjunctive combination of these three agents of evidence.

As suggested by the plausibility curves of the conjunctive combination in figure 6 and the corresponding values of K , there is no linear relation between the maximum of the plausibility of this combination and the conflict. Examples illustrated in figures 6(c), 6(d) and 6(e) show also the sensitivity of the conjunctive combination to the agreement of precise sources of information.

Table 1. Partial and global conflicts amounts

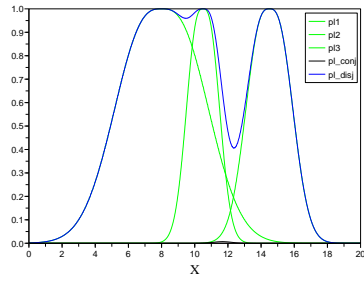
	μ_1	μ_2	μ_3	$k_{1,2}$	$k_{1,3}$	$k_{2,3}$	K
a)	8	10.5	14.5	0.092	0.867	0.929	0.976
b)	8	10.5	12.5	0.092	0.451	0.195	0.522
c)	8	10.5	10.5	0.092	0.050	0	0.109
d)	8	10.5	8.5	0.092	0.000	0.195	0.257
e)	8	10.5	8	0.092	0	0.415	0.454
f)	8	8	8	0	0	0	0

5.2 Conflict management by RCR-S adaptive combination

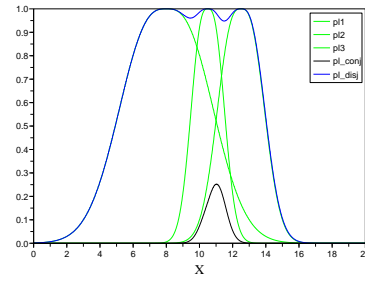
Many authors propose adaptive combination rules weighting conjunctive and disjunctive rules of combination. Florea *et al.* [6] give a general formulation of most of them and propose robust combination rules referred as RCR. To illustrate our work, we apply our relations to a RCR-S combination rule defined as:

$$Pl_{RCR-S_{1\dots n}}(x) = \frac{K}{1-K+K^2} Pl_{1\odot\dots\odot n}(x) + \frac{1-K}{1-K+K^2} Pl_{1\ominus\dots\ominus n}(x) \quad . \quad (23)$$

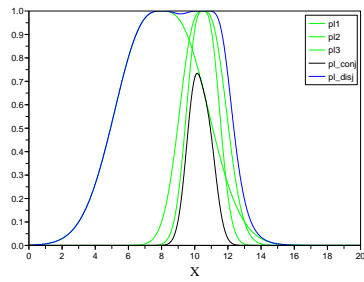
Figure 7 illustrates from left to right, the RCR-S plausibility curves obtained for the examples shown in respectively figures 6(b) and 6(c). We observe that



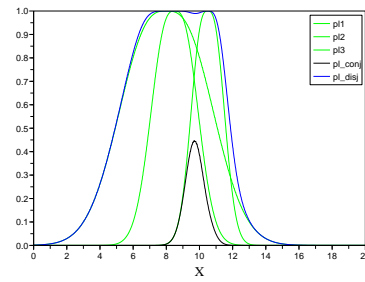
(a) $K = 0.976$



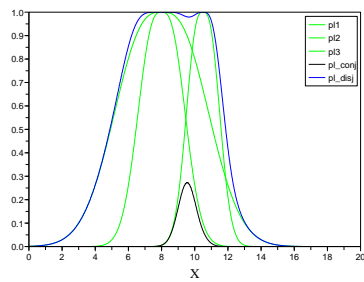
(b) $K = 0.522$



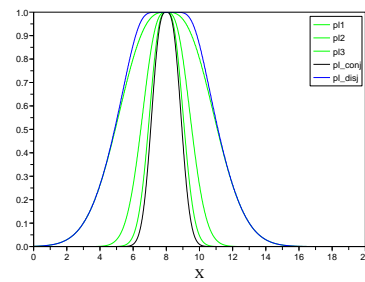
(c) $K = 0.109$



(d) $K = 0.257$



(e) $K = 0.454$



(f) $K = 0$

Fig. 6. Basic combinations of 3 Gaussian based agents of evidence (plausibility curves).

the trend in conjunctive combination is preserved even if $K > 0.5$. When applied in a very high conflicting situation as illustrated in figure 6(a) ($K = 0.976$), the RCR-S combination operation is equivalent to the disjunctive one as expected.

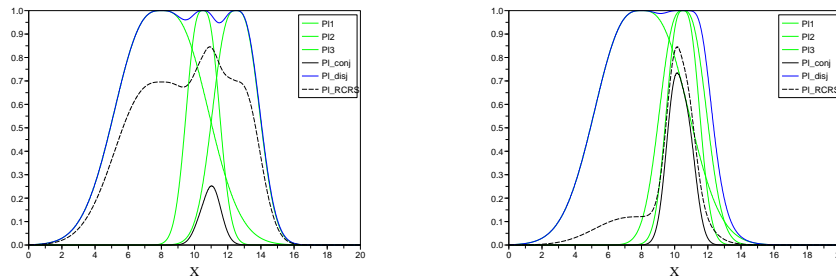


Fig. 7. Plausibilities of Conjunctive, disjunctive and RCR-S combinations.

6 Conclusions

Much existing adaptive combination rules can be applied on the plausibilities of conjunctive and disjunctive combinations presented here, mixed according to the global conflict value. But such a conflict's management is inefficient when one source of information is in total conflict with the other ones. Coherent sets of information sources could then be defined using partial conflicts values to perform more accurate combinations.

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